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Fuzzy Control of a Toroidal Thermosyphon for Known Heat Flux Heating Conditions

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Abstract **-** In this paper, we develop intelligent control strategies based on the Fuzzy Logic technique to stabilize fluid velocity and temperature in a toroidal thermosyphon. The goal is on extending the analysis of our previous work [1], which focused on building a proportional-type fuzzy controller for the fluid flow, and corresponding temperature, of the thermal loop. The natural convection loop has a toroidal shape of a torus. A known influx of heat occurs in some parts of the loop whereas heat efflux takes place in others. By using space-averaged values of the fluid velocity and one-dimensional approximation for the fluid temperature, the resulting integro-differential equations for linear momentum and thermal energy are converted to a nonlinear dynamical system. Three possible scenarios, namely, stable, limit cycles and chaos, arise naturally in the flow and thermal dynamics of the device. Two types of fuzzy controllers, each built with an increasing amount of information about the fluid velocity in the system are built and tested. For them, triangular membership functions along with if-then rules are used to stabilize the system dynamics under different conditions of operation. Since the tilt angle for the loop and the heat flux are used as the parameters characterizing its dynamic behavior, these are the manipulated variables, whereas the control variables are average fluid velocity and temperatures inside the loop. MATLAB is used to implement the fuzzy controller, along with the corresponding control actions, while numerical experiments are conducted to assess its relative performance. Results demonstrate that all fuzzy controllers can effectively stabilize the thermosyphon system. However, as more information about the system is supplied to the controller the better it performs.

Keywords**:** Thermal control; Thermosyphon; Fuzzy logic; Stability analysis.

1. Introduction

Toroidal thermosyphon devices are natural convection loops, with the shape of a torus, that provide transfer of energy from one region of the loop by buoyant flow of the working fluid to another without need of external fluid pumps. These thermal devices are key in application areas such as: geothermal energy, energy storage, solar heating, and electronic and nuclear reactor cooling [2–4]. Thus, understanding the time-dependent behavior of these systems is important for both performance prediction and system control. In this context, experimental and numerical investigations have reported the existence of either constant, cyclic, or chaotic behavior of these systems, depending on the values of parameters such as heat input, wall temperature or tilt angle, in the open literature [5–9]. Therefore, depending on the objective, often the system would need to be controlled in some fashion.

Building reliable controllers for a given design, in a specific application, is difficult due to the dynamic nature of the fluid flow and the related energy transfer, and though the proportional-integral-derivative (PID) scheme has been widely used in industry, its lack of robustness [10] requires alternative control strategies. Preliminary work on intelligent control schemes of thermosyphon models has been recently reported by Lopez and Pacheco-Vega [1], where the focus was on building controllers based on fuzzy logic. This technique has the ability of describing complex systems via linguistic variables and expert-based rules derived from human experience [11, 12], and has been successful in control applications of complex thermal systems [13–16]. The results by Lopez and Pacheco-Vega [1], demonstrated that a fuzzy controller based on information about the fluid velocity only, is capable of stabilizing the device by manipulating the tilt angle to a region where the system is naturally stable.

Here we are interested in extending the analysis of Lopez and Pacheco-Vega [1], on the development of robust fuzzy controllers for a toroidal thermosyphon system by increasing the amount of information about the system is provided to the controller. Thus, we first briefly describe the device, its mathematical model, and a set of numerical tests for different conditions of the parameters. Then, we provide a short introduction to fuzzy logic along with details on the development of the controller. Finally, numerical experiments are conducted to assess the relative performance, and the results and corresponding analysis are discussed.

2. System Description and Mathematical Equations

Consider a loop filled with a single-phase fluid, as depicted in Figure 1. The tube diameter is d and the length from the center of the loop to the midpoint of the tube is R, with $R>>d$. The angle θ describes the position along the circumference of the loop. The regions where heat leaves and enters the device are $0 \le \theta \le \pi$ ($0° \le \theta \le 180°$), and $\pi \le \theta \le 2\pi$ ($180° \le \theta \le 360°$), respectively. Differences in temperature within the fluid cause differences in fluid density generating its motion. Though three possible heating conditions, namely known heat flux, known wall temperature and mixed conditions [8] may exist, in the present study we will focus on the "known heat flux" heating condition.

