Linear Response of Optical Systems With Exceptional Points

A. Hashemi[1,2,†], **K. Busch**[3,4], **D. N. Christodoulides**[5], **S. K. Ozdemir**[6], **and R. El-Ganainy**[1,2,‡]

[1] *Department of Physics, Michigan Technological University, Houghton, Michigan, 49931, USA*
[2] *Henes Center for Quantum Phenomena, Michigan Technological University, Houghton, Michigan, 49931, USA*
[3] *Humboldt-Universität zu Berlin, Institut für Physik, AG Theoretische Optik & Photonik, D-12489 Berlin, Germany*
[4] *Max-Born-Institut, Max-Born-Straße 2A, 12489 Berlin, Germany*
[5] *CREOL/College of Optics and Photonics, University of Central Florida, Orlando, Florida 32816, USA*
[6] *Department of Engineering Science and Mechanics, and Materials Research Institute, The Pennsylvania State University, University Park, Pennsylvania 16802, USA*

† *hashemis@mtu.edu*, ‡ *ganainy@mtu.edu*

Abstract: We develop a linear theory for non-Hermitian optical systems having exceptional points. In contrast to previous studies, our analysis results in an exact expression for the resolvent operator without the need to use perturbation expansions.

Linear response analysis of optical systems is crucial for understanding their behavior at all levels (linear, nonlinear, classical, quantum, etc.). Under linear conditions, the steady-state response of a resonant photonic structure can be expressed in terms of the frequency domain resolvent (sometimes also called Green's operator or function) $\hat{G}(\omega) \equiv (\omega\hat{I} - \hat{H})^{-1}$, where $\hat{H}$ is the system's Hamiltonian, and $\hat{I}$ is the unit operator. In the absence of exceptional points (EPs) from the spectrum associated with H, the resolvent $\hat{G}$ can be expressed as a series expansion of the left and right eigenvectors of H. However, the situation is quite different when the spectrum exhibits EPs, in which case we will denote the Hamiltonian by H_{EP}. In this case, the dimensionality of the eigenspace associated with H_{EP} is reduced, and the Hamiltonian is said to be defective. In previous studies, it was assumed that under this condition, the system can be analyzed only within the scope of perturbation theory [1].

In this work, we show that this is not correct, and we obtain an exact expression for the system's linear response using the generalized left and right eigenvectors of the defective Hamiltonian H_{EP}. Moreover, our analysis also reveals a regular pattern that governs the frequency response function, showing that it follows a Lorentzian or

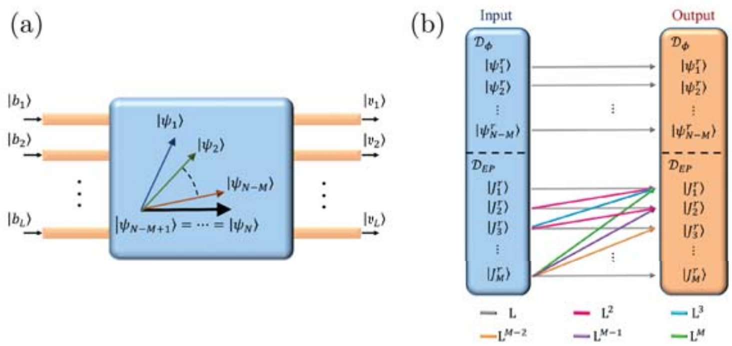

Fig. 1. (a) A schematic of an N-dimensional open resonant photonic structure with an EP of order M that could arise due to gain/absorption or coupling to L input/output ports. (b) Pictorial representation of the linear response of a non-Hermitian system having an EP. Input and output channels belonging to the same modal class excite Lorentzian response. However, a super-Lorentzian response emerges if the output signal matches a generalized (or ordinary) eigenvector with an order lower than the order of the generalized eigenvector corresponding to the input signal. The symbol L^m indicates a super-Lorentzian response of order m.

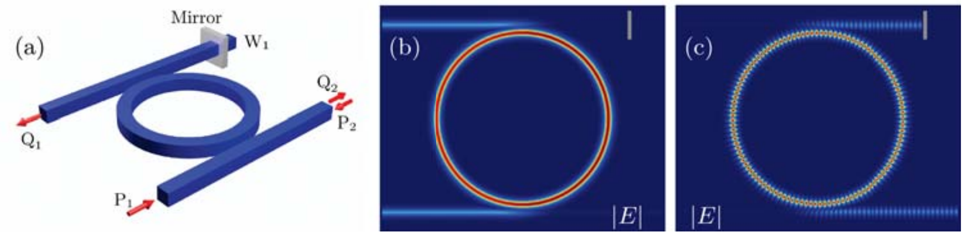

Fig. 2. (a) A schematic of a photonic structure implementing second-order exceptional surface. (b), (c) Plots of the steady state electric field distribution under excitation from ports $P_{1,2}$, respectively.

super-Lorentzian line shape in the same system depending on the input/output channel configuration. As an example, we consider the system shown in Fig. 1(a), which has an EP of order M as well as L input and L output channels. Fig. 1(b) presents a pictorial representation of the system's frequency response based on our exact analytic expression for the resolvent operator. For instance, when the input/output channels overlap only with the eigenfunctions of the system, the response is a simple Lorentzian. On the other hand, a driving signal that excites one of the canonical Jordan vectors will lead to a more complicated response that involves multiple terms with different super-Lorentzian functions. To illustrate this point, consider an excitation that overlaps with the canonical vector $|J_2^r\rangle$, described by $(\hat{H}_{EP} - \Omega_{EP}\hat{I})^2 |J_2^r\rangle = 0$. By referring to Fig. Fig. 1(b), it is clear that such an excitation will lead to both first (gray arrow) and second (red arrow) order Lorentzian shapes. These results explain the complex lineshape predicted recently for the process of spontaneous emission from a single quantum dot in the vicinity of EPs [2, 3].

Another intriguing feature predicted by our analysis is that the mode matching condition between the driving signal and the resonant eigenstates does not necessarily lead to the best excitation efficiency in open systems with EPs. To illustrate this point, we consider the photonic structure shown 2(a), which consists of an optical ring resonator coupled symmetrically to two waveguides with an end mirror at one port, as shown in the figure. As has been shown in [4], this structure exhibits an exceptional surface. The exceptional eigenvector of this device, $|J_1^r\rangle$ and the generalized eigenvector $|J_2^r\rangle$, which correspond to clockwise and counterclockwise waves, can be excited from ports $P_{1,2}$, respectively. Our analysis reveals that, under steady state conditions, excitation from port P_2 leads to higher optical energy storage inside the resonator than that from P_1. These predictions are in perfect agreement with full-wave simulations. Figure 2(b) depicts the electric field distribution corresponding to these two cases. Notice that in the first case, the field is purely a traveling wave, whereas in the second, it contains a standing wave component. Importantly, in agreement with our linear response theory, the scattering coefficient between ports P_1 and Q_1 demonstrates a Lorentzian response while that between P_2 and Q_2 features a super-Lorentzian of order two.

In conclusion, in this work, we present a detailed analysis for resonant optical systems having exceptional points and develop a linear response theory to describe their behavior. Our analysis shows that the resolvent operator associated with defective Hamiltonians can be obtained exactly without resorting to perturbation theory. Our formalism explains the mixed lineshape responses predicted in previous studies for systems operating close to exceptional points and presents a clear algorithmic approach for engineering the spectral response by controlling the input/output channel configurations. Finally, we also show that maximization of the optical energy stored inside a photonic system can be achieved by violating the mode matching condition- an effect analogous to adjoint coupling in bulk resonator systems [5].

References

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