Global Sensitivity Analysis based Design of Input Shapers

Adrian Stein, * **Tarunraj Singh** *

* *Department of Mechanical and Aerospace Engineering, University at Buffalo (SUNY), Buffalo, NY 14260-4400, USA*

Abstract: This paper focuses on the development of a global sensitivity based design of robust input shapers. The Derivative-based Global Sensitivity Measure (DGSM) which consists of the expected value of the absolute value of the gradient of the terminal residual energy with respect to the uncertain parameters, is used to tradeoff performance to robustness. A multi-objective cost function which is a convex combination of the terminal residual energy for rest-to-rest maneuvers and the DGSM, is used to design robust input shapers. The proposed approach is illustrated on a single spring-mass-dashpot system and the benchmark floating oscillator. As compared to the traditional Zero Vibration Derivative Input Shaper, the proposed approach is demonstrated to reduce the average residual energy over the domain of uncertainty.

Keywords: Global Sensitivity Analysis, Input Shaper, Controller Design

1. INTRODUCTION

Precision motion control in applications such as wafer scanners, atomic force microscopes, hard disk drives, cranes etc. are demanding higher speed of operation and smaller residual vibrations to increase productivity. Increasing speed can be achieved with light weight structures, which are then burdened with low frequency vibrations which is deleterious to the desirable functioning of the device. For applications where there are significant uncertainties in the model parameters, it is imperative that the issue of robustness be addressed in the design of controllers. A popular approach for precision motion control of vibratory systems is by shaping the command input using a technique which is referred to as Input Shaping (Singer and Seering (1990)). The survey papers (Singh and Singhose (2002)) and (Singh and Vyhlídal (2020)) provide an overview of the early development and the latest developments in Input Shaping, respectively. There is a large body of research addressing the questions of robustness by forcing the state sensitivities to the uncertain parameters to be zero at the terminal time of the maneuver (Liu and Singh (1997)). The sensitivities are typically evaluated at the nominal model parameters and are only guaranteed to provide locally robust controllers. To address the issues of robustness of the controller over a support of the uncertainty, a minimax problem formulation has been addressed in the past (Singhose et al. (1996), Singh (2002)).

Global sensitivity analysis (GSA) refers to approaches to analyze the impact of uncertain variables on outputs of interest over the entire domain of uncertainty (Saltelli (2008)). These metrics attempt to apportion uncertainties in the output to individual uncertain variables and as a consequence, provide a rank order of the impact of the uncertain variables on the output of interest. There are numerous global sensitivity metrics including the Sobol indices (Sobol' (1990)) which are variance based measures, derivative based metrics which are relative to the Morris method and the non-moment based metrics such as the δ metrics (Borgonovo (2007), Nandi and Singh (2021)).

In this paper, we propose the use of global sensitivity measures, specifically the Derivative-based Global Sensitivity Measure (DGSM) (Kucherenko and Song (2016)) which is the expected value of the absolute value of the gradient of the output to the uncertain parameters. Since this paper deals with precision motion control of vibratory systems, the output of interest is the residual energy at the terminal time. A multi-objective cost function which weights the residual energy against the DGSM is used to arrive at a robust Input Shaper.

2. METHOD

Fig. 1 illustrates a single mass system, where the mass is connected to a rigid wall with a spring of stiffness k and a damper with a damping coefficient c. The equations of

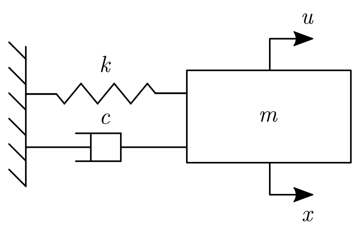

Fig. 1. Single mass-spring-damper model where the input u is controlling the position of the first mass x_1.

motion can be written as:

$$\ddot{x} = -\frac{k}{m}x - \frac{c}{m}\dot{x} + \frac{u}{m} \tag{1}$$

⋆ This material is based upon work supported through National Science Foundation (NSF) under Awards No. CMMI-2021710.

