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Building Mathematics Professional Development With an Explicit Attention to Concepts and Student Opportunities to Struggle Framework

AUTHOR NAMES:

Hughes, Gwyneth; Carney, Michele B.; Champion, Joe; Yundt, Lindsey

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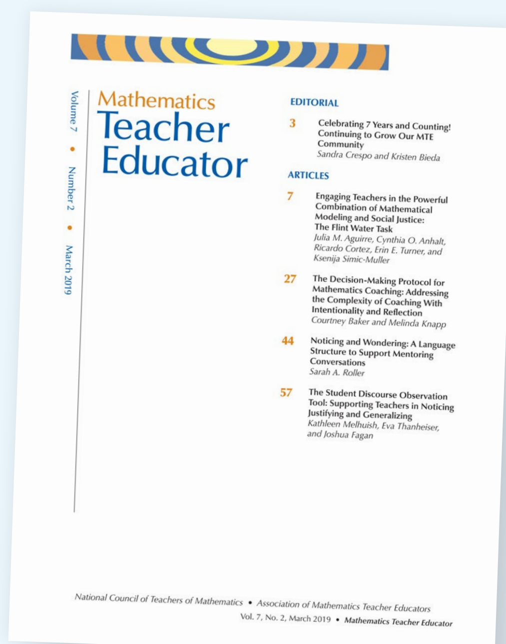
CONTACT: mte@nctm.org



NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Building Mathematics Professional Development With an Explicit Attention to Concepts and Student Opportunities to Struggle Framework

Gwyneth Hughes
Boise State University

Michele B. Carney
Boise State University

Joe Champion
Boise State University

Lindsey Yundt
Boise State University

Two broad categories of instructional practices, (a) explicitly attending to concepts and (b) fostering students' opportunities to struggle, have been consistently linked to improving students' mathematical learning and achievement. In this article, we describe an effort to build these practices into a framework that is useful for a diverse set of professional development (PD) offerings. We describe three examples of how the framework is used to support teacher learning and classroom instructional practice: a state-mandated course, lesson studies, and a large-scale teacher–researcher alliance. Initial findings suggest that consistently emphasizing this framework provides both content and structural guidance during PD development and gives coherence and focus to teachers' PD experiences.

Keywords: Teacher professional development; instructional practice; productive struggle; conceptual understanding

Introduction

During the past several years, our university-based network of mathematics teacher educators has been attempting to foreground instructional practices that improve students' mathematics learning in all of our teacher professional development (PD) offerings. As many teacher educators know, placing student learning at the center of PD is easier said than done. Some difficulties include the facts that there are many purposes for PD, resources are always limited, students' learning emerges from a complex web of factors, and teacher–participants have diverse backgrounds, contexts, needs, preferences, and practical constraints. Nonetheless, our primary goal as teacher educators is to offer coherent PD centered on student learning, and we have addressed that goal in two main ways. First, we design PD programs around instructional practices identified in research as positively affecting student achievement. Second, we focus our work with teachers on coexploring ways to adapt and implement those instructional practices within teachers' local contexts. To these ends, we offer several interconnected K–12 mathematics PD programs, including a state-mandated course, direct collaborations with individual and groups of teachers, topic-specific learning modules, and a multidistrict teacher/researcher alliance with a grade-band focus.

As our work has expanded, we have faced a central problem of practice: the need for an overarching PD framework to ensure coherence and focus across and within diverse programs. Our definition of a PD framework is a set of research-based practices or beliefs that summarize and elucidate what we mean by "impactful" mathematics education. Such a PD framework supports us as developers and facilitators in creating programs using the same basic foundation, regardless of content, format, or grade level. In addition, a framework gives participants a structure on which to organize and connect their learning within and across PD experiences. Based on our goals and collective professional experiences in education, we wanted our PD framework to address six priorities:

- grounded in research on practices that positively influence student achievement
- accessible and relatable to educators

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- respectful of educators' expertise
- supportive of educators with varied mathematical knowledge for teaching
- conceptually aligned across content and formats
- applicable to many educational contexts (e.g., urban vs. rural and elementary vs. secondary)

This article describes how we developed the framework, how it has shaped our programs, and some preliminary evidence of its influence on educators and student achievement. The framework, which we call the EAC/SOS framework, takes its name from two clusters of instructional practices with robust research evidence for supporting students' success in mathematics: *Explicit Attention to Concepts* (EAC) and *Student Opportunities to Struggle* (SOS) (Hiebert & Grouws, 2007; Stein et al., 2017). Our intention in sharing the EAC/SOS framework is two-fold. First, we hope that some mathematics teacher educators will be able to apply the framework to their own context, as a way to guide the creation of PD. Second, and more generally, we hope readers can draw from our experiences as they engage in similar journeys toward creating and refining overarching frameworks that steer their own deeply situated PD work.

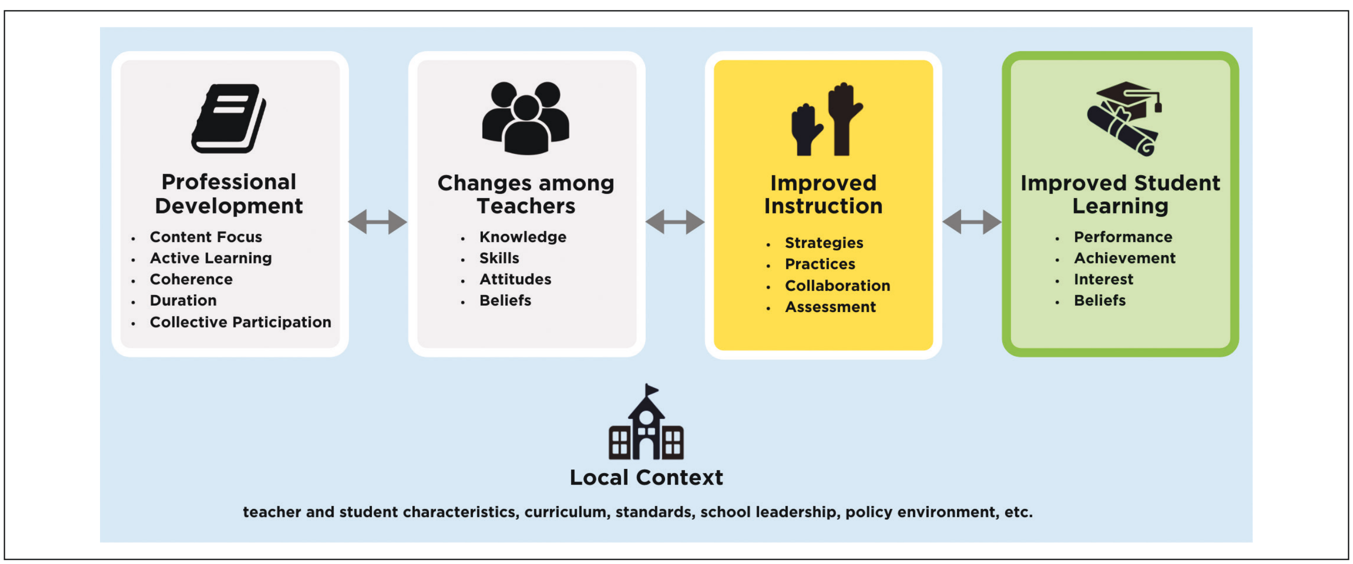
Why Develop Another PD Framework?

Large-scale analyses (Hattie, 2003) suggest teacher factors account for approximately 30% of variation in achievement. However, the field is continuing to understand how to close the loop between efforts to improve teacher factors and more distal goals for improving student learning. Desimone (2009) put forward a framework for how specific attributes of PD can influence teachers and their instruction, and, in turn, improve student learning. Figure 1 outlines our theory-based conceptual model based on Desimone's work for how PD can improve student learning. The figure emphasizes the driving goals of our PD offerings: Improved Instruction and Improved Student Learning, and it is in these areas where we have added to Desimone's original framework. We operationalize student learning as evidenced by a combination of four intertwined elements: performance, achievement, interests, and beliefs. As mathematics educators, we are concerned about all these elements of learning, but much of our PD addresses achievement because of general statewide agreement among teachers, sponsors, and stakeholders regarding the measures and objectives for students' mathematics achievement. We included the overarching component of local context in our version of the framework to ensure our work considers the realities teachers face in the day-to-day functioning of American public schools.

Building from our model of PD's influence on student learning, we set out to connect our programs with research-based frameworks that offer structure for the goals, activities, and resources. In reviewing published descriptions of mathematics PD frameworks, many published in *Mathematics Teacher Educator*, we found a few general types. Many focus on a specific aspect of mathematics *pedagogy*—a classroom practice or set of practices—that is independent of specific content

Figure 1

Conceptual Model for the Effects of PD on Teachers, Instruction, and Student Learning (Adapted From Desimone, 2009).



or grade level. Examples include classroom discourse (Herbel-Eisenmann et al., 2013), connecting representations (Hughes et al., 2015), or professional noticing (Fukawa-Connelly et al., 2018; Jilk, 2016; van Es et al., 2015). Other programs use *content* to guide PD, such as a focus on learning trajectories (Edgington et al., 2016) or content standards (Seago et al., 2013). Finally, some PD programs use frameworks specific to *curriculum*, such as task development or modification (Aguirre et al., 2019). These categories of PD frameworks are neither mutually exclusive nor exhaustive. Panorkou and Kobrin (2017), for instance, combined both professional noticing and learning trajectories into their PD, noting that understanding learning trajectories helps teachers understand student thinking and make instructional decisions.

As teacher educators, we use several frameworks within specific PD settings. For example, both learning trajectories and professional noticing are foundational to PD in which we look at student work with groups of teachers. Classroom discourse frameworks are also central to how we model mathematical discussions with teachers. On the basis of the scale and context of our work, we needed an overarching PD framework to connect across the variety of specific math PD frameworks we use in different offerings. In addition, a singular framework would support coherence for both us, as teacher educators, and the teachers we serve across diverse settings.

Adopting the EAC/SOS Framework

The Math Education Collective (MEC) at Boise State University offers research-based mathematics support for roughly 6,000 educators and roughly 90 school districts and charters in Idaho. The service area spans most of the urbanized areas in Idaho as well as many rural, small-enrollment districts. The MEC houses several state-level grants, provides fee-based math PD services, and houses an NSF-funded project, Researching the Order of Teaching (ROOT), which conducts research on effective instructional strategies in Grades 6–8. The goal of the MEC is to increase student mathematical learning—with a focus on achievement—across grade levels, curricula, and variable district needs. Therefore, any underlying framework for our PD needs to be relatively easy to communicate and perceived as relevant among teachers and school leaders.

