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Temporal dynamics of faculty hiring in mathematics

Cody FitzGerald^{1,2}, Yitong Huang^{2,3}, Katelyn Plaisier Leisman¹ & Chad M. Topaz^{4,5,6}

University faculty hiring networks are known to be hierarchical and to exacerbate various types of inequity. Still, a detailed, historical understanding of hiring dynamics lacks in many academic fields. We focus on the field of mathematics, analyzing over 120,000 records from 150 institutions over seven decades to elucidate the temporal dynamics of hiring doctoral-granting (DG) faculty at the individual and departmental levels. We demonstrate that the disparity between the number of mathematics Ph.D.s awarded and the number of DG faculty positions filled has grown over time. Even institutions with the best records of DG faculty placement have experienced a temporal decline in the probability of their graduates obtaining a DG faculty position. By quantifying the mathematical prestige of each department with a network statistic, authority centrality, we find an approximately linear relationship between the log of the prestige of one's Ph.D. institution and the log of the probability of obtaining a faculty position. Moreover, we observe associations suggesting that the probability of DG faculty placement has decreased over time and is smaller for women than for men. On the departmental level, a group of 14 elite departments dominated the authority centrality of the entire network between 1950 and 2019. Strikingly, one department within this elite group increased its centrality scores consistently, which hints at the possibility for a department to improve its prestige. This analysis highlights the challenges of transitioning from Ph.D. holder to faculty member in mathematics.

¹Department of Engineering Sciences and Applied Mathematics, Northwestern University, Evanston, IL, USA. ²NSF-Simons Center for Quantitative Biology, Evanston, IL, USA. ³Department of Molecular Biosciences, Northwestern University, Evanston, IL, USA. ⁴Institute for the Quantitative Study of Inclusion, Diversity, and Equity, Williamstown, MA, USA. ⁵Williams College, Williamstown, MA, USA. ⁶University of Colorado-Boulder, Boulder, CO, USA.

email: cody.fitzgerald@northwestern.edu; yitong.huang@northwestern.edu

Introduction

An estimated two thousand mathematics Ph.D.s are awarded annually in the United States (Reys et al., 2022). At the same time, only around 300 tenure-track positions in doctoral degree-granting mathematics departments are under recruitment each cycle. Moreover, upwards of 70% of these positions are filled by faculty who are not new Ph.D.s (Jahan et al., 2019). The hiring process is not just competitive but is opaque and poorly understood by applicants (Fernandes et al., 2020). The result of a search is eventually known publicly, but the list of applicants and the selection criteria used are not revealed. Success in acquiring a faculty position may not only depend on an individual scholar's research productivity and academic credentials (Fernandes et al., 2020), but also on demographic factors such as race, gender, and childhood socioeconomic status (Clauset et al., 2015; Morgan et al., 2022; Wapman et al., 2022; White-Lewis, 2020).

In recent years there has been growing interest in quantifying the faculty hiring process in various fields (Barnett et al., 2010; Clauset et al., 2015; Cowan and Rossello, 2018; Fernandes et al., 2020; Fowler et al., 2007; Hanneman, 2001; Lee et al., 2021; Mai et al., 2015; Wapman et al., 2022; Zuo et al., 2019). For instance, Clauset et al. (2015) used a network science approach to study the trajectories of 19,000 faculty members in business, computer science, and history. They found that faculty hiring networks have a particularly hierarchical structure. They also found that doctoral program prestige is a strong predictor of faculty placement and that there exists a bias towards men as compared to women with the same training, especially in the fields of business and computer science (Clauset et al., 2015). A follow-up study in 2021, which used the same data set, utilized adaptive rewiring network models to test mechanisms that give rise to the hierarchical nature of the faculty hiring observed in the 2015 study (Lee et al., 2021). It concludes that a mixture of two mechanisms, total faculty production and local homophily, likely drives the dynamics of real-world faculty hiring. Another recent study combined a survey of tenure-track faculty members in eight fields, US census data, and the NSF Survey of Earned Doctorates to study the impact of socioeconomic status on faculty position acquisition and found that faculty members tended to grow up in wealthier homes and were 25 times more likely to have a parent who held a Ph.D., as compared to the general US population (Morgan et al., 2022). An extensive study of faculty hiring between 2011 and 2020 encompassed 295,089 US faculty members in 10,612 departments across 107 fields and eight domains (including mathematics and computing) using data from the Academic Analytics Research Center (Wapman et al., 2022). This study confirmed the hierarchical structure observed in previous studies extends across many fields of academic hiring. The study also uncovered populations of US faculty members with high attrition rates: US faculty members trained outside the US, Canada, and the UK; faculty members that are trained and hired by the same university; and faculty members that are trained at universities from which relatively few graduates acquire faculty positions. Finally, this study found that recent changes in the gender parity of faculty members are explained mainly by the retirement of older faculty members, who tend to be men (Wapman et al., 2022). A survey-based statistical analysis of academic job seekers primarily in the life sciences identified significant associations between receiving an academic job offer and the number of job applications filed, winning a career transition award, total citation count, authorship of high-profile papers, and postdoctoral fellowships (Fernandes et al., 2020). The same study also found significant negative associations between receiving an academic job offer and the number of years an applicant is on the job market and joint industry and academic job searches

(Fernandes et al., 2020). Despite these efforts spent investigating academic hiring, a comprehensive understanding of the evolution of the mathematics faculty hiring process over many decades is lacking. Our analysis moves towards filling this gap in understanding.

We complement the existing work on faculty hiring by studying doctoral-granting (DG) faculty hiring in mathematics over the past 70 years at the individual and departmental scale using the Mathematics Genealogy Project (MGP, <http://www.genealogy.ams.org>), which is an extensive database of graduate advisor–advisee relationships. We consider academic hiring as a complex system at two different scales: the individual level and the department level. At the individual level, we examine over 120,000 records from MGP to uncover characteristics that lead to successful DG faculty placements in mathematics. Here, we set out to understand which factors, academic or otherwise, allow a mathematics graduate degree holder to transition to a DG faculty member in mathematics and how generally difficult this transition is to make. We will refer to this transition as the graduate-to-faculty transition (GFT) throughout the paper. We begin our investigation into the GFT by studying 150 US math departments and show that the GFT is decreasing over time. This finding even holds for historically “well-placing” departments, which annually graduated at least one student who eventually acquired a DG faculty position in mathematics between 1950 and 2015. We also find statistically significant academic-based and gender-based factors that influence the GFT.