The residual energy for a rest-to-rest maneuver, is:

$$V_1 = T_1 + U_1 = \frac{m}{2}\dot{x}_f^2 + \frac{k}{2}(x_f - 1)^2 \quad (2)$$

where T_1 and U_1 are the kinetic and potential energy respectively. Note that the final desired position is 1 and therefore just point-to-point maneuver are considered in this paper. The subscript "f" marks the state at the final time t_f. The initial conditions are: $x(0) = \dot{x}(0) = 0$.

2.1 Closed form Time Delay Filter

Considering the nominal model with $k = 1$ and $c = 0.1$, an input shaped could be used to shape the reference as illustrated in Fig. 2, where $r(t) = 1$, the unit step, throughout the paper. In this paper a time delay filter (TDF) is used as an input shaper with a transfer function:

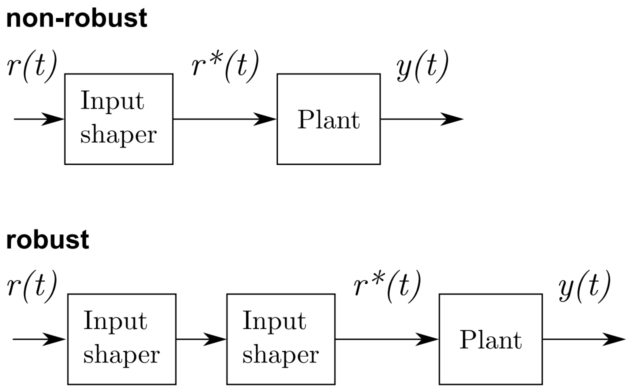

Fig. 2. Block diagram of a nonrobust and robust input shaper.

$$G(s) = A_0 + A_1 e^{-s\tau}, \quad (3)$$

where the gains A_0 and A_1 can be calculated in closed form (Singer and Seering (1990), Singh (2010)):

$$A_0 = \frac{e^{\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}}{e^{\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} + 1} \quad (4)$$

$$A_1 = \frac{1}{e^{\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} + 1} \quad (5)$$

and τ is delay time and can be calculated as (Singh (2010)):

$$\tau = \frac{\pi}{\omega\sqrt{1-\zeta^2}} \quad (6)$$

It should be noted that we consider a damped system and therefore $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$, where ω_n and ζ are the natural frequency and the damping ratio ($c = 2\zeta\omega_n$). The robust input shaper, which consists of a cascade of two time-delay filters, is chosen as a benchmark to compare the performance of the proposed controller.

2.2 Sensitivity based design

The aim here is to reduce the sensitivity of the residual energy with respect to the spring stiffness k and damping coefficient c. Therefore, the sensitivities dV_1/dk and dV_1/dc are included in the controller design. Taking the derivative of Eq. (1) with respect to k and c results in:

$$\frac{d\ddot{x}}{dk} = -\frac{1}{m}x - \frac{k}{m}\frac{dx}{dk} - \frac{c}{m}\frac{d\dot{x}}{dk} \quad (7)$$

$$\frac{d\ddot{x}}{dc} = -\frac{k}{m}\frac{dx}{dc} - \frac{1}{m}\dot{x} - \frac{c}{m}\frac{d\dot{x}}{dc}. \quad (8)$$

To trade-off performance for robustness, a cost function F can be written as a combination of the residual energy V and the sensitivities with respect to k and c,

$$F = \alpha(T_1 + U_1) + \frac{(1-\alpha)}{10}\left(\left|\frac{dV_1}{dk}\right| + \left|\frac{dV_1}{dc}\right|\right) \quad (9)$$

dV_1/dk and dV_1/dc can be calculated from Eq. (2) and a scaling factor of 1/10 is used to weight the cost appropriately. α is used as a weighing parameters to penalize the cost either on the residual energy ($\alpha \to 1$) or on the sensitivities in k and c ($\alpha \to 0$). For simplicity we further set $m = 1$ and Eq. (9) becomes:

$$F = \alpha\left(\frac{\dot{x}(t_f)^2}{2} + \frac{k}{2}(x(t_f) - 1)^2\right) + (1-\alpha)\ldots$$

$$\ldots\left(\left|\dot{x}(t_f)\frac{d\dot{x}(t_f)}{dk} + \frac{(x(t_f)-1)^2}{2} + k(x(t_f)-1)\frac{dx(t_f)}{dk}\right|\ldots\right.$$

$$\left.\ldots + \left|\dot{x}(t_f)\frac{d\dot{x}(t_f)}{dc} + k(x(t_f)-1)\frac{dx(t_f)}{dc}\right|\right) \quad (10)$$