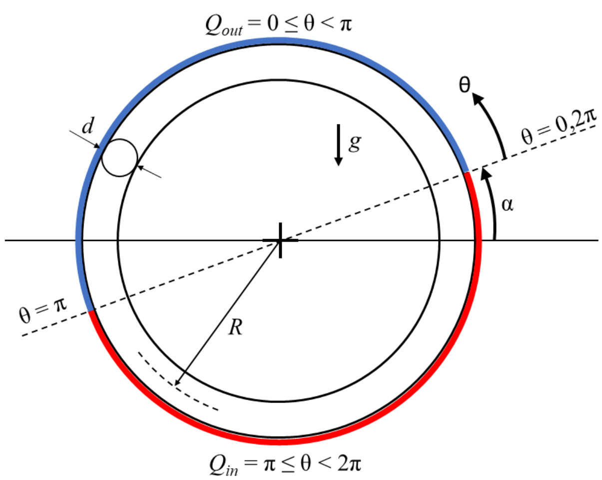

Fig. 1: Schematic of a toroidal thermosyphon.

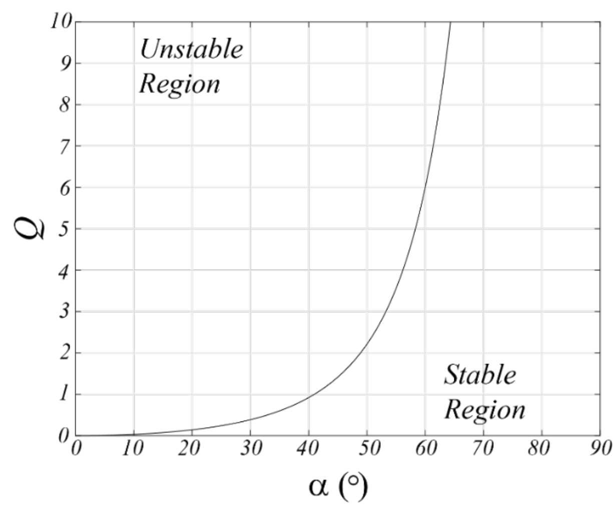

Fig. 2: Linear stability curve.

Though a detailed account of the governing equations, is in [8] and the references therein, here we use the same one-dimensional versions of the momentum and energy equations of [1, 8]. Mass conservation provides a velocity independent of the spatial coordinate; i.e., $u = u(t)$, while temperature is $T = T(t,\theta)$. Here, time t and the angle θ are the independent variables whereas u and T are the dependent variables of the problem. The tilt angle, α, is one of the parameters, while the heat flux, Q, is the other. Thus, under the Boussinesq approximation for the buoyancy term, and absent axial conduction within the fluid, the integral of the momentum equation over the loop and the energy equation, both in nondimensional form, are given as [8]

$$\frac{du}{dt} + u = \frac{1}{\pi}\int_0^{2\pi} T \cos(\theta + \alpha)\, d\theta, \tag{1}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial \theta} = Q; \text{ with } Q = -\hat{Q}\sin\theta, \tag{2}$$

where Q is a prescribed non-dimensional heat flux of strength $\hat{Q}$ either going *in* or *out* of the system.

By following Pacheco-Vega et al. [8], Eqs. (1) and (2), can be transformed into a system of first-order ODEs. Thus, by expanding the temperature as

$$T(t,\theta) = T_0^c(t) + \sum_{m=1}^{\infty} [T_m^c(t)\cos m\theta + T_m^s(t)\sin m\theta], \tag{3}$$

using the orthogonality conditions, integrating around the loop, and by defining $x \equiv u$, $y \equiv T_1^c$ and $z \equiv T_1^s$, Eqs. (1) and (2), become

$$\frac{dx}{dt} = -x + y\cos\alpha - z\sin\alpha, \tag{4}$$

$$\frac{dy}{dt} = -xz, \tag{5}$$

$$\frac{dz}{dt} = -Q + xy, \tag{6}$$

where $x(t)$ is the fluid velocity, and $y(t)$ and $z(t)$ are the coefficients - for the first mode - of the nondimensional temperature distribution in Eq. (3). By finding x, y and z, for a given Q and α, the behavior of the thermosyphon can be predicted.