In addition to our individual-level examination of the GFT, we use network analysis methods to investigate the department level. Myers et al. (2011) analyzed the MGP database between 1973 and 2011 and showed that department authority scores correlate with departmental rankings from US News & World Reports and the National Research Council. Hub and authority centrality were originally developed in the context of ranking web pages (Kleinberg, 1999). We join Myers et al. (2011) in using these centralities as proxies for departmental prestige. Similar to the definitions given in Myers et al. (2011), we define a department with a high authority value as one where Ph.D. recipients go on to become DG faculty at prestigious schools and a department with a high hub value as one containing many DG faculty who received their Ph.D. at prestigious schools. Expanding on Myers et al. (2011), we conduct a fine-grained analysis that explores gains and losses in the hub and authority centrality of US math departments between 1950 and 2019.

In our analysis, we find that a subset of 14 “elite” departments holds approximately 70% of the authority centrality of the entire network. Moreover, the total centrality held by these elite departments remains relatively constant between 1950 and 2019. However, the share of centrality held individually by each of these elite departments varies over that time frame. Strikingly, we find that one of the 14 elite departments dramatically increased its share of both hub and authority centrality.

In summary, our investigations into the GFT and the temporal dynamics of centrality at the departmental scale make three contributions:

- We perform an analysis of the evolution of academic hiring within the field of mathematics between 1950 and 2019.
- We begin to uncover academic and demographic factors associated with successfully acquiring a DG faculty position at the individual level in the field of mathematics.
- We find that an elite group of departments comprise a large portion of network centrality between 1950 and 2019. Furthermore, the portion of centrality held by the elite group remains relatively stable throughout that time span.

However, we find one math department that has dramatically and consistently increased its share of both hub and authority centrality, the latter of which is a proxy for rank.

Data collection

We begin with a sample of 150 Ph.D.-granting institutions based on the U.S. News Graduate Schools Top Mathematics Programs rankings. The union of three archived U.S. News rankings from 1998, 2010, and 2018 comprise a final sample of 150 institutions that we analyze.

We then proceed to gather data from the Mathematics Genealogy Project (MGP, <http://www.genealogy.ams.org>). We collected all data on October 4, 2022. For each of the 150 schools, we search MGP for all individuals who ever obtained a Ph.D. from that school. From the search results, we retain each person's name, the year of their Ph.D., and a hyperlink to their full record within MGP. We acquire 121,521 records of graduates from 150 schools. Of these, 466 records are missing a year, accounting for 0.4% of the data. For the remaining records, the year ranges from 1792 to 2022, with a median year of 1997 and a mean year of approximately 1993.

Next, we gather information about which individuals have served as DG faculty. From MGP data, the only means we have to infer whether the target individual became a DG faculty member is to check whether they advised Ph.D. students who also appear in MGP. For each person in our database, we see if there are any students listed on their individual MGP record. If there are no students listed, we assume the individual did not serve as DG faculty. If there are students listed, we list all the schools at which the students received their Ph.D.s and take the mode as the primary school at which the advisor was DG faculty. For example, if an individual advised 8 Ph.D. students at the University of Iowa and 2 at the University of Indiana, we would take the University of Iowa as the institution where the individual was DG faculty. If the mode is not unique, we record the first (in chronology) institution. It is a limitation of our approach that we cannot identify faculty in departments that do not grant Ph.D.'s, nor DG faculty who do not have any Ph.D. students appearing in MGP, either through omission of data or through not having served as advisor. Overall, we identify a DG faculty institution for 24,928 individuals. Finally, in our construction of hiring networks, we restrict attention to those who became DG faculty at one of the 150 US News-ranked schools, which results in a data set of 19,372 individuals.

Next, if available, we gather information about the school where each individual's advisor received a Ph.D., as well as how many students the advisor is listed on MGP as having mentored.

Finally, for each record, we extract the individual's first name and run it through the *genderize.io* algorithm (Demografix ApS, 2022) to infer the individual's gender. From the outset, it is important to recognize the limitations of this approach. Most critically, inferred gender is not the same as actual gender. The only accurate source for information on an individual's gender is the individual themselves. However, many of the MGP records correspond to deceased individuals, and even for living individuals, a survey methodology is unlikely to produce much data. Moreover, an additional severe limitation of our approach is the algorithm's unfortunate use of binary gender. Overall, it would be strongly preferable to have self-identified gender information, and information unrestricted to a binary scale. Lacking this data, we proceed with using the gender inference algorithm, keeping in mind its serious shortcomings.

For each name, the algorithm outputs an inferred binary gender based on its internal database of names with self-identified

gender as scraped from online sources. For example, consider the first name Kelsey. Suppose that the *genderize.io* database contains 900 instances of this name, 765 coming from self-identified women, and 135 coming from self-identified men. Then the algorithm would infer the gender of "woman" and would report a probability of 0.85, based on the frequency 765/900. An additional limitation of the algorithm is now apparent: there is no way to know the extent to which the underlying data might be biased. For instance, if we had access to self-identified gender information for all individuals in the world with the name Kelsey, would the observed frequency of women be close to the 0.85 reported by the algorithm?

Keeping all of the limitations in mind, we proceed by choosing a gender inference probability below which we are unwilling to accept the algorithm's inference. We initially experimented with a high threshold, $p = 0.95$, but manual examination of the excluded data suggested that an overwhelming majority of discarded names were of East or South Asian origin. Discarding so many of these names would result in the erasure of those groups from our study. Thus, to achieve more inclusion, we seek a cutoff probability threshold p^* such that the frequencies of inferred men and women in the excluded data are equal to the average probabilities of being a man and being a woman in that same excluded data; that is to say, we strive for aggregate internal consistency in the excluded data. We find a single value of p^* satisfying this criterion, namely $p^* \approx 0.6$, and we exclude data where $p < p^*$. We find 93,882 inferred men (77.2%) and 22,410 inferred women (18.4%). For 5229 individuals (4.3%), our cutoff for probability precluded an inference.

Results

Focusing on individuals: the graduate to faculty transition rate.

