

Evaluating Human Locomotion Safety in Mobile Robots Populated Environments

Boyi Hu

Industrial & Systems Engineering
University of Florida
Gainesville, FL, 32611
boyihu@ise.ufl.edu

Yue Luo

Industrial & Systems Engineering
University of Florida
Gainesville, FL, 32611
yueluo@ufl.edu

Yuhao Chen

Industrial & Systems Engineering
University of Florida
Gainesville, FL, 32611
yuhaochen@ufl.edu

Abstract— The overarching goal of this work is to understand how human locomotion adapts to mobile collaborative robots (cobots) that are designed to complement human well-being. This understanding will provide relevant inherent safe and human-centered design guidance for future mobile cobot systems. In this study, we will focus on the warehousing, wholesale, and retail trade (WRT) industry, where in general human workers are exposed to extensive experience working with mobile cobots, investigating the human locomotion safety in this environment. Eight participants were recruited to simulate a grocery shopping task with and without the mobile robot nearby. The walking trajectory of all participants revealed that the mobile robot complicated participants walking path selection, compared to the baseline “No Robot” condition. Meanwhile, participants lowered their walking speed and showed a proactive reaction to the approaching robot by initiating and ceasing the walking actions more smoothly. In conclusion, findings confirmed the values of mobile cobots in complex occupational settings and suggested more a systematic approach to ensure these intelligent systems’ inherent safety.

Keywords—*Mobile Robots; Human-Robot Interaction; Locomotion Safety; Human-centered Path Planning*

I. INTRODUCTION

The declaration “a robot in every home” by Bill Gates a decade and a half ago [1] is becoming a reality: ground-based autonomous transportation systems, exoskeleton suits, and airborne smart drones are deployed ubiquitously. While our focus has been drawn on risks and injuries arising from autonomous cars crashing [2], [3] or muscle strains and fractures caused by improper exoskeleton designs [4]–[6], much less attention has been paid to dangers arising from the imminent arrival of mobile collaborative robots (cobots) that share the floor with us. These mobile cobots are invented to improve human well-being by simplifying task flow or partially taking over human operations, such as service assistants [7], surveillance patrolling [8], and delivery [9]. Traditionally, safety in robotics applications (e.g., manufacturing assembly lines) can be maintained by separating the working space between humans and robots. Nevertheless, mobile cobots, which perform work side-by-side with people, have created unprecedented incidents which pose new safety challenges. Previous efforts on mobile cobots’ safety have been primarily focused on improving robotics control methods that avoid direct physical contact. In this work, the transient influence of mobile cobot deployment on surrounding pedestrians’ locomotion safety (i.e., risk of slips, trips, and falls) will be systematically investigated.

During locomotion (i.e., walking), a dynamic interplay is established between the (1) perception systems such as the vision, vestibular, and proprioception, (2) neuromuscular

systems that control posture, gait and balance, and (3) the environment. A prompt and accurate response to this information stream appears to be indispensable for falling injury prevention. Worldwide, falls represent one of the leading causes of disability [10]–[12], and the second leading global cause of accidental death [10], [13], [14]. In developed countries, slips, trips, and falls (STFs) contribute between 20 and 40% of disabling workplace injuries [15]–[17]. STFs were also the leading reason for unintentional injury emergency department visits, comprising 21% of such visits [18], [19]. Furthermore, based on a recent nationwide large-scale study, despite the general improvements in medical treatments, as a driver of morbidity and mortality, STFs continue to pose an unresolved threat to society [10]. A similar trend can also be observed in occupational settings, despite the fact that automation and robotics have been consistently reducing the incident rates of workplace injuries caused by overexertion and stuck by/against, the incident rate of STFs has been largely unchanged throughout the years [20], [21]. All of these indicate that a gap needs to be filled in terms of how robotics and Artificial Intelligence (AI) methods can be leveraged for STF prevention, especially in the coming ubiquitous robot era. In this work, the purpose of this study was to assess individuals’ movement and gait responses to mobile cobots that work closely with them in a complex and unstructured environment. Findings will be of great importance for advancing our understanding of assured robotics as well as how inherent safety can be guaranteed from the robotics design and Human-Robot Interaction (HRI) point of view. As an emerging technology, despite the huge potential, not too many business sectors have accumulated a large critical mass that has long-term experience with mobile cobots. In this study, we will focus on the warehousing, wholesale, and retail trade (WRT) industry, as WRT workers, in general, have extensive experience working with mobile cobots compared to other business sectors (such as Amazon Kiva robots and Walmart AlphaBots), investigating the human locomotion safety in this environment.