3. System Dynamics

As it was the case in our previous work [1], the heat in-flux and out-flux are modeled using $Q = \hat{Q}\sin\theta$, over the entire loop ($0 < \theta < 2\pi$). Thus heat flux Q, and the inclination angle α, are parameters that determine the behavior of the system. Previous work [1, 7, 8] has shown that the system can have a stable, cyclic, or chaotic flow and heat transfer. The two critical points [1] of the system, P_1 and P_2, of Eqs. (4)–(6), are $P_{1,2} = (\bar{x}, \bar{y}, \bar{z}) = \left(\pm\sqrt{Q\cos\alpha}, \pm\sqrt{Q/\cos\alpha}, 0\right)$, both of which exist if $-90° < \alpha < 90°$, with $\bar{x}, \bar{y}, \bar{z}$ representing the critical points. For our purposes, a linear stability analysis of $P_1 = \left(\sqrt{Q\cos\alpha}, \sqrt{Q/\cos\alpha}, 0\right)$, indicates that the critical point is stable as long as $Q \leq \sin^2\alpha / \cos^3\alpha$, with $\alpha \geq 0$. The neutral stability curve, shown in Figure 2, shows the stable and unstable regions in the $Q - \alpha$ parameter plane. This is the plane we will use to decide which control tests will be carried out. In agreement with the information of Figure 2, examples of the aforementioned dynamic behavior for P_1, are shown in Figures 3, 4, and 5, respectively, for stable-, limit cycle-, and chaotic-behavior for a heat flux value of $Q = 5$, and tilt angles of $\alpha = 60°, 50°, 30°$. The relationship between α and Q lays the groundwork for developing the fuzzy controller.

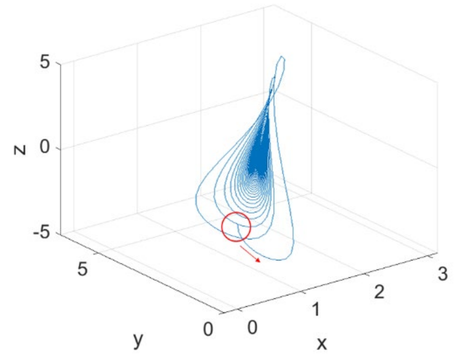

Fig 3: Phase space for $Q = 5$, $\alpha = 60°$.

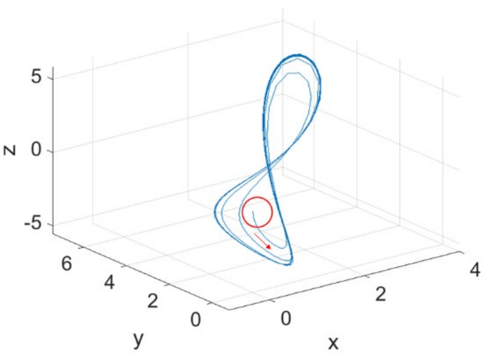

Fig 4: Phase space for $Q = 5$, $\alpha = 50°$.

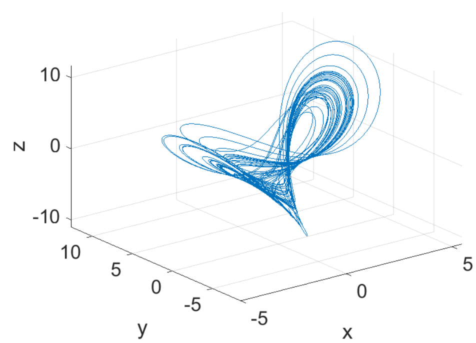

Fig. 5: Phase space for $Q = 5$, $\alpha = 30°$.