We begin our analysis by computing the probability that a mathematics Ph.D. holder becomes a faculty member who advises one or more mathematics Ph.D. students to graduation. For this analysis, we use all 121,055 records in our dataset that have a graduation year listed. Figure 1 shows the total number of mathematics Ph.D. graduates each year from 1900 to 2019, and the number of those Ph.D. graduates who are listed on MGP as having eventually advised students. Both the number of

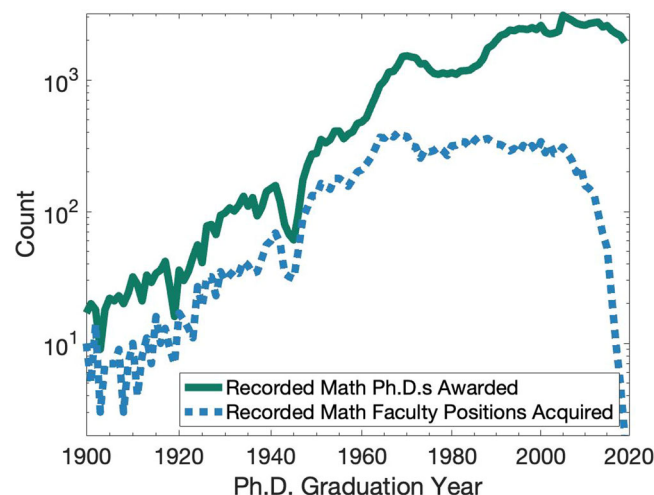


Fig. 1 The number of mathematics Ph.D. degrees awarded from 150 US math departments (green solid curve) and the number of people who later became doctoral-granting (DG) faculty members from this pool (blue dotted curve) between the years 1900 and 2019. The steep decline beginning in 2010 is simply an indication of the necessary time lag between receiving one's Ph.D. and graduating one's first Ph.D. student.

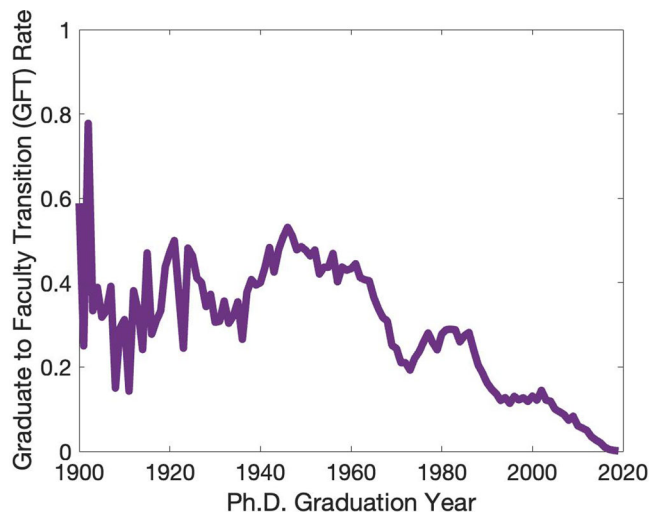


Fig. 2 The combined graduate to doctoral-granting (DG) faculty transition (GFT) rate for 150 DG US math departments from 1900 to 2019. The GFT rate decays nonlinearly over time.

mathematics Ph.D. degrees awarded and the number of people who receive a Ph.D. and go on to become a DG faculty member in mathematics increase between 1900 and 1970, but after roughly 1970 the number of new DG faculty members saturates, and even declines slightly, while total annual graduates continue to rise. Most of the steep decline evident in eventual advisors after about 2010 is due to the fact that many of those who graduated more recently may not have advised students yet (but perhaps will eventually).

By dividing the number of people who receive a Ph.D. and go on to become a DG faculty member in mathematics (blue dotted curve in Fig. 1) by the total number of mathematics Ph.D. degrees awarded (green solid curve in Fig. 1), we obtain the graduate to DG faculty transition (GFT) rate.

The annual GFT rate for all records, shown in Fig. 2, can be interpreted as a global measure of the relative ease of acquiring a DG faculty position in mathematics. The GFT rate decays nonlinearly from 1900 through 2019. In the past 30 or so years, the GFT rate has trended closer to zero than it previously had over the time scale we considered, though we again note that the data is incomplete for recent graduates who may eventually train future students but have not yet done so.

To further investigate the GFT rate, we look at what happens when we restrict the data to graduates from a list of “well-placing” schools between the years of 1950 and 2015. We choose these schools to determine if a subset of math departments were immune from the global behavior of the GFT rate over this time span. We choose these years because the data are less complete before 1950, and graduates after 2015 may not have had enough time to acquire a DG faculty position and advise their first trainee. We define a well-placing school as a school that between these years annually graduated at least one mathematics Ph.D. student who eventually mentored a student at one of the 150 departments we included in our analysis. Ten schools fall into this category, including Harvard University, Massachusetts Institute of Technology, Princeton University, Stanford University, University of Chicago, University of North Carolina–Chapel Hill, University of California–Berkeley, University of Michigan, University of Pennsylvania, and University of Wisconsin–Madison. The minimum, median, and maximum GFT rates for schools in this group are shown in Fig. 3. We can see that the transition rate of “well-placing” mathematics departments declines in a similar manner to the global trend

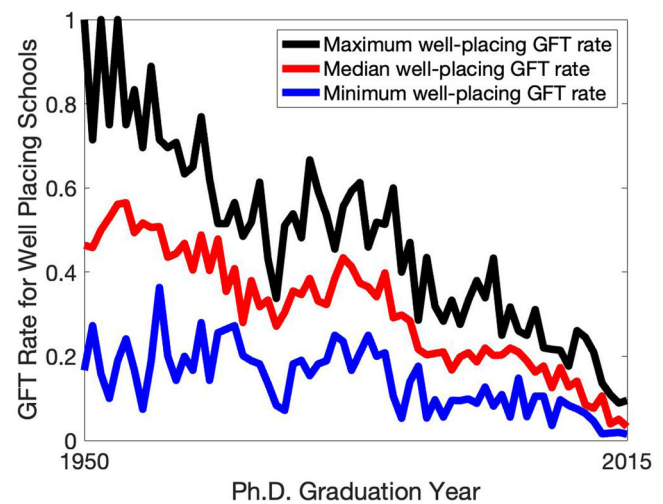


Fig. 3 The minimum, median, and maximum graduate-to-faculty transition (GFT) rates for 10 schools that have annually graduated at least one mathematics Ph.D. student who became a doctoral-granting (DG) faculty member in a department in one of the 150 departments we considered between 1950 and 2015. All three measures decrease, but the maximum GFT rate most sharply declines between the years of 1950–2015.

seen in Fig. 2. Note the spread that exists in Fig. 3 in the 1950s and 1960s dies off over time as the minimum, median, and maximum GFT rates each trend downward over time.