Therefore, the purpose of this study is to prove the concept that human behavior is physically affected by the presence of a mobile cobot during grocery shopping, further impacting human locomotion safety in the environment. The research question and proposed hypothesis are listed as follows:

Research Question: Does the presence of a mobile cobot influence the motion behavior of a customer in a retail environment?

Research Hypothesis: The presence of a mobile cobot changes the motion behavior of a human in a retail environment, as indicated by the spatiotemporal walking parameters.

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II. METHODS

A. Participants

A total of eight healthy adults (five males and three females, 19.4 ± 2.0 years old, 66.0 ± 10.1 kg, and 176.7 ± 10.2 cm in stature) were recruited from the University of Florida. Seven of the participants self-reported being right-handed, whereas one claimed to be ambidextrous. All participants provided written informed consent after they confirmed they had no neurological or musculoskeletal disorders that would affect their walking behaviors. The study was approved by the University of Florida Institutional Review Board (IRB Project #: IRB202002765).

B. Experimental Site and Instruments

To simulate a high-fidelity retail environment, a research facility with configurable shelves and a selection of commercial items was employed (Fig. 1). The experiment site has a dimension of 6.41 m by 8.62 m, with six shelves, one checkout machine, and one shopping cart in it. To track human motion behavior, a full set of seventeen inertial measurement unit (IMU) sensors (MVN Awinda, Xsens Technologies BV, Enschede, The Netherlands) were attached to participants' full body, from head to toe (Fig. 2), following the manufacturer's instruction manual[22], [23]. These sensors are small in size and light in weight (36mm x 24.5 mm x 10 mm, 10 g), making them non-intrusive for participants during grocery shopping tasks [24]. The sensor on the pelvis was selected in this study because it is nearly equivalent to the center of mass of the human body [25]. The sampling frequency of the motion recordings was set at 60 Hz. A customized mobile robot platform was utilized in this study to represent the cleaning robot present in the retail environment (Fig. 2). This robot platform is comprised of a Fetch Freight Base (Fetch Robotics, Inc., San Jose, California) and a UR5 robot manipulator (Universal Robots, Odense, Denmark) that runs on Robot Operating System (ROS). The dimension of the robot platform is 0.508 m in length, 0.559 m in width, and

1.295 m in height, whereas the maximum speed of the mobile robot was limited to 1.0 m/s for safety concerns.

C. Procedures

The experiment was conducted in a simulated retail environment with high fidelity, with one mobile robot, six grocery shelves, one checkout machine, and one shopping cart present simultaneously.

Upon arrival, the participant signed the informed consent after they were briefed on the details of the study. The researchers then inquired about demographic data (age, gender, weight, height & dominance), measured body dimensions, finished sensor attachment, and calibrated the motion tracking system in sequence. Following that, the participant was instructed to perform the grocery shopping tasks, which are frequent among customers and employees in a retail context. The grocery shopping tasks were essentially item picking and sorting tasks, in which the participant pushed a shopping cart (45 kg) and retrieved eight specified items from shelves using their dominant hand. These eight specified items were chosen at random from the shelf and were documented in a list for participants' reference during tasks. After collecting all items, the participant returned to the checkout machine and sorted all items into two bins.