A gradient-based method finds a control input u for a point-to-point maneuver where the final time t_f is prescribed by the user and is assumed to be 2τ to provide a fair comparison to the TDF. Uncertainties in k and c are assumed to be uniformly distributed and represented as:

$$k = 1 + \tilde{k} \quad (11)$$

$$c = 0.1 + \tilde{c} \quad (12)$$

where $\tilde{k}$ and $\tilde{c}$ range from -0.3 to 0.3, and -0.03 to 0.03 respectively. The cost for the optimizer is the Expected value of the residual energy over the 2D uncertain domain and is represented as:

$$f = E[F(k, c, \alpha)]. \quad (13)$$

2.3 Floating oscillator

To illustrate the GSA on a multimode system a floating oscillator with a PD controller is chosen which is illustrated in Fig. 3. The equations of motion are:

$$m_1\ddot{x}_1 - k(x_2 - x_1) - c(\dot{x}_2 - \dot{x}_1) = u \quad (14)$$

$$m_2\ddot{x}_2 + k(x_2 - x_1) + c(\dot{x}_2 - \dot{x}_1) = 0 \quad (15)$$

where m_1 and m_2 are the first and second mass respectively. A PD-controller

$$u = -k_p(x_1 - r^*) - k_d\dot{x}_1, \quad (16)$$

is applied on the position error of the first mass x_1, making the closed loop system one with two underdamped modes.

The residual energy of the system is:

$$V_2 = T_2 + U_2 = \frac{m_1\dot{x}_{1f}^2 + m_2\dot{x}_{2f}^2}{2}\ldots$$

$$\ldots + \frac{1}{2}\left[x_{1f} - r^*, x_{2f} - r^*\right]\begin{bmatrix} k + k_p & -k \\ -k & k \end{bmatrix}\begin{bmatrix} x_{1f} - r^* \\ x_{2f} - r^* \end{bmatrix}$$

where the cost function remains the same as in Eq.(9), and k and c are uncertain as prescribed in Eq. (11) and

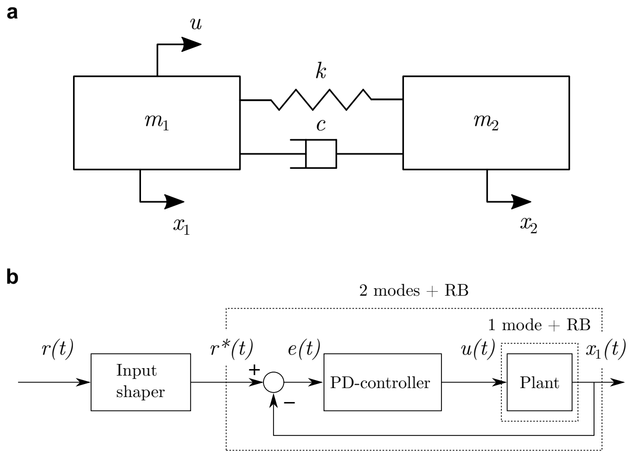

Fig. 3. a) Floating oscillator with a PD controller where the input u is controlling the position x_1 of the first mass. b) Block diagram of the model, showing the location of the input shaper.

(12). Further we set $k_p = k_d = 1$ and $r^* = 1$. Robustness (2 cascaded TDFs per mode) is applied to both modes resulting in a cascade of two time-delay filter with three delays each.