Centrality scores: hubs and authorities. Here, we apply the network science approach described in Myers et al. (2011) to a larger data set and incorporate temporal dynamics into the centrality analysis. Myers et al. constructed a doctoral-granting faculty hiring network based on data from the MGP database between the years 1973 and 2011. In their network, the nodes represent US math departments and the edges are representative of DG faculty hiring. The edges are weighted by the number of people who were trained as graduate students in a department and acquire DG faculty positions in another department. The edges point from the DG faculty department towards the graduate training department, and the graduate training and DG faculty departments can be the same if the person was trained and hired by the same department. As an example, a professor in the Purdue University Department of Mathematics who received a Ph.D. from the University of Wisconsin–Madison Department of Mathematics is represented as an edge pointing from the Purdue University Department of Mathematics node to the University of Wisconsin–Madison Department of Mathematics node.

In Myers et al. (2011), network centrality was analyzed for 58 math departments using hub and authority centrality scores. Hub and authority centrality scores were originally developed in the context of ranking web pages on the internet (Kleinberg, 1999) and were used in the search engine for Ask.com (Newman, 2018). Mathematically, centrality authority and centrality hub vectors x and y are the left and right singular vectors corresponding to the largest singular value of the adjacency matrix, A , scaled so their elements each sum to 1. Thus, they satisfy

$$x = \alpha A y,$$

and

$$y = \beta A^T x,$$

where constants α and β satisfy $\alpha\beta = 1/\sigma^2$ and σ is the largest singular value of A (Newman, 2018).

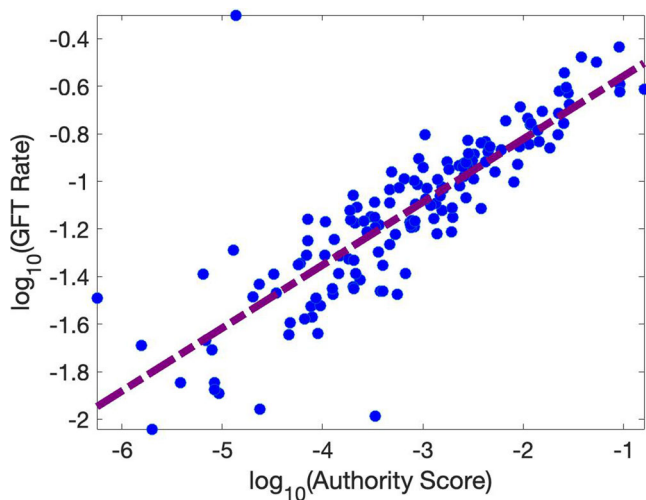


Fig. 4 Graduate-to-faculty transition (GFT) rate by average authority score of graduate training department between the years of 1950 and 2019 on a log-log plot. The dashed line represents a least-squares fit of the data, yielding an r^2 value of 0.7 on the log-log axes.

In Myers et al. (2011), departments with high authority scores are defined as “a strong source of prestigious Ph.D. students,” and departments with high hub scores are defined as “a strong destination.” We refine their definitions for the reader’s clarity. We take a department with a high authority score to mean an institution from which Ph.D. recipients go on to become DG faculty at prestigious schools, and we define a department with a high hub score as a department that hires many DG faculty who received a Ph.D. from a prestigious school. Next, we compute the GFT rate as a function of authority score for each of the 150 schools in our set that graduated students between the years of 1950 and 2019 (see Fig. 4).

As expected, higher authority scores, suggesting higher prestige, correlate with higher GFT rates. The data appear linear on a log-log plot of the GFT rate versus the authority score of the Ph.D. training departments ($r^2 = 0.7$ via least-squares fitting on the log-log axes). It is worth pointing out that San Diego State University (GFT rate = 0.5, authority score $\approx 1.36 \times 10^{-5}$) appears to be an outlier in the group because only two graduates in our dataset obtained a Ph.D. from San Diego State University, and one of them eventually became a DG faculty member.

Furthermore, we look at the probability of moving up (higher authority score) when transitioning from Ph.D. holder to DG faculty. Of the graduates who eventually go on to advise students at one of the 150 schools of interest, we compute the fraction of those who have advised students at an institution with a greater authority score than that of their Ph.D. granting institution. This fraction is shown in Fig. 5. It remains relatively stable between 1950 and 2019, largely hovering between 15% and 25%. Note that graduates from 2019 may not have enough time to advise students yet, and in fact, our data only contains two individuals who graduated in 2019 and already have advisees listed.

Logistic model: probability of faculty placement. To identify factors associated with a target individual obtaining a doctoral-granting (DG) faculty position, we construct a logistic regression model. The dichotomous outcome variable is whether or not the target individual became a DG faculty. As discussed previously, we infer DG faculty status from the individual’s record of student advising as listed in MGP, should it exist. We include several explanatory variables in our model: the target’s year of receipt of

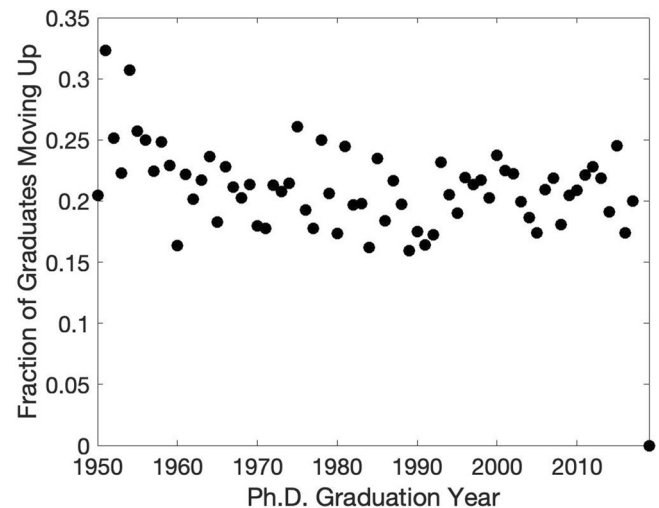


Fig. 5 The fraction of doctoral-granting (DG) faculty members who acquire a faculty position in a DG department with a higher authority score than the department where they completed their graduate studies versus the year of Ph.D. graduation. The fraction hovers between 15% and 25% and is relatively stable over seven decades.

