

MetaRadar: Multi-Target Detection for Reconfigurable Intelligent Surface Aided Radar Systems

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Abstract—As a widely used localization and sensing technique, radars will play an important role in future wireless networks. However, the wireless channels between the radar and the targets are passively adopted by traditional radars, which limits the performance of target detection. To address this issue, we propose to use the reconfigurable intelligent surface (RIS) to improve the detection accuracy of radar systems due to its capability to customize channel conditions by adjusting its phase shifts, which is referred to as MetaRadar. In such a system, it is challenging to jointly optimize both radar waveforms and RIS phase shifts in order to improve the multi-target detection performance. To tackle this challenge, we design a waveform and phase shift optimization (WPSO) algorithm to effectively solve the multi-target detection problem, and also analyze the performance of the proposed MetaRadar scheme theoretically. Simulation results show that the detection performance of the MetaRadar scheme is significantly better than that of the traditional radar schemes.

Index Terms—Multi-target detection, reconfigurable intelligent surface, radar systems, waveform design.

NOMENCLATURE

- ($\cdot$)^{*} Conjugate operator.
($\cdot$)^H Conjugate transpose operator.

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- ($\cdot$)^T Transpose operator.
($\theta_{m,n}^r, \varphi_{m,n}^r$) Direction from the m -th element to the n -th antenna.
(θ_k, φ_k) Direction towards the k -th target.
 α Solution accuracy of the interior-point algorithm.
 $\beta_{j,j'}^{c+1}$ Weighting factor for predicted distance $d_{j,j'}^{c+1}(\mathcal{P}^{c+1})$.
 $\mathbf{0}^{M \times N}$ Matrix with all the elements being 0 and size $M \times N$.
 γ Vector of responses of all the targets.
 Ξ Gain of the direct path from the antenna array to the targets.
 $\mathbf{a}_k$ Steering vector of the RIS for the k -th target.
 $B(s)$ Gain of the reflection path from the antenna array to the directions of targets under phase shift vector s .
 $b_k(s)$ Gain of the reflection path from the antenna array to the direction of the k -th target under phase shift vector s .
 $\mathbf{I}^{M \times N}$ Identity matrix with size $M \times N$.
 $\mathbf{J}_k$ Shift matrix of the k -th target which shifts signals from $\mathbf{y}_k^{t,c}$ to $\mathbf{y}_k^{d,c}$.
 $\mathbf{p}^A$ Position of the antenna array's center.
 $\mathbf{p}^R$ Position of the RIS's center.
 $\mathbf{s}$ Phase shift vector that contains the phase shifts of all the RIS elements.
 $\mathbf{s}^{r,c}$ RIS phase shift vector in the reception step in the c -th cycle.
 $\mathbf{s}^{t,c}$ RIS phase shift vector in the transmission step in the c -th cycle.
 $\mathbf{V}^c$ Residual term in the c -th cycle.
 $\mathbf{v}^c$ Vectorization of the residual term $\mathbf{V}^c$.
 $\mathbf{W}^c$ Radar waveform matrix in the c -th cycle.
 $\mathbf{w}^c$ Vectorization of the radar waveform $\mathbf{W}^c$.
 $\mathbf{Y}^c$ Signal received by the antenna array in the c -th cycle.
 $\mathbf{y}^c$ Vectorization of the matrix $\mathbf{Y}^c$.
 $\mathbf{y}^{(c)}$ Received signals from the 1-st to the c -th cycle.

$\mathbf{y}_k^{d,c}$	Delayed signal from the k -th target in the c -th cycle.
$\mathbf{Y}^{t,c}$	Transmitted signal towards the directions of the targets in the c -th cycle.
$\mathbf{y}_k^{t,c}$	Transmitted signal towards the direction of the k -th target in the c -th cycle.
γ_k	Response of the k -th target characterizing the reflection and channel propagation effects related to the k -th target.
$\hat{\gamma}^{(c),j}$	Estimated responses given hypothesis U_j in the c -th cycle.
λ	Wavelength of the carrier signal.
$\mathbb{C}^{M \times N}$	Set of all complex $M \times N$ matrices.
$\mathbb{E}(\cdot)$	Stochastic expectation.
$\mathbb{H}^{M \times N}$	Set of all Hermitian $M \times N$ matrices.
$\mathcal{P}^{c+1}$	Set of variables to be optimized in the $(c+1)$ -th cycle.
ω	Threshold of the target response amplitude.
$\otimes$	Kronecker product.
σ^2	Variance of the residual term.
$\{\hat{l}_k\}^{(c),j}$	Estimated delays given hypothesis U_j in the c -th cycle.
d^s	Separation between adjacent antennas.
d^u	An upper bound of (22a).
$d_{j,j'}^{c+1}(\mathcal{P}^{c+1})$	Predicted distance between hypotheses U_j and $U_{j'}$ in the $(c+1)$ -th cycle given $\mathcal{P}^{c+1}$.
d_k	Range of the k -th target.
G^A	Gain of an antenna.
$G_P^A(\theta, \varphi)$	Normalized radiation pattern of an antenna towards direction (θ, φ) .
G^R	Gain of an RIS element.
$G_P^R(\theta, \varphi)$	Normalized radiation pattern of an RIS element towards direction (θ, φ) .
$h_{m,n}$	Path gain between the n -th antenna and the m -th RIS element.
I	Number of angular grids.
J_K	Number of hypotheses when the number of targets is K .
K	Number of targets.
L	Number of snapshots of the radar waveform.
l_k	Delay of the echo signals received by the antenna array from the k -th target compared with the transmitted signals.
L_R	Number of snapshots of the received signal.
$l_{m,n}$	Distance between the n -th antenna and the m -th RIS element.
M	Number of elements in the RIS.
N	Number of antennas in the MIMO array.
N_s	Number of phase shifts of an RIS element.
$p^c(U_j)$	Prior probability of hypothesis U_j in the c -th cycle.
P_M	Maximum transmission power.
$r(s_m)$	Reflection coefficient of the m -th RIS element with phase shift s_m .
S^e	Area of an RIS element.
s_m	Phase shift of the m -th RIS element.

U	A hypothesis representing that there are $N(U)$ targets and the k -th target is in the $i_k(U)$ -th grid.
v^l	Speed of light.
$\text{Re}(\cdot)$	Real part of a complex variable.
$\text{tr}(\cdot)$	Trace operator.
$\text{vec}(\cdot)$	Vector operator.

I. INTRODUCTION

DRIVEN by the demand to support applications such as virtual reality (VR) and autonomous driving, localization and sensing become crucial functions for future wireless systems [1], [2]. To put this vision into practice, various sensing techniques which enable the perception of the surrounding environment are developed. Among various sensing techniques, radar technique which uses radio frequency (RF) signals to detect and locate targets through reflected signals has attracted much attention due to many advantages such as environmental robustness, direct measurement of velocity, and low cost [3]. Specifically, radars can directly measure the velocity of the target by leveraging the Doppler effect, which is especially useful for autonomous driving [4]. In addition, with the development of RF CMOS and multiple-input-multiple-output (MIMO) technology, radars are becoming more cost-efficient, which will lead to greater economic interests [5].

In the literature, various works which optimize the waveforms of radar signals for detection performance enhancement have been proposed [6], [7]. Based on the optimization criterions, these works can be broadly classified into three types: detection probability, signal-to-interference-plus-noise ratio (SINR), and relative entropy based schemes [8]. The detection probability based schemes optimize the radar waveforms to directly maximize the detection probability given the constraint of false alarm probability, while the optimization problem is difficult to tackle due to the complicated relationship between the detection probability and the radar waveforms [9]. In the SINR based schemes, the SINRs of the echo signals from the targets are maximized because a large SINR usually leads to a high detection accuracy [10]. The relative entropy is another widely used criterion, which has been shown to be effective to evaluate the detection performance [11], [12]. In [13], the authors considered the detection of a single target in colored noise, and derived the optimal radar waveforms that maximized the relative entropy. The multi-target detection scenario was investigated in [14]. However, the propagation channels between the radar system and the targets are passively adopted in the aforementioned schemes, which limits the performance of the radar. Specifically, if the channel conditions are unfavorable, the radar signals will suffer heavy attenuation when propagating through the channels, resulting in a smaller SINR/relative entropy and a lower target detection accuracy.

To address this issue and improve the detection performance of the radar system, the RIS is a promising solution. The RIS is a type of planar metamaterial that can be used to effectively control the propagation channels [15], [16]. It is composed of a large number of elements with electrically tunable phase

shifts, and thus the characteristics of reflection channels via the RIS elements can be changed by adjusting the phase shifts of these elements [17], [18]. By properly designing the phase shifts of RIS elements, the channel conditions between the radar antennas and the targets can be optimized to promote the detection performance of the radar systems.¹

Several RIS-aided radar schemes have been discussed in the literature. In [19], the authors considered a scenario where there was no line-of-sight (N-LOS) link between the radar antennas and the target, and an RIS was deployed to enable the radar to detect the target in the NLOS areas. The authors in [20] utilized an RIS to promote the received signal-to-noise ratio (SNR) for enhanced detection capabilities of a single-antenna radar. The MIMO radar case was considered in [21] and [22], where the phase shifts of the RIS were optimized to improve the radar performance. However, the aforementioned schemes all consider a single target scenario, which cannot be directly applied to the multi-target scenario. Besides, the radar waveforms are not optimized in these schemes, which results in limited improvement of radar performances. Different from these schemes, in this paper, we investigate the multi-target detection using an RIS-assisted radar, and jointly optimize the radar waveforms and RIS phase shifts in our proposed scheme.

Two challenges need to be addressed in the proposed scheme. *First*, to fully realize the benefits of the RIS in the multi-target detection scenario, the operations of the RIS need to be coordinated with the transceiver antennas, which complicates the system design. *Second*, since radar waveforms and RIS phase shifts are jointly optimized for multi-target detection, the optimization problem is challenging because the two variables are coupled with each other. To handle these challenges, we propose the multi-target detection protocol for the MetaRadar, and formulate the multi-target detection problem which optimizes the radar waveform and RIS phase shifts based on the relative entropy criterion. The waveform and phase shift optimization (WPSO) algorithm is designed to efficiently tackle the formulated problem. In general, our contributions can be summarized as follows:

- We propose a multi-target detection protocol which coordinates the operations of the RIS and the radar antennas. The protocol runs in a cognitive manner, where the detection performance of the MetaRadar can be adaptively improved cycle by cycle.
- The multi-target detection problem is formulated using the criterion of relative entropy, where the radar waveforms and the RIS phase shifts are jointly optimized. The WPSO algorithm is designed to tackle the formulated multi-target detection problem, which can be used to efficiently derive the optimized radar waveforms and the RIS phase shifts.
- The convergence and complexity of the proposed WPSO algorithm are analyzed, and the relationship between the RIS phase shifts, radar configuration, and the detection performance is also discussed. We also verify the

effectiveness of the proposed MetaRadar scheme through simulation.

The rest of this paper is organized as follows. In Section II, we describe the target detection scenario and introduce the model of the MetaRadar. The multi-target detection protocol is proposed in Section III. In Section IV, we formulate the waveform and RIS phase shift optimization problem. The WPSO algorithm is designed in Section V to solve the formulated problem. In Section VI, we provide the analysis of the proposed scheme. The simulation results are presented in Section VII. Finally, we draw the conclusions in Section VIII.

II. SYSTEM MODEL

In this section, we first introduce the considered scenario in Subsection II-A, and then model the RIS, path gains, and the radar receiver in Subsections II-B, VI-B, and II-D, respectively.

A. Scenario Description

We consider a multi-target detection scenario using the MetaRadar. As shown in Fig. 1, the MetaRadar is composed of a transmitter (Tx), a receiver (Rx), a MIMO antenna array with N antennas connected with the Tx and Rx, and an RIS with M elements. By deploying an RIS in the radar system, we can create reflection paths between the MIMO antenna array and the targets, and thus the overall channel conditions between the array and targets can be improved by optimizing the phase shifts of the RIS.

The MetaRadar functions in two modes, i.e., the transmission and reception modes. In the transmission mode, the Tx first generates signals according to designed waveforms, and then radiates the signals through the MIMO antenna array towards the targets via both direct and reflection paths, as illustrated in Fig. 1 (a). Then, the MetaRadar converts to the reception mode, where the antenna array receives the echo signals reflected by the targets. The received signals will be delivered to the Rx in order to detect and locate targets.

B. Reconfigurable Intelligent Surface

The RIS is a type of planar material made up of many homogenous RIS elements. A programmable RIS element is illustrated in Fig. 2. In an element, several metal patches are connected by the pin diodes and printed on the dielectric substrates. Each pin diode can be tuned to two states, i.e., *ON* and *OFF* states, leading to different states and reflection coefficients of the RIS element [23].

Suppose each RIS element has N_s different reflection coefficients with same amplitude gain η and N_s different phase shifts which are uniformly distributed in the range $[0, 2\pi)$ with interval $\Delta s = \frac{2\pi}{N_s}$ [23]. Therefore, the reflection coefficient of the m -th RIS element with phase shift s_m can be expressed as

$$r(s_m) = \eta e^{-js}, s \in \{i\Delta s | i = 1, \dots, N_s\}. \quad (1)$$

¹The RIS-aided radar scheme can be utilized in the scenario where radar signals suffer heavy attenuation when propagating through the wireless channels. For example, the proposed scheme is especially suitable to extend the detection range of millimeter wave radars.

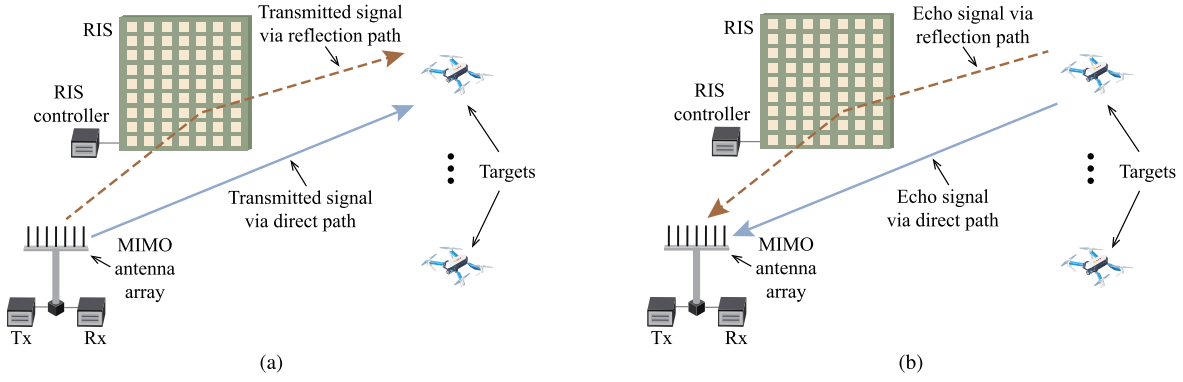


Fig. 1. A MetaRadar system: (a) Transmission mode; (b) Reception mode.