Building From Literature

A theoretical framework for PD is a set of guiding principles, typically based in prior literature, that makes clear the underlying beliefs and assumptions that guide large-scale, multifaceted PD work. In a sense, a theoretical framework for PD serves both to guide the PD developers in their creation process and to ground the PD participants in the overarching assumptions and practices in which the developers are working. In practice, such a framework elucidates the focus of work at the programmatic level while guiding the content and structure of specific offerings. A PD framework can also be shared with participants to emphasize the important takeaways or a schema for thinking about the content of the PD.

In looking to ground our PD framework in past research, we started with Hiebert and Grouws' (2007) review of the literature of dozens of empirical studies on how teaching factors affect students' mathematical learning. The learning outcomes they investigated—skill efficiency and conceptual understanding in mathematics—cut across grade levels and topics, and their criteria for robust evidence was appropriately expansive.

Teachers in our PD consistently report that teaching for conceptual understanding is more difficult than building skill efficiency; something they often attribute to curriculum and their own schooling experience. This challenge is recognized in mathematics education literature as well (e.g., Ma, 2010; Philipp, 2007; Stigler & Hiebert, 1997). Consequently, Hiebert and Grouws' (2007) analysis of teaching practices that influence students' conceptual understanding, defined as "the mental connections among mathematical facts, procedures, and ideas" (p. 382), is especially useful. Ultimately, Hiebert and Grouws identified in the literature two clusters of effective instructional practices in mathematics that cut across study design, teaching formats, audience, and even historical time:

- EAC—Teachers and students are involved in bringing out and explicitly connecting between mathematical concepts through activities such as questioning, discussion, and comparison.
- SOS—Students need regular opportunities and time to grapple with key mathematical concepts. Here, struggle is used in the sense of "productive struggle" as described later by Warshauer (2015), rather than as acute frustration.

It is through combining these elements that students build mental connections between mathematical procedures and ideas that define conceptual understanding. On the one hand, explicitly attending to concepts can be seen as an externally mediated process of connection, where the teacher is designing interactions or activities that explicitly and publicly bring out such connections. By having opportunities to struggle mathematically, on the other hand, students develop those connections by drawing on their own knowledge. Moreover, these two elements have the potential to reinforce one another. For example, when students see mathematics as a series of interconnected concepts, they are better prepared to take responsibility for their learning, struggling through a novel problem by drawing from their prior math experience (e.g., Carpenter et al., 1996; Fosnot and Dolk, 2001). Likewise, when students are given the opportunity to struggle in mathematics, they will be better able to take on the cognitive work of learning and connecting new concepts (Boaler, 2015; Warshauer, 2015).

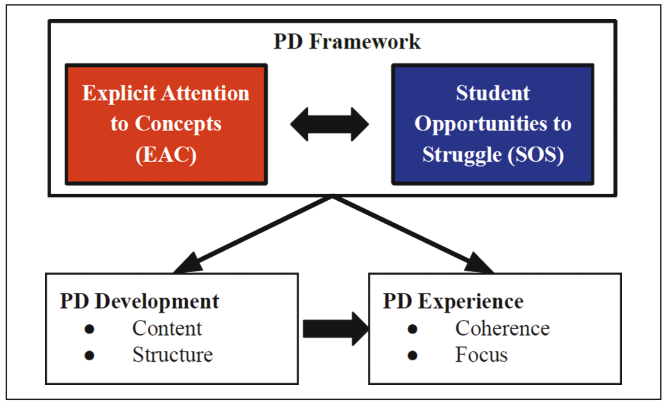
Stein et al. (2017) reported on a large-scale study of EAC and SOS, and we use their naming convention herein. Using survey responses, video, and artifacts, they categorized teachers' instructional practices on the basis of presence or absence of EAC and SOS. Group means on both traditional and problem-based assessments were significantly higher for students of teachers who demonstrated both high EAC and high SOS, especially on the more conceptually-based tasks. Students of teachers who identified with neither element of teaching scored lowest on both. Students of teachers who incorporated *either* EAC or SOS scored in between those two end-members. Collectively, the findings suggest incorporating EAC and SOS into teaching may be helpful for designing PD, because EAC and SOS teaching practices appear to promote both skill proficiency and conceptual understanding, particularly when paired.

Several empirical studies support an interconnection between EAC and SOS (Kapur, 2014; Loehr et al., 2014; Song & Kapur, 2017). For example, research by Schwartz et al. (2011) found providing students with opportunities to struggle to develop their own measure of density before being explicitly taught the concept improved students' ability to transfer the concept to new situations. The combination of EAC paired with SOS in the form of project-based learning and problem solving has also supported traditionally marginalized communities in accessing challenging mathematics (e.g., Gutierrez, 2000; Kitchen et al., 2017). Perseverance and self-efficacy have likewise been connected to SOS (Boaler, 2016; Morales & DiNapoli, 2018). Additionally, the way students perceive themselves as mathematically capable has a large influence on their problem-solving ability (Pajares & Miller, 1994) and whether they pursue majors and/or careers that incorporate mathematics or even see mathematics as relevant to their lives and interests (Boaler & Selling, 2017; Martin, 2012).

Figure 2 features the connections between the EAC/SOS framework elements. First, we use the two constructs to develop PD programs, thinking about where we explicitly discuss the EAC/SOS practices and engage participants in activities that make use of those practices. This planning process is further elucidated in the upcoming PD examples.

Figure 2

A Model of How the EAC/SOS Framework Impacts Both the Development of the PD and the Participants' Experience With the Material



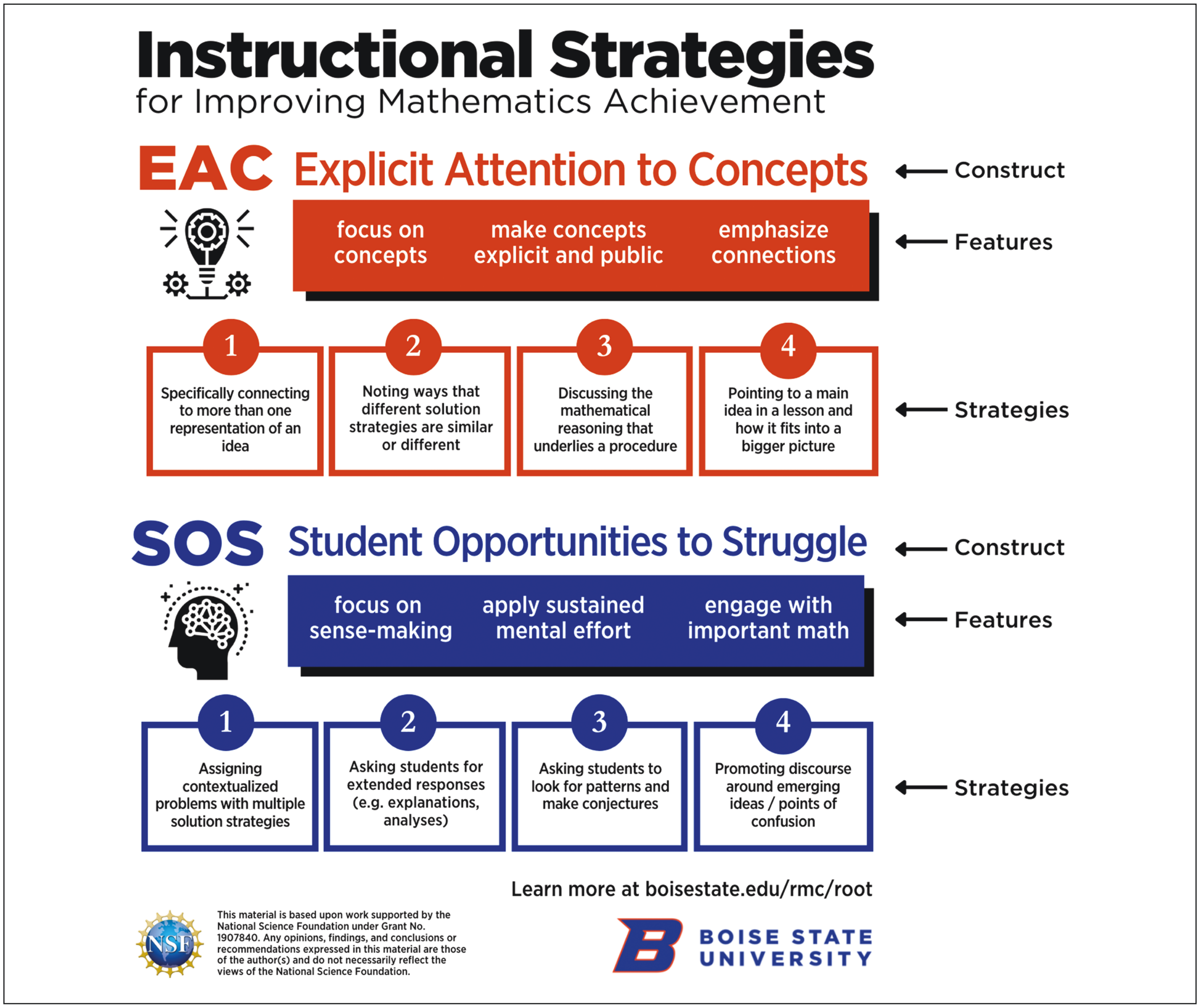
Definitions and Practices in the Framework

On the basis of our cumulative experience as teachers and teacher educators, we know that, when looking at education research, teachers understandably move quickly to the question of "What can this realistically look like in my practice?" To assist in connecting research to practice, we developed a variety of materials to support teachers' implementation of instructional strategies associated with EAC and SOS in the classroom. Figure 3 is an example of a one-page document we created to scaffold from the constructs of EAC and SOS to specific instructional strategies. Three characterizing instructional features are indicated under each construct. For EAC, these feautures are *focus on concepts*, *make concepts explicit and public*, and *emphasize connections*. For SOS, these features are *focus on sense making*, *apply sustained mental effort*, and *engage with important mathematics*. Last, four example strategies are provided for each construct to assist teachers in connecting these ideas to classroom practice. Further explanation of the framework and a more detailed handout are available through Champion et al. (2020).