Ph.D.; their inferred gender; prestige measures of their Ph.D. granting institution and their advisor’s Ph.D. granting institution; and finally, the number of Ph.D. students overseen by their advisor. We now explain these choices in detail.

Since the academic job market is tighter in some years and less so in others, we include the year of Ph.D. receipt as an explanatory variable. To address differential outcomes depending on gender, we include inferred gender in the model. We also include the interaction of the inferred gender with the year of Ph.D. receipt in order to allow for the possibility of a shifting hiring landscape for women over time.

To account for the prestige of the target individual’s Ph.D. granting institution, we (initially) include its hub and authority scores as computed over a 10-year time period up to but not including the Ph.D. year. For that same time period, to account for the influence of the target’s Ph.D. advisor as part of the job search process, we include the hub and authority scores of that person’s Ph.D. granting institution. Finally, to account for the target’s Ph.D. advisor’s advising patterns, albeit in a crude manner, we include the total number of students who received Ph.D.s from the target’s advisor over the same 10-year window. It is important to recall that the target Ph.D. and advisor Ph.D. hub and authority scores are all calculated from the same network and thus may be correlated. In fact, the correlation between hub and authority for the target is $r \approx 0.66$ and for the advisor is $r \approx 0.51$. Among the four network scores, the two least correlated are the target’s authority score and the advisor’s hub score ($r \approx 0.16$), so we include these in our model. While including the other two network scores might increase model fit, it could be at the expense of interpretability, especially if estimates for the effects of strongly correlated variables have opposite signs.

Concretely, the model described above is

$$\begin{aligned} \text{Log Odds} \sim & \beta_0 + \beta_1(\text{Year}) + \beta_2(\text{Inferred Gender Woman}) \\ & + \beta_3(\text{Year} \cdot \text{Inferred Gender Woman}) \\ & + \beta_4(\text{Target's Ph.D. Authority}) + \beta_5(\text{Advisor's Ph.D. Hub}) \\ & + \beta_6(\text{Number of Students Advised by Advisor}), \end{aligned} \quad (1)$$

where since we use a logistic regression framework, Log Odds is the log odds of obtaining a DG faculty position.

Table 1 Estimates obtained from fitting model (1), which describes the log odds of an individual becoming a DG faculty member, based on $N = 81,691$ records (see the section “Logistic model: probability of faculty placement” for details, including important limitations of the modeling approach).

Coefficient of	Estimate	Standard error	z	Significant
(Intercept)	72.9	1.28	57.1	***
Year	−0.0377	0.000644	−58.6	***
Inferred gender woman	−13.9	3.81	−3.65	***
Year · inferred gender woman	0.00688	0.00192	3.59	***
Target’s Ph.D. authority	1.37	0.0310	44.3	***
Advisor’s Ph.D. hub	0.0714	0.0521	1.37	
Students advised by advisor	−0.0479	0.00229	−20.9	***

Significance column is coded as * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

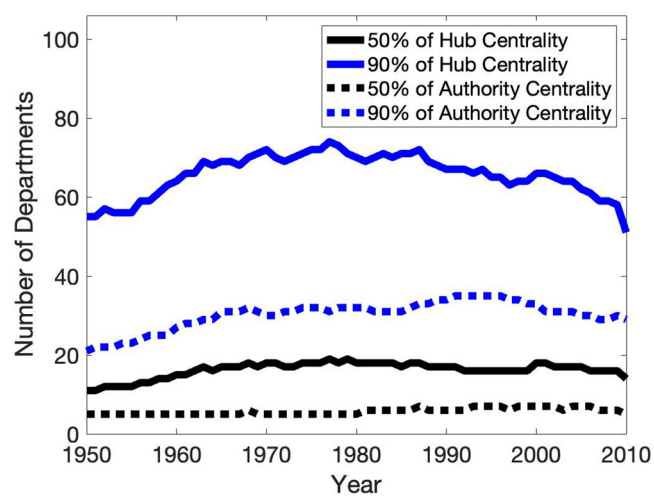


Fig. 6 The number of departments that comprise 50% and 90% of the hub and authority centrality between the years 1950 and 2019. Centrality scores are computed using a rolling window of 10 years, indicated on the x-axis by the first year of the decade. Black and blue curves correspond to 50% and 90% of centrality, respectively. Solid and dashed curves correspond to hub and authority centrality, respectively. This analysis includes 106 schools.

Before fitting the regression, we perform several data filtering steps. First, to be consistent with the time period we have studied previously, we filter out target individuals who received their Ph.D. prior to 1950 or after 2015, resulting in a data set of 104,674 records. Second, to be consistent with whom we include in our network, we further restrict attention to individuals whose advisor received their Ph.D. at one of the 150 schools we study, reducing the data set to 85,323. While we have not formally coded the geography of the advisor Ph.D. institutions for the excluded records, inspection suggests that the vast majority are schools outside of the United States. Third, we are forced to eliminate any records missing a gender inference. Removing these 3,569 records leaves 81,754 remaining. Fourth and last, in 63 cases (constituting 0.1% of the remaining data) we are unable to compute prestige scores because no one from the target individual’s Ph.D. granting institution became DG faculty during the 10-year time window prior to the year of Ph.D. receipt. We must eliminate these records, resulting in a final data set of 81,691 records.

Next, to again be consistent with our previous decisions, we take the outcome variable to be not merely whether the target individual became a DG faculty member, but whether they became a DG faculty member at one of the 150 schools we study. There are 4032 individuals in the data set who did become DG faculty but not at one of the 150 schools, and so along with 64,381

individuals who did not become DG faculty at any school in our data set, these individuals have an outcome variable coded as false.

In summary, our logistic model investigates whether individuals who received a Ph.D. at one of the 150 schools and whose advisor received their Ph.D. at one of those same schools ended up becoming DG faculty at still one of those same schools. We have 81,691 records, and the observed frequency of a positive outcome (becoming DG faculty at one of the 150 schools) is 0.16.