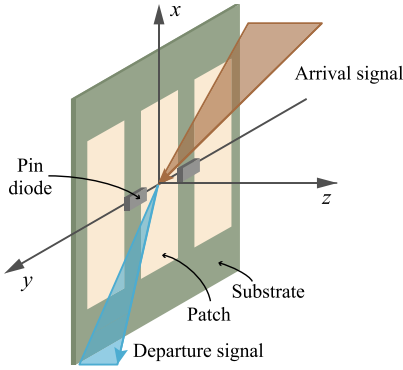


Fig. 2. Reflection characteristics of an RIS element.

C. Path Gains

The reflection paths between the antenna array and the targets consists of two parts: the antenna-RIS path and the RIS-target path. More specifically, the path gain between the n -th antenna and the m -th RIS element is given by [24], [25]

$$h_{m,n} = \frac{1}{\sqrt{4\pi}} \times \frac{\sqrt{G^A G_P^A(\theta_{m,n}^r, \varphi_{m,n}^r) G_P^R(\theta_{m,n}^r, \varphi_{m,n}^r) S^e}}{l_{m,n}} \times e^{-j 2\pi l_{m,n}/\lambda}, \quad (2)$$

where G^A is the gain of an antenna. $(\theta_{m,n}^r, \varphi_{m,n}^r)$ is the direction from the m -th element to the n -th antenna, as illustrated in Fig. 3. $G_P^A(\theta_{m,n}^r, \varphi_{m,n}^r)$ is the normalized radiation pattern towards direction $(\theta_{m,n}^r, \varphi_{m,n}^r)$. $G_P^R(\theta_{m,n}^r, \varphi_{m,n}^r)$ is normalized radiation pattern of an RIS element towards direction $(\theta_{m,n}^r, \varphi_{m,n}^r)$. S^e is the area of an RIS element. $l_{m,n}$ is the distance between the n -th antenna and the m -th RIS element. λ is the wavelength of the carrier signal. Based on [26], the normalized radiation pattern $G_P^R(\theta, \varphi)$ can be modeled as

$$G_P^R(\theta, \varphi) = \begin{cases} \cos^3(\theta), & \theta \in [0, \pi/2], \varphi \in [0, 2\pi], \\ 0, & \theta \in (\pi/2, \pi], \varphi \in [0, 2\pi]. \end{cases} \quad (3)$$

Since targets are in the farfield of RIS,² signals reflected by the RIS can be viewed as a plane wave at the location of each target [27]. Therefore, the amplitude gain and the relative

²The distance between the targets and the MetaRadar is greater than $2D^2/\lambda$, where D is the size of the RIS.

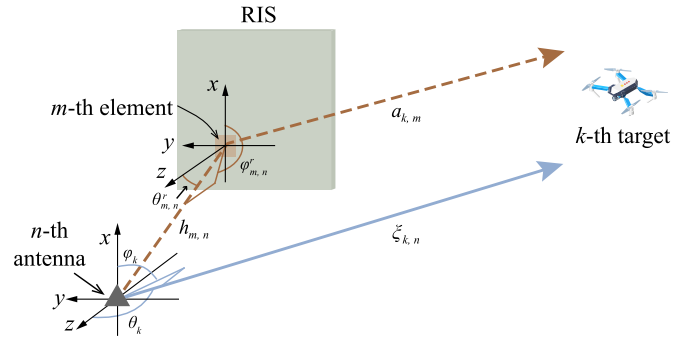


Fig. 3. Illustration of the main channel parameters.

phase delay of the m -th RIS element comparing with the first element towards the k -th target is given by

$$a_{k,m} = \sqrt{G^R G_P^R(\theta_k, \varphi_k)} e^{je_k \mathbf{p}_m^e}, \quad (4)$$

where G^R is the gain of an RIS element, (θ_k, φ_k) is the direction of the k -th target, $\mathbf{e}_k$ is the wave vector with wavelength λ and direction towards the k -th target, and $\mathbf{p}_m^e$ is the relative position of the m -th RIS element compared with the first element. The vector $\mathbf{a}_k = (a_{k,1}, \dots, a_{k,M})$ is referred to as the steering vector of the RIS for the k -th target [28].

Therefore, the gain of the reflection path from the antenna array to the directions of targets can be expressed as

$$\mathbf{B}(s) = \mathbf{A}\mathbf{R}(s)\mathbf{H}, \quad (5)$$

where $\mathbf{B}(s) = (\mathbf{b}_1(s), \dots, \mathbf{b}_K(s))^T \in \mathbb{C}^{K \times N}$, with K being the number of targets. $\mathbf{A} = (\mathbf{a}_1, \dots, \mathbf{a}_K)^T \in \mathbb{C}^{K \times M}$ is the steering vector of the RIS. $\mathbf{H} = [h_{m,n}] \in \mathbb{C}^{M \times N}$ is the matrix of channel gain between the antenna array and the RIS. $\mathbf{R}(s) = \text{diag}(\mathbf{r}(s)) \in \mathbb{C}^{M \times M}$ is the reflection coefficient matrix of the RIS under phase shift vector $\mathbf{s} = (s_1, \dots, s_M)$, and $\mathbf{r}(s) = (r(s_1), \dots, r(s_M))$ is the reflection coefficient vector of the RIS.

Similarly, the gain of the reflection path from the targets to the antenna array under phase shift vector $\mathbf{s}$ can be expressed as

$$\mathbf{B}^T(s) = \mathbf{H}^T \mathbf{R}^T(s) \mathbf{A}^T. \quad (6)$$

In addition, we assume the antenna array and the RIS are closely spaced, and the targets are in the farfield of both the antenna array and the RIS. In other words, the distance

between the antenna array and the RIS is much smaller than that between the antenna array (or RIS) and the targets. Thus, the direction of a target is the same for both the RIS element and the MIMO element,³ and the gain of the direct path from the n -th antenna to the k -th target can be given by

$$\xi_{k,n} = \sqrt{G^A G_P^A(\theta_k, \varphi_k)} e^{j\mathbf{e}_k \mathbf{p}_n^a}, \quad (7)$$

where $G_P^A(\theta_k, \varphi_k)$ is the normalized radiation pattern of the antenna in the MIMO array towards direction (θ_k, φ_k) , $\mathbf{p}_n^a$ is the relative position of the n -th antenna compared with the first antenna. Consequently, the steering vector of the antenna array towards the direction of the k -th target is $\boldsymbol{\xi}_k = (\xi_{k,1}, \dots, \xi_{k,N})$, and the steering vector of the antenna array towards the directions of targets is $\boldsymbol{\Xi} = (\boldsymbol{\xi}_1, \dots, \boldsymbol{\xi}_K)^T$.

D. Receiver Model

The functionality of the Rx is to detect targets using the received echo signals, and the multi-target detection is performed using the multiple hypotheses testing techniques. Specifically, we first form multiple hypothesis to represent different detection results. During the detection process, we update the probability of each hypothesis using the received signals. When the detection process ends, the hypothesis with the highest probability will be selected as the detection result.

Before defining the hypotheses, we first introduce some assumptions on the number and locations of targets. Specifically, we consider a practical scenario, where the directions and the number K of targets are unknown, and the number K is in the range $[0, K_M]$, where K_M is a positive constant. Besides, the location of the k -th target is represented by the direction (θ_k, φ_k) and range d_k . We discretize the space of interest into I angular grids denoted by $\mathcal{I} \in \{1, \dots, I\}$, and a target is located in one of the grids. Given the number of targets and the direction of each target, the range of each target can be estimated based on the received signals, which will be discussed in Section III. This means that a hypothesis only needs to contain the information of the number and directions of targets. Consequently, we introduce indicative vector $\mathbf{i}(U) = (i_1(U), \dots, i_{N(U)}(U))$ to represent hypothesis U , which means that there are $N(U)$ targets, and the k -th target is in the $i_k(U)$ -th grid.

The prior probabilities of hypotheses have to be initialed before the detection process. Let $p^1(U)$ denote the prior probability of hypothesis U . Since K is unknown and K is the range $[0, K_M]$, without loss of generality, we assume that K follows uniform distribution in range $[0, 1, \dots, K_M]$, which is a commonly used assumption when there is no prior

knowledge of the number of targets [14]. Thus, the prior probability of hypothesis U can be given by

$$p^1(U) = \frac{1}{K_M + 1} \times \frac{1}{J(U)}, \quad (8)$$

where $J(U)$ denotes the number of hypotheses with $N(U)$ targets.

Based on Bayes' theorem, the prior probabilities of hypotheses can be updated by exploiting the information in the received signals, and the decisions of target detection can be made using these prior probabilities. Details of the probability updating process will be introduced in Section III.

III. MULTI-TARGET DETECTION PROTOCOL

In this section, we propose a multi-target detection protocol to coordinate the operations of Tx, Rx, antenna array, and RIS in the detection process. We divide the timeline in the detection process into cycles with duration δ_C , and the probabilities of all the hypotheses will be updated cycle by cycle. After C cycles, the detection process will terminate, and the hypothesis with the highest probability will be chosen as the correct hypothesis [32].

As illustrated in Fig. 4, each cycle contains four steps, i.e., the optimization, transmission, reception, and the detection steps.

A. Optimization

In this step, radar waveforms and the RIS phase shifts are optimized by the Tx based on the signals received in previous cycles. Let $\mathbf{W}^c \in \mathbb{C}^{N \times L}$ denote the generated waveform matrix in the c -th cycle, with L being the number of snapshots. Besides, the optimized vectors of RIS phase shifts in the transmission and reception steps in the c -th cycle are denoted by $\mathbf{s}^{t,c} = (s_1^{t,c}, \dots, s_M^{t,c})$ and $\mathbf{s}^{r,c} = (s_1^{r,c}, \dots, s_M^{r,c})$, respectively.⁴ Details of the optimization problem will be introduced in Section IV. As for the first cycle, the waveform matrix $\mathbf{W}^1$ and RIS phase shift vectors $\mathbf{s}^{t,1}$ and $\mathbf{s}^{r,1}$ are randomly generated.

B. Transmission

The generated radar waveforms $\mathbf{W}^c$ are emitted by the antenna array towards the RIS in this step. The RIS phase shifts are set as $\mathbf{s}^{t,c}$ during the whole step.⁵ Based on (5) and (7), the transmitted signals towards the directions of the targets can be expressed as

$$\mathbf{Y}^{t,c} = (\mathbf{B}(\mathbf{s}^{t,c}) + \boldsymbol{\Xi})\mathbf{W}^c, \quad (9)$$

where $\mathbf{Y}^{t,c} = (\mathbf{y}_1^{t,c}, \dots, \mathbf{y}_K^{t,c})^T \in \mathbb{C}^{K \times L}$, with $\mathbf{y}_k^{t,c}$ being the transmitted signal towards the direction of the k -th target.

⁴The RIS phase shifts $\mathbf{s}^{t,c}$ and $\mathbf{s}^{r,c}$ are sent to the RIS controller in the optimization step in order to avoid the signaling cost in the transmission and reception steps.

⁵The duration of the adjustment of RIS phases is much smaller than that of the transmission step δ_T [33], [34]. For example, when the target is 150m away from the radar, the duration δ_T is 1 μ s. In contrast, the phases of the RIS can be changed within 12.5ns [33], which is much smaller than δ_T . The price to adjust the phases is also acceptable because the control circuit of the RIS is very simple, and the main cost of the circuit is the low-cost FPGA module.

³In practice, the farfield constraint $d_{n,k} > 2D_{m,n}^2/\lambda$ is easy to be satisfied. For example, suppose the working frequency of the radar is 3GHz, the size of the RIS is 1m $\times$ 1m (20 $\times$ 20 elements), the size of the MIMO array is 0.2m $\times$ 0.2m (4 $\times$ 4 elements), and the distance between the RIS center and the MIMO array center is 1m (10 λ). Thus, the maximum distance between a MIMO antenna and an RIS element is about 2m, which indicates that when $d_{n,k} > 80$ m, the direction of a target is the same for all the RIS elements and the MIMO antennas. Since the radars working at 3GHz are typically used for surveillance with a few hundreds of meter detection range [30], [31], this constraint is satisfied in practice, indicating that it is reasonable to assume the direction of a target is the same for both the RIS element and the MIMO element.

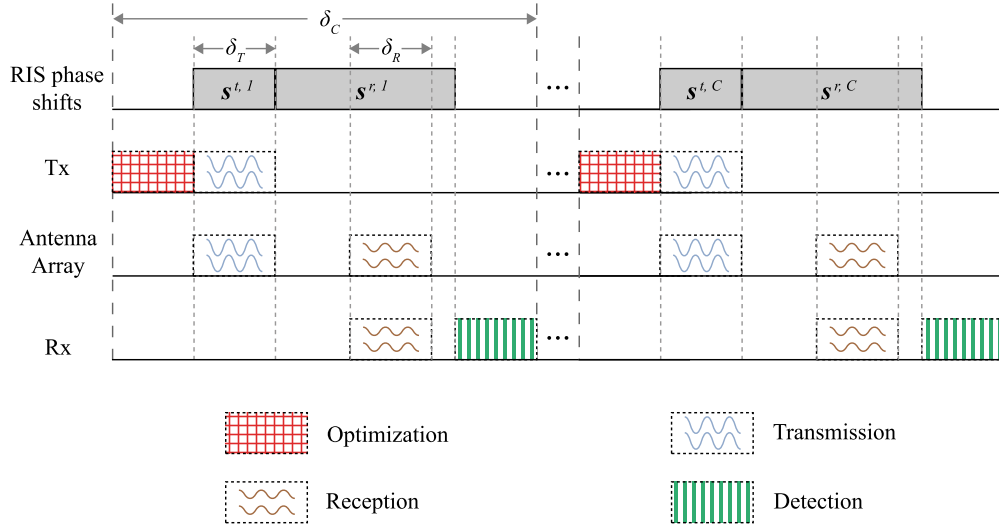


Fig. 4. Multi-target detection protocol.