3. RESULTS

A Pareto front is established by sweeping α from 1 to 0. For each α, MATLAB add-on SNOPT7 (Gill et al. (2005)) is used to determine the shaped reference profile. The optimization process gets initialized with $\alpha = 1$ and then progressively reduced to $\alpha = 0$, where the initial guess for the following α sample is the solution of the previous one.

3.1 Single mass system

For the optimization process the final time for the GSA is $t_f = 6.2911$. Fig. 4 illustrates the position x and velocity $\dot{x}$ of the mass over time. Each graph represents a different combination for k and c when $\alpha = 1$. It can be clearly seen that the controller based on the GSA design is outperforming the TDF in terms of the final position ($x_f = 1$) whereas both perform about the same for the final velocity $\dot{x}_f$. The different control structure for a GSA based design compared to a TDF can be seen in Fig. 5. The input for the GSA is restricted to 5 samples and for simplicity just the two extreme cases $\alpha = 1$ and $\alpha = 0$ are shown. It is obvious that the control input is less aggressive for $\alpha = 0$ than for $\alpha = 1$. Note that $\alpha = 0$ represents the most desensitized cost function. The residual energy can be illustrated as a surface plot over the 2D uncertain domain of k and c as it can be seen in Fig. 6. Setting $\alpha = 1$, the cost function purely minimizes the mean of the expected value of the residual energy over the uncertain space. The GSA design (blue) shows on average a better performance than the robust TDF (red) especially in the regions of most uncertainty. Just around the nominal stage and along its uncertainty in c, the robust TDF outperforms the GSA which is the tradeoff for the advantage of the GSA. The main goal of the GSA is to desensitize the residual energy for uncertainties in k and c. This can be best expressed in terms of variance of the residual energy

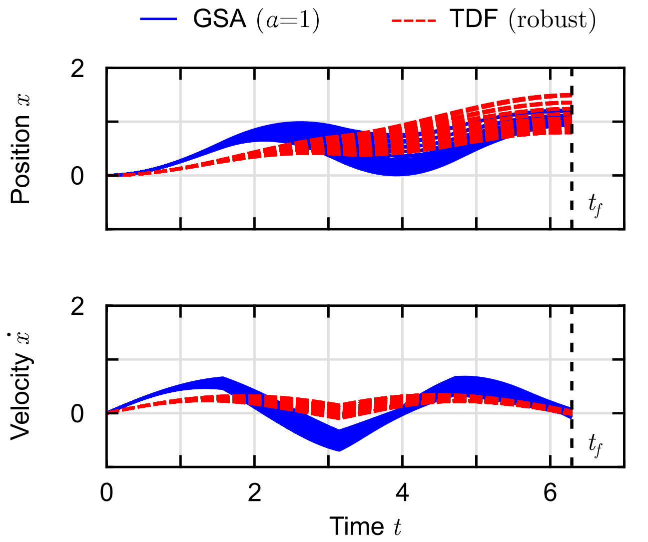

Fig. 4. Position x_1 and velocity $\dot{x}_1$ for $k = [0.7,\ 1.3]$ and $c = [0.07,\ 0.13]$. The solid blue and dashed red graphs refer to the global sensitivity based design ($\alpha = 1$) and a robust time delay filter respectively. tf marks the final time of the maneuver.

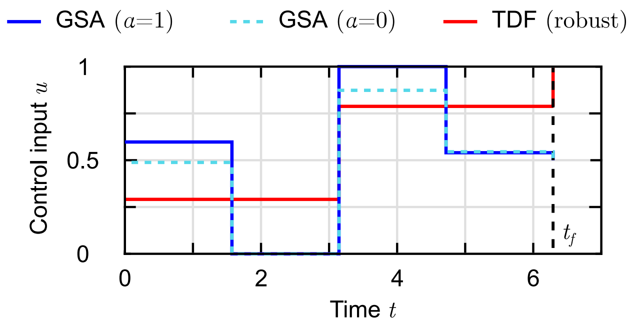

Fig. 5. Control input for a global sensitivity based design and robust time delay filter. The solid blue, dashed teal and solid red line represent the global sensitivity based design for $\alpha = 1$, $\alpha = 0$ and robust time delay filter respectively.