We fit the model using the standard `glm` command in the RStudio statistical computing environment. In terms of evaluating our model, we steer clear of measures of predictive accuracy because our intention is not to build a classifier. Indeed, with such a limited set of explanatory variables, we have no expectation that our model could be used in this way. However, we can still hope to assess if there are important associations between the outcome variable and the explanatory variables that we do have. Loosely, our situation could be compared to a linear regression that produces a low coefficient of determination but statistically significant coefficients. Thus, to diagnose our model, we set a relatively low bar and compare it to a null model. A likelihood ratio test of our model results in $p < 0.001$, suggesting that the model is preferable to the null one. Table 1 provides estimates from our model.

The signs of the significant coefficients have the following interpretations. The negative coefficient on year means that overall, there is an association between time and a decreased probability of obtaining a DG faculty position. Similarly, the negative coefficient on inferred gender woman indicates an association between being a woman and a decreased probability of obtaining a DG faculty position. The positive coefficient on the interaction of year and inferred gender woman suggests that the decreased probability of hiring for inferred women is becoming less severe over time. The positive coefficient on the authority score of the individual’s Ph.D. granting institution shows an association between that institution’s prestige and its students being hired as DG faculty. This result is perhaps not surprising given the way the authority score is constructed. There is one additional significant coefficient, namely, the negative coefficient on the number of students advised by the individual’s Ph.D. advisor. This coefficient indicates an association that is less intuitive to us and could perhaps be a target for further research. We could hypothesize that when an advisor advises more students, they are able to provide less individualized attention to each student. This reduced attention might decrease the probability of a DG faculty placement, either because the student receives insufficient guidance or because the situation discourages the student from wanting to be in academia. Regardless, these are merely conjectures, and ethnographic data might shed further light on plausible explanations.

Temporal dynamics of US math departments

To begin our department-level analysis, we compute the hub and authority centrality scores of a subset of 106 of the US mathematics departments we considered, starting in 1950 and going through 2019, using a rolling window of 10 years for each network construction, thereby extending the analysis presented in Myers et al. (2011). We determine this subset by computing the centrality scores of our entire set of 150 departments for each rolling decade starting with 1950 and ending with 2019. Any school that does not appear as an institution with DG faculty during each rolling 10-year period returns an authority (hub) score of zero. We omit any school which returned a zero centrality score for any of the rolling decades and re-compute the centrality scores for the remaining 106 departments. From this network analysis, we find that a minority of departments hold the majority of the hub and authority centrality. Figure 6 shows that between five and seven departments hold 50% of the authority centrality and between 11 and 19 departments hold 50% of the hub centrality between 1950 and 2019, calculated over rolling decades, and plotted by the first year of each decade. Roughly one-third of departments hold 90% of the authority centrality and two-thirds of departments hold 90% of the hub centrality between the years of 1950 and 2019.

Furthermore, we explore the temporal dynamics of hub and authority centrality associated with elite math departments between the years of 1950 and 2019. To select elite departments to study, we compute the departments with the seven highest authority scores across seven network constructions spanning 1950–1959, 1960–1969, 1970–1979, 1980–1989, 1990–1999, 2000–2009, and 2010–2019 and take the union of the department sets. As a result of this computation, the following departments meet our definition of elite: California Institute of Technology, Carnegie Mellon University, Columbia University, Cornell University, Harvard University, Massachusetts Institute of Technology, Princeton University, Stanford University, University of Chicago, University of California–Berkeley, University of Michigan, University of Washington, University of Wisconsin–Madison, and Yale University. Together this group of departments consistently holds approximately 72% of the authority centrality and approximately 43% of the hub centrality between 1950 and 2019 (see Fig. 7).

While the share of hub and authority centrality held by this group of elite departments remains relatively constant between 1950 and 2019, the individual share of hub and authority centrality held by each elite department changes more dramatically over time. We compute the Kendall rank correlation coefficient (Kendall, 1948) of authority and hub centrality time series spanning decades beginning in 1950 and ending in 2019 for pairs of elite departments. In this context, a Kendall rank correlation coefficient close to 1 would indicate that the centrality score trajectories of two departments between 1950 and 2019 move in the same direction. Similarly, a Kendall rank correlation coefficient close to -1 would indicate that the centrality trajectories of two departments between 1950 and 2019 move in opposite directions. Large and significant Kendall rank correlation coefficients associated with pairs of elite departments' hub and authority centrality time series are summarized in Fig. 8.

Strikingly, we find consistent gains between 1950 and 2019 in both hub and authority centrality by the Carnegie Mellon University Departments of Mathematics while the Massachusetts Institute of Technology Department of Mathematics and the Yale University Department of Mathematics fall in both measures of centrality over the same time course, as shown in Figs. 8 and 9.

To understand what caused the gains in hub and authority centrality between 1950 and 2019, we plot the rolling averages (using a 10-year window) of graduates from Carnegie Mellon

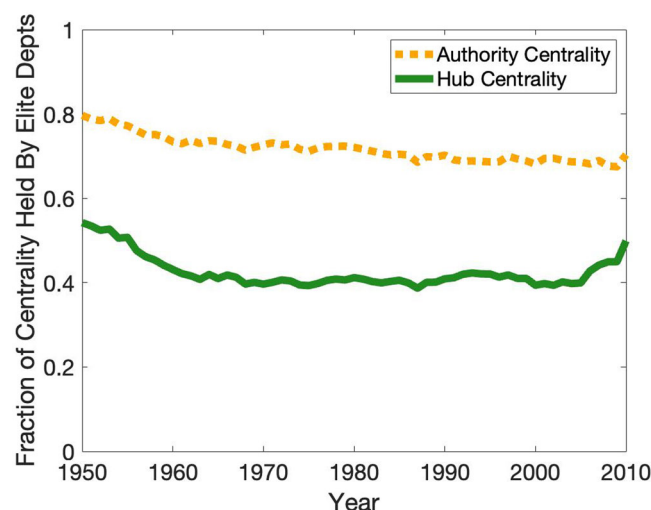


Fig. 7 The fraction of hub (green solid curve) and authority (orange dotted curve) centrality held in total by California Institute of Technology, Carnegie Mellon University, Columbia University, Cornell University, Harvard University, Massachusetts Institute of Technology, Princeton University, Stanford University, University of Chicago, University of California–Berkeley, University of Michigan, University of Washington, University of Wisconsin–Madison, and Yale University between the years of 1950 and 2019. Centrality scores are computed using a rolling window of 10 years, indicated on the x-axis by the first year of the decade. To select elite departments to study, we compute the departments with the seven highest authority scores across seven network constructions spanning 1950–1959, 1960–1969, 1970–1979, 1980–1989, 1990–1999, 2000–2009, and 2010–2019 and take the union of the department sets. Both the hub and authority centrality held by this group of elite departments remains relatively stable over this time frame.