At least in our formulation, practices associated with EAC and SOS are not specific to any one pedagogical style. For example, in the Munter et al. (2015) article that examines and compares direct (teacher-centered) and dialogic (student-centered) instruction, we find examples of both EAC and SOS in the descriptions of both models. For example, in a class based largely on lecture, the teacher can still explicitly draw attention to key concepts and ask students to highlight them across different problems and topics. Assignments in such a classroom can also require students to struggle by applying new concepts to novel problems. Likewise, in a largely project-based math class, students are likely to

Figure 3

Handout to Briefly Describe EAC and SOS Constructs, Features, and Strategies



engage in struggle as they investigate and make decisions. The teacher can also structure projects, evaluation, and class discussion around explicit concepts. We thus see EAC and SOS as being key instructional practices for teacher participants to recognize and bolster in their own classrooms.

Framework Implementation and Evidence of Effectiveness

The EAC/SOS framework is central to planning our PD programs. Figure 4 illustrates how the two constructs generally guide the development of PD content and structure. Generally, we identify key concepts and specific activities that emphasize those concepts (EAC). We then consider how the PD (a) gives participants examples of how to engage students in productive struggle; and (b) engages participants themselves in productive struggle as part of the course assignments.

Viewing our entire program and specific PD offerings through this lens allows us to ask questions as we design, review, and revise. Such a framework helps us not "lose the forest for the trees" by ensuring we do not overfocus on, for example, wanting to do a specific problem or use a new technology. Rather, we want to maintain our focus on influencing teachers' knowledge and beliefs and, in turn, improving student achievement.

Figure 4

Example of How the EAC/SOS PD Framework Guides PD Development.

PD	Explicit Attention to Concepts (EAC)	Student Opportunity to Struggle (SOS)
Content	What are the key concepts that underlie the mathematics of focus, based on the literature?	What **resources** are provided to support teachers in facilitating productive struggle in their classrooms?
Structure	What problems or experiences will engage participants with those key concepts?	How do we engage participants in productive struggle themselves to model SOS facilitation?

We also use specific activities to increase teacher participants' understanding of EAC and SOS during their PD experiences and to support teachers working to translate the practices into their classrooms, schools, and districts. To illustrate this work further, we describe three specific PD programs and how EAC/SOS is embedded. We make an argument for the effectiveness of the EAC/SOS framework by examining a sample of PD participants' survey responses, artifacts, and/or reflections, and, for one program, initial student achievement data.

Example 1: State-Mandated Course

Teaching Mathematical Thinking (TMT[1]) is a course sponsored by the Idaho State Department of Education as part of a 2008 legislative mandate connected to Idaho teacher certification to strengthen educators' understanding of the development of number, operations, and algebraic thinking across the grades. The 3-credit (45-hr) PD course is facilitated in both in-person and online versions, but all require participants to examine mathematical topics through the lenses of central mathematics concepts, student thinking, and pedagogical techniques. Past research has shown this course significantly shifts teachers' self-efficacy and mathematical knowledge for teaching (Carney et al., 2016).

Since 2018, Boise State University's version of the TMT course has included EAC and SOS as specific pedagogical techniques for TMT. The constructs are introduced during the first day of the course in relation to connecting between multiple representations of a visual problem-solving task related to systems of equations. Specifically, participants are asked to engage in productive struggle by solving the initial task in a way that makes sense to them. Although algebra teachers tend to use a system of equations, most others use combinations of guess and check, tables, or pictures to solve the problem. Participants then work in small groups and in a facilitated whole-class discussion to identify commonalities across the different representations, explicitly attending to concepts such as minimum, maximum, and rate of change. The EAC/SOS framework is then introduced formally, and the constructs are directly tied to the activity. Throughout the rest of the course, methods of implementing EAC and SOS are connected to particular mathematical content. Discussion prompts also ask participants to further analyze practices or concepts through the lens of the EAC/SOS framework.

In addition to being a focus within the TMT course, the construction of each course module is tied to the EAC/SOS framework. Figure 5 shows how the framework guides the design of an asynchronous place value module. Through the lens of EAC, we identified using one as a base unit and iterating and partitioning by 10 to create new units (e.g., tens and tenths) as a key concept for the unit. Participants engage in activities that highlight this key concept by analyzing and translating between base-10 representations (see **Appendix A**—an excerpt from the module presentation). In terms of SOS, participants are provided with examples of a student engaging in productive struggle around those key concepts during a presentation (**Appendix A**, slide 7), and in a teacher-friendly reading (Brickwedde, 2018). In creating and sharing problems with their group members, participants are also actively engaged in productive struggle themselves. Through the interplay of activities, readings, minilectures, and shared group work, all participants, regardless of instructional level or experience, engage in both EAC and SOS.

Participant Reflections

Teachers in the course frequently report a deepening of their own mathematical understanding (Carney et al., 2016;

1 Previously titled as Mathematical Thinking for Instruction (MTI), renamed in 2018.

Figure 5

Example of How the EAC/SOS Framework Guides TMT Module Content and Structure.

Place Value Module	Explicit Attention to Concepts (EAC)	Student Opportunity to Struggle (SOS)
Content	*Key concepts:* • 1 as the base unit • Iterating & partitioning by 10 to create new units • Brickwedde (2018) article: connecting multiplicative thinking to place value	*Provide examples of engaging students in productive struggle:* • Video of student thinking • Example problems and vignettes in Brickwedde (2018)
Structure	*Asynchronous activities in workbook and recorded lecture:* • Analyze base-10 block representations • Translate into different units (tenths v. tens)	*Assignment presented to group via Voicethread:* • Create and present place value "riddles." • Connect between visual representations of fraction, decimal, and percent.

Hughes et al., 2015), and it seems the EAC/SOS framework has given some participants language for identifying these shifts in their own thinking, as evidenced by their reflections.

In the Place Value Module cited above, participants were then given the following discussion board prompt:

> **Choose a quote from the reading from this module and connect to the idea of Explicitly Attending to Concepts. Choose any two of the activities from this module and compare how they demonstrate explicit attention to concepts.**

All 54 course participants described EAC because it was specifically asked for in the prompt. However, 18 participants went beyond, giving evidence of a change in their own conceptual understanding or reflecting on how they could incorporate EAC techniques in their own classrooms.

For example:

- What I like about this explicitly attending to concepts is that the teacher is drawing the connection from place value to multiplication. Creating the understanding and visually showing how the place value increases within the operation of multiplication. I like how the teacher begins with a challenge to challenge the students' thinking. This is a conversation that I will be having with my fourth graders.
- Thinking of my 5th grade curriculum, there are honestly very few examples of visualizing such large quantities in a grid or table form. Students may see up to 1,000 to represent the thousandths place in decimals. Having students explain how they see the connections between iconic [visual] and numerical [representations] in activities such as a quick write or partner share could be great methods to reinforce this concept.

Survey Data

Initial results from a pre–post EAC/SOS survey administered to all TMT participants includes responses from n = 155 participants in 2019–2020 TMT courses. The Priorities for Mathematical Instruction (PMI) Survey aims to locate teachers' thinking about instruction based on the continua described in Carney et al. (2021) using composite scores from their responses to priority-based questions about different instructional practices and beliefs. Two example items are given in Figure 6.

Figure 7 shows the pre–post shift we see in participants standardized gain scores on the EAC (median pre = .08, post = .92) and SOS (median pre = .01, post = .89) scales. These shifts provide evidence to support the claim that implementation of the EAC/SOS framework in the course influences teachers' beliefs. This evidence, combined with the above qualitative examination, indicates the potential efficacy of the framework for moving participants toward an instructional orientation that places greater priority on EAC and SOS in mathematics teaching.

Figure 6

Example EAC/SOS Survey Items.

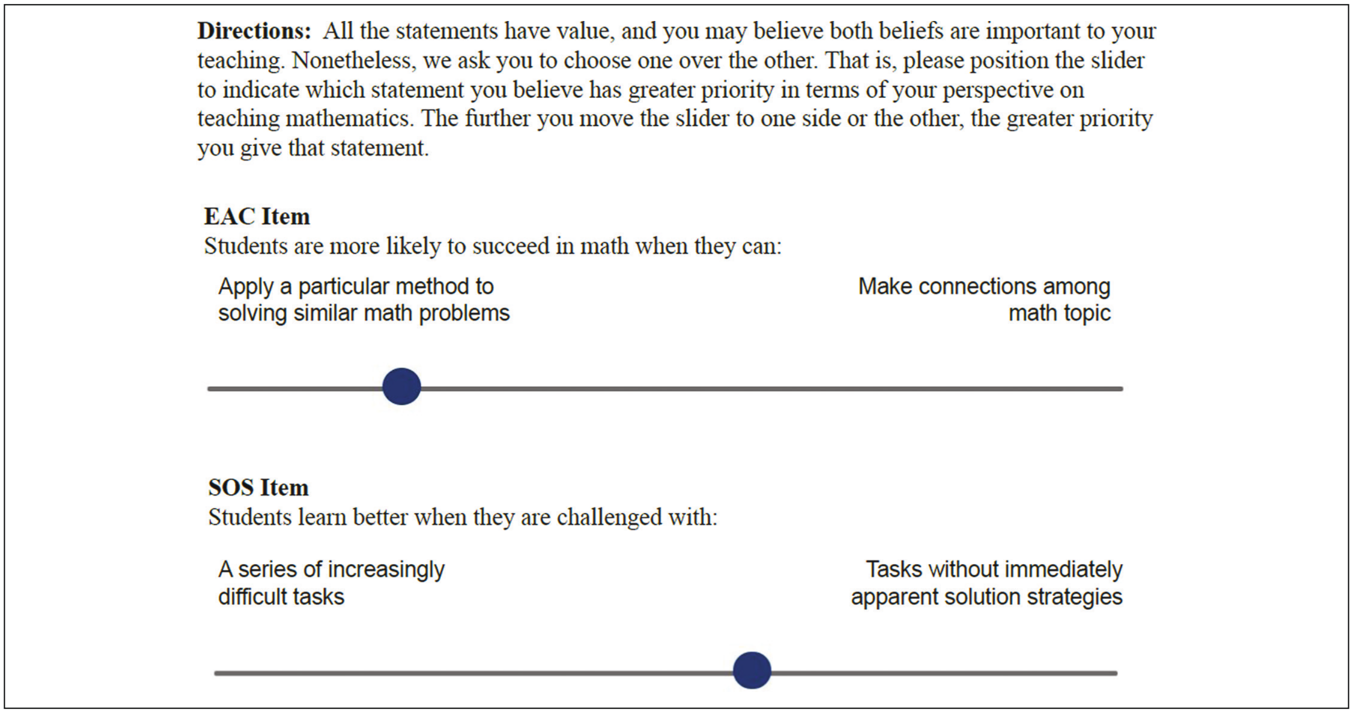
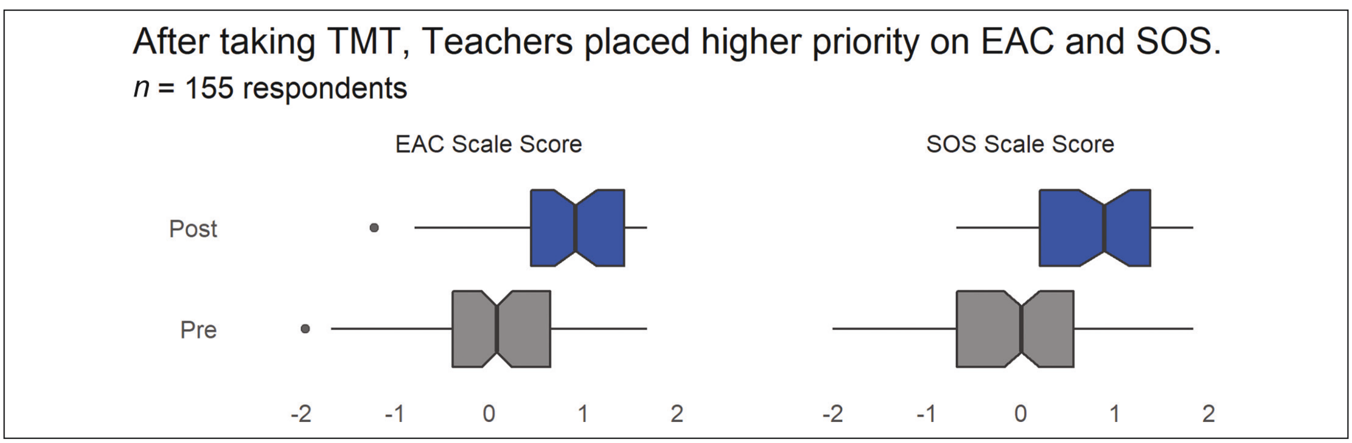