C. Reception

After the waveforms $\mathbf{W}^c$ are transmitted, the phase shifts of the RIS are set as $\mathbf{s}^{r,c}$, and the antenna array listens for the echo signals from the targets. Since the distances between the targets and the radar can be different, the echo signals from different targets may have different delays. Suppose that the echo signals received by the antenna array from the k -th target delays l_k snapshots compared with the transmitted signals. The delayed signal from the k -th target is given by

$$\mathbf{y}_k^{d,c} = \mathbf{y}_k^{t,c} \mathbf{J}_k, \quad (10)$$

where $\mathbf{J}_k$ is a shift matrix with size $L \times L_R$, with L_R being the number of snapshots of the received signal. The $(l \times l')$ -th element of $\mathbf{J}_k$ satisfies

$$\mathbf{J}_k(l, l') = \begin{cases} 1, & l' - l - l_k + L^m = 0, \\ 0, & \text{otherwise,} \end{cases} \quad (11)$$

where L^m is the minimum delay. Besides, we assume that $L^m > L$ so that the received signals will not overlap with the transmitted signals. Consequently, the signals received by the antenna array can be expressed as⁶

$$\mathbf{Y}^c = \sum_{k=1}^K \gamma_k (\mathbf{b}_k^T(\mathbf{s}^{r,c}) + \boldsymbol{\xi}_k^T) (\mathbf{b}_k(\mathbf{s}^{t,c}) + \boldsymbol{\xi}_k) \mathbf{W}^c \mathbf{J}_k + \mathbf{V}^c, \quad (12)$$

⁶The gain of the double cascaded channel $\mathbf{b}_k^T(\mathbf{s}^{r,c})\mathbf{b}_k(\mathbf{s}^{t,c})$ can be comparable with that of the LOS channel $\boldsymbol{\xi}_k^T\boldsymbol{\xi}_k$ from the antenna array to the antenna array via the targets. According to [26], the gain of the double cascaded channel can be roughly expressed as $A/(d_1^2 d_2^2)$, where A is a parameter related to the gain of the RIS, d_1 denotes the distance between the antenna and the RIS, and d_2 denotes the distance between the RIS and the targets. Besides, the gain of the LOS channel can be approximated by B/d_3^2 , where B is a parameter related to the gain of the antenna, and d_3 denote the distance between the antenna and the targets. Since we assume that the distance between the antenna and the RIS is much smaller than the distance between the RIS/antenna array and the target, we have $d_1 \ll d_2 \approx d_3$. In addition, as we assume that the antenna array are close to the RIS, by carefully designing the phase shifts of the RIS, the gain of the RIS can be promoted to satisfy $A/d_1^2 \geq B$. Thus, we can assure that the gain of the double cascaded channel is the same or even larger than that of the LOS channel.

where γ_k is the response of the k -th target characterizing the reflection and channel propagation effects related to the k -th target [10], and $\mathbf{V}^c \in \mathbb{C}^{L \times L_R}$ denotes the residual term which includes noise and interferences from the environment [35], [36]. For simplicity, we assume that the rows of $\mathbf{V}^c$ follow independent and identically distributed circularly symmetric complex Gaussian distribution with mean zero and covariance matrix $\sigma^2 \mathbf{I}^{L \times L_R}$ [14]. Let $\mathbf{y}^c = \text{vec}(\mathbf{Y}^c)$ denote the received signal vector, which is given by

$$\mathbf{y}^c = \mathbf{F}^c \boldsymbol{\gamma} + \mathbf{v}^c, \quad (13)$$

where the response vector $\boldsymbol{\gamma} = (\gamma_1, \dots, \gamma_K)^T \in \mathbb{C}^{K \times 1}$, the residual vector $\mathbf{v}^c = \text{vec}(\mathbf{V}^c) \in \mathbb{C}^{N L_R \times 1}$, and $\mathbf{F}^c = (\mathbf{Q}_1^c(\mathbf{s}^{t,c}, \mathbf{s}^{r,c})\mathbf{w}^c, \dots, \mathbf{Q}_K^c(\mathbf{s}^{t,c}, \mathbf{s}^{r,c})\mathbf{w}^c) \in \mathbb{C}^{N L_R \times K}$ with $\mathbf{Q}_k^c(\mathbf{s}^{t,c}, \mathbf{s}^{r,c}) = \mathbf{J}_k^T \otimes ((\mathbf{b}_k^T(\mathbf{s}^{r,c}) + \boldsymbol{\xi}_k^T)(\mathbf{b}_k(\mathbf{s}^{t,c}) + \boldsymbol{\xi}_k)) \in \mathbb{C}^{N L_R \times N L}$ and $\mathbf{w}^c = \text{vec}(\mathbf{W}^c) \in \mathbb{C}^{N L \times 1}$.

D. Detection

The received signal $\mathbf{y}^c$ will then be processed by the Rx to update the probabilities of hypotheses for multi-target detection. Suppose the number of targets is K , we can form J_K hypotheses, and we have

$$j_{K,I} = \binom{I + K - 1}{K}. \quad (14)$$

Since $0 \leq K \leq K_M$, there are $J = J_0 + J_1 + \dots + J_{K_M}$ hypotheses, denoted by $\{U_0, U_1, \dots, U_{J-1}\}$. Given signal $\mathbf{y}^{(c)} = (\mathbf{y}^1, \dots, \mathbf{y}^c)^T$ received in previous c cycles, the prior probability of hypothesis U_j in the $(c+1)$ -th cycle can be expressed as [37]

$$p^{c+1}(U_j) = \frac{p^1(U_j)p^{(c)}(\mathbf{y}^{(c)}|U_j)}{\sum_{j=0}^J p^1(U_j)p^{(c)}(\mathbf{y}^{(c)}|U_j)}. \quad (15)$$

where $p^{(c)}(\mathbf{y}^{(c)}|U_j)$ denote the probability to receive $\mathbf{y}^{(c)}$ given hypothesis U_j , which can be given by

$$\begin{aligned} p^{(c)}(\mathbf{y}^{(c)}|U_j) &= \prod_{i=1}^c p^i(\mathbf{y}^i|U_j) \\ &= \prod_{i=1}^c \frac{1}{(\pi\sigma^2)^{NL_R}} \exp\left(-\frac{\|\mathbf{y}^i - \bar{\mathbf{y}}^i(U_j, \{\hat{l}_k\}^{(c),j}, \hat{\gamma}^{(c),j})\|^2}{\sigma^2}\right), \end{aligned} \quad (16)$$

where $\{\hat{l}_k\}^{(c),j}$ and $\hat{\gamma}^{(c),j}$ denote the estimated delays and responses given hypothesis U_j , respectively. $\bar{\mathbf{y}}^i(U_j, \{\hat{l}_k\}^{(c),j}, \hat{\gamma}^{(c),j})$ denotes the mean signals received by the antenna array under hypothesis U_j , delays $\{\hat{l}_k\}^{(c),j}$, and responses $\hat{\gamma}^{(c),j}$ in the i -th cycle. Note that U_0 denotes the hypothesis when $K = 0$, and thus $\bar{\mathbf{y}}^i(U_j, \{\hat{l}_k\}^{(c),j}, \hat{\gamma}^{(c),j}) = \mathbf{0}, \forall i$. When $j \neq 0$, delays $\{\hat{l}_k\}^{(c),j}$ and responses $\hat{\gamma}^{(c),j}$ can be estimated jointly using the maximum likelihood estimation method. Specifically, given delays $\{\hat{l}_k\}$, the maximum likelihood estimation of responses $\hat{\gamma}^{(c),j}(\{\hat{l}_k\})$ can be expressed as

$$\hat{\gamma}^{(c),j}(\{\hat{l}_k\}) = \left((\mathbf{F}^{(c)})^H \mathbf{F}^{(c)} \right)^{-1} (\mathbf{F}^{(c)})^H \mathbf{y}^{(c)}, \quad (17)$$

where $\mathbf{F}^{(c)} = (\mathbf{F}^1, \dots, \mathbf{F}^c)^T$ is calculated using delays $\{\hat{l}_k\}$. Next, based on (17), the maximum likelihood estimation of the delays can be given by

$$\begin{aligned} \{\hat{l}_k\}^{(c),j} &= \arg \max_{\{\hat{l}_k\}} p^{(c)}(\mathbf{y}^{(c)}|U_j, \{\hat{l}_k\}) \\ &= \prod_{i=1}^c \frac{1}{(\pi\sigma^2)^{NL_R}} \exp\left(-\frac{\|\mathbf{y}^i - \bar{\mathbf{y}}^i(U_j, \{\hat{l}_k\}, \hat{\gamma}^{(c),j})\|^2}{\sigma^2}\right), \end{aligned} \quad (18)$$

where $p^{(c)}(\mathbf{y}^{(c)}|U_j, \{\hat{l}_k\})$ denotes the probability to receive $\mathbf{y}^{(c)}$ given hypothesis U_j , delays $\{\hat{l}_k\}$, and responses $\hat{\gamma}^{(c),j}(\{\hat{l}_k\})$. Finally, using the estimated delays $\{\hat{l}_k\}^{(c),j}$, the estimated responses can be expressed as $\hat{\gamma}^{(c),j} = \hat{\gamma}^{(c),j}(\{\hat{l}_k\}^{(c),j})$. Note that if multiple targets are in the same direction, their delays need to be different. Using the relation

$$\hat{d}_k = \frac{v^l \hat{l}_k}{2}, \quad (19)$$

where v^l denotes the speed of light, the range of the k -th target can be determined if the delay of echo signal $\hat{l}_k$ is decided.

To reduce the mis-detection probability caused by the noise and interference, a *threshold detection* will be conducted for each target defined in hypothesis U_j using the estimated target response $\hat{\gamma}^{(c),j}$ [28]. To be specific, if $|\gamma_k^{(c),j}|$ is greater than a pre-determined positive threshold ω , the k -th target is viewed to be present. Otherwise, the response $\gamma_k^{(c),j}$ is viewed to be caused by the noise and interference, and thus hypothesis U_j will be rejected.

IV. JOINT OPTIMIZATION PROBLEM OF WAVEFORM AND PHASE SHIFTS

In this section, we formulate the waveform and RIS phase shift optimization problem in each cycle in Subsection IV-A. Since the waveform and RIS phase shifts are coupled which are difficult to be simultaneously optimized, we decouple the formulated problem into three subproblems to solve it efficiently in Subsection IV-B.

A. Waveform and RIS Phase Shift Optimization Problem Formulation

The aim to optimize the radar waveforms $\mathbf{w}$ and RIS phase shift vectors $\mathbf{s}^t$ and $\mathbf{s}^r$ in each cycle is to improve the detection performance. In this paper, we use the relative entropy as the evaluation criterion, which is shown to be an effective tool to study the performance of multiple hypotheses testing and is widely used for radar detection applications [38]. The relative entropy indicates the “distance” between the probability functions of two different hypotheses. When the “distance” is maximized, the hypotheses are more likely to be distinguished, and thus we can obtain a higher detection accuracy.

According to [14], we define the predicted distance between hypotheses U_j and $U_{j'}$ in the $(c+1)$ -th cycle as

$$\begin{aligned} d_{j,j'}^{c+1}(\mathcal{P}^{c+1}) &= D(p^{c+1}(\mathbf{y}|U_j, \mathcal{P}^{c+1}), p^{c+1}(\mathbf{y}|U_{j'}, \mathcal{P}^{c+1})) \\ &\quad + D(p^{c+1}(\mathbf{y}|U_{j'}, \mathcal{P}^{c+1}), p^{c+1}(\mathbf{y}|U_j, \mathcal{P}^{c+1})), \end{aligned} \quad (20)$$

where $\mathcal{P}^{c+1} = \{\mathbf{w}^{c+1}, \mathbf{s}^{t,c+1}, \mathbf{s}^{r,c+1}\}$ is the set of variables to be optimized in the $(c+1)$ -th cycle. $p^{c+1}(\mathbf{y}|U_j, \mathcal{P}^{c+1})$ denotes the probability function given hypothesis U_j , variables $\mathcal{P}^{c+1}$, estimated target responses $\hat{\gamma}^{c+1}$, and delays $\{\hat{l}_k\}^{c+1}$. Besides, $D(p^{c+1}(\mathbf{y}|U_j, \mathcal{P}^{c+1}), p^{c+1}(\mathbf{y}|U_{j'}, \mathcal{P}^{c+1}))$ is the relative entropy between probability functions $p^{c+1}(\mathbf{y}|U_j, \mathcal{P}^{c+1})$ and $p^{c+1}(\mathbf{y}|U_{j'}, \mathcal{P}^{c+1})$, where

$$\begin{aligned} D(p^{c+1}(\mathbf{y}|U_j, \mathcal{P}^{c+1}), p^{c+1}(\mathbf{y}|U_{j'}, \mathcal{P}^{c+1})) &= \int p^{c+1}(\mathbf{y}|U_j, \mathcal{P}^{c+1}) \log \frac{p^{c+1}(\mathbf{y}|U_j, \mathcal{P}^{c+1})}{p^{c+1}(\mathbf{y}|U_{j'}, \mathcal{P}^{c+1})} d\mathbf{y}. \end{aligned} \quad (21)$$

The objective of the optimization problem is to maximize the weighted sum of predicted distances between every two hypotheses. Therefore, the optimization problem in the $(c+1)$ -th cycle can be formulated as

$$\begin{aligned} \text{P1: } \max_{\mathbf{w}^{c+1}, \mathbf{s}^{t,c+1}, \mathbf{s}^{r,c+1}} \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \beta_{j,j'}^{c+1} d_{j,j'}^{c+1} & \\ (\mathbf{w}^{c+1}, \mathbf{s}^{t,c+1}, \mathbf{s}^{r,c+1}), & \end{aligned} \quad (22a)$$

$$\text{s.t. } \|\mathbf{w}^{c+1}\|^2 = P_M, \quad (22b)$$

$$\mathbf{s}_m^{t,c+1} \in \{i\Delta s | i = 1, \dots, N_s\}, \quad \forall m, \quad (22c)$$

$$\mathbf{s}_m^{r,c+1} \in \{i\Delta s | i = 1, \dots, N_s\}, \quad \forall m, \quad (22d)$$

where $\beta_{j,j'}^{c+1}$ is the weighting factor for the predicted distance. Constraint (22b) is the power constraint for antennas [39], with constant P_M being the maximum transmission power. Constraints (22c) and (22d) confine the available states of

RIS elements [10]. The weighting factor is set as the prior probability product of two hypotheses, which is given by

$$\beta_{j,j'}^{c+1} = p^{c+1}(U_j)p^{c+1}(U_{j'}). \quad (23)$$

The intuition of (23) is that if the probabilities of two hypotheses are higher, the predicted distance of these two probabilities needs to have larger weight for better discrimination.