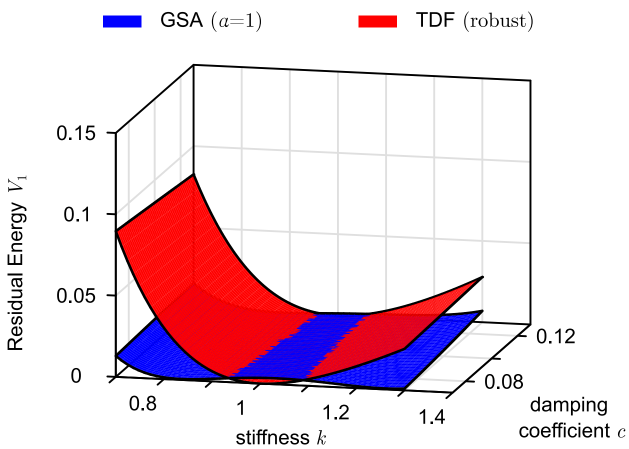

Fig. 6. Residual energy of a single mass-spring-damper system over the uncertain spring stiffness and damping constant. The blue and red surface show the residual energy referring to the global sensitivity ($\alpha = 1$) based and the robust time delay filter design respectively.

over the uncertainties. Shifting α towards 0 increases the penalty on the sensitivity part of the cost function relative to the residual energy. Fig. 7 illustrates two histograms of the residual energy, when $\alpha = 1$ where the cost function is purely the mean of the residual energy and $\alpha = 0$ where the cost depends solely on the sensitivities of the residual energy with respect to k and c. As expected for $\alpha = 1$ the mean is low but the variance is high, whereas for $\alpha = 0$ the mean is higher than for $\alpha = 1$ but the variance is smaller. This observation matches the expectations when analyzing Eq. (9). One could imagine that the blue surface of Fig. 6 gets lifted up but flattened out when $\alpha = 0$ representing a higher mean but less variance. Another way of illustrating Eq. (9) is the so called Pareto frontier. From there one could pick the optimal tradeoff point (knee point) when weighing the mean of the residual energy compared to the sensitivities with respect to k and c. This is shown in Fig. 8. Besides the histograms, Fig. 9 presents the variation of the expected value of the residual energy and its variance as a function of α. From Fig. 9a) it can be seen that increasing α increases the penalty on the mean of the residual energy, reducing the mean residual energy, but increases the variance of the residual energy. Fig. 9b) illustrates the variance of the residual energy over α.

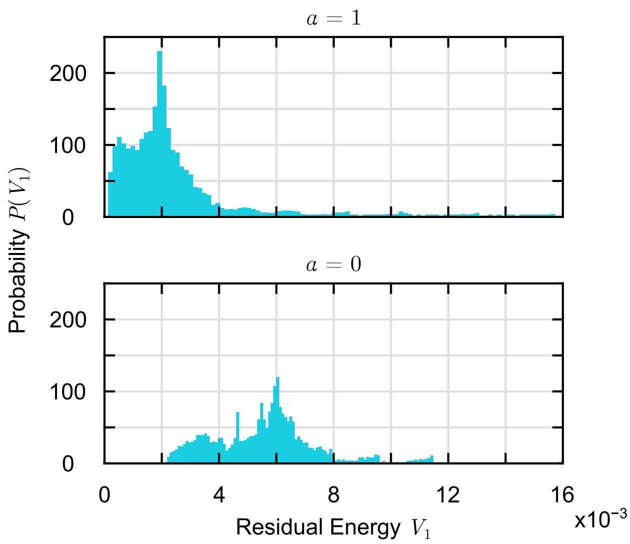

Fig. 7. Histogram of the residual energy of a single Mass-Spring-Damper system over uncertain spring stiffness and damping constant when a global sensitivity based controller design is applied for choosing $\alpha = 1$ and $\alpha = 0$.