Figure 7

TMT Course Participants' Pre- and Post-Scores on the PMI Survey.



Note: Scales are standardized, z = (composite score–overall mean)/(overall mean), with nonoverlapping middle notches indicating statistically different group medians.

Example 2: Intensive Small-Scale Collaboration Through Lesson Study

In contrast to the large-scale work represented by the TMT course, we also engage in school-based collaboration with teachers. An example in the MEC is Lesson Study. In Lesson Study, teachers engage in cycles of planning, implementation, and reflection facilitated by a math specialist. Within these cycles, teachers focus on a question or topic related to student-centered learning. Specifically, participants research and plan a lesson. One teacher then teaches the lesson while the others carefully observe and collect data on students' participation. The team reflects and revises on the basis of the lesson observation. The same day, another teacher teaches the revised lesson while the others observe, allowing the team to reflect on how the revisions affected student learning. A team typically completes two to three lesson study cycles per year. Lesson Study has been a common practice in Japan for decades (Murata & Takahashi, 2002) and has gained traction in North America (Takahashi & McDougal, 2016). Research points to the association of increased student achievement with sustained lesson study practice (Lewis et al., 2006).

One math specialist at the MEC has been engaged in year-round lesson study cycles for the past 5 years. The description below refers to her experience facilitating lesson study through the lens of the EAC/SOS framework. Data presented come from the program assignments and evaluations of the 2020 cohort of 22 K– Grade 4 math teachers. Participants in the lesson study program begin each year with a kickoff workshop, in which the facilitator introduces EAC and SOS as two components to consider when developing a central focus question. For example, in this group of teachers, 15 of their 17 questions (some have common questions) focused on practices associated with SOS (8), EAC (4), or both (3). For example:

- SOS: "How can we build a class culture that will encourage students to pose questions and justify their reasoning?"
- EAC: "How can we use the progression of mathematical concepts to make instructional decisions that bridge learning gaps?"
- Both: "What routines can we embed in our lessons that support students in exploring the relationships between representations?"

The facilitator also uses the EAC/SOS framework to support teachers in developing, observing, reflecting on, and revising the lesson. The intentional incorporation of the EAC/SOS framework into a lesson study cycle is outlined in Figure 8.

In their reflections after their first cycle, of the 22 respondents, 20 referred specifically to realizations about practices associated with EAC or SOS in their reflections. For example:

- SOS: "I am learning a lot from this study. Wording questions in order to solicit thinking without directly teaching the concept is something that I am working on. The old-school teacher in me wants to use different modalities to demonstrate the concept vs. asking questions and allowing the students to struggle towards self-discovery. I am improving."
- EAC: "Making connections between representations is imperative in order for students to be willing to try to understand a new representation they didn't create. Connecting these visuals may require modeling of the strategy or even getting manipulatives out."

Figure 8

Example of How the EAC/SOS Framework Guides Lesson Study Content and Structure.

Lesson Study Cycle	Explicit Attention to Concepts (EAC)	Student Opportunity to Struggle (SOS)
Content	Facilitator and participants identify key concept(s) based on the needs of participating teachers and identify activities & readings for meetings.	Facilitator chooses activities, practitioner papers, and/or videos that illustrate SOS for meetings.
Structure *Lesson Plan*	Participants plan the lesson around identified key concepts with facilitator support.	Participants look for opportunities for SOS throughout the lesson structure.
Implementation & Reflection	Participants observe how concepts were made explicit and how students engaged with them.	Participants observe whether all students engaged in SOS.
Revision & Reteaching	Participants revise and reteach the lesson to make concepts more explicit.	Participants revise & reteach the lesson to ensure all students engage in SOS.

We see the EAC/SOS Framework as grounding lesson study PD in a shared vision of what mathematics instruction should include, while empowering teachers to learn and experiment.

Example 3: Large-Scale Teacher–Researcher Alliance

The final PD program discussed in this centers on the EAC/SOS framework, the interplay between those two constructs in middle school classrooms, and its effect on student learning and achievement. ROOT is a large-scale NSF DRK-12 sponsored teacher–researcher alliance in which the MEC brings together researchers at Boise State and 100 middle grades (6–8) mathematics teachers to study effective mathematics instruction for improving middle grades students' achievement in modeling and problem solving. The EAC/SOS framework is at the forefront of the project, with PD sessions and data collection centered around adaptation and implementation of the framework elements across a wide range of instructional contexts. The first year of the project focused on PD sessions to familiarize participants with the elements of the EAC/SOS framework. The second year of the project implemented classroom teaching studies in which student achievement was compared with respect to teachers' implementation of specific EAC and SOS strategies (Figure 3) in a classroom setting. The project is ongoing but has already helped to refine and articulate the features, strategies, and cues for building EAC and SOS routines. The handout in Figure 3 (ROOT Project, 2021) was developed in the ROOT project to assist teachers with their implementation of classroom studies around EAC and SOS during year 2.

Figure 9 provides an example of how the EAC/SOS framework guided the creation of PD sessions on several key concepts, including developing some group consensus around what EAC and SOS look like in the classroom. This activity made use of the handout featured in Figure 3. **Appendix B** provides the slides associated with this activity. (The videos are not provided due to IRB requirements.)

In an early phase of the ROOT research, 83 participating teachers completed classroom studies of EAC and SOS strategies. Because the project focuses on students' mathematics achievement, we asked teachers to use pre–post assessments of student learning, with items scored on a standardized scale, and to implement a research design that afforded comparisons of EAC and SOS strategies

Figure 9

Example of How the EAC/SOS Framework Guided ROOT Module Content and Structure.

Intro to EAC and SOS	Explicit Attention to Concepts (EAC)	Student Opportunity to Struggle (SOS)
Content	• Understand the impact of the numerator and the denominator in fraction multiplication. • Identify the features and strategies associated with EAC and SOS within a classroom instructional context. • Begin to develop group consensus about how these practices 'look' in a classroom.	Participants watch a classroom video of a teacher engaging in an EAC strategy - pressing connections between representations. While the strategy was EAC-focused, the video also highlights the student struggle that can accompany EAC-focused activities, something we were explicitly focusing on with participants.
Structure	• Participants engaged in a math task and responded to prompts designed to highlight the role of the numerator and denominator in fraction multiplication. • Participants discussed their responses and pressed connections between the symbolic and bar model representations.	Small- and whole-group discussions followed the video using the EAC/SOS handout. Participants identified features and strategies associated with EAC/SOS within the classroom video. Our goal was to see the variety in their interpretations, and begin developing community ideas about EAC and SOS in practice.

across groups of students. Unfortunately, the teaching studies occurred during the 2020–2021 school year, which was highly affected by the COVID-19 pandemic, so teachers worked with mathematics specialists to produce common data formats under extraordinarily challenging conditions. In all, we had 46 single-group pre–post designs (potential for between teacher comparisons), 17 two-group pre–post designs (potential for within teacher comparisons), nine cluster crossover designs (improved within teacher comparisons), and four blended designs. Teachers chose one or two of eight EAC or SOS instructional strategies (Figure 3) to study, then planned and implemented instruction focused on their strategy/strategies (M = 7.9 teaching days, SD = 3.1 days) and collected pre-(mid-)post student assessment data (M = 50 students, SD = 30). Of the combined n = 3,923 students with research quality achievement data, the mean pre–post gain (normalized) among students was M = 34.7% (SD = 24.2%). Students' mathematics achievement was similar across teaching studies, with moderate variability, but the data suggested student achievement varied greatly within strategies. Interestingly, as shown in Figure 10, students' mathematics achievement in the teaching studies differed moderately across the strategies (as evidenced by the length of the box and whiskers), with studies of SOS strategies tending to outperform studies of EAC strategies (as evidenced by the location of the median for each plot). The evidence of the effectiveness of EAC and SOS strategies generally comes from comparing to regular instruction on the far left of the graph.