B. Problem Decomposition

It is difficult to tackle problem (P1) because variables $\mathbf{w}^{c+1}$, $\mathbf{s}^{t,c+1}$, and $\mathbf{s}^{r,c+1}$ are coupled in the objective function (22a). To solve problem (P1) efficiently, we decouple it into three subproblems, i.e., waveform optimization subproblem, RIS phase shift optimization subproblems for transmission and reception steps.

1) *Optimization Subproblem of Waveform*: Given the fixed RIS phase shift vectors $\mathbf{s}^t$, and $\mathbf{s}^r$, this subproblem is given by

$$\begin{aligned} P_w: \max_{\mathbf{w}^{c+1}} & \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \beta_{j,j'}^{c+1} d_{j,j'}^{c+1}(\mathbf{w}^{c+1}, \mathbf{s}^t, \mathbf{s}^r), \\ \text{s.t. (22b).} \end{aligned} \quad (24a)$$

2) *Optimization Subproblem of RIS Phase Shift Vector in the Transmission Step*: In this subproblem, the RIS phase shift vector $\mathbf{s}^{t,c+1}$ is optimized given $\mathbf{w}$ and $\mathbf{s}^r$, which can be written as

$$\begin{aligned} P_t: \max_{\mathbf{s}^{t,c+1}} & \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \beta_{j,j'}^{c+1} d_{j,j'}^{c+1}(\mathbf{w}, \mathbf{s}^{t,c+1}, \mathbf{s}^r), \\ \text{s.t. (22c).} \end{aligned} \quad (25a)$$

3) *Optimization Subproblem of RIS Phase Shift Vector in the Reception Step*: Similarly, the optimization subproblem of the RIS phase shift vector in the reception step can be formulated as

$$\begin{aligned} P_r: \max_{\mathbf{s}^{r,c+1}} & \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \beta_{j,j'}^{c+1} d_{j,j'}^{c+1}(\mathbf{w}, \mathbf{s}^t, \mathbf{s}^{r,c+1}), \\ \text{s.t. (22d).} \end{aligned} \quad (26a)$$

V. ALGORITHM DESIGN

In this section, we propose a waveform and phase shift optimization (WPSO) algorithm, which solves problem (P1) by iteratively solving the aforementioned three subproblems. The techniques to solve these three subproblems are introduced first in Subsections V-A and V-B, and then the description of the overall WPSO algorithm is presented in Subsection V-C. The superscript $c+1$ for cycles are omitted for brevity in the following part of this paper.

A. Optimization of Waveform

It is challenging to solve subproblem (P_w) due to the complicated expression of the predicted distance $d(U_j, U_{j'}|\mathcal{P})$ in the objective function (22a). In Proposition 1, a simplified expression of $d(U_j, U_{j'}|\mathcal{P})$ is provided as follows:

Proposition 1: The predicted distance $d(U_j, U_{j'}|\mathcal{P})$ between hypotheses U_j and $U_{j'}$ given $\mathcal{P}$ can be expressed as

$$d(U_j, U_{j'}|\mathcal{P}) = \frac{1}{\sigma^2} \text{Re}(\text{tr}(\mathbf{w}^H \mathbf{Z}'(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r) \mathbf{w})), \quad (27)$$

where

$$\begin{aligned} \mathbf{Z}'(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r) &= \mathbf{Z}''(U_j, U_j, \mathbf{s}^t, \mathbf{s}^r) + \mathbf{Z}''(U_{j'}, U_{j'}, \mathbf{s}^t, \mathbf{s}^r) \\ &\quad - 2\mathbf{Z}''(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r), \end{aligned} \quad (28)$$

$$\begin{aligned} \mathbf{Z}''(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r) &= \sum_{i=1}^{N(U_j)} \sum_{i'=1}^{N(U_{j'})} \hat{\gamma}_i^H(U_j) \hat{\gamma}_{i'}(U_{j'}) \mathbf{Q}_i^H(U_j, \mathbf{s}^t, \mathbf{s}^r) \\ &\quad \times \mathbf{Q}_{i'}(U_{j'}, \mathbf{s}^t, \mathbf{s}^r). \end{aligned} \quad (29)$$

Proof: See Appendix A. ■

Therefore, problem (P_w) can be reformulated as

$$P'_w: \max_{\mathbf{w}} \text{Re}(\text{tr}(\mathbf{w}^H \mathbf{Z}(\mathbf{s}^t, \mathbf{s}^r) \mathbf{w})), \quad (30a)$$

$$\text{s.t. } \text{tr}(\mathbf{w}^H \mathbf{w}) = P_M, \quad (30b)$$

where $\mathbf{Z}(\mathbf{s}^t, \mathbf{s}^r) = \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \frac{\beta_{j,j'}}{\sigma^2} \mathbf{Z}'(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r)$, and constraint (30b) corresponds to constraint (22b). Since problem (P'_w) is a quadratically constrained quadratic program (QCQP), we can use the semidefinite relaxation (SDR) technique [40] to efficiently solve it. Specifically, problem (P'_w) is equivalent to the following problem:

$$P_{X_w}: \max_{\mathbf{X}_w} \text{Re}(\text{tr}(\mathbf{X}_w \mathbf{Z}(\mathbf{s}^t, \mathbf{s}^r))), \quad (31a)$$

$$\text{s.t. } \text{tr}(\mathbf{X}_w) = P_M, \quad (31b)$$

$$\mathbf{X}_w \succcurlyeq 0, \quad (31c)$$

$$\mathbf{X}_w \in \mathbb{H}^{NL \times NL}, \quad (31d)$$

$$\text{rank}(\mathbf{X}_w) = 1, \quad (31e)$$

where $\mathbf{X}_w = \mathbf{w} \mathbf{w}^H$, and $\mathbf{X}_w \succcurlyeq 0$ indicates that $\mathbf{X}_w$ is a positive semidefinite matrix. To solve problem (P_{X_w}), the rank constraint in (31e) is first dropped to derive a relaxed version of problem (P_{X_w}), denoted by (P'_{X_w}), which can be efficiently solved using existing optimization techniques [41]. Then, the randomization method in [40] can be adopted to convert the solution of problem (P'_{X_w}) to a feasible solution of problem (P_{X_w}), which is also the solution of problem (P_w).

B. Optimization of RIS Phase Shift Vector in the Transmission and Reception Steps

Due to the similar structures of problems (P_t) and (P_r), these two problems can be solved by the same optimization methods. For brevity, in this subsection we only show how to solve problem (P_t).

To tackle problem (P_t) where phase shift $\mathbf{s}^t$ is optimized, in Proposition 2, we provide another simplified expression of distance $d(U_j, U_{j'}|\mathcal{P})$ which separates variable $\mathbf{r}(\mathbf{s}^t)$ from other parameters as follows:

Proposition 2: The predicted distance $d(U_j, U_{j'}|\mathcal{P})$ between hypotheses U_j and $U_{j'}$ given $\mathcal{P}$ can be expressed as

$$\begin{aligned} d(U_j, U_{j'}|\mathcal{P}) &= \frac{1}{\sigma^2} \text{Re}(\mathbf{r}^H(\mathbf{s}^t) \mathbf{Z}'(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) \mathbf{r}(\mathbf{s}^t) \\ &\quad + \mathbf{r}^H(\mathbf{s}^t) \mathbf{z}'_1(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) \\ &\quad + \mathbf{z}'_2(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) \mathbf{r}(\mathbf{s}^t) + \mathbf{z}'_3(U_j, U_{j'})), \end{aligned} \quad (32)$$

where parameter $\mathbf{Z}'(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r)$, $\mathbf{z}'_1(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r)$, $\mathbf{z}'_2(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r)$, and $\mathbf{z}'_3(U_j, U_{j'})$ are defined in Appendix B.

Proof: See Appendix B. ■

Consequently, we can reformulated problem (P_t) as

$$\begin{aligned} P'_t: \max_{\mathbf{r}} & \text{Re}(\mathbf{r}^H \mathbf{Z}(\mathbf{w}, \mathbf{s}^r) \mathbf{r} + \mathbf{r}^H \mathbf{z}_1(\mathbf{w}, \mathbf{s}^r) \\ & + \mathbf{z}_2(\mathbf{w}, \mathbf{s}^r) \mathbf{r} + \mathbf{z}_3), \end{aligned} \quad (33a)$$

$$\text{s.t. } r_m \in \{\eta e^{ji\Delta s} | i = 1, \dots, N_s\}, \forall m, \quad (33b)$$

where

$$\mathbf{Z}(\mathbf{w}, \mathbf{s}^r) = \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \frac{\beta_{j,j'}}{\sigma^2} \mathbf{Z}'(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r), \quad (34)$$

$$\mathbf{z}_1(\mathbf{w}, \mathbf{s}^r) = \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \frac{\beta_{j,j'}}{\sigma^2} \mathbf{z}'_1(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r), \quad (35)$$

$$\mathbf{z}_2(\mathbf{w}, \mathbf{s}^r) = \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \frac{\beta_{j,j'}}{\sigma^2} \mathbf{z}'_2(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r), \quad (36)$$

$$\mathbf{z}_3 = \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \frac{\beta_{j,j'}}{\sigma^2} \mathbf{z}'_3(U_j, U_{j'}), \quad (37)$$

And constraint (33b) corresponds to constraint (22c). To efficiently solve problem (P'_t) , we first relax the discrete phase shifts to continuous ones, and obtain the following problem:

$$\begin{aligned} P''_t: \max_{\mathbf{r}} & \text{Re}(\mathbf{r}^H \mathbf{Z}(\mathbf{w}, \mathbf{s}^r) \mathbf{r} + \mathbf{r}^H \mathbf{z}_1(\mathbf{w}, \mathbf{s}^r) \\ & + \mathbf{z}_2(\mathbf{w}, \mathbf{s}^r) \mathbf{r} + \mathbf{z}_3), \end{aligned} \quad (38a)$$

$$\text{s.t. } |r_m|^2 = \eta^2, \quad m = 1, \dots, M. \quad (38b)$$

Since problem (P''_t) is also a QCQP, the SDR technique can also be applied to solve problem (P''_t) . Let $\mathbf{r}''$ denote the solution of problem (P''_t) . Next, all the elements in vector $\mathbf{r}''$ are quantized to the nearest values in the set $\{\eta e^{ji\Delta s} | i = 1, \dots, N_s\}$ to obtain $\mathbf{r}'$. Vector $\mathbf{r}'$ is a feasible solution of problem (P'_t) because it satisfies constraint (33b). Finally, the solution of problem (P_t) can be derived from $\mathbf{r}'$ using the relation in (1).

C. Joint Optimization Algorithm

As shown in Fig. 5, the index of iteration x is set as 0 at the beginning of the WPSO algorithm. Besides, we randomly initial variables $\mathbf{w}_x$, $\mathbf{s}_x^t$, and $\mathbf{s}_x^r$ within the feasible region of problem (P1). Then, the value of the objective function (22a) d_x given $\mathbf{w}_x$, $\mathbf{s}_x^t$, and $\mathbf{s}_x^r$ is calculated. Following that, we optimize the radar waveform and RIS phase shifts in an iterative manner by solving the three subproblems. Specifically, in the x -th iteration, the WPSO algorithm first solves subproblem

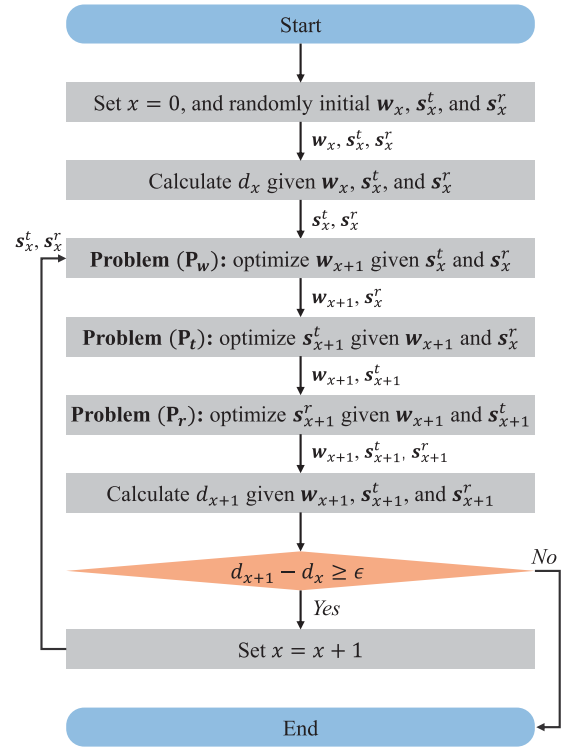


Fig. 5. Flow chart of the waveform and phase shift optimization algorithm.

(P_w) to obtain $\mathbf{w}_{x+1}$ given $\mathbf{s}_x^t$ and $\mathbf{s}_x^r$. Then, subproblem (P_t) is solved to derive $\mathbf{s}_{x+1}^t$ given $\mathbf{w}_{x+1}$ and $\mathbf{s}_x^r$. The variable $\mathbf{s}_{x+1}^r$ is finally derived by solving subproblem (P_r) given $\mathbf{w}_{x+1}$ and $\mathbf{s}_{x+1}^t$. The algorithm will terminate if the value difference of the objective function (22a) in two adjacent iterations is smaller than a predefined threshold ϵ .