University, Massachusetts Institute of Technology, and Yale University who eventually became a doctoral-granting (DG) math faculty, as well as the number of DG math faculty hires at each university, starting with the year 1950 and ending with the year 2019 (see Fig. 10). For Carnegie Mellon University, we observe that the average number of graduates that eventually became a DG faculty and the number of DG faculty hires both initially rise, and then remain somewhat constant. In contrast, for Yale University, we observe an initial rise in the average number of graduates that eventually became DG faculty followed by a decline starting slightly after 1980. We also see a decline in the average number of DG faculty hires at Yale University over time. For 10-year windows near the end of the range considered, such as 2010 to 2019, the graduate student and DG faculty averages are slightly lower, likely due to the fact that graduate students may not have secured a DG faculty position yet, and similarly for faculty members who have been hired but have not yet had a student graduate.

Discussion

In this study, we analyze the MGP database to understand how academic faculty selection operates in the field of mathematics. In particular, we seek to elucidate the factors that influence the transition from mathematics graduate degree holder to doctoral-granting (DG) faculty member and understand temporal trends related to this transition. We first show a growing disparity between the number of mathematics Ph.D.s awarded and the number of DG faculty positions acquired over time. This disparity is most extreme after 1970. The GFT rate has reached its lowest point during the last 30 years, and even historically well-placing departments exhibit the same trend. Additionally, we find

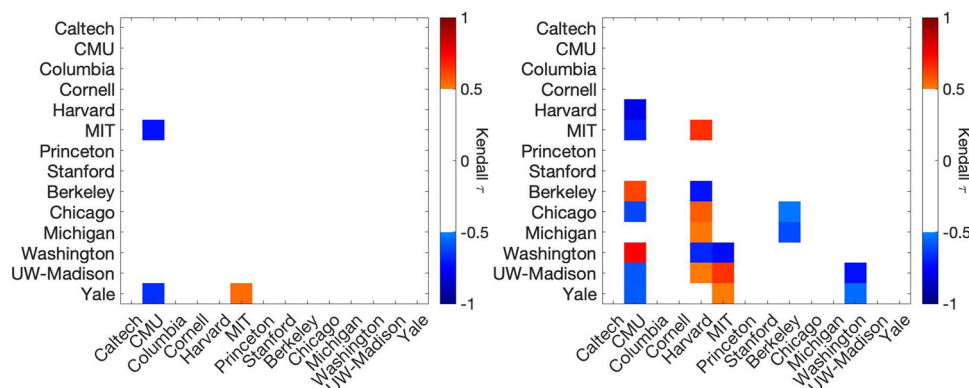


Fig. 8 Significant ($p \leq 0.05$) and large ($|\tau| > 0.5$) Kendall rank correlation coefficients for time series of the hub (left) and authority (right) scores of elite math departments. We focus mainly on the relationships between the Carnegie Mellon University, Massachusetts Institute of Technology, and Yale University Departments of Mathematics.

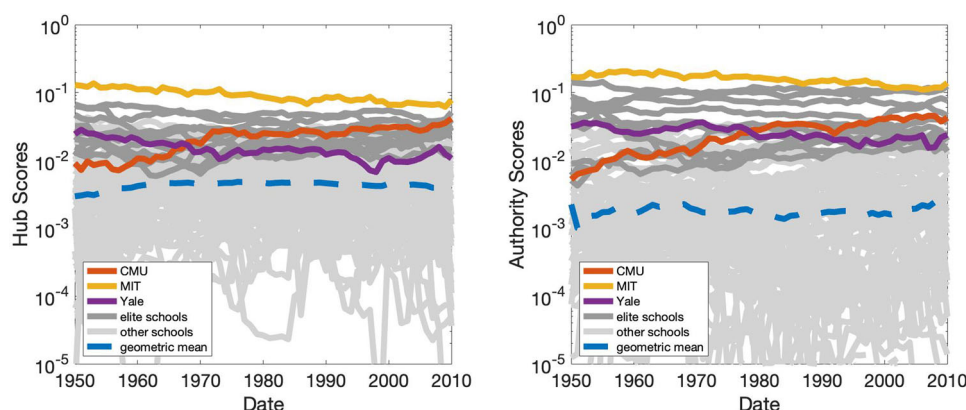


Fig. 9 Time series of hub and authority centrality scores associated with 106 math departments between the years 1950 and 2019. Centrality scores are computed using a rolling window of 10 years, indicated on the x-axis by the first year of the decade. The hub and authority centrality score time series for Carnegie Mellon University, Massachusetts Institute of Technology, and Yale University math departments are highlighted in red, yellow, purple, respectively. The geometric mean of all 106 departments' temporal hub and authority score is plotted as a dashed blue curve. Other elite departments are plotted in dark gray and all other departments are plotted in light gray. The hub and authority centrality time series for the Carnegie Mellon University Department of Mathematics increases as the hub and authority centrality time series associated with the Massachusetts Institute of Technology and Yale University Departments of Mathematics decrease in a significant manner.