Each participant completed a total of ten trials, five of which were under "No Robot" conditions and the other five under "With Robot" conditions. During the "No Robot" condition, participants traversed between shelves to pick and sort items according to the item lists (Fig. 1, blue dotted line). During the "With Robot" condition, participants were instructed to finish the same tasks, but with a robot looped around the shelves (Fig. 1, red dotted line). The robot simulated a mobile platform in the retail environment to disinfect, clean, and inspect its surroundings. It was programmed to move between waypoints automatically in such a way that the participant and the robot frequently interacted. Obstacle avoidance and path replanning

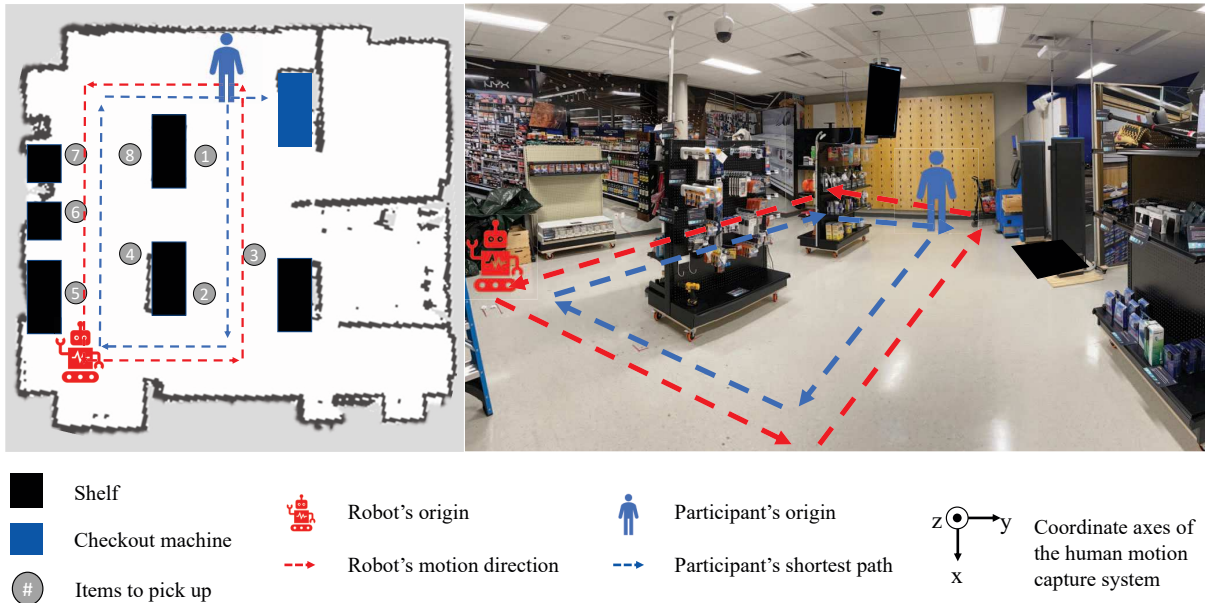


Fig. 1. The grocery shopping task and the motion directions of participants and the robot.

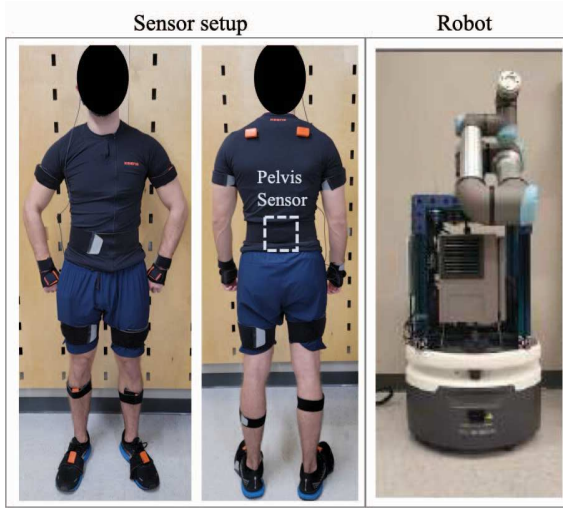


Fig. 2. Sensor setup and demonstration of the customized mobile robot platform.

algorithms, as well as the researchers' necessary manual remote control of the robot, were applied to ensure that no collision injuries would occur throughout the experiment. In each robot condition, participants utilized identical item lists. And each robot condition was set to be repeated five times with the aim to prevent potential mental or physical fatigue. The order of robot conditions was randomly presented across participants.