VI. PERFORMANCE ANALYSIS

In this section, we first analyse the convergence and the complexity of the proposed WPSO algorithm in Subsection VI-A, and then discuss the detection performance of the proposed scheme in Subsection VI-B.

A. Convergence and Complexity

1) *Convergence:* In each iteration of the WPSO algorithm, the subproblems (P_w) , (P_t) , and (P_r) are solved sequentially using the interior-point algorithm, which is guaranteed to converge [41]. Hence, the WPSO algorithm will converge if the number of iterations of the WPSO algorithm is limited. Since the objective function is increased by at least ϵ in each iteration and the objective function has an upper bound d^u which is provided in Proposition 3, the number of iterations is bounded and the convergence of the WPSO algorithm is guaranteed.

Proposition 3: An upper bound of the objective function (22a) is given by

$$d^u = \frac{J(J-1)P_M}{2}. \quad (39)$$

Proof: See Appendix C. ■

2) *Complexity*: In each iteration of the WPSO algorithm, the SDR technique is utilized for 3 times to solve the three subproblems. According to [40], the complexity of the SDR techniques for the subproblems (P_w) , (P_t) , and (P_r) are $O((NL)^{4.5} \log(1/\alpha))$, $O((M+1)^{4.5} \log(1/\alpha))$, and $O((M+1)^{4.5} \log(1/\alpha))$, respectively, where α is the solution accuracy of the interior-point algorithm. Thus, the complexity of each iteration of the WPSO algorithm is $O(((NL)^{4.5} + (M+1)^{4.5}) \log(1/\alpha))$.

Based on (50), the objective function (22a) is nonnegative. According to the discussion in Section VI-A.1, the maximum number of iterations of the WPSO algorithm is $\lceil d^u/\epsilon \rceil$. Therefore, the complexity of the WPSO algorithm is $O(((NL)^{4.5} + (M+1)^{4.5}) \log(1/\alpha) \lceil d^u/\epsilon \rceil)$.

B. Detection Performance Analysis

In this subsection, we compare the detection performance of MetaRadar with traditional MIMO radar. For simplicity, we consider the case where $K = 1$ and $I = 1$. There are 2 hypotheses in this case denoted by U_0 and U_1 . Based on (50), the predicted distance between hypotheses U_0 and U_1 of MetaRadar in this case is given by

$$\begin{aligned} d(U_1, U_0 | \mathcal{P}) &= \frac{1}{\sigma^2} \|\bar{\mathbf{y}}(U_1, \mathcal{P}) - \bar{\mathbf{y}}(U_0, \mathcal{P})\|^2 \\ &= \frac{1}{\sigma^2} \|\gamma(\mathbf{b}_1^T(s^r) + \boldsymbol{\xi}_1^T)(\mathbf{b}_1(s^t) + \boldsymbol{\xi}_1) \mathbf{W}\|^2 \\ &= \frac{|\gamma|^2}{\sigma^2} \|\mathbf{b}_1(s^r) + \boldsymbol{\xi}_1\|^2 \|\mathbf{b}_1(s^t) + \boldsymbol{\xi}_1\|^2 \|\mathbf{W}\|^2 \\ &= \frac{|\gamma|^2 P_M}{\sigma^2} \|\mathbf{b}_1(s^r) + \boldsymbol{\xi}_1\|^2 \|\mathbf{b}_1(s^t) + \boldsymbol{\xi}_1\|^2. \end{aligned} \quad (40)$$

As for the MIMO radar, its received signals can be expressed as [14]

$$\mathbf{Y}_{MIMO} = \sum_{k=1}^K \gamma_k \boldsymbol{\xi}_k^T \boldsymbol{\xi}_k \mathbf{W} \mathbf{J}_k + \mathbf{V}, \quad (41)$$

Similar to the proof in Appendix A, the predicted distance of traditional MIMO radar between hypotheses U_0 and U_1 when $K = 1$ is given by

$$\begin{aligned} d_{MIMO}(U_1, U_0 | \mathbf{W}) &= \frac{1}{\sigma^2} \|\gamma \boldsymbol{\xi}_1^T \boldsymbol{\xi}_1 \mathbf{W} \mathbf{J}_1\|^2 \\ &= \frac{|\gamma|^2 P_M}{\sigma^2} \|\boldsymbol{\xi}_1\|^4 \\ &= \frac{(N|\gamma|G^A G_P^A(\theta_1, \varphi_1))^2 P_M}{\sigma^2}. \end{aligned} \quad (42)$$

It can be observed from (40) and (42) that the predicted distances of both MetaRadar and MIMO radar are positively related to $|\gamma|^2$ and the power of waveform P_M , and are negatively related to the variance of the residual term σ^2 . Besides, we can conclude that if $\|\mathbf{b}_1(s^r) + \boldsymbol{\xi}_1\|^2 \|\mathbf{b}_1(s^t) + \boldsymbol{\xi}_1\|^2 > (NG^A G_P^A(\theta_1, \varphi_1))^2$, the predicted distance of the MetaRadar is larger than that of the traditional MIMO radar. In this circumstance, hypotheses U_0 and U_1 can be better distinguished by the MetaRadar, indicating that MetaRadar has a higher detection accuracy.

Since $\mathbf{b}_1(s^t) + \boldsymbol{\xi}_1$ and $\mathbf{b}_1(s^r) + \boldsymbol{\xi}_1$ denote the channel gain from the antenna array to the direction of the target and vice versa, respectively, $\|\mathbf{b}_1(s^r) + \boldsymbol{\xi}_1\|^2 \|\mathbf{b}_1(s^t) + \boldsymbol{\xi}_1\|^2$ can be viewed as the power gain of the MetaRadar. As the power gain of the MetaRadar is determined by the phase shift vectors and the configuration of the MetaRadar, in the following, we first analyse the optimal phase shift vectors which maximize the power gain of the MetaRadar given the RIS configuration, and then discuss the relationship between the configuration of the MetaRadar and the power gain when RIS phase shift vectors are optimized.

1) *Optimal RIS Phase Shift Vectors*: The maximum power gain given the configuration of the MetaRadar when $N = 1$ is provided in Proposition 4 as follows:

Proposition 4: Assume that all the RIS elements have continuous phase shifts, the maximum power gain given the configuration of the MetaRadar when $N = 1$ is

$$\begin{aligned} B &= \|\mathbf{b}_1(s^{r,*}) + \boldsymbol{\xi}_1\|^2 \|\mathbf{b}_1(s^{t,*}) + \boldsymbol{\xi}_1\|^2 \\ &= (G^A)^2 \left(\sum_{m=1}^M \frac{\rho \sqrt{G_P^A(\theta_{1,m}^r, \varphi_{1,m}^r) G_P^R(\theta_{1,m}^r, \varphi_{1,m}^r)}}{l_{1,m}} \right. \\ &\quad \left. + \sqrt{G_P^A(\theta_1, \varphi_1)} \right)^4, \end{aligned} \quad (43)$$

where

$$s_m^{t,*} = s_m^{r,*} = \text{mod}(-\mathbf{e}_1 \mathbf{p}_m^e - \mathbf{e}_1 \mathbf{p}_1^a - \frac{2\pi l_{1,m}}{\lambda}, 2\pi), \quad \forall m, \quad (44)$$

and

$$\rho = \frac{\eta \sqrt{G^R S^e G_P^R(\theta_1, \varphi_1)}}{\sqrt{4\pi}}. \quad (45)$$

Proof: The term $\|\mathbf{b}_1(s) + \boldsymbol{\xi}_1\|^2$ can be expressed as

$$\begin{aligned} \|\mathbf{b}_1(s) + \boldsymbol{\xi}_1\|^2 &= \|\mathbf{a}_1 \mathbf{R}(s) \mathbf{H} + \boldsymbol{\xi}_1\|^2 \\ &= \left| \sum_{m=1}^M a_{1,m} r_m(s_m) h_{m,1} + \xi_{1,1} \right|^2 \\ &= G^A \left| \sum_{m=1}^M \frac{\rho \sqrt{G_P^A(\theta_{1,m}^r, \varphi_{1,m}^r) G_P^R(\theta_{1,m}^r, \varphi_{1,m}^r)}}{l_{1,m}} \right. \\ &\quad \left. \times e^{-j(\mathbf{e}_1 \mathbf{p}_m^e + s_m + 2\pi l_{1,m}/\lambda)} + \sqrt{G_P^A(\theta_1, \varphi_1)} e^{j\mathbf{e}_1 \mathbf{p}_1^a} \right|^2 \\ &\leq G^A \left(\sum_{m=1}^M \frac{\rho \sqrt{G_P^A(\theta_{1,m}^r, \varphi_{1,m}^r) G_P^R(\theta_{1,m}^r, \varphi_{1,m}^r)}}{l_{1,m}} \right. \\ &\quad \left. + \sqrt{G_P^A(\theta_1, \varphi_1)} \right)^2, \end{aligned} \quad (46)$$

where

$$\rho = \frac{\eta \sqrt{G^R S^e G_P^R(\theta_1, \varphi_1)}}{\sqrt{4\pi}}, \quad (47)$$

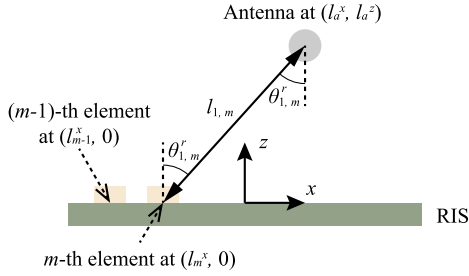


Fig. 6. Top view of the MetaRadar.

and $\|b_1(s) + \xi_1\|^2$ is maximized when

$$s_m = \text{mod}(-e_1 p_m^e - e_1 p_1^a - \frac{2\pi l_{1,m}}{\lambda}, 2\pi). \quad (48)$$

Consequently, the maximum power gain of the MetaRadar when $N = 1$ is (43), and the optimal RIS phase shift vectors satisfy (44). ■

2) *Configuration of the MetaRadar*: The configuration of the MetaRadar when $N = 1$ is shown in Fig. 6. For simplicity, we consider a two-dimensional (2D) configuration, and the extension to 3D configuration is feasible. The RIS is parallel to the x -axis and its center is at the origin $(0, 0)$. In the following, we discuss how the number of RIS elements and the antenna position affect the power gain B .

a) *Number of RIS elements*: The power gain is positively related to the number of RIS elements. This is because if we add an element at the edge of the RIS and keep the positions of existing M elements fixed, according to (43), the power gain will increase because the term $\sqrt{G_P^A(\theta_{1,m}^r) G_P^R(\theta_{1,m}^r)} / l_{1,M+1} > 0$. This indicates that an RIS with a larger size can provide a higher detection accuracy.

b) *Antenna position*: In the following proposition, we show that the antenna should be placed near the y -axis to promote the detection performance.

Proposition 5: Suppose the antenna is isotropic ($G^A = 1$ and $G_P^A(\theta, \varphi) = 1, \forall \theta, \varphi$). The power gain of the MetaRadar where the antenna is close to the z -axis is higher than that of the MetaRadar where the antenna is far away from the z -axis given the number of the RIS elements M and the z -coordinate of the antenna l_a^z . Specifically, if $l_a^x \notin [-l^e/2, l^e/2]$, there exists a $l' \in [-l^e/2, l^e/2]$ with higher power gain than that of l_a^x , where l^e is the distance between two adjacent RIS elements.

Proof: See Appendix D. ■

When $l_a^z \gg l^e/2$, the power gain when l_a^x is optimized is given by Proposition 6:

Proposition 6: When $l_a^z \gg l^e/2$, the power gain when l_a^x is optimized can be expressed as

$$B(l^*, l_a^z) = \left(\sum_{m=1}^M \frac{\rho(l_a^z)^{1.5}}{((l_a^z)^2 + ((M+1)l^e/2 - ml^e)^2)^{1.25}} + 1 \right)^4. \quad (49)$$

Proof: If $l_a^z \gg l^e/2$, we have $l_a^z \gg |l'|$, which indicates that the term l' in the denominator in (73) can be omitted.

TABLE I
SIMULATION PARAMETERS

Parameters	Values
Position of the RIS's center $\mathbf{p}^R$	$(0, 0, 0)$
Number of RIS elements M	64
Area of an RIS element S^e	$(\lambda/2)^2$
Position of the antenna array's center $\mathbf{p}^A$	$(0, 3\lambda, 0)$
Number of antennas N	4
Separation between adjacent antennas d^s	$\lambda/2$
Gain of an antenna G^A	1
Normalized radiation pattern of an antenna $G_P^A(\theta, \varphi)$	1
Maximum transmission power of the antenna array P_M	12W
Variance of the residual term σ^2	-50dBw
Number of snapshots of the radar waveform L	10
Number of snapshots of the received signal L_R	15
Number of angular grids I	4
Number of targets K	2
Amplitude of target response $ \gamma_k $	-40dB

Therefore, the maximum power gain can be expressed as (49) based on (73). ■

As for the z -coordinate of the antenna l_a^z , since both the numerator $(l_a^z)^{1.5}$ and the denominator $((l_a^z)^2 + ((M+1)l^e/2 - ml^e)^2)^{1.25}$ in (49) increase with l_a^z , its relationship with the power gain is much more complicated which makes it difficult to provide a closed-form expression of the optimal z -coordinate of the antenna.

VII. SIMULATION RESULTS

In this section, the performance of the MetaRadar is provided. The configuration of the MetaRadar is shown in Fig. 1, and the simulation parameters are listed in Table I. The RIS is located at the plane $z = 0$, and the position of its center is $(0, 0, 0)$. The RIS contains 64 elements. Each element has 8 different phase shifts, and the area of an element is $(\lambda/2)^2$ [42]. The antennas are arranged as a 2×2 array, and the separation between adjacent antennas is $\lambda/2$ [43]. The center of the antenna array is at $(0, 3\lambda, 0)$. Each antenna is assumed to be isotropic. That is, $G^A = 1$ and $G_P^A(\theta, \varphi) = 1, \forall \theta, \varphi$. The maximum transmission power of the antenna array is set as 12W. The variance of the residual term σ^2 is -50dBw, and threshold $\omega = \sigma/60$. The number of snapshots of the radar waveform is 10, and the number of snapshots of the received signal is 15. The angular range of interest is $\theta = \pi/6$, $\varphi \in [0, 2\pi)$, and the range $[0, 2\pi)$ is uniformly divided into 4 grids. We assume that the number of targets is 2. The two targets are located at $(10\Delta d, \pi/6, \pi/4)$ and $(15\Delta d, \pi/6, 3\pi/4)$. Here, Δd denotes the length of a range cell, which is normalized as 1. The correct hypothesis is denoted by U_{j^*} . The amplitude of target response $|\gamma_k|$ is -40dB, and the phase of the target response is randomly selected in $[0, 2\pi)$ in each Monte Carlo run.

