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Lattice QCD equation of state at finite chemical potential from an alternative resummation: strangeness neutrality and beyond

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Abstract. In this contribution we present a resummation of the Quantum Chromodynamics (QCD) equation of state from lattice simulations at imaginary chemical potentials. We generalize the scheme introduced in a previous work [1], to the case of non-zero strangeness chemical potential. We present continuum extrapolated results for thermodynamic observables in the temperature range $130\,\text{MeV} \leq T \leq 280\,\text{MeV}$, for chemical potentials up to $\mu_B/T = 3.5$, along the strangeness neutral line. Furthermore, we relax the constraint of strangeness neutrality, by extrapolating to small values of the strangeness-to-baryon-number ratio $R = n_S/n_B$.

1 Introduction

The knowledge of the QCD equation of state is of crucial importance, besides its intrinsic fundamental interest, for the modeling of heavy-ion collisions. At vanishing baryon density – where the transition is a crossover [2] – the equation of state has been known for about a decade from lattice QCD simulations [3–5]. On the other hand, results at finite baryon density are limited by the sign problem in lattice simulations. Though direct simulations have been performed recently with a novel reweighting approach [6, 7], on large lattices the use of indirect (extrapolation) techniques is necessary. These rely on Taylor expansion or analytical continuation from imaginary chemical potential [8–13].

Recently, different forms of reorganization of the Taylor series have been proposed [1, 14, 15], with the aim of improving its convergence. In particular, in [1] we introduced a scheme to extrapolate the equation of state of QCD based on the following ansatz:

$$F(T, \hat{\mu}_B) = F(T', 0) \ , \tag{1}$$

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whereby the $\hat{\mu}_B$-dependence[1] of an observable $F(T, \hat{\mu}_B)$ is described by a rescaling of the temperature:

$$T' = T\left(1 + \kappa_2^F(T)\hat{\mu}_B^2 + \kappa_4^F(T)\hat{\mu}_B^4 + O(\hat{\mu}_B^6)\right) , \quad (2)$$

in turn described by the alternative coefficients $\kappa_n^F(T)$. In essence, we move to a framework in which we expand in the shift

$$\Delta T = T - T' = \left(\kappa_2^F(T)\hat{\mu}_B^2 + \kappa_4^F(T)\hat{\mu}_B^4 + O(\hat{\mu}_B^6)\right) . \quad (3)$$

In these proceedings we illustrate a generalization of this scheme, to the case of non-vanishing strangeness chemical potential, studying in particular the case of strangeness neutrality $n_S = 0$, which has phenomenological relevance for heavy-ion collisions. Moreover, we perform an initial extrapolation to non-zero strangeness density, through an expansion in the strangeness-to-baryon ratio $R = n_S/n_B$.

2 Observables

We consider three different observables:

$$c_1^B(\hat{\mu}_B, T)\,, \qquad \mathrm{M}(\hat{\mu}_B, T) = \frac{\mu_S}{\mu_B}(\hat{\mu}_B, T)\,, \qquad \chi_2^S(\hat{\mu}_B, T)\,, \quad (4)$$

where

$$c_n^B = \frac{d^n}{d\hat{\mu}_B^n}\frac{p}{T^4} = \left(\frac{\partial}{\partial\hat{\mu}_B} + \frac{d\hat{\mu}_S}{d\hat{\mu}_B}\frac{\partial}{\partial\hat{\mu}_S}\right)^n \frac{p}{T^4} = \left(\frac{\partial}{\partial\hat{\mu}_B} - \frac{\chi_{11}^{BS}}{\chi_2^S}\frac{\partial}{\partial\hat{\mu}_S}\right)^n \frac{p}{T^4}$$

are the Taylor coefficients of the pressure along the strangeness neutral line, μ_S is such to realize strangeness neutrality, and $\chi_2^S(\hat{\mu}_B, T) = \partial^2(p/T^4)/\partial\hat{\mu}_S^2$. Note that, because of strangeness neutrality, $c_1^B = \chi_1^B = \partial(p/T^4)/\partial\hat{\mu}_B$. With respect to [1], we introduce an additional change in the case of c_1^B: for the observable F in Eq.(1), we consider the ratio $c_1^B(T, \hat{\mu}_B)/\overline{c}_1^B(\hat{\mu}_B)$, where $\overline{c}_1^B$ is the ($\hat{\mu}_B$-dependent) Stefan-Boltzmann limit of c_1^B. This is done in order to avoid the increase of κ_2 at large temperatures. With this modification, the expansion parameters are effectively changed, and we thus refer to the corrected ones by λ_n^F.

We show in the left panel of Fig.1 results for the ratio $c_1^B(T, \hat{\mu}_B)/\overline{c}_1^B(\hat{\mu}_B)$ at different values of the imaginary baryon chemical potential, in the case of strangeness neutrality (left). In the right panel, we show the same curves, with the x-axis replaced by $T \rightarrow T(1 + \lambda\hat{\mu}_B^2)$, with $\lambda = 0.0165$. We see in this case, that the difference between the different curves is minimal around the transition temperature, as well as in the high temperature limit. We also note that this value is different from the value $\kappa = 0.0205$ used in [1]. This is because the coefficients λ_n are defined after dividing by the Stefan-Boltzmann limit, but more importantly because the curvature in the case of strangeness neutrality seems to be smaller. A similar scenario appears in the curvature of the QCD transition line, where the numerical value in the case when $\mu_Q = \mu_S = 0$ is slightly larger [16].