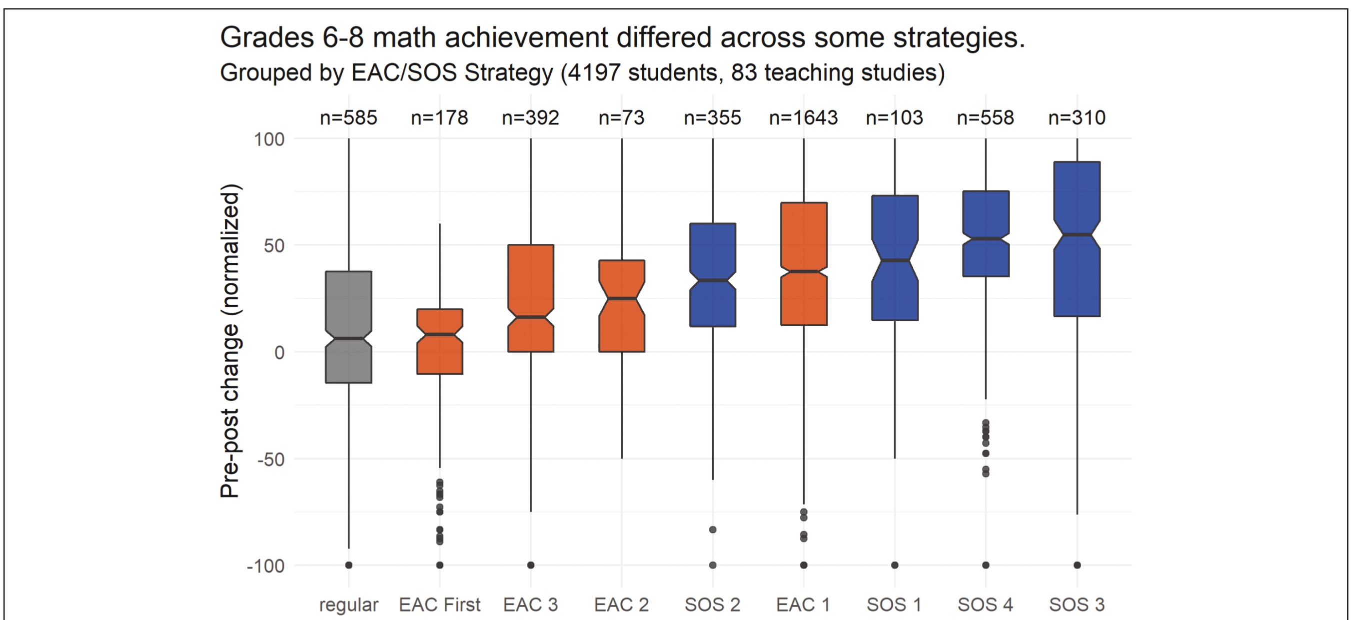
Discussion

Mathematics teacher PD can adopt a range of forms and goals, but creating sustainable PD programs that connect research and practice to influence student learning is a point of emphasis for many mathematics teacher educators. As we collectively work to expand teachers' access to quality PD across districts, states, and even larger scales, maintaining coherence can be difficult. In our own work, which stretches across different scales, formats, contexts, and grade levels, we sought a framework that connects all of our programs while still leaving room to use more specific math PD frameworks. The EAC/SOS framework meets these criteria because it foregrounds instructional practices that promote student achievement in ways that are general enough to be applicable in a variety of teacher support and education settings for both preservice and in-service teachers, and clear and focused enough for educators to synthesize and implement. We see both the EAC/SOS framework itself and the general process of connecting PD to a common framework as applicable to mathematics teacher educators who are similarly creating or refining impactful math PD programs.

In our setting, honing the lens through which we design and implement PD programs to the EAC/SOS framework has allowed us to put research on student learning at the forefront of all of our PD activities with

Figure 10

Distributions of Students' Pre–Post Change on Grades 6–8 Mathematics Assessments in ROOT Teaching Studies of EAC/SOS Strategies.



Note. Nonoverlapping middle notches indicate statistically different group medians.

Table 1

Potential Uses for the EAC/SOS Framework in Mathematics Teacher Education

Context	Potential uses
Mathematics methods class	Lesson/unit planning with EAC and SOS as overarching practices
Professional learning community	Focused discussions around planning for and reflection on the use of EAC and SOS instructional practices with a common math content topic
Classroom-based research	Investigation of the utility of a specific EAC or SOS instructional practice for a particular context/math topic
Content-focused workshop	Reflecting upon the use of rich mathematical tasks to encourage SOS or pressing connections between representations to encourage EAC through the lens of a particular math content/topic

teachers. All our PD offerings can now include common elements: (a) engaging teachers with rich mathematical tasks; (b) sharing the EAC/SOS framework, with teachers experiencing the practices as reflective learners; and (c) discussing how instructional practices associated with EAC and SOS can be implemented in the classroom. PD facilitators draw teachers' attention to their own opportunities to struggle with mathematical content and how those opportunities allow for mathematical sense making. Facilitators use EAC to develop participants' understandings of key concepts by drawing attention to similarities, differences, and connections among participants' approaches, strategies, and models for solving complex problems. Further, the EAC/SOS framework helps facilitators support teachers in considering how EAC and SOS instructional practices can be implemented in their classrooms, which has produced some encouraging outcomes data. We think this serves to support teacher participants' understanding of the constructs from a learner's perspective and their transferring of practices from PD to their own classrooms, two of the key features of effective PD (Desimone, 2009; Zaslavsky, 1995).

This general framework, tied to our overarching PD goals, also brings clarity to prioritizing requests for PD support, helping to define the scope of our work, and to managing limited resources effectively. We hope the description of our own process of identifying, situating, and implementing our PD framework supports other mathematics teacher educators in doing so as well.

Recommendations and Future Directions

The usefulness of the EAC/SOS framework in our PD projects has led us to consider creative ways to build it into future work. As outlined in Table 1, we have identified several possible additional contexts for implementing the framework in both preservice and in-service mathematics teacher education settings. In fact, two of the authors have already started using the EAC/SOS framework as a consistent touchstone for an elementary mathematical methods course.

One area of potential future research is investigating which aspects of the EAC/SOS framework are most useful for teachers, depending on their background and context. Communication is a key challenge, and on the basis of our informal observations, providing very specific examples of instruction may cause participants to lose sight of features of the EAC/SOS framework. For example, by providing the example of a card sort as a tool that is useful for making concepts explicit across representations, it seemed some educators assumed that simply doing a card sort engaged their students in EAC. By refocusing on the broader features of EAC, participants were able to think more deeply about how to create and implement a card sort in a way that supports students' ability to identify and connect concepts across representations. Toward that end, we have recently developed a highly visual poster (ROOT Project, 2021) with "cues"—short, easy-to-remember actions that can help teachers effectively implement EAC and SOS—that we are eager to assess as a way to promote effective daily instructional practices (see **Appendix C**).

The EAC/SOS framework may also serve as the foundation for a kind of professional learning progression for teachers working to implement effective instructional practices. In observing teachers' engagement with the framework, we are hopeful that PD that connects back to EAC and SOS can move nearly every teacher a little farther along their professional path, regardless of their current point in the journey.

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Authors

Gwyneth Hughes, Boise State University, ID 83725; gwynethhughes@boisestate.edu

Michele B. Carney, Boise State University, ID 83725; michelecarney@boisestate.edu