A. Performance Comparison

In this subsection, we compare the performance of the proposed scheme with those of the random scheme and traditional MIMO radar scheme based on the relative entropy criterion [14]. In the random scheme, the radar waveforms have fixed envelope and random phases. The phase shift

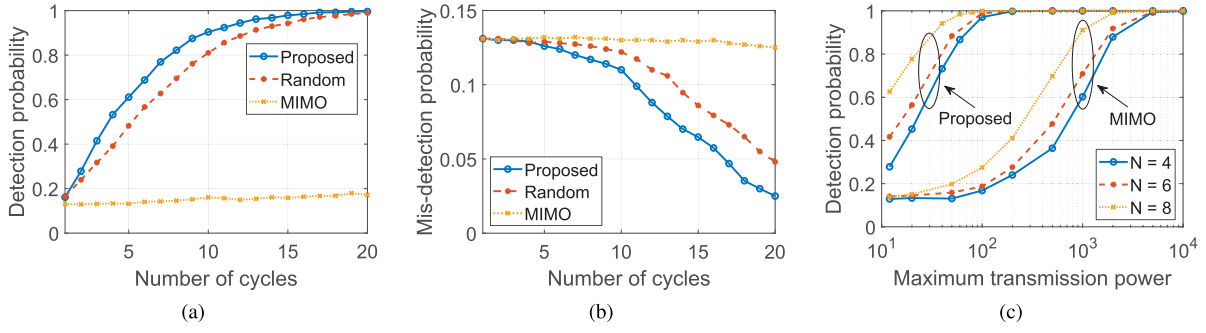


Fig. 7. (a) The detection probability $p(U_{j^*}|U_{j^*})$ versus the number of cycles C in the detection process; (b) The mis-detection probability $\sum_{j \neq j^*} p(U_{j^*}|U_j)p^1(U_j)$ versus the number of cycles C in the detection process; (c) The detection probability $p(U_{j^*}|U_{j^*})$ versus the maximum transmission power P_M with different numbers of antennas N .

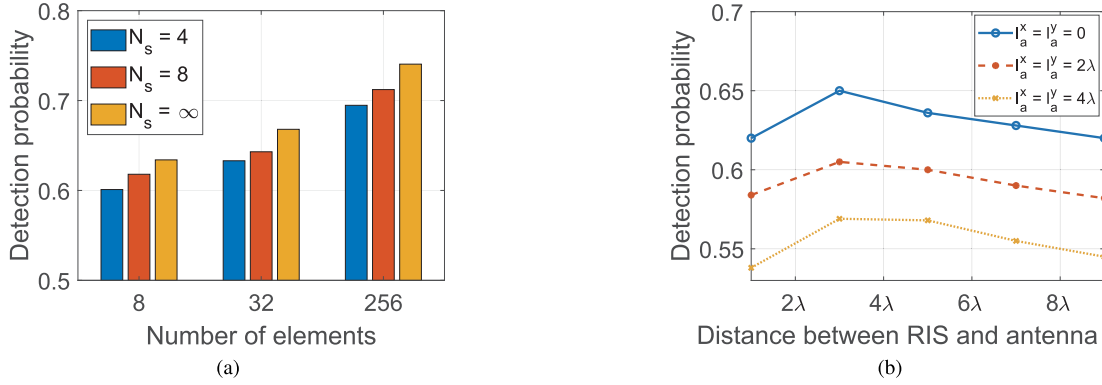


Fig. 8. (a) The detection probability $p(U_{j^*}|U_{j^*})$ versus the number of elements M with different number of phase shifts N_s ; (b) The detection probability $p(U_{j^*}|U_{j^*})$ versus the distance between the RIS and the antenna array l_a^z with different x and y coordinates of the antenna array center.

vectors of the RIS in the transmission and reception steps are also randomly generated.

Fig. 7 (a) and (b) show the detection probability $p(U_{j^*}|U_{j^*})$ and the mis-detection probability $\sum_{j \neq j^*} p(U_{j^*}|U_j)p^1(U_j)$ versus the number of cycles C in the detection process, respectively. It can be observed that the detection probability obtained by the proposed scheme is higher than those obtained by the random and MIMO schemes, and the mis-detection probability obtained by the proposed scheme is smaller than those obtained by the other two schemes, which verify the effectiveness of the proposed scheme. Besides, we can also observe that the performances of the random and proposed schemes are much higher than that of the MIMO scheme. Specifically, the detection probability obtained by the MIMO scheme increase slowly with the number of cycles and is lower than 0.2 when $C = 20$, while the probabilities of detection obtained by the proposed and random schemes with RIS can approach 1 after sufficient number of cycles. As for mis-detection, the probability obtained by the MIMO scheme when $C = 20$ is also much higher than those obtained by the random and proposed schemes. This is because by incorporating the RIS, the power gain of the radar can be significantly promoted even the phase shifts of the RIS are not optimized, leading to a rapid increase/decrease of detection/mis-detection probabilities of the random and proposed scheme.

Fig. 7 (c) presents the detection probability $p(U_{j^*}|U_{j^*})$ versus the maximum transmission power P_M when $C = 3$.

It can be seen that for both the proposed and the MIMO schemes, the detection probability first increases and then remains close to 1 when the maximum transmission power P_M increases. Besides, the detection probability p also increases with the number of antennas N . However, to reach the same detection probability, the maximum transmission power P_M and the number of antennas N required by the proposed scheme are much smaller than that of the MIMO scheme, which indicates a tradeoff between the cost of deploying an RIS and the cost of increasing the size and the power of MIMO antenna array.

B. Radar Configuration

Fig. 8 (a) shows the detection probability $p(U_{j^*}|U_{j^*})$ versus the number of elements M with different number of phase shifts N_s when $C = 6$. We can observe that the detection probability increases with the number of elements, which is in accordance with the conclusion in Section VI-B.2. Besides, the detection probability also increases with the number of phase shifts N_s , since the reflection coefficients of the RIS with more number of phase shifts can be adjusted more precisely to obtain a larger relative entropy and higher accuracy. This also indicates a trade-off between the number of elements and the number of phase shifts given the detection performance.

Fig. 8 (b) depicts the detection probability $p(U_{j^*}|U_{j^*})$ versus the distance between the RIS and the antenna array l_a^z .

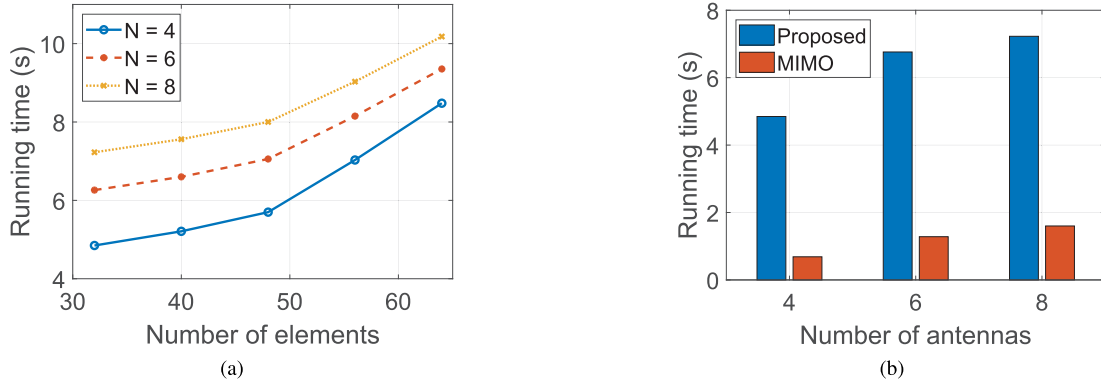


Fig. 9. (a) The running time t^r versus the number of RIS elements M with different numbers of antennas N ; (b) The running time t^r versus the number of elements N with different schemes.

with different x and y coordinates of the antenna array center when $C = 6$. It can be observed that the detection probability when the center of the antenna array is close to the z axis ($l_a^x = l_a^y = 0$) is higher than those when the center of the antenna array is far away from the z axis ($l_a^x = l_a^y = 2\lambda, 4\lambda$), which is in accordance with **Proposition 5**. Besides, we can observe that the detection probability first increases and then decreases when the distance between the RIS and the antenna array l_a^z increases, and there exists an optimal distance between the RIS and the antenna array with the best detection performance.

C. Complexity

Fig. 9 (a) shows the running time t^r of the WPSO algorithm in each cycle versus the number of RIS elements M , where the running time is obtained using a computer with Intel Core i5-8250U CPU (1.6GHz), 8 GB RAM, and Matlab 2019b. We can observe that the running time increases with the number of RIS elements M and the number of antennas N , which matches the results in Section VI-A.2.

The running time t^r in each cycle versus the number of elements N with different schemes is shown in Fig. 9 (b). We can observe that the running time increases with the number of antennas for both schemes, and the running time of the proposed scheme is significantly longer than that of the traditional MIMO radar scheme. This is because the traditional MIMO radar scheme only optimizes the radar waveforms, while the proposed scheme iteratively optimizes the radar waveforms and the RIS phase shifts, which leads to a higher detection accuracy compared with the traditional MIMO radar scheme.

VIII. CONCLUSION

In this paper, we have investigated the multi-target detection using the RIS-assisted radar systems. A multi-target detection protocol has been designed to adaptively promote the detection performance by coordinating the operations of the antenna arrays and the RIS. We have also formulated the optimization problem for target detection based on the relative entropy criterion. To tackle the formulated problem, we have proposed the WPSO algorithm which jointly optimizes the radar waveforms and the RIS phase shifts. The detection performance

of the proposed scheme has been analyzed theoretically, and the superiority of the proposed scheme has been verified through simulation. We can conclude from the results of analysis and the simulation that: 1) by incorporating an RIS in the radar systems which creates reflection paths between the antennas and the targets, the power gain of the radar can be enhanced, leading to a significant performance improvement of the MetaRadar compared with traditional radar schemes; 2) to meet the same detection performance, we can increase the number of RIS elements to reduce the demand for the number of phase shifts, and vice versa; 3) to maximize the detection accuracy, it is preferred to put the RIS and the antenna array coaxially, and the distance between the RIS and the antenna array needs to be carefully chosen.

APPENDIX A PROOF OF PROPOSITION 1

Based on (16) and (21), the predicted distance between hypotheses U_j and $U_{j'}$ can be expressed as

$$\begin{aligned}
 d(U_j, U_{j'} | \mathcal{P}) &= \int p(\mathbf{y} | U_j, \mathcal{P}) \log \frac{p(\mathbf{y} | U_j, \mathcal{P})}{p(\mathbf{y} | U_{j'}, \mathcal{P})} d\mathbf{y} \\
 &\quad + \int p(\mathbf{y} | U_{j'}, \mathcal{P}) \log \frac{p(\mathbf{y} | U_{j'}, \mathcal{P})}{p(\mathbf{y} | U_j, \mathcal{P})} d\mathbf{y} \\
 &= \frac{1}{2\sigma^2} \int p(\mathbf{y} | U_j, \mathcal{P}) (||\mathbf{y} - \bar{\mathbf{y}}(U_{j'}, \mathcal{P})||^2 - ||\mathbf{y} - \bar{\mathbf{y}}(U_j, \mathcal{P})||^2) d\mathbf{y} \\
 &\quad + \frac{1}{2\sigma^2} \int p(\mathbf{y} | U_{j'}, \mathcal{P}) (||\mathbf{y} - \bar{\mathbf{y}}(U_j, \mathcal{P})||^2 - ||\mathbf{y} - \bar{\mathbf{y}}(U_{j'}, \mathcal{P})||^2) d\mathbf{y} \\
 &= \frac{1}{\sigma^2} ||\bar{\mathbf{y}}(U_j, \mathcal{P}) - \bar{\mathbf{y}}(U_{j'}, \mathcal{P})||^2. \quad (50)
 \end{aligned}$$

According to (13), $\bar{\mathbf{y}}(U_j, \mathcal{P}) = \mathbf{F}(U_j, \mathcal{P})\hat{\gamma}(U_j)$, and thus we have

$$\begin{aligned}
 d(U_j, U_{j'} | \mathcal{P}) &= \frac{1}{\sigma^2} ||\mathbf{F}(U_j, \mathcal{P})\hat{\gamma}(U_j) - \mathbf{F}(U_{j'}, \mathcal{P})\hat{\gamma}(U_{j'})||^2 \\
 &= \frac{1}{\sigma^2} \text{Re} \left(\hat{\gamma}^H(U_j) \mathbf{F}^H(U_j, \mathcal{P}) \mathbf{F}(U_j, \mathcal{P}) \hat{\gamma}(U_j) \right. \\
 &\quad \left. + \hat{\gamma}^H(U_{j'}) \mathbf{F}^H(U_{j'}, \mathcal{P}) \mathbf{F}(U_{j'}, \mathcal{P}) \hat{\gamma}(U_{j'}) \right. \\
 &\quad \left. - 2\hat{\gamma}^H(U_j) \mathbf{F}^H(U_j, \mathcal{P}) \mathbf{F}(U_{j'}, \mathcal{P}) \hat{\gamma}(U_{j'}) \right) \quad (51)
 \end{aligned}$$

The term $\hat{\gamma}^H(U_j)\mathbf{F}^H(U_j, \mathcal{P})\mathbf{F}(U_{j'}, \mathcal{P})\hat{\gamma}(U_{j'})$ in (51) can be expressed as

$$\begin{aligned} & \hat{\gamma}^H(U_j)\mathbf{F}^H(U_j, \mathcal{P})\mathbf{F}(U_{j'}, \mathcal{P})\hat{\gamma}(U_{j'}) \\ &= \text{tr}\left(\mathbf{w}\mathbf{w}^H \sum_{i=1}^{N(U_j)} \sum_{i'=1}^{N(U_{j'})} \hat{\gamma}_i^H(U_j)\hat{\gamma}_{i'}(U_{j'})\mathbf{Q}_i^H(U_j, \mathbf{s}^t, \mathbf{s}^r) \right. \\ & \quad \left. \times \mathbf{Q}_{i'}(U_{j'}, \mathbf{s}^t, \mathbf{s}^r)\right) \\ &= \text{tr}(\mathbf{X}_w \mathbf{Z}''(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r)), \end{aligned} \quad (52)$$

where $\mathbf{X}_w = \mathbf{w}\mathbf{w}^H$ and $\mathbf{Z}''(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r) = \sum_{i=1}^{N(U_j)} \sum_{i'=1}^{N(U_{j'})} \hat{\gamma}_i^H(U_j)\hat{\gamma}_{i'}(U_{j'})\mathbf{Q}_i^H(U_j, \mathbf{s}^t, \mathbf{s}^r)\mathbf{Q}_{i'}(U_{j'}, \mathbf{s}^t, \mathbf{s}^r)$.