The procedure to determine the coefficients λ_n^F makes use of results at both zero and non-zero chemical potential. The temperature T' for which Eq. (1) is satisfied is determined for different values of T, $\hat{\mu}_B$ and on lattices with different numbers of timeslices N_τ. A global fit for each temperature T is then performed, and continuum extrapolated results for the λ_n^F are obtained. For more details on the procedure, we refer the reader to [1, 15]. Once the coefficients are determined, the use of Eq. (1) for the observables in Eq. (4) allows us to determine the latter at finite (real) values of the chemical potential.

[1] We use the following notation for the dimensionless chemical potentials: $\hat{\mu}_i = \mu_i/T$.

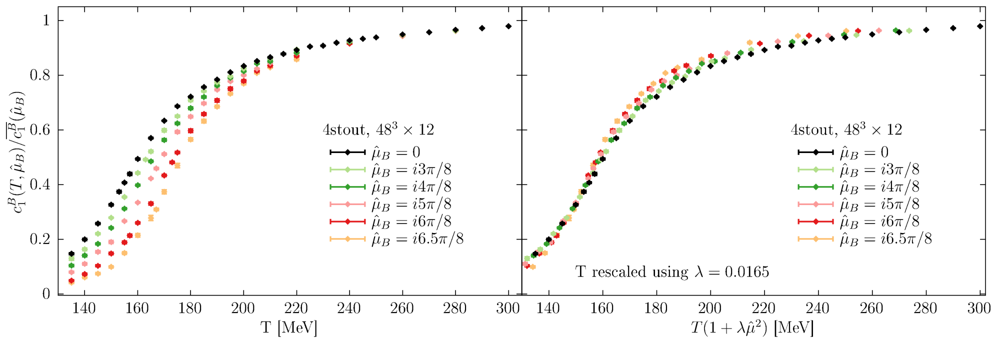

Figure 1. Left panel: results for the ratio $c_1^B(T, \hat{\mu}_B)/\overline{c}_1^B(\hat{\mu}_B)$ at different values of the imaginary baryon chemical potential, in the case of strangeness neutrality, on a $48^3 \times 12$ lattice. Right panel: same as on the left panel, with the x-axis rescaled according to $T \to T(1 + \lambda\hat{\mu}_B^2)$, with $\lambda = 0.0165$.

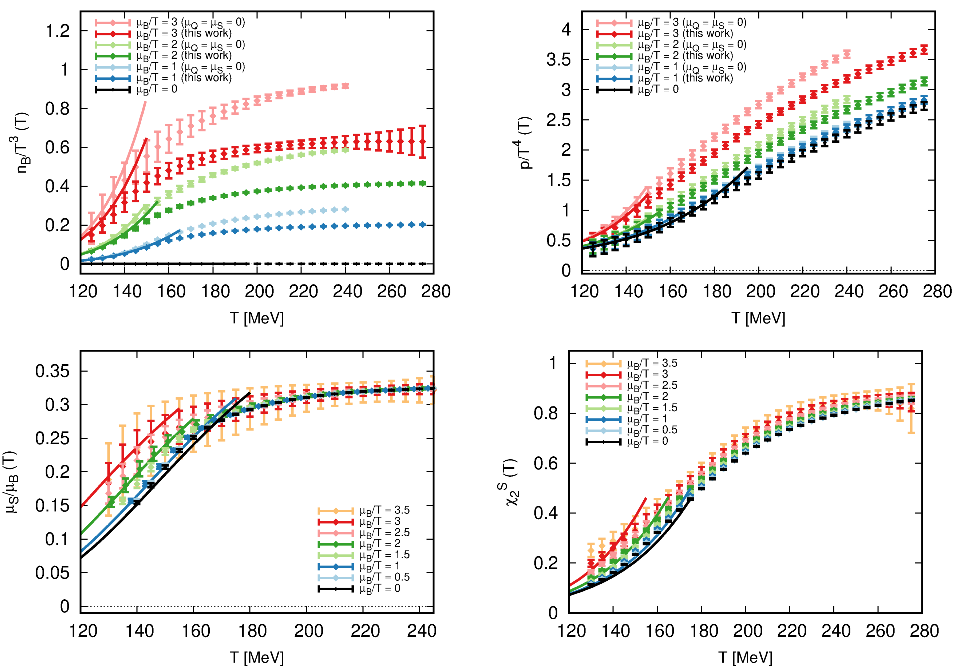

Figure 2. From top to bottom, left to right: baryon density, pressure, strangeness-to-baryon chemical potential ratio, and strangeness second susceptibility. The results are shown at increasing real values of $\hat{\mu}_B$. With solid lines we show the results from the hadron resonance gas (HRG) model. For baryon density and pressure, we also show (in lighter shades), the results for the non-strangeness neutral case.