4. Fuzzy Control

4. 1. Fuzzy logic Background

The fuzzy logic (FL) technique uses linguistic variables to develop rules, based on external "expert" knowledge, along with so-called membership functions of fuzzy sets, which enable handling vagueness and imprecision in the data to solve a particular problem [18]. The concept of fuzzy sets (which includes a sliding scale of membership of an element belonging to a specific set) establishes a generalization of that of a strict binary crisp set, where an element can either belong to the set or does not belong in it. In the context of crisp sets, for example, the fluid temperature, T_f "is" either hot or it "is not". This is defined mathematically as

$$\mu_A(T_f) = \begin{cases} 1, & T_f \in A \\ 0, & T_f \notin A \end{cases}, \quad (7)$$

where μ_A is the membership function of the crisp set A; i.e., the set of water temperature being hot. In the context of fuzzy sets, on the other hand, an element can have a varying degree of membership to a specific set; so, in fact, it can partially belong to several sets. Thus, in the same example of the fluid temperature, T_f, in a fuzzy set a fluid can be described anywhere in between "very hot", "hot", "warm", "cold", or "very cold". This notion of degree of belonging allows for a smooth transition among membership functions of a specific variable. This is defined, mathematically, as

$$\mu_A(T_f) = \epsilon[0,1], \quad (8)$$

with the set of water temperature now being a fuzzy set A, defined as

$$A(T_f) = \sum_i \frac{\mu_A(T_{f,i})}{T_{f,i}}, \quad (9)$$

with a membership function μ_A is for the i-th fuzzy water temperature set. After a so-called fuzzification process, in which a crisp value is mapped into fuzzy sets via their membership functions, the inference engine uses knowledge about the process or system from the expert, and generates a cumulative fuzzy output – via output membership functions – which then is mapped back (by a so-called defuzzification process), into a crisp value. This inference engine is outlined in Figure 6.

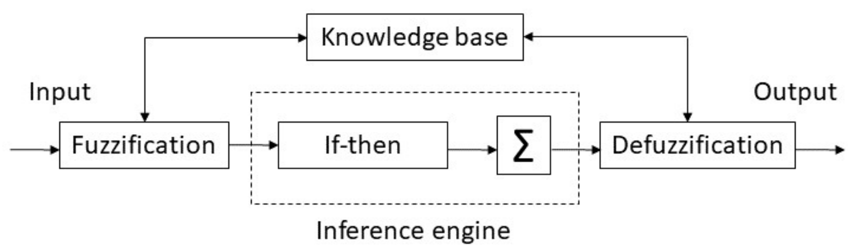

Fig. 6: Fuzzy logic inference engine.

In the context of thermal control, although most control applications use PID controllers, significant improvements in fuzzy logic controllers have shown to be promising in controlling these complex devices [19–21]. Thus, this is the type of controller that will be used to manipulate the tilt angle α, to control the fluid flow and temperature in the thermosyphon device.

4.2. Controller Development

The goal of the fuzzy control strategies is to achieve specific fluid flow and temperature values inside the thermosyphon system while, at the same time, maintaining its stability. This is done by controlling the value of nondimensional velocity x, with respect to a predefined setpoint, by adjusting the tilt angle α, in a closed single-input single-output feedback loop. In this context, we note that the three dependent variables, the non-dimensional velocity x, and the two Fourier coefficients of fluid temperature, $y(t)$ and $z(t)$, are interconnected by the heat transfer by convection physics. Thus, the three variables will not separate into different behaviors but if one is stable, then the others will also be. From this fact, only x will be used to determine how much the tilt angle would need to be adjusted. To this end, in the present work we focus on developing two different fuzzy controllers, namely FLC1 and FLC2, the difference between them being the information provided to the controller. For the first controller (FLC1), for instance, the error value $E_{\Delta x}$, between the setpoint x_{set}, and the actual x, fluid velocity, will dictate the how much the tilt angle α, will change until the system stabilizes. For the numerical tests, the ranges for the fuzzy sets of both the input and output variables have been set to $E_{\Delta x} \in [-1,1]$ and $\Delta\alpha \in [-0.5,0.5]°$, respectively. Using these ranges, the corresponding fuzzy sets, along the membership functions, for $E_{\Delta x}$ and the Δα, are shown for clarity in Figures 7 and 8. Although not included here, the set of linguistic rules, for this FLC1 controller, were also established. For the second controller, FLC2, not only $E_{\Delta x}$ was considered as an input to the controller but also the information on its derivative $dE_{\Delta x}/dt$, for which the respective fuzzy sets and membership functions, in the range of [−0.06,0.06], are shown in Figure 9. For this FLC2 controller the resulting response-control surface, illustrated in Figure 10, provides a map between the two inputs, $E_{\Delta x}$ and $dE_{\Delta x}/dt$, to the controller and the controller output (which is the input to the thermosyphon system), i.e., the tilt angle α.