D. Data Processing

Data processing was performed using MATLAB scripts (MATLAB R2019b, MathWorks, Natick, MA, USA). The sensor on the pelvis was selected in this study because it is nearly equivalent to the center of mass of the human body [25]. Before parameter computation, the pelvis position data were filtered with a low-pass Butterworth filter (2nd order, 6

Hz cut-off frequency, zero lag) [26]–[28]. The coordinate system (i.e., the origin and axis directions) of the pelvis position data was calibrated to be the same across all participants (Fig. 1).

In this study, only the subtask of traversing between shelves was included to be analyzed; other subtasks, such as item picking, were not targeted and were excluded by identifying critical events (i.e., both feet on the ground near shelves & hands reaching to items) with the help of the visualization from the IMU data collection software (Xsens MVN Analyze 2019, Xsens Technologies BV, Enschede, Netherlands). Eight parameters were calculated to quantify human motion behavior, which is a potential indicator of human locomotion safety, when participants traversed between shelves using methods described by Rudenko and colleagues [29]: 1) walking time, 2) walking distance, 3) motion speed, 4) motion acceleration, 5) trajectory curvature, and 6-8) the trial-wise standard deviation values of the motion speed, motion acceleration, and trajectory curvature. The descriptions of each human motion parameter are shown in Table 1.

E. Statistical Analysis

The changes in dependent variables, eight human motion parameters, were evaluated by one-way analyses of variance (ANOVA) using R studio. The model included the robot condition ("No Robot" vs. "With Robot") as the within-subject factor and the participant as the random effect. The significance level was set at $\alpha = 0.05$.

III. RESULTS

A. Trajectory Comparison

Fig. 3 shows the comparison of walking trajectories of all participants finishing the grocery shopping task. In general, when participants walked in the presence of the robot, their walking path was observed to be more complicated and less cluttered, compared to the baseline ("No Robot" condition).

TABLE I. DESCRIPTION OF EIGHT MOTION PARAMETERS

Motion Parameters	Description
Walking time (sec)	Walking time participants used to traverse between shelves. The time spent on item browsing and order picking was not taken into account.
Walking distance (m)	Walking distance participants took to traverse between shelves.
Motion speed (m/s)	Motion speed, calculated by the pelvis sensor, average values of speed measured at one-second time intervals in a trial.
Motion acceleration (m/s ²)	Motion acceleration, calculated by the pelvis sensor, average values of resultant acceleration when traversing between shelves in a trial.
Trajectory curvature (m ⁻¹)	The average trajectory curvature of the pelvis in a trial, trajectory curvature was calculated by the first, middle, & last points of the four-second time intervals.
Motion speed STD (m/s)	The standard deviation of the motion speed in a trial.
Motion acceleration STD (m/s ²)	The standard deviation of the motion acceleration in a trial.
Trajectory curvature STD (m ⁻¹)	The standard deviation of the trajectory curvature in a trial.

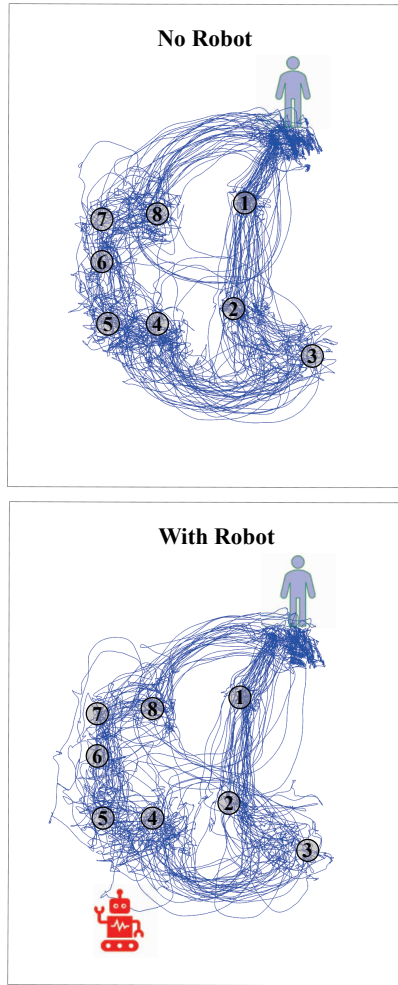


Fig. 3. Trajectory comparison between the “No Robot” and “With Robot” conditions.