Joe Champion, Boise State University, ID 83725; joechampion@boisestate.edu

Lindsey Yundt, Boise State University, ID 83725; lindseyyundt@boisestate.edu

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Appendix A

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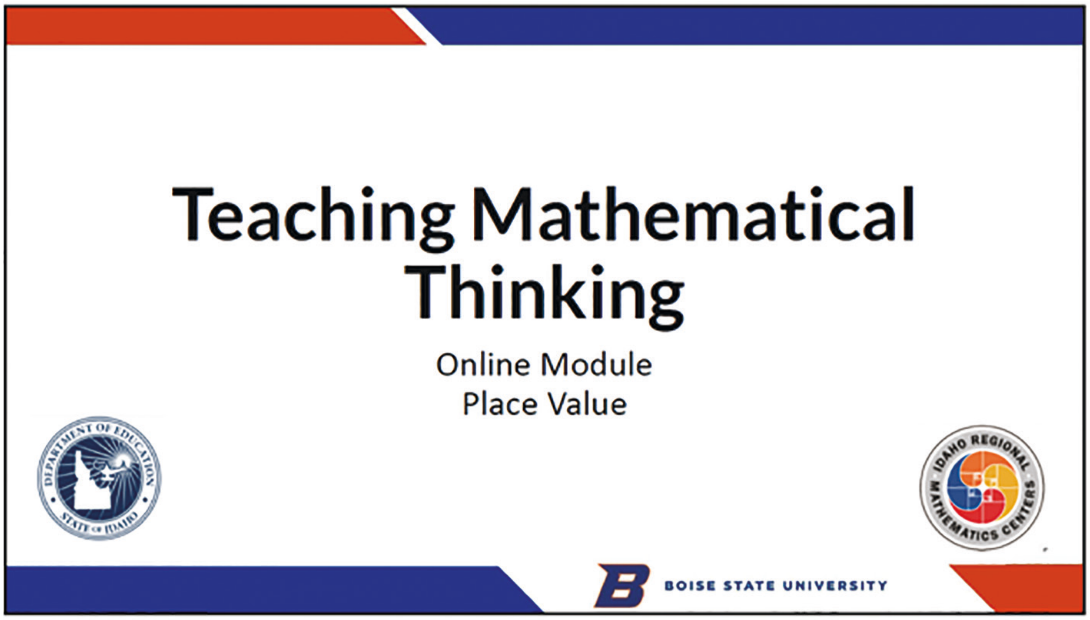
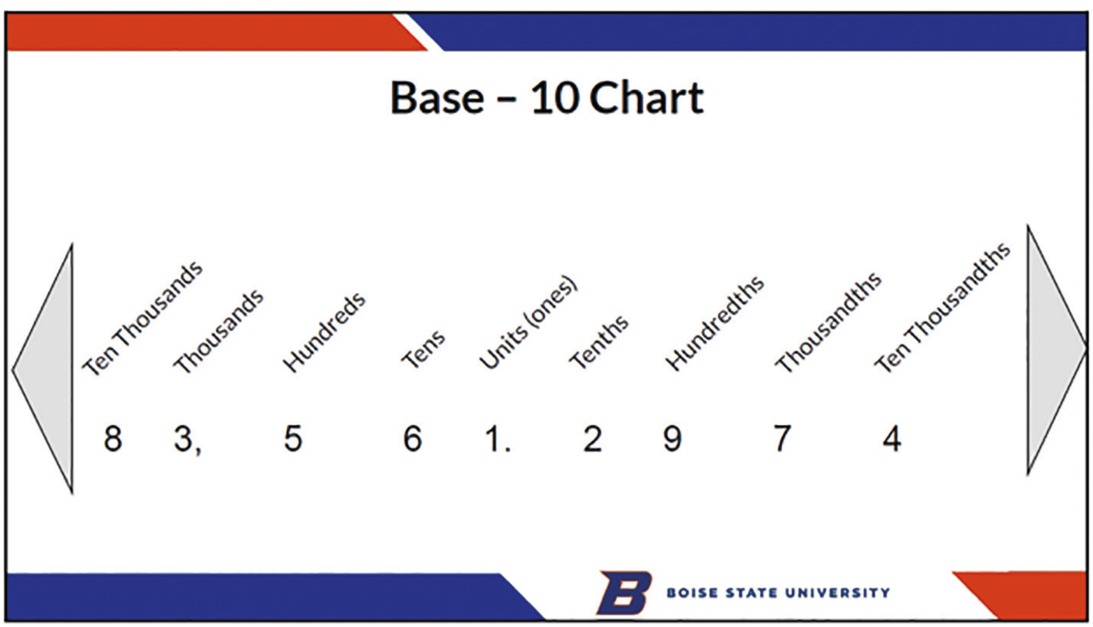
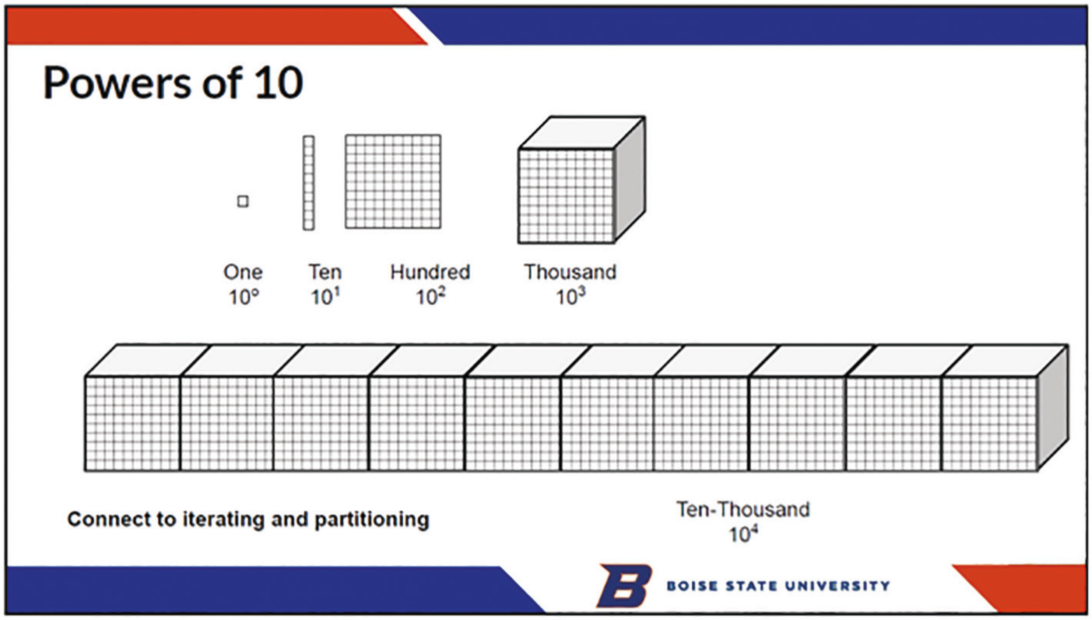
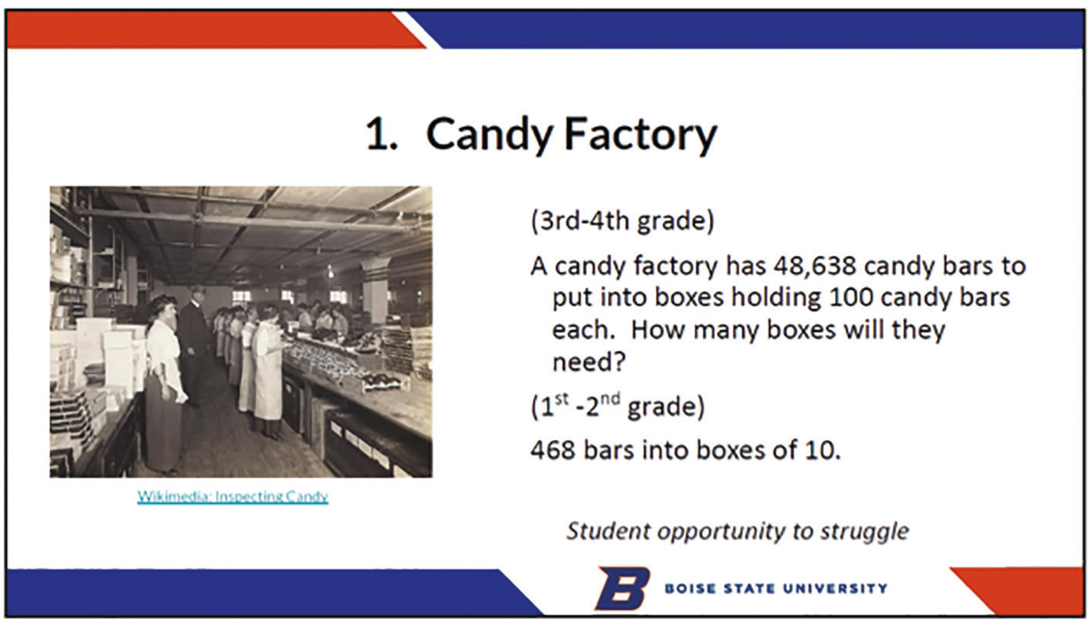





Powers of 10
One
10^0
Ten
10^1
Hundred
10^2
Thousand
10^3
Connect to iterating and partitioning
Ten-Thousand
10^4
BOISE STATE UNIVERSITY


1. Candy Factory
Wikimedia: Inspecting Candy
(3rd-4th grade)
A candy factory has 48,638 candy bars to put into boxes holding 100 candy bars each. How many boxes will they need?
(1st -2nd grade)
468 bars into boxes of 10.
Student opportunity to struggle
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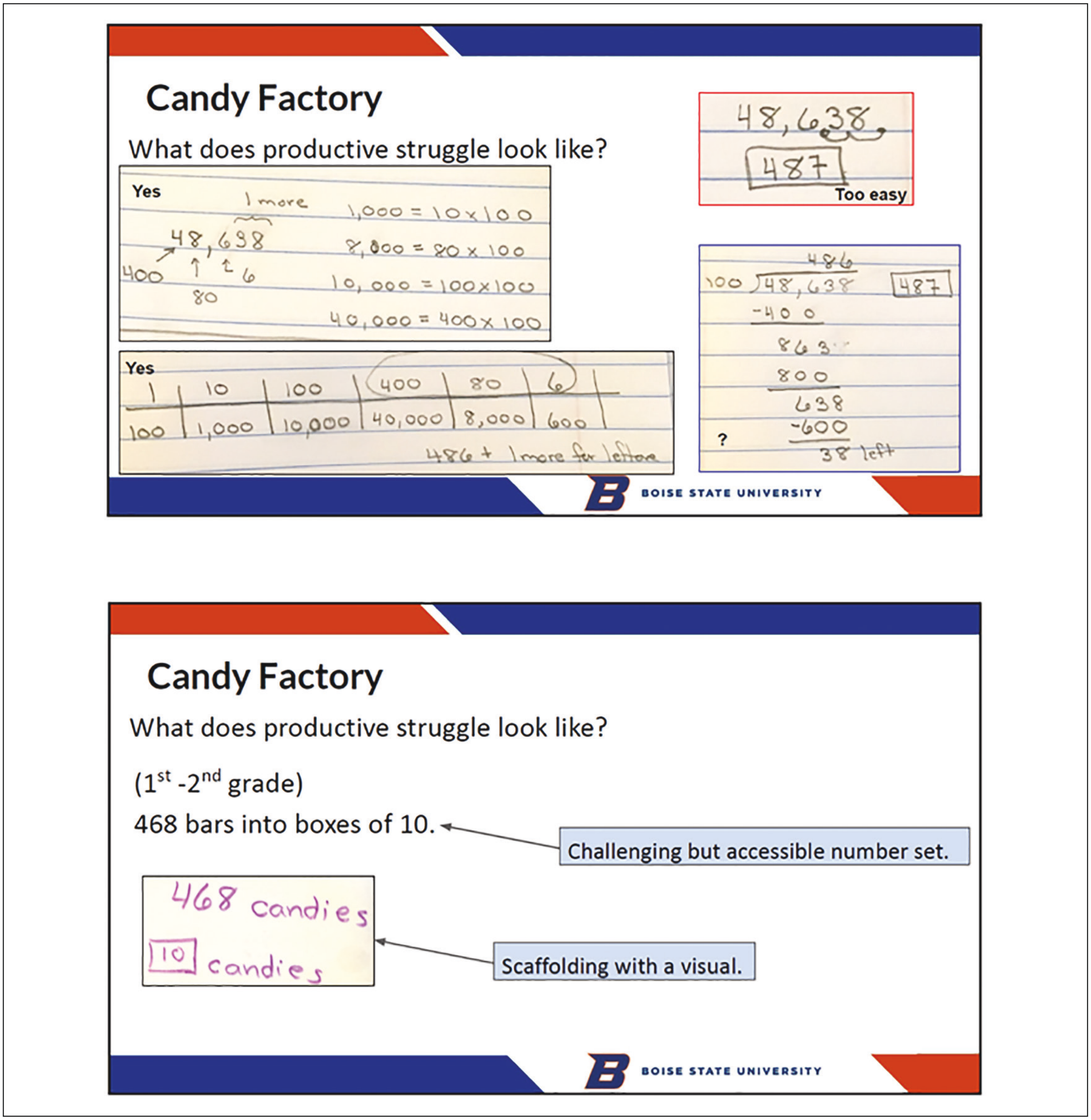
Candy Factory
What does productive struggle look like?
Yes
1 more
48,638
400
80
6
1,000 = 10 × 100
8,000 = 80 × 100
10,000 = 100 × 100
40,000 = 400 × 100
Yes
1 | 10 | 100 | 400 | 80 | 6
100 | 1,000 | 10,000 | 40,000 | 8,000 | 600
486 + 1 more for leftover
48,638
487
Too easy
486
100) 48,638
487
−400
863
800
638
−600
38 left
?
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Candy Factory
What does productive struggle look like?
(1st -2nd grade)
468 bars into boxes of 10.
Challenging but accessible number set.
468 candies
10 candies
Scaffolding with a visual.
BOISE STATE UNIVERSITY