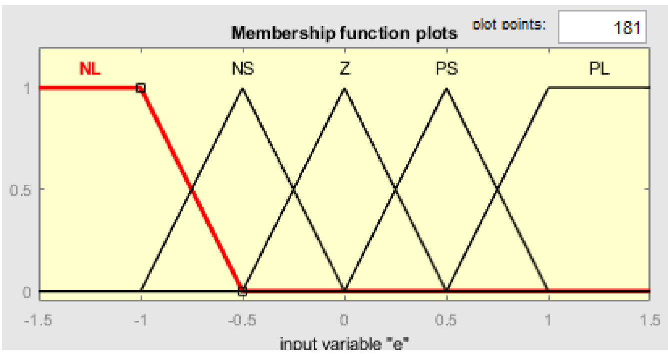

Fig. 7: Fuzzy sets and membership functions for $E_{\Delta x}$.

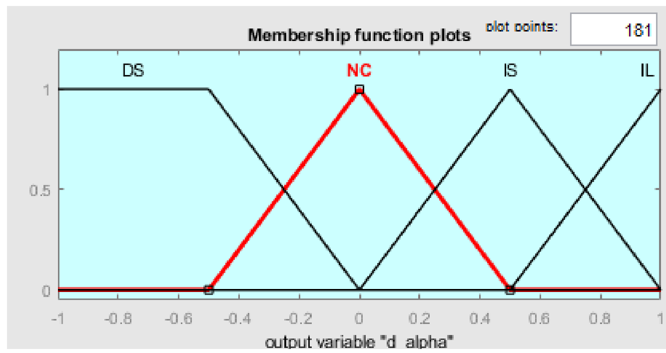

Fig. 8: Fuzzy sets and membership functions for Δα(°).

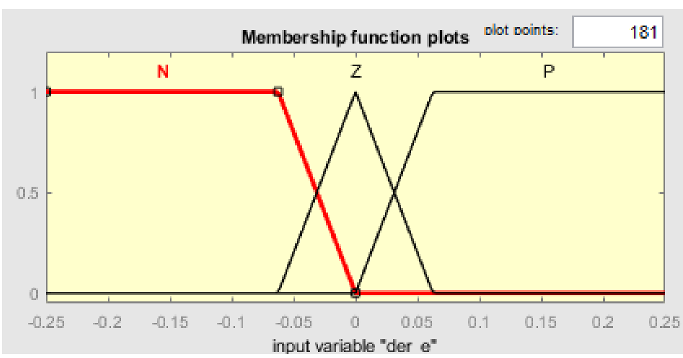

Fig. 9: Fuzzy sets and membership functions for $dE_{\Delta x}/dt$.

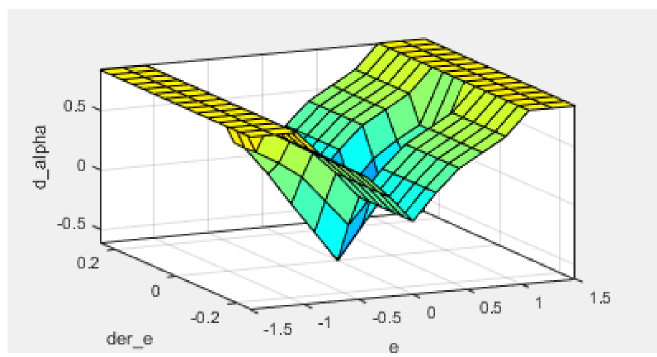

Fig. 10: Response control surface for FLC 2 controller.