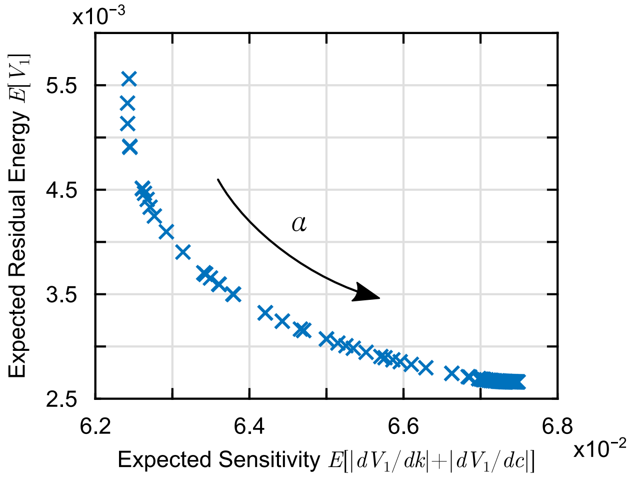

Fig. 8. Pareto frontier of the expected values of the residual energy and the summation of the absolute sensitivities with respect to the spring stiffness and damping constant when parameter α is changed from 0 to 1 in 100 samples.

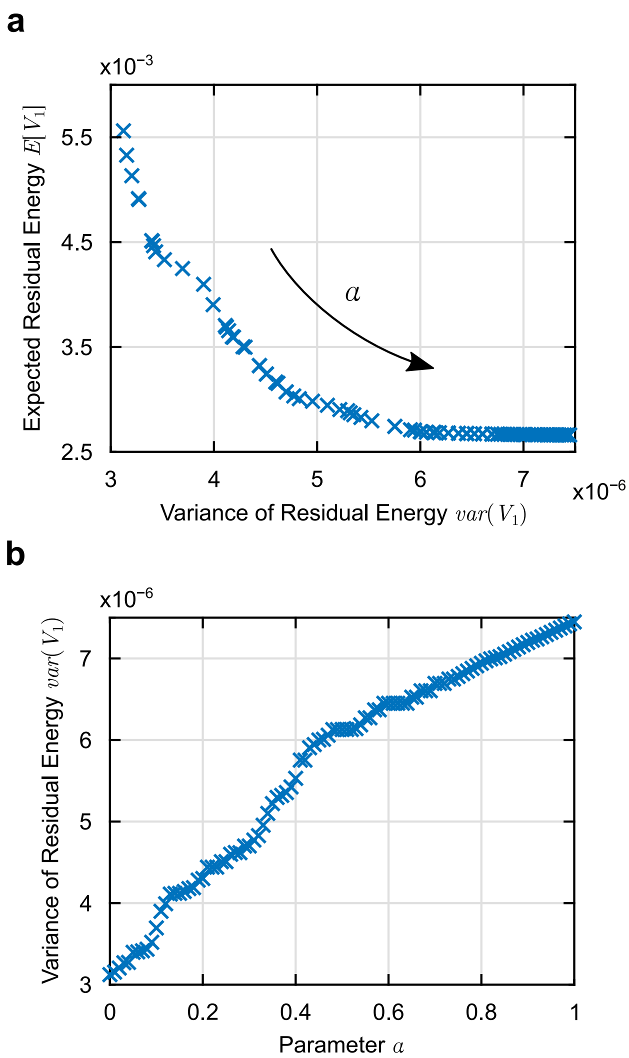

Fig. 9. a) Expected value over variance of the residual energy when parameter α is varied from 0 to 1 in 100 samples.

3.2 Floating oscillator

For the floating oscillator, the maneuver time t_f is 14.2672. Here the GSA controller is discretized with 15 samples. The trajectories for the positions x_1 and x_2 and the velocities $\dot{x}_1$ and $\dot{x}_2$ for a floating oscillator can be seen in Fig. 10. Picking $\alpha = 1$, it should be noted that the GSA performs as well as the robust TDF and both methods seem to fulfill the final goal $x_1 \approx 1$ and $\dot{x}_{1f} \approx 0$ over multiple realizations over the uncertain domain.