468 bars into boxes of 10.
How many boxes do we need?
Student Video
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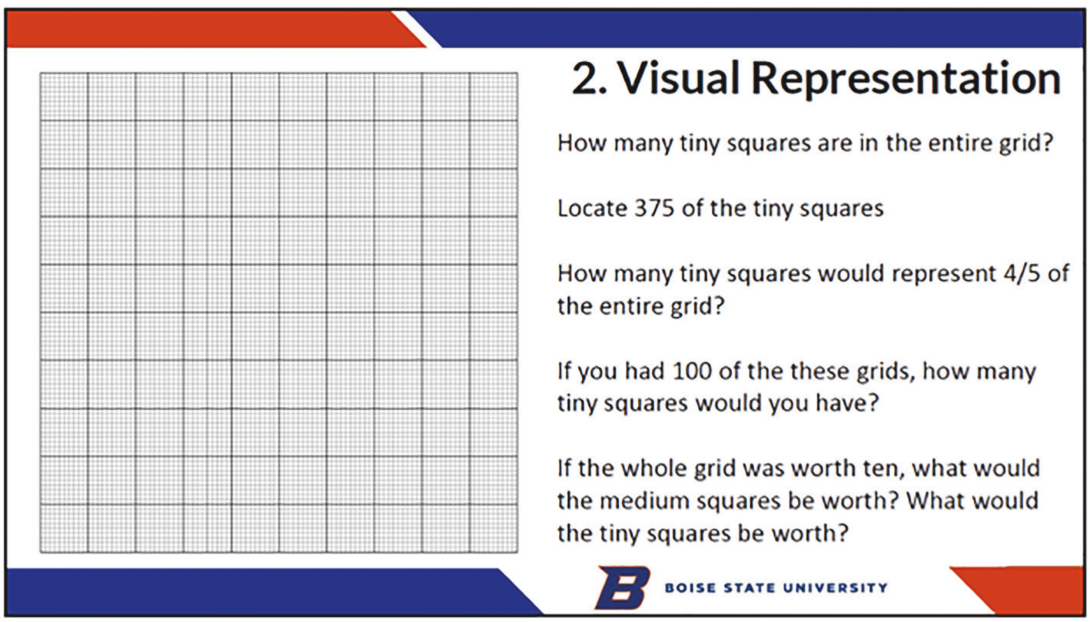
2. Visual Representation
How many tiny squares are in the entire grid?
Locate 375 of the tiny squares
How many tiny squares would represent 4/5 of the entire grid?
If you had 100 of the these grids, how many tiny squares would you have?
If the whole grid was worth ten, what would the medium squares be worth? What would the tiny squares be worth?
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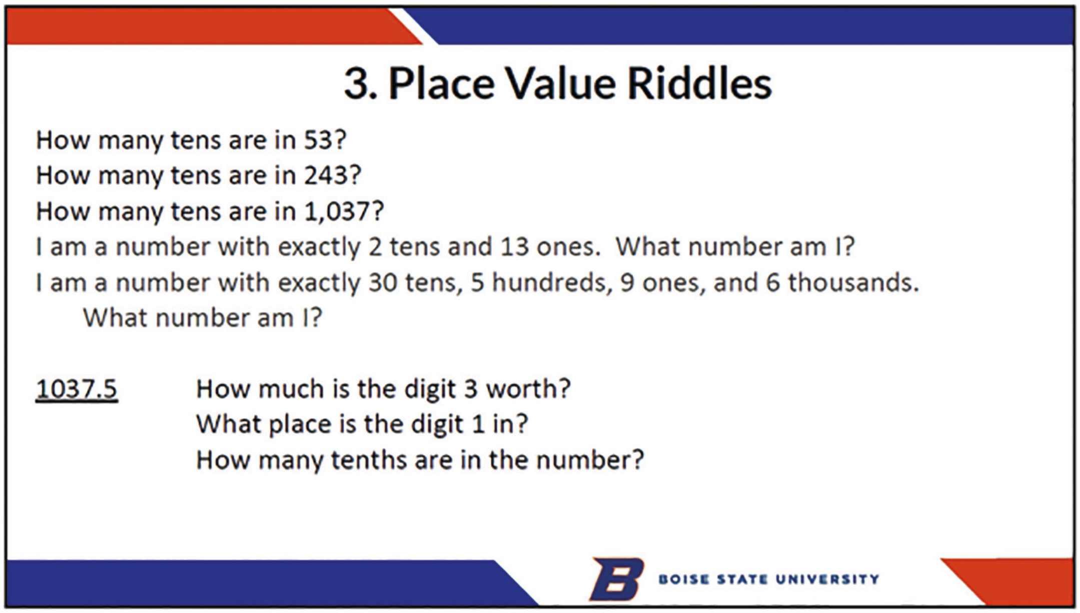
3. Place Value Riddles
How many tens are in 53?
How many tens are in 243?
How many tens are in 1,037?
I am a number with exactly 2 tens and 13 ones. What number am I?
I am a number with exactly 30 tens, 5 hundreds, 9 ones, and 6 thousands. What number am I?
1037.5
How much is the digit 3 worth?
What place is the digit 1 in?
How many tenths are in the number?
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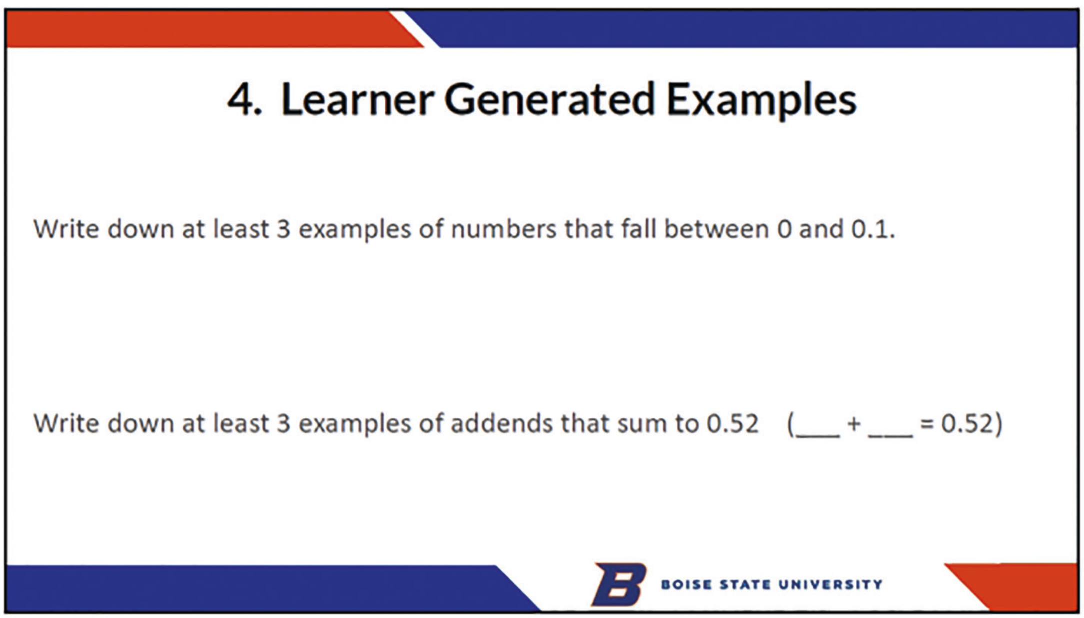
4. Learner Generated Examples
Write down at least 3 examples of numbers that fall between 0 and 0.1.
Write down at least 3 examples of addends that sum to 0.52 (___ + ___ = 0.52)
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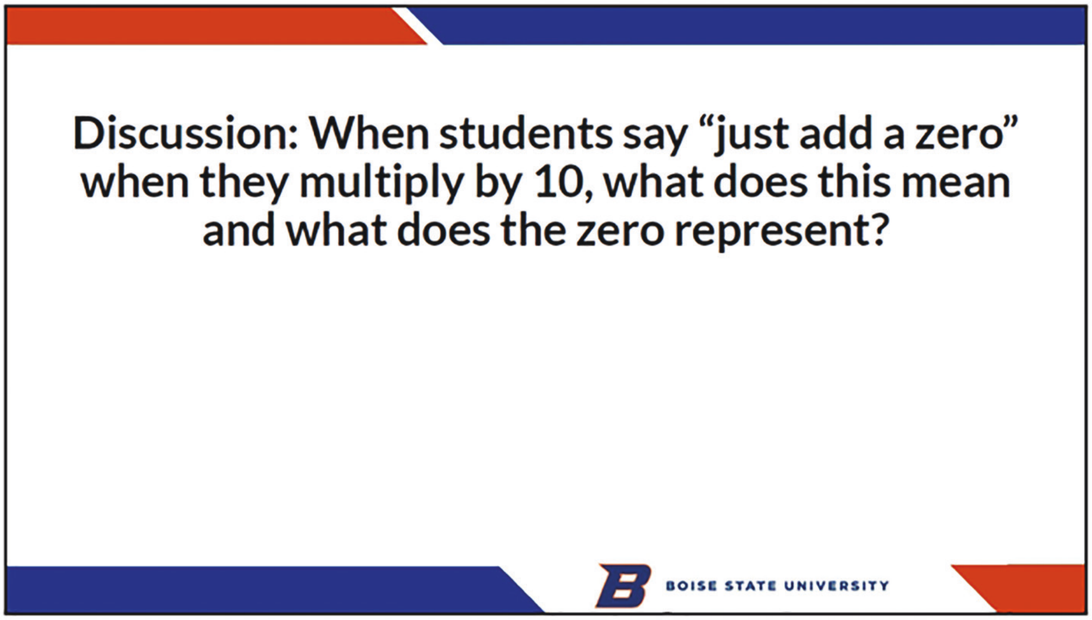
Discussion: When students say “just add a zero” when they multiply by 10, what does this mean and what does the zero represent?
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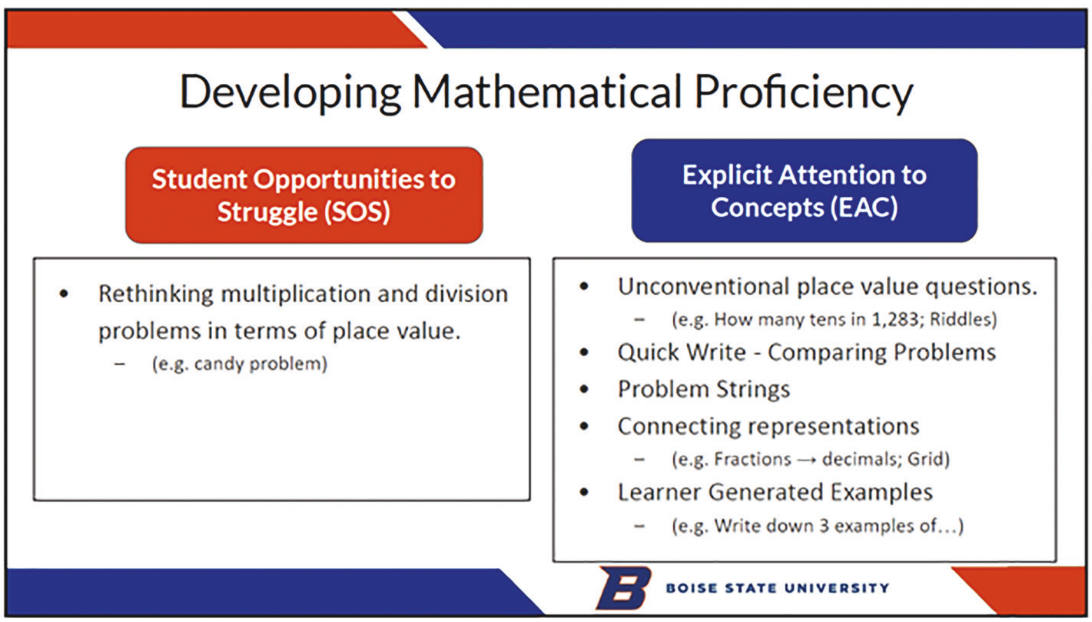
Developing Mathematical Proficiency
Student Opportunities to Struggle (SOS)
Explicit Attention to Concepts (EAC)
• Rethinking multiplication and division problems in terms of place value.
– (e.g. candy problem)
• Unconventional place value questions.
– (e.g. How many tens in 1,283; Riddles)
• Quick Write - Comparing Problems
• Problem Strings
• Connecting representations
– (e.g. Fractions → decimals; Grid)
• Learner Generated Examples
– (e.g. Write down 3 examples of…)
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Appendix B