Therefore, the predicted distance between hypotheses U_j and $U_{j'}$ is given by

$$\begin{aligned} d(U_j, U_{j'}|\mathcal{P}) &= \frac{1}{\sigma^2} \text{Re}\left(\text{tr}(\mathbf{X}_w \mathbf{Z}''(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r) \right. \\ & \quad \left. + \mathbf{X}_w \mathbf{Z}''(U_{j'}, U_j, \mathbf{s}^t, \mathbf{s}^r) \right. \\ & \quad \left. - 2\mathbf{X}_w \mathbf{Z}''(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r))\right) \\ &= \frac{1}{\sigma^2} \text{Re}\left(\text{tr}(\mathbf{X}_w \mathbf{Z}'(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r))\right) \\ &= \frac{1}{\sigma^2} \text{Re}\left(\text{tr}(\mathbf{w}^H \mathbf{Z}'(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r) \mathbf{w})\right), \end{aligned} \quad (53)$$

where $\mathbf{Z}'(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r) = \mathbf{Z}''(U_j, U_j, \mathbf{s}^t, \mathbf{s}^r) + \mathbf{Z}''(U_{j'}, U_{j'}, \mathbf{s}^t, \mathbf{s}^r) - 2\mathbf{Z}''(U_j, U_{j'}, \mathbf{s}^t, \mathbf{s}^r)$.

APPENDIX B PROOF OF PROPOSITION 2

Based on (5), (12), and (13), the parameter $\mathbf{F}$ can be expressed as

$$\begin{aligned} \mathbf{F}(U_j, \mathcal{P}) &= (\mathbf{Q}'_1(U_j, \mathbf{w}, \mathbf{s}^r)\mathbf{r}'(\mathbf{s}^t) + \zeta_1(U_j), \\ & \quad \dots, \mathbf{Q}'_K(U_j, \mathbf{w}, \mathbf{s}^r)\mathbf{r}'(\mathbf{s}^t) + \zeta_K(U_j)) \in \mathbb{C}^{N_{LR} \times K}, \end{aligned} \quad (54)$$

where $\mathbf{Q}'_k(U_j, \mathbf{w}, \mathbf{s}^r) = (\mathbf{J}_k^T \mathbf{W}^T \mathbf{H}^T) \otimes ((\mathbf{b}_k^T(\mathbf{s}^r) + \xi_k^T) \mathbf{a}_k) \in \mathbb{C}^{N_{LR} \times M^2}$, $\mathbf{r}'(\mathbf{s}^t) = \text{vec}(\mathbf{R}(\mathbf{s}^t)) \in \mathbb{C}^{M^2 \times 1}$, $\zeta_k(U_j) = \text{vec}((\mathbf{b}_k^T(\mathbf{s}^r) + \xi_k^T) \xi_k \mathbf{W}^c \mathbf{J}_k)$, and $K = N(U_j)$. Since $\mathbf{R}(\mathbf{s}^t)$ is a diagonal matrix with diagonal elements being $\mathbf{r}(\mathbf{s}^t) = (r(s_1), \dots, r(s_M))$, the i -th element of vector $\mathbf{r}'(\mathbf{s}^t)$ is given by

$$\mathbf{r}'(\mathbf{s}^t)_i = \begin{cases} r_m(s_m), & i = (m-1)(M+1) + 1 \\ 0, & \text{otherwise.} \end{cases} \quad (55)$$

Thus, we have

$$\mathbf{Q}'_k(U_j, \mathbf{w}, \mathbf{s}^r)\mathbf{r}'(\mathbf{s}^t) = \mathbf{Q}_k(U_j, \mathbf{w}, \mathbf{s}^r)\mathbf{r}(\mathbf{s}^t), \quad (56)$$

where $\mathbf{Q}_k(U_j, \mathbf{w}, \mathbf{s}^r)$ is given by

$$\mathbf{Q}_k(U_j, \mathbf{w}, \mathbf{s}^r) = (\mathbf{Q}'_{k,1}(U_j, \mathbf{w}, \mathbf{s}^r), \mathbf{Q}'_{k,M+2}(U_j, \mathbf{w}, \mathbf{s}^r), \dots, \mathbf{Q}'_{k,M^2}(U_j, \mathbf{w}, \mathbf{s}^r)), \quad (57)$$

where $\mathbf{Q}'_{k,i}(U_j, \mathbf{w}, \mathbf{s}^r)$ denote the i -th column of $\mathbf{Q}'_k(U_j, \mathbf{w}, \mathbf{s}^r)$. Consequently, parameter $\mathbf{F}$ can be expressed as

$$\mathbf{F}(U_j, \mathcal{P}) = (\mathbf{Q}_1(U_j, \mathbf{w}, \mathbf{s}^r)\mathbf{r}(\mathbf{s}^t) + \zeta_1(U_j), \dots, \mathbf{Q}_K(U_j, \mathbf{w}, \mathbf{s}^r)\mathbf{r}(\mathbf{s}^t) + \zeta_K(U_j)). \quad (58)$$

Similar to the proof in Appendix A, (51), the predicted distance between hypotheses U_j and $U_{j'}$ can be expressed as

$$\begin{aligned} d(U_j, U_{j'}|\mathcal{P}) &= \frac{1}{\sigma^2} \text{Re}\left(\hat{\gamma}^H(U_j)\mathbf{F}^H(U_j, \mathcal{P})\mathbf{F}(U_{j'}, \mathcal{P})\hat{\gamma}(U_j) \right. \\ & \quad \left. + \hat{\gamma}^H(U_{j'})\mathbf{F}^H(U_{j'}, \mathcal{P})\mathbf{F}(U_j, \mathcal{P})\hat{\gamma}(U_{j'}) \right. \\ & \quad \left. - 2\hat{\gamma}^H(U_j)\mathbf{F}^H(U_j, \mathcal{P})\mathbf{F}(U_{j'}, \mathcal{P})\hat{\gamma}(U_{j'})\right), \end{aligned} \quad (59)$$

where $\hat{\gamma}^H(U_j)\mathbf{F}^H(U_j, \mathcal{P})\mathbf{F}(U_{j'}, \mathcal{P})\hat{\gamma}(U_{j'})$ can be expressed as

$$\begin{aligned} & \hat{\gamma}^H(U_j)\mathbf{F}^H(U_j, \mathcal{P})\mathbf{F}(U_{j'}, \mathcal{P})\hat{\gamma}(U_{j'}) \\ &= \sum_{i=1}^{N(U_j)} \sum_{i'=1}^{N(U_{j'})} \hat{\gamma}_i^H(U_j)\hat{\gamma}_{i'}(U_{j'}) (\mathbf{r}^H(\mathbf{s}^t)\mathbf{Q}_i^H(U_j, \mathbf{w}, \mathbf{s}^r) \\ & \quad + \zeta_i^H(U_j)) (\mathbf{Q}_{i'}(U_{j'}, \mathbf{w}, \mathbf{s}^r)\mathbf{r}(\mathbf{s}^t) + \zeta_{i'}(U_{j'})) \\ &= \mathbf{r}^H \mathbf{Z}''(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r)\mathbf{r}(\mathbf{s}^t) + \mathbf{r}^H \mathbf{z}_1''(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) \\ & \quad + \mathbf{z}_2''(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r)\mathbf{r}(\mathbf{s}^t) + \mathbf{z}_3''(U_j, U_{j'}), \end{aligned}$$

where

$$\begin{aligned} \mathbf{Z}''(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) &= \sum_{i=1}^{N(U_j)} \sum_{i'=1}^{N(U_{j'})} \hat{\gamma}_i^H(U_j)\hat{\gamma}_{i'}(U_{j'}) \\ & \quad \mathbf{Q}_i^H(U_j, \mathbf{w}, \mathbf{s}^r)\mathbf{Q}_{i'}(U_{j'}, \mathbf{w}, \mathbf{s}^r), \end{aligned} \quad (60)$$

$$\begin{aligned} \mathbf{z}_1''(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) &= \sum_{i=1}^{N(U_j)} \sum_{i'=1}^{N(U_{j'})} \hat{\gamma}_i^H(U_j)\hat{\gamma}_{i'}(U_{j'}) \\ & \quad \mathbf{Q}_i^H(U_j, \mathbf{w}, \mathbf{s}^r)\zeta_{i'}(U_{j'}), \end{aligned} \quad (61)$$

$$\begin{aligned} \mathbf{z}_2''(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) &= \sum_{i=1}^{N(U_j)} \sum_{i'=1}^{N(U_{j'})} \hat{\gamma}_i^H(U_j)\hat{\gamma}_{i'}(U_{j'})\zeta_i^H(U_j) \\ & \quad \mathbf{Q}_{i'}(U_{j'}, \mathbf{w}, \mathbf{s}^r), \end{aligned} \quad (62)$$

$$\begin{aligned} \mathbf{z}_3''(U_j, U_{j'}) &= \sum_{i=1}^{N(U_j)} \sum_{i'=1}^{N(U_{j'})} \hat{\gamma}_i^H(U_j)\hat{\gamma}_{i'}(U_{j'}) \\ & \quad \zeta_i^H(U_j)\zeta_{i'}(U_{j'}). \end{aligned} \quad (63)$$

Therefore, the predicted distance between hypotheses U_j and $U_{j'}$ is given by

$$\begin{aligned} d(U_j, U_{j'} | \mathcal{P}) &= \frac{1}{\sigma^2} \text{Re} \left(\hat{\gamma}^H(U_j) \mathbf{F}^H(U_j, \mathcal{P}) \mathbf{F}(U_{j'}, \mathcal{P}) \hat{\gamma}(U_{j'}) \right. \\ &\quad + \hat{\gamma}^H(U_{j'}) \mathbf{F}^H(U_{j'}, \mathcal{P}) \mathbf{F}(U_j, \mathcal{P}) \hat{\gamma}(U_j) \\ &\quad \left. - 2 \hat{\gamma}^H(U_j) \mathbf{F}^H(U_j, \mathcal{P}) \mathbf{F}(U_{j'}, \mathcal{P}) \hat{\gamma}(U_{j'}) \right) \\ &= \frac{1}{\sigma^2} \text{Re} \left(\mathbf{r}^H(\mathbf{s}^t) \mathbf{Z}'(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) \mathbf{r}(\mathbf{s}^t) \right. \\ &\quad + \mathbf{r}^H(\mathbf{s}^t) \mathbf{z}'_1(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) \\ &\quad \left. + \mathbf{z}'_2(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) \mathbf{r}(\mathbf{s}^t) + \mathbf{z}'_3(U_j, U_{j'}) \right), \end{aligned} \quad (64)$$

where

$$\begin{aligned} \mathbf{Z}'(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) &= \mathbf{Z}''(U_j, U_j, \mathbf{w}, \mathbf{s}^r) + \mathbf{Z}''(U_{j'}, U_{j'}, \mathbf{w}, \mathbf{s}^r) \\ &\quad - 2 \mathbf{Z}''(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r), \end{aligned} \quad (65)$$

$$\begin{aligned} \mathbf{z}'_1(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) &= \mathbf{z}''_1(U_j, U_j, \mathbf{w}, \mathbf{s}^r) + \mathbf{z}''_1(U_{j'}, U_{j'}, \mathbf{w}, \mathbf{s}^r) \\ &\quad - 2 \mathbf{z}''_1(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r), \end{aligned} \quad (66)$$

$$\begin{aligned} \mathbf{z}'_2(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r) &= \mathbf{z}''_2(U_j, U_j, \mathbf{w}, \mathbf{s}^r) + \mathbf{z}''_2(U_{j'}, U_{j'}, \mathbf{w}, \mathbf{s}^r) \\ &\quad - 2 \mathbf{z}''_2(U_j, U_{j'}, \mathbf{w}, \mathbf{s}^r), \end{aligned} \quad (67)$$

$$\begin{aligned} \mathbf{z}'_3(U_j, U_{j'}) &= \mathbf{z}''_3(U_j, U_j) + \mathbf{z}''_3(U_{j'}, U_{j'}) - 2 \mathbf{z}''_3(U_j, U_{j'}). \end{aligned} \quad (68)$$

APPENDIX C PROOF OF PROPOSITION 3

Based on (23) and (50), the objective function (22a) can be expressed as

$$\begin{aligned} &\sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \beta_{j,j'} d(U_j, U_{j'} | \mathcal{P}) \\ &= \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \frac{\beta_{j,j'}}{\sigma^2} \|\bar{\mathbf{y}}(U_j, \mathcal{P}) - \bar{\mathbf{y}}(U_{j'}, \mathcal{P})\|^2 \\ &\leq \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \frac{\beta_{j,j'}}{\sigma^2} (\|\bar{\mathbf{y}}(U_j, \mathcal{P})\|^2 + \|\bar{\mathbf{y}}(U_{j'}, \mathcal{P})\|^2), \\ &\leq \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \frac{1}{\sigma^2} (\|\bar{\mathbf{y}}(U_j, \mathcal{P})\|^2 + \|\bar{\mathbf{y}}(U_{j'}, \mathcal{P})\|^2). \end{aligned} \quad (69)$$