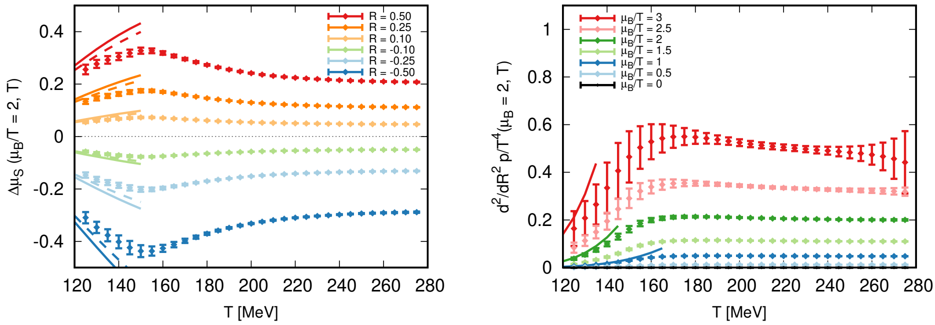

Figure 3. Left panel: shift in strangeness chemical potential for different values of the ratio R, at $\hat{\mu}_B = 2$. The lines indicate results from the HRG model obtained from Eq. (6) (dashed), or by solving $R = n_s/n_B$ for $\hat{\mu}_S$ (solid). Right panel: first non-vanishing coefficient for the expansion of the pressure in R around $n_S = 0$, for increasing values of $\hat{\mu}_B$. Solid lines show results from the HRG model.

We show in Fig. 2 the baryon density (top left), strangeness-to-baryon chemical potentials ratio (bottom left) and second strangeness susceptibility (bottom right) at increasing real values of $\hat{\mu}_B$. We also show (top right) the pressure, obtained from the baryon density through simple integration. For the baryon density and pressure, we also show the results in the case without strangeness neutrality. We see that all results, up to $\hat{\mu}_B = 3.5$ show very reasonable uncertainties, and do not hint at unphysical, non- monotonic behavior typical of traditional Taylor expansions.

3 Beyond strangeness neutrality

We wish to expand beyond strangeness neutrality in terms of the ratio $R = n_S/n_B$. Defining $\hat{\mu}_S^\star$ as the strangeness chemical potential realizing strangeness neutrality, we can write:

$$n_S \equiv \chi_1^S(\hat{\mu}_S) \approx \chi_2^S(\hat{\mu}_S^\star)\Delta\hat{\mu}_S \ , \qquad n_B \equiv \chi_1^B(\hat{\mu}_S) \approx \chi_1^B(\hat{\mu}_S^\star) + \chi_{11}^{BS}(\hat{\mu}_S^\star)\Delta\hat{\mu}_S \ , \tag{5}$$

where $\Delta\hat{\mu}_S \equiv \hat{\mu}_S - \hat{\mu}_S^\star$. Hence, we obtain:

$$R = \frac{\chi_1^S}{\chi_1^B} = \frac{\chi_2^S(\hat{\mu}_S^\star)\Delta\hat{\mu}_S}{\chi_1^B(\hat{\mu}_S^\star)\Delta\hat{\mu}_S + \chi_{11}^{BS}(\hat{\mu}_S^\star)} \ , \qquad \Delta\hat{\mu}_S = \frac{R\hat{\chi}_1^B(\hat{\mu}_S^\star)}{\chi_2^S(\hat{\mu}_S^\star) - R\chi_{11}^{BS}(\hat{\mu}_S^\star)} \ . \tag{6}$$

In the left panel of Fig. 3 we show the displacement $\Delta\hat{\mu}_S$ that realizes $n_S = Rn_B$, for $\hat{\mu}_B = 2$ and different values of R.

The correction to the pressure for small values of R vanishes at first order, because $n_S = 0$ by construction. The first non-zero contribution comes at the second order in R:

$$\frac{\mathrm{d}^2\hat{p}}{\mathrm{d}R^2}(T,\hat{\mu}_B) = \frac{\left(\chi_1^B(T,\hat{\mu}_B)\right)^2}{\chi_2^S(T,\hat{\mu}_B)} \ . \tag{7}$$

We show this correction coefficient for different values of $\hat{\mu}_B$ in the right panel of Fig. 3.

In these proceeding we have shown our continuum extrapolated results for thermodynamic quantities at finite real chemical potential, up to $\hat{\mu}_B = 3.5$. We obtained these with a generalization of the novel resummation scheme introduced in [1], applied to the case of strangeness neutrality. We showed that our results, for all chemical potentials, show no hints of pathological behavior, and display uncertainties well under control. Finally, we calculated the first terms for an expansion *around* the strangeness neutral line, in terms of the ratio $R = n_S/n_B$. These results can be improved systematically with larger statistics, and thus in the future provide guidance for the knowledge of the QCD equation of state for different chemical compositions of the QCD medium.

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