To test the ability of the fuzzy controllers in stabilizing the thermosyphon system, single-input single-output (SISO) feedback loops for both FLC1 and FLC2 – shown in Figures 11 and 12, respectively – were designed and implemented in Simulink-MATLAB. The input to the system is the tilt angle α, while the system output is the fluid velocity x (which is linked to the Fourier fluid temperature modes y and z). In reference to Figures 11 and 12, once x is set to x_{set}, the inputs to the controller, $E_{\Delta x}$ and $dE_{\Delta x}/dt$, go through the controller to determine the change in α, which then becomes an input to the plant (represented by the system of non-linear ODEs [Eqs. (4)-(6)]), thus computing the values of x, y and z. For the next time increment, the feedback value (where the output value x, is compared to the reference value x_{ref}), is fed to the controller until stability is achieved. A number of different initial conditions can also be used to determine if, when provided a value that should result in chaotic behavior, the controller can stabilize the system.

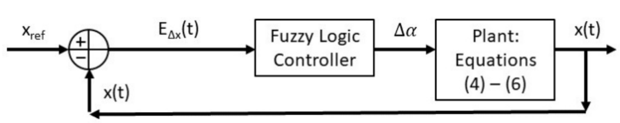

Fig. 11: SISO feedback loop system with one input to the controller (FLC1).

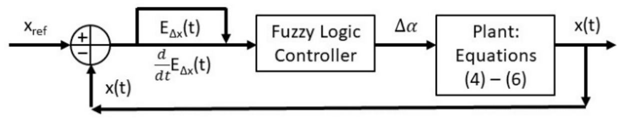

Fig. 12: SISO feedback loop system with two inputs to the controller (FLC2).

5. Results of the Thermal Control

5.1 Open loop tests

Numerical simulations were ran in MATLAB using a 4th order Runge-Kutta method to solve the system of ODEs (4)-(6), for a heat input of $Q = 5$ and values of the tilt angle α = 60°, 50°, and 30°, shown, respectively, in Figures 3, 4 and 5. As seen from these figures, respectively, the system either arrives to a stable (constant) flow, a sustained periodic flow or a totally chaotic flow. These results show that the nonlinear system has a wide range of fluid flow and temperature behavior which stem from the different combinations of inputs to the thermosyphon device.

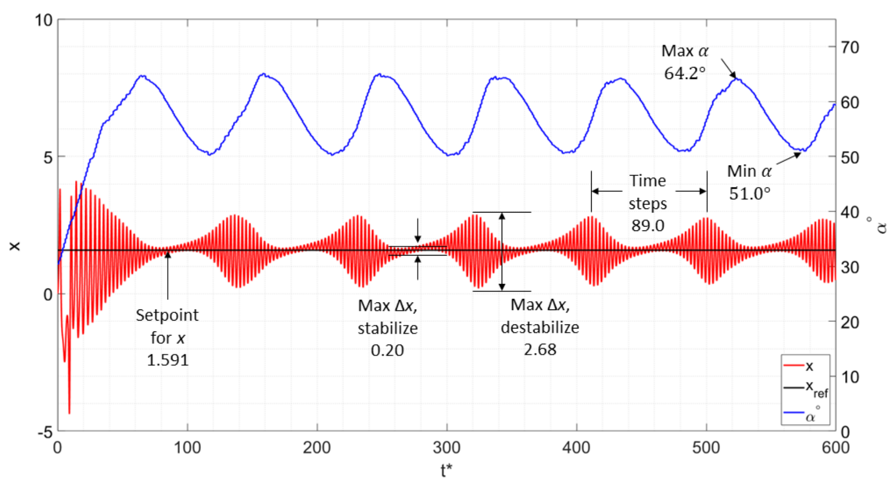

Fig. 13: FLC1 control actions with starting condition of $Q = 5$ and α = 30°.