Body Movement Pattern Comparison

Significant differences were observed when participants traversed between shelves under two robot conditions (Table 2 & Fig. 4). On average, participants spent more period of time ($F(1, 68) = 8.13$, $\eta_p^2 = 0.11$, $p\text{-value} = 0.01$) traversing between shelves with the presence of a robot (mean: 44.36 seconds), when compared to the “No Robot” condition (mean: 40.08 seconds). The presence of a robot also brought on decreases in participants’ motion speed ($F(1, 68) = 10.96$, $\eta_p^2 = 0.14$, $p\text{-value} < 0.001$) and motion acceleration ($F(1, 68) = 11.28$, $\eta_p^2 = 0.14$, $p\text{-value} < 0.001$) (mean speed: 0.49 m/s; mean acceleration: 1.08 m/s²), compared to the “No Robot” condition (mean speed: 0.53 m/s; mean acceleration: 1.14 m/s²). The standard deviation measure of participants’ motion acceleration was also affected, indicated by a reduction of values from 0.68 m/s² to 0.66 m/s² when there was a robot present in the experiment site against the “No Robot” condition ($F(1, 68) = 6.08$, $\eta_p^2 = 0.08$, $p\text{-value} = 0.02$). Walking distance, trajectory curvature, and the other two standard deviation (STD) measurements were not significantly affected by the presence of the robot.

IV. DISCUSSION

This study aimed to prove the concept that human motion is physically affected by the presence of a mobile cobot during grocery shopping, and the hypothesis is confirmed by the results in Table 2. With a robot looped around in the retail environment, participants spent more time traversing between shelves ($p\text{-value} = 0.01$), accompanied by significant decreases in motion speed ($p\text{-value} < 0.001$), motion acceleration ($p\text{-value} < 0.001$), and the standard deviation values of motion acceleration ($p\text{-value} = 0.02$), when compared to the “No Robot” condition.

The 11 % increase in walking time (40.08 seconds vs. 44.36 seconds) and 8 % decrease in walking speed (0.53 m/s vs. 0.49 m/s) under the “With Robot” condition indicate that participants changed their walking pattern to a slower pace, either proactively or reactively, when there was a robot in the scene. Combined with the fact that the standard deviation of motion speed did not change significantly under the “With Robot” condition ($p\text{-value} = 0.68$), it is highly likely that changes in participants’ motion speed were smooth and global

TABLE II. MEAN (STANDARD DEVIATION) OF EIGHT MOTION PARAMETERS AND THE EFFECT OF ROBOT CONDITION ON THESE MOTION PARAMETERS.

Motion Parameters	Robot Conditions		
	No Robot	With Robot	$p\text{-value}$
Walking time (sec)	40.08 (7.02)	44.36 (8.00)	0.01
Walking distance (m)	21.41 (2.83)	22.17 (3.24)	0.15
Motion speed (m/s)	0.53 (0.05)	0.49 (0.05)	< 0.001
Motion acceleration (m/s ²)	1.14 (0.12)	1.08 (0.11)	< 0.001
Trajectory curvature (m ⁻¹)	1.56 (1.05)	1.69 (1.63)	0.56
Motion speed STD (m/s)	0.24 (0.02)	0.23 (0.03)	0.68
Motion acceleration STD (m/s ²)	0.68 (0.10)	0.66 (0.08)	0.02
Trajectory curvature STD (m ⁻¹)	2.15 (2.49)	3.21 (6.69)	0.34