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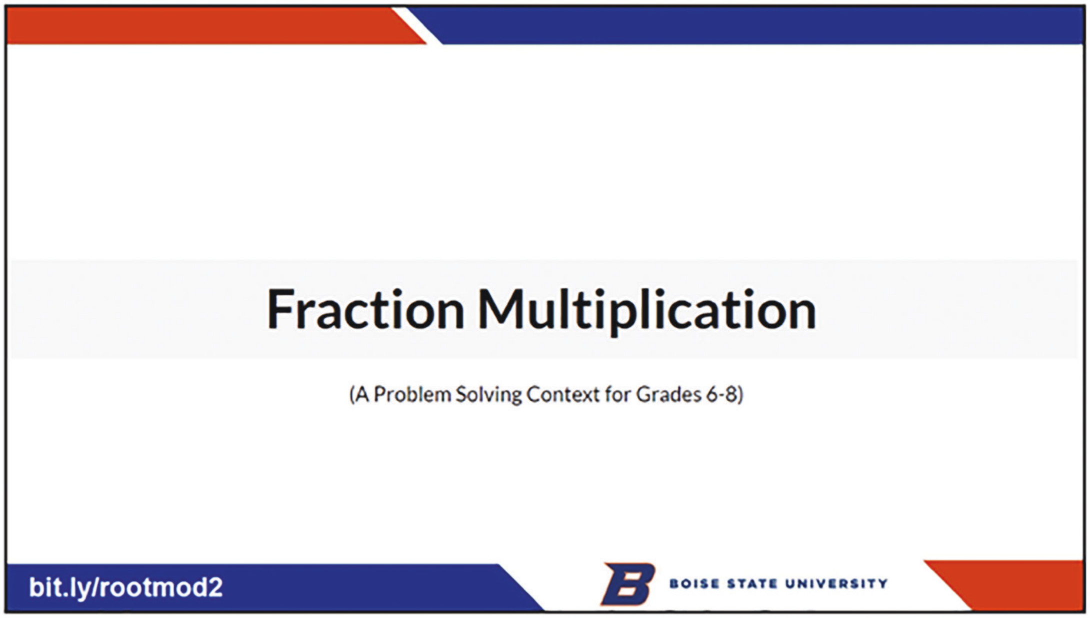
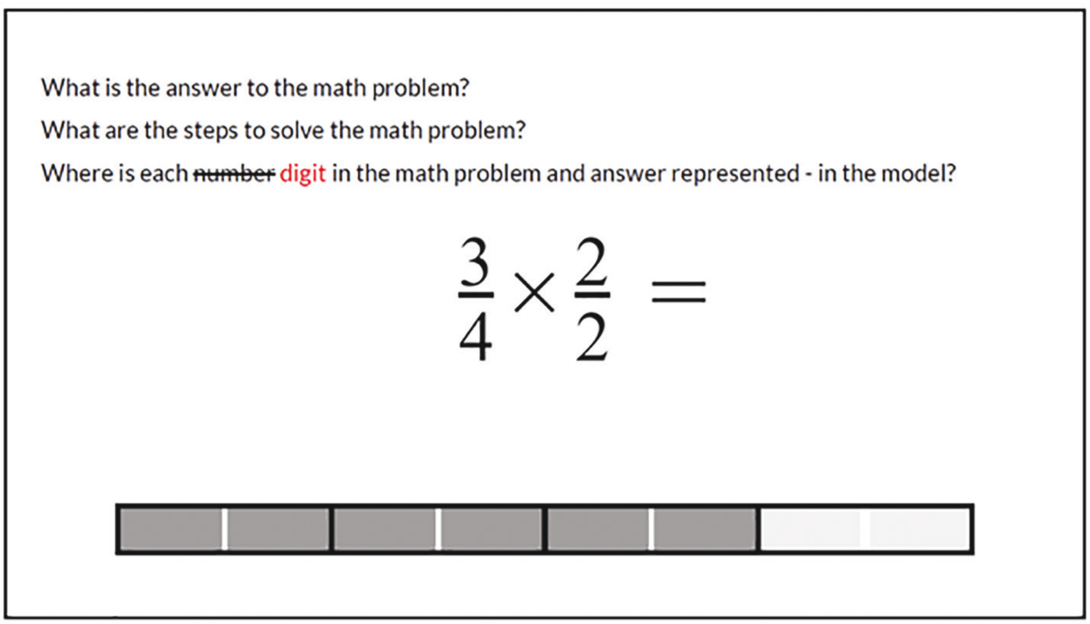
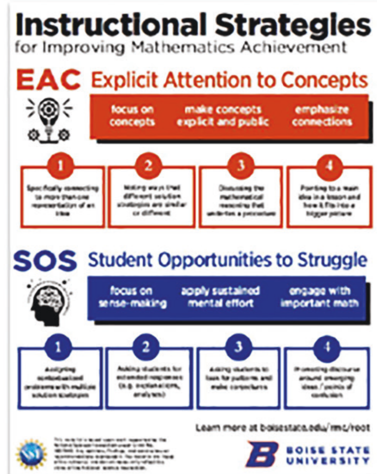




Classroom Video 1

Watch **two** clips from a classroom lesson.

- Look for and identify EAC **features**, **strategies**, and **routines**.



bit.ly/rootmod2

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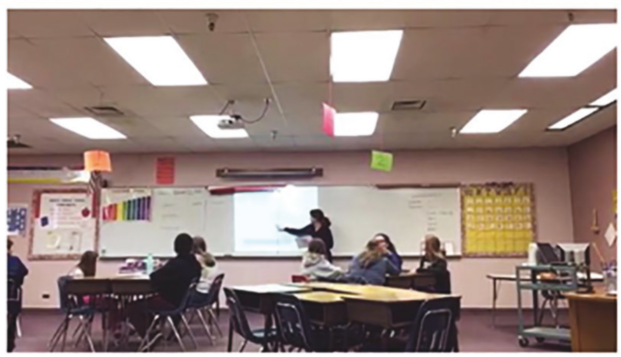
What is the answer to the math problem?

What are the steps to solve the math problem?

Where is each number in the math problem and answer represented - in the model?

$$\frac{3}{4} \times \frac{2}{2} =$$



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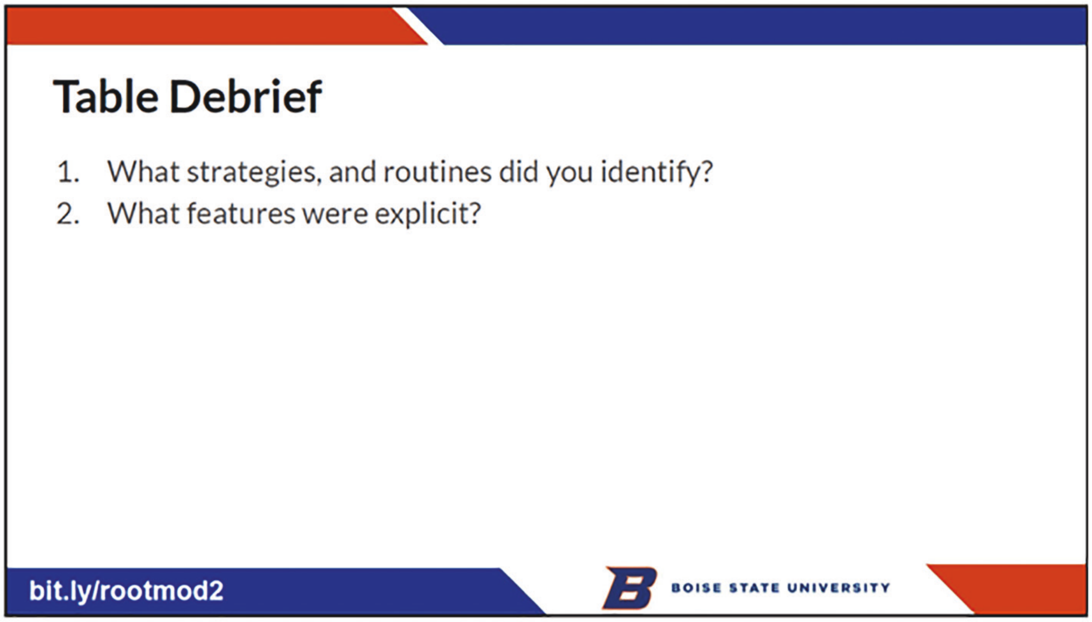
Table Debrief
1. What strategies, and routines did you identify?
2. What features were explicit?
bit.ly/rootmod2
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Appendix C

A full downloadable version of this file can be found at this link.