Therefore, the objective function (22a) has an upper bound if the energy of the received signals $\|\bar{\mathbf{y}}(U_j, \mathcal{P})\|^2$ is limited. Since the energy of signals will decrease due to the path loss and the reflection by the RIS and the targets, the energy of the received signals cannot be greater than the that of the

transmitted signals P_M . Therefore, we have

$$\begin{aligned} \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \beta_{j,j'} d(U_j, U_{j'} | \mathcal{P}) &\leq \sum_{j=0}^{J-1} \sum_{j'=j+1}^{J-1} \frac{2P_M}{\sigma^2} \\ &= \frac{J(J-1)P_M}{2}. \end{aligned} \quad (70)$$

APPENDIX D PROOF OF PROPOSITION 5

The power gain of the MetaRadar when the isotropic antenna is at the location (l_a^x, l_a^z) is given by

$$\begin{aligned} B(l_a^x, l_a^z) &= \left(\sum_{m=1}^M \frac{\rho G_P^R(\theta_{1,m}^r)}{l_{1,m}} + 1 \right)^4 \\ &= \left(\sum_{m=1}^M \frac{\rho \cos^{1.5}(\theta_{1,m}^r)}{l_{1,m}} + 1 \right)^4 \\ &= \left(\sum_{m=1}^M \frac{\rho (l_a^z)^{1.5}}{((l_a^z)^2 + (l_a^x - l_m^x)^2)^{1.25}} + 1 \right)^4 \\ &= \left(\sum_{m=1}^M \frac{(\rho l_a^z)^{1.5}}{((l_a^z)^2 + (l_a^x + (M+1)l^e/2 - ml^e)^2)^{1.25}} + 1 \right)^4 \end{aligned} \quad (71)$$

Suppose $l_a^x = nl^e + l'$, where n is a non-zero integer and $l' \in [-l^e/2, l^e/2]$. It can be proved that $B(l', l_a^z) > B(l_a^x, l_a^z)$ always holds. Specifically, the power gain when the antenna is at (l_a^x, l_a^z) can be expressed as

$$\begin{aligned} B(l_a^x, l_a^z) &= \left(\sum_{m=1}^M \frac{\rho (l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - (m-n)l^e)^2)^{1.25}} + 1 \right)^4, \end{aligned} \quad (72)$$

and the power gain when the antenna is at (l', l_a^z) can be expressed as

$$\begin{aligned} B(l', l_a^z) &= \left(\sum_{m=1}^M \frac{\rho (l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - ml^e)^2)^{1.25}} + 1 \right)^4. \end{aligned} \quad (73)$$

To prove $B(l_a^x, l_a^z) > B(l', l_a^z)$, it is equivalent to prove the following relationship hold.

$$\begin{aligned} &\sum_{m=1}^M \frac{(l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - ml^e)^2)^{1.25}} \\ &> \sum_{m=1}^M \frac{(l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - (m-n)l^e)^2)^{1.25}}. \end{aligned} \quad (74)$$

When $M = 1$, it is obvious that (74) holds. When $n \geq \lfloor M/2 \rfloor$ and $M > 1$, it is also easy to prove (74). When $0 < n < \lfloor M/2 \rfloor$ and $M > 1$, we have

$$\begin{aligned} &\sum_{m=1}^M \frac{(l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - ml^e)^2)^{1.25}} \\ &- \sum_{m=1}^M \frac{(l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - (m-n)l^e)^2)^{1.25}} \end{aligned}$$

$$\begin{aligned}
&= \sum_{m=1}^{M-n} \frac{(l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - ml^e)^2)^{1.25}} \\
&\quad - \sum_{m=n+1}^M \frac{(l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - (m-n)l^e)^2)^{1.25}} \\
&\quad + \sum_{m=M-n+1}^M \frac{(l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - ml^e)^2)^{1.25}} \\
&\quad - \sum_{m=1}^n \frac{(l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - (m-n)l^e)^2)^{1.25}} \\
&= \sum_{m=1}^n \frac{(l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - (m-n+M)l^e)^2)^{1.25}} \\
&\quad - \frac{(l_a^z)^{1.5}}{((l_a^z)^2 + (l' + (M+1)l^e/2 - (m-n)l^e)^2)^{1.25}}. \quad (75)
\end{aligned}$$

Since $0 < n < \lfloor M/2 \rfloor$, we have

$$\begin{aligned}
&|(l_a^z)^2 + (l' + (M+1)l^e/2 - (m-n+M)l^e)^2| \\
&< |(l_a^z)^2 + (l' + (M+1)l^e/2 + (n-m)l^e)^2|, \quad (76)
\end{aligned}$$

and thus (75) is greater than 0. Similar process can be applied to prove the case when $n < 0$, which is omitted here for brevity. Since $B(l_a^x, l_a^z) > B(l_a^x, l_a^z)$ holds, the $l' \in [-l^e/2, l^e/2]$ with higher power gain than that of l_a^x has been found.

REFERENCES

- [1] S. Dang, O. Amin, B. Shihada, and M.-S. Alouini, "What should 6G be?" *Nature Electron.*, vol. 3, no. 1, pp. 20–29, Jan. 2020.
- [2] W. Saad, M. Bennis, and M. Chen, "A vision of 6G wireless systems: Applications, trends, technologies, and open research problems," *IEEE Netw.*, vol. 34, no. 3, pp. 134–142, Oct. 2020.
- [3] S. Campbell *et al.*, "Sensor technology in autonomous vehicles: A review," in *Proc. 29th Irish Signals Syst. Conf. (ISSC)*, Belfast, U.K., Jun. 2018, pp. 1–4.
- [4] J. Kocic, N. Jovicic, and V. Drndarevic, "Sensors and sensor fusion in autonomous vehicles," in *Proc. 26th Telecommun. Forum (TELFOR)*, Belgrade, Serbia, Nov. 2018, pp. 420–425.
- [5] I. Bilik, O. Longman, S. Villeval, and J. Tabrikian, "The rise of radar for autonomous vehicles: Signal processing solutions and future research directions," *IEEE Signal Process. Mag.*, vol. 36, no. 5, pp. 20–31, Sep. 2019.
- [6] J. Tang, J. Luo, B. Tang, and J. Du, "Target models and waveform design for detection in MIMO radar," *Sci. China Inf. Sci.*, vol. 57, no. 1, pp. 1–12, Jan. 2014.
- [7] Z. Feng *et al.*, "Joint radar and communication: A survey," *China Commun.*, vol. 17, no. 1, pp. 1–27, Jan. 2020.
- [8] G. Cui, X. Yu, J. Yang, Y. Fu, and L. Kong, "An overview of waveform optimization methods for cognitive radar," *J. Radars*, vol. 8, no. 5, pp. 537–557, Oct. 2019.
- [9] F. Gini, A. D. Maio, and L. Patton, *Waveform Design and Diversity for Advanced Radar Systems*. London, U.K.: IET Press, 2012.
- [10] X. Yu, K. Alhujaili, G. Cui, and V. Monga, "MIMO radar waveform design in the presence of multiple targets and practical constraints," *IEEE Trans. Signal Process.*, vol. 68, pp. 1974–1989, 2020.
- [11] S. Kay, "Waveform design for multistatic radar detection," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 45, no. 3, pp. 1153–1166, Jul. 2009.
- [12] J. Tang, N. Li, Y. Wu, and Y. Peng, "On detection performance of MIMO radar: A relative entropy-based study," *IEEE Signal Process. Lett.*, vol. 16, no. 3, pp. 184–187, Mar. 2009.
- [13] B. Tang, J. Tang, and Y. Peng, "MIMO radar waveform design in colored noise based on information theory," *IEEE Trans. Signal Process.*, vol. 58, no. 9, pp. 4684–4697, Sep. 2010.
- [14] L. Wang, W. Zhu, Y. Zhang, Q. Liao, and J. Tang, "Multi-target detection and adaptive waveform design for cognitive MIMO radar," *IEEE Sensors J.*, vol. 18, no. 24, pp. 9962–9970, Dec. 2018.
- [15] H. Zhang, B. Di, L. Song, and Z. Han, "Reconfigurable intelligent surfaces assisted communications with limited phase shifts: How many phase shifts are enough?" *IEEE Trans. Veh. Technol.*, vol. 59, no. 3, pp. 1307–1318, Mar. 2020.
- [16] Y. Zhang *et al.*, "Beyond cell-free MIMO: Energy efficient reconfigurable intelligent surface aided cell-free MIMO communications," *IEEE Trans. Cognit. Commun. Netw.*, vol. 7, no. 2, pp. 412–426, Jun. 2021.
- [17] M. D. Renzo *et al.*, "Smart radio environments empowered by reconfigurable AI meta-surfaces: An idea whose time has come," *EURASIP J. Wireless Commun. Netw.*, vol. 2019, no. 129, pp. 1–20, May 2019.
- [18] J. Hu *et al.*, "Reconfigurable intelligent surface based RF sensing: Design, optimization, and implementation," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2700–2716, Nov. 2020.
- [19] A. Aubry, A. De Maio, and M. Rosamilia, "Reconfigurable intelligent surfaces for N-LOS radar surveillance," *IEEE Trans. Veh. Technol.*, vol. 70, no. 10, pp. 10735–10749, Oct. 2021.
- [20] S. Buzzi, E. Grossi, M. Lops, and L. Venturino, "Radar target detection aided by reconfigurable intelligent surfaces," *IEEE Signal Process. Lett.*, vol. 28, pp. 1315–1319, 2021.
- [21] S. Buzzi, E. Grossi, M. Lops, and L. Venturino, "Foundations of MIMO radar detection aided by reconfigurable intelligent surfaces," 2021, *arXiv:2105.09250*.
- [22] W. Lu, B. Deng, Q. Fang, X. Wen, and S. Peng, "Intelligent reflecting surface-enhanced target detection in MIMO radar," *IEEE Sensors Lett.*, vol. 5, no. 2, pp. 1–4, Feb. 2021.
- [23] B. Di, H. Zhang, L. Song, Y. Li, Z. Han, and H. V. Poor, "Hybrid beamforming for reconfigurable intelligent surface based multi-user communications: Achievable rates with limited discrete phase shifts," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 8, pp. 1809–1822, Aug. 2020.
- [24] M. A. Elmoosallamy, H. Zhang, L. Song, K. Seddik, Z. Han, and G. Y. Li, "Reconfigurable intelligent surfaces for wireless communications: Principles, challenges, and opportunities," *IEEE Trans. Cogn. Commun. Netw.*, vol. 6, no. 3, pp. 990–1002, Sep. 2020.
- [25] D. Parsons, *The Mobile Radio Propagation Channel*. New York, NY, USA: Wiley, 1994.
- [26] W. Tang *et al.*, "Wireless communications with reconfigurable intelligent surface: Path loss modeling and experimental measurement," *IEEE Trans. Wireless Commun.*, vol. 20, no. 1, pp. 421–439, Jan. 2020.
- [27] M. Skolnik, *Radar Handbook*. New York, NY, USA: McGraw-Hill, 2008.
- [28] M. I. Skolnik, *Introduction to Radar Systems*. New York, NY, USA: McGraw-Hill, 2001.
- [29] H. Zhang, H. Zhang, B. Di, K. Bian, Z. Han, and L. Song, "Towards ubiquitous positioning by leveraging reconfigurable intelligent surface," *IEEE Commun. Lett.*, vol. 25, no. 1, pp. 284–288, Jan. 2021.
- [30] F. Liu, C. Masouros, A. P. Petropulu, H. Griffiths, and L. Hanzo, "Joint radar and communication design: Applications, state-of-the-art, and the road ahead," *IEEE Trans. Commun.*, vol. 68, no. 6, pp. 3834–3862, Jun. 2020.
- [31] J. H. Reed *et al.*, "On the co-existence of TD-LTE and radar over 3.5 GHz band: An experimental study," *IEEE Wireless Commun. Lett.*, vol. 5, no. 4, pp. 368–371, Aug. 2016.
- [32] B. C. Levy, *Principles of Signal Detection and Parameter Estimation*. New York, NY, USA: Springer, 2008.
- [33] L. Zhang *et al.*, "A wireless communication scheme based on space- and frequency-division multiplexing using digital metasurfaces," *Nature Electron.*, vol. 4, no. 3, pp. 218–227, Mar. 2021.
- [34] W. Tang *et al.*, "MIMO transmission through reconfigurable intelligent surface: System design, analysis, and implementation," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2683–2699, Nov. 2020.
- [35] H. Zhang, B. Di, L. Song, and Z. Han, *Reconfigurable Intelligent Surface-Empowered 6G*. New York, NY, USA: Springer, 2021.
- [36] J. Li, L. Xu, P. Stoica, K. W. Forsythe, and D. W. Bliss, "Range compression and waveform optimization for MIMO radar: A Cramér-Rao bound based study," *IEEE Trans. Signal Process.*, vol. 56, no. 1, pp. 218–232, Jan. 2008.
- [37] H. Zhang *et al.*, "MetaRadar: Indoor localization by reconfigurable meta-materials," *IEEE Trans. Mobile Comput.*, early access, Dec. 14, 2020, doi: [10.1109/TMC.2020.3044603](https://doi.org/10.1109/TMC.2020.3044603).
- [38] N. A. Goodman, P. R. Venkata, and M. A. Neifeld, "Adaptive waveform design and sequential hypothesis testing for target recognition with active sensors," *IEEE J. Sel. Topics Signal Process.*, vol. 1, no. 1, pp. 105–113, Jun. 2007.
- [39] X. Liu, T. Huang, N. Shlezinger, Y. Liu, J. Zhou, and Y. C. Eldar, "Joint transmit beamforming for multiuser MIMO communications and MIMO radar," *IEEE Trans. Signal Process.*, vol. 68, pp. 3929–3944, 2020.

- [40] Z.-Q. Luo, W.-K. Ma, A. M.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.
- [41] Y. Ye, *Interior Point Algorithms: Theory and Analysis* (Wiley-Interscience Series in Discrete Mathematics and Optimization). New York, NY, USA: Wiley, 1997.
- [42] H. Zhang, H. Zhang, B. Di, K. Bian, Z. Han, and L. Song, "MetaLocalization: Reconfigurable intelligent surface aided multi-user wireless indoor localization," *IEEE Trans. Wireless Commun.*, vol. 20, no. 12, pp. 7743–7757, Dec. 2021.
- [43] R. Deng *et al.*, "Reconfigurable holographic surfaces for future wireless communications," *IEEE Wireless Commun.*, vol. 28, no. 6, pp. 126–131, Dec. 2021.



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