5.2 Closed-loop tests

The objective of the two FL controllers was to maintain this system at a stable operating condition. To this end, various initial conditions and starting values of the parameters were used to test the robustness of controllers FCL1 and FCL2. However, for brevity, only starting values of $Q = 5$ and $\alpha = 30°$, which correspond to a point in the unstable region of the stability curve of Figure 2, with natural chaotic behavior of the system, are reported here. Figure 13 presents the results for both the fluid velocity x, and the tilt angle α for FLC1 while Figure 14 shows those of the FCL2 controller. From Figure 13 it can be seen that the system immediately shows unstable behavior; however the FCL1 controller begins to stabilize the system by increasing the tilt angle α and reducing the difference between x and x_{set}. As the system enters the stable region and $E_{\Delta x}$ is minimal, the controller then begins to decrease α to return the system to the unstable region while maintaining a fixed x value. As the system destabilizes, the controller starts to act again until the oscillatory behavior of the system decreases to a value close to zero. This cycle repeats as the controller manipulates the tilt angle in the 51° to 64° range with the system achieving a smaller-amplitude oscillatory behavior. On the other hand, from Figure 14, with the same starting conditions for the test as before, shows a similar overall performance of the FLC2 than that of the FLC1, with a couple of marked differences, the first being the larger timeframe that the controller maintains the system under stable conditions along with the smaller oscillatory behavior of the system under those conditions, and the smoother manipulation of the tilt angle, which is reflected in the way the fluid velocity evolves with time. This is a result from increasing the amount of information supplied to the FCL2 controller, as now $E_{\Delta x}$ and $dE_{\Delta x}/dt$, is used to control the system. This test clearly illustrates that the performance of the FL controller improves as more information about the system is provided to it.

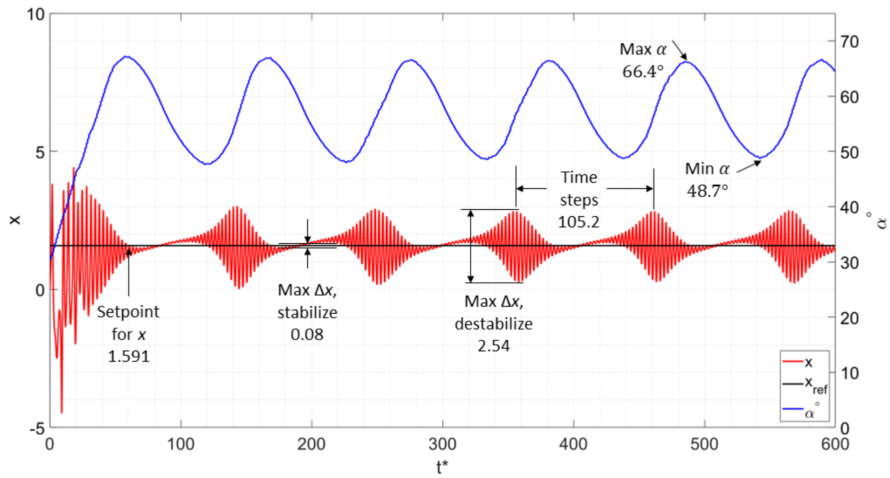

Fig. 14: FLC2 control actions with starting condition of $Q = 5$ and α = 30°.

6. Conclusions

Robust and efficient controllers are important to ensure thermal stability of complex systems, while operating under off-design conditions, as it is the case of natural convection loops, also known as thermosyphons. Although PID controllers are common in industry, they lack robustness. In this work we have developed two fuzzy-based controllers which use information about the difference in the fluid velocity, and its time derivative, to provide adequate input values of the tilt angle in order to stabilize the fluid velocity and its corresponding temperatures. The numerical tests show that both fuzzy controllers successfully perform the control actions, and they are able to stabilize the system under different operating conditions. The controllers have been tested against different starting conditions to simulate experimental data of a complex system. Furthermore, the FCL2 controller illustrates that as more information about the system is provided to the controller, the better it performs. This work has clearly demonstrated that a fuzzy logic based controller is an accurate and efficient alternative to the control of these complex applications. Future work will include the implementation of a third fuzzy controller that uses not only $E_{\Delta x}$, the derivative of $E_{\Delta x}$, and the corresponding integral of $E_{\Delta x}$ to further improve its performance.

Acknowledgments

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