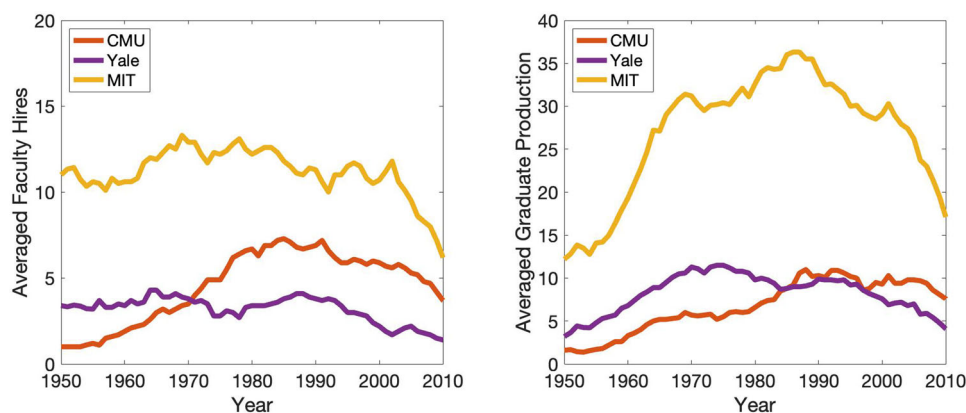


Fig. 10 Averaged DG faculty hires and averaged graduate production at the Carnegie Mellon University, Massachusetts Institute of Technology, and Yale University Departments of Mathematics over time. (Left): Rolling averages using a 10-year window of DG math faculty hires at Carnegie Mellon University, Massachusetts Institute of Technology, and Yale University, by the year of their first graduated student. (Right): Rolling averages using a 10-year window of graduates from Carnegie Mellon University, Massachusetts Institute of Technology, and Yale University who eventually became a doctoral-granting (DG) math faculty, by year of graduation. Both subplots start with the year 1950 and end with the year 2019.

that the log of the GFT rate correlates with the log of the authority score of the graduate training institution. A logistic regression model reveals that both time and the authority score of an individual's Ph.D. granting department are significantly associated with the probability of obtaining a DG faculty position in a department. Moreover, individuals inferred to be women appear to be disadvantaged as compared to men, though the disadvantage narrows over time.

At the department level, we find that the Carnegie Mellon University Department of Mathematics increased its hub and authority centrality consistently over time, while the Massachusetts Institute of Technology and Yale University Departments of Mathematics fell in both measures over the same time frame. These gains and losses in centrality correspond to the temporal dynamics of the average number of graduate students produced (who eventually become a DG math faculty) and averaged DG math faculty hires at Carnegie Mellon University and Yale University, respectively. Increasing graduate student output (specifically those that become DG faculty members), and DG faculty hiring, would have resulted in increasing the authority and hub scores, respectively. However, the underlying details of what made these universities more or less attractive to graduate students and faculty remain unclear but may involve some combination of department culture, department research interests, and its relevance to funding initiatives over this period, amongst a suite of other unknown factors.

Our study has several limitations regarding the data we used. We do not have information about the number of job applications individuals filed, career transition award recipient status, total citation counts, high-impact journal authorship records, postdoctoral fellowship recipient status, the length of time an individual spends on the job market, or if individuals conducted joint academic-industry job searches. It was previously shown that these factors are significantly associated with receiving an academic job offer in a recent survey analysis (Fernandes et al., 2020). We also do not have information pertaining to an individual's desire and decision-making process regarding pursuing an academic career versus another type of employment. Additionally, we are only studying the transition from mathematics Ph.D. graduate to DG faculty member in mathematics. It is possible that mathematics departments have hired DG faculty members who are not listed in the MGP database due to their graduate training field. Our study also does not include DG faculty members who do not advise graduate students, for example, mathematics Ph.D. holders who teach at liberal arts colleges, community colleges, or even in non-mathematics departments.

Our study also has important limitations regarding the centrality metric we used. Our method for computing temporal trajectories of network centrality is rather straightforward, and other more sophisticated frameworks have been designed for this purpose (see, e.g., Taylor et al., 2017, 2019, 2021; Kawakatsu et al., 2021). In our study, we used hub and authority centrality, which are eigenvector-based centrality metrics, to quantify the importance of math departments in our faculty hiring network. Following Myers et al. (2011), we used hubs and authorities because of their interpretable meaning in the context of math departments. However, eigenvector-based centralities can exhibit a phenomenon known as "localization," in which centrality is concentrated in a handful of nodes relative to the rest of the network (Martin et al., 2014). This property is known to be related to the structure of the network and "is particularly visible in networks with high-degree hubs¹ or power-law degree distributions" (Martin et al., 2014). To address this challenge, the recent work of Taylor et al. (2017) has proposed a generalization of eigenvector centrality for temporal networks. They find that

"the strength of the coupling between layers is important for determining multiscale properties of centrality, such as localization phenomenon." They tested their method on the MGP database in the weak and strong coupling limits and noted, "We believe this weak-coupling regime to be inappropriate for the MGP Ph.D. exchange network, as mathematics department prestige should not fluctuate wildly from one year to the next." They also comment that "For scenarios in which exploring various [couplings] is not computationally feasible, we highlight that restricting one's attention to the [strong coupling] limit can still yield very informative results." While we do not use the exact method in Taylor et al. (2017), we do expect the layers of our temporal network to be relatively strongly coupled due to our use of a 10-year rolling window for each network construction. For example, the network that spans 1950–1959 shares much of the same information with the network that spans 1951–1960.

In summary, we find that the Graduate Faculty Transition rate, an imperfect indicator of the relative ease of acquiring a DG faculty position in mathematics, is decaying over time, even for historically well-placing departments. We also uncover statistically significant factors (academic and otherwise) that influence the transition, including gender, year, target's Ph.D. authority, and the number of students advised by Ph.D. advisor. At the department level, we find that the Carnegie Mellon University Department of Mathematics consistently gained both hub and authority centrality between 1950 and 2019, a rare behavior in our network analysis. Taken together, we find that acquiring a DG faculty position is becoming more difficult and overall the rankings of departments remain relatively constant, though it is not impossible for departments to improve their ranking over time.

Data availability

The code and an anonymized version of the Mathematics Genealogy Project dataset (<https://mathgenealogy.org/index.php>) we analyzed is posted here https://github.com/cefitzg/MGP_plots.

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Note

- 1 Martin and coworkers state the definition of "hubs" as "nodes of unusually high degree," not to be confused with the technical definition of hubs and authority centralities in the section "Centrality scores: hubs and authorities".

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Competing interests

The authors declare no competing interests.

Ethical approval

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Informed consent

This article does not contain any studies with human participants performed by any of the authors.

Additional information

Correspondence and requests for materials should be addressed to Cody FitzGerald or Yitong Huang.

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