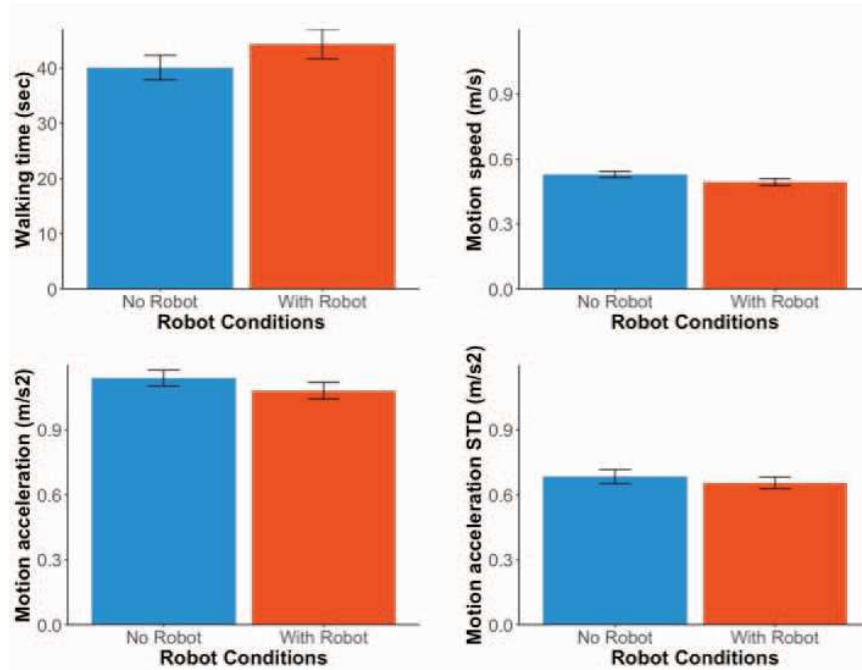


Fig. 4. Motion comparison between two robot conditions.

throughout the trial, rather than sudden and local at certain time points. The smooth and global changes in the motion speed can be attributed to participants' proactive reaction to reduce their motion speed after they noticed the approaching robot from a distance. Even if the robot was not that close, they may intentionally walk slower to avoid any potential collisions. The proactive human reaction when interacting with challenges in the environment, not necessarily the mobile cobots, has been observed in other studies [30], [31].

When compared to the "No Robot" condition, the reduction in the average (p -value < 0.001) and standard deviation (p -value = 0.02) values of participants' motion acceleration was also observed in the "With Robot" condition. Motion acceleration is a measure to depict the change rate of motion speed [32]. A lower motion acceleration implies that participants changed their motion speed more slowly. With that being said, the reduction of motion acceleration, whether its average or the standard deviation value, once again revealed that participants took a proactive reaction to the approaching robot and initiated and ceased their walking actions more smoothly, maybe to prevent collisions with the robot.

There are a few limitations recognized in this study that need to be noted. Firstly, as a pilot study, a limited sample size of participants was involved. Only healthy college students were recruited for this study. Individuals from varied backgrounds (e.g., less educated populations & robotics professionals) may have different reactions to the presence of robots. The recruitment of participants from a more diverse background could help the generalizability of this study. Secondly, human-robot interaction was only compared to the "No Robot" condition, although the comparison to human-human interaction was likewise worthwhile. In the retail environment, some interactions between customers and employees may be reformed by interactions between customers and serving robots. And the reactions of customers

to those adjustments may be of interest to the majority of retail stores. Therefore, to further investigate the physical reactions of retail customers to robot-related innovations, our future research plans include a comparison of human-robot interaction and human-human interaction among participants from a wider range of backgrounds.

V. CONCLUSIONS

In summary, this work is expected to establish and characterize the user's short-term gait adaptation and voluntary response with mobile cobots nearby. Successful completion of this work will enable the modeling and validation of the potential influence of mobile cobots on locomotion safety. Furthermore, it could serve as an estimate of the impact and effectiveness of a human-centered robot control algorithm in the following future experiments.

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