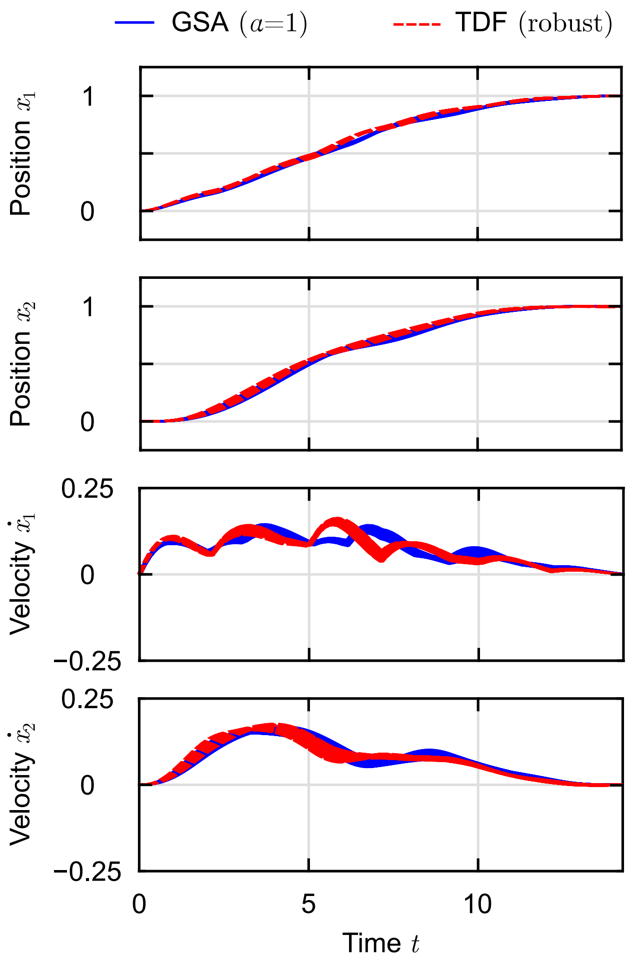

Fig. 10. Position x_1 and x_2, and velocity $\dot{x}_1$ and $\dot{x}_2$ for $k = [0.7, 1.3]$ and $c = [0.07, 0.13]$. The solid blue and dashed red graphs refer to the global sensitivity based design ($\alpha = 1$) and a robust time delay filter respectively.

A closer look at the residual energy in Fig. 11 reveals that the GSA for $\alpha = 1$ performs on average better than the robust TDF. Only around the nominal stage where $k = 1$ and $c = 0.01$ and along the uncertainty in c the robust TDF slightly outperforms the GSA based design. It is clearly visible that the uncertainty in the damping coefficient c doesn't impact the residual energy as much as the spring stiffness k.

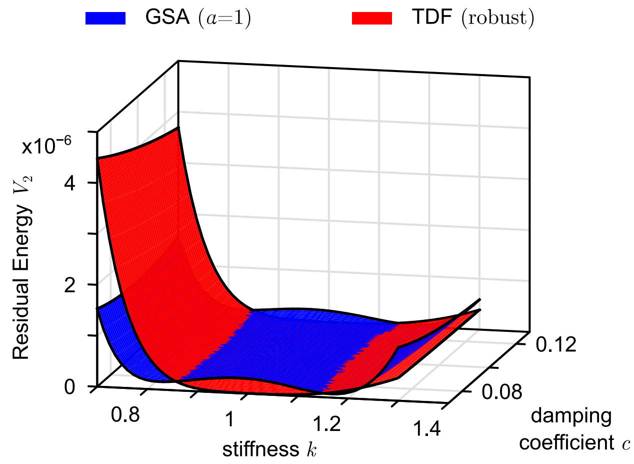

Fig. 11. Residual energy of a double mass-spring-damper system over the uncertain spring stiffness and damping constant. The blue and red surface show the residual energy referring to the global sensitivity ($\alpha = 1$) based and the robust time delay filter design respectively.

A closer look at the histograms (see Fig. 12) reveals the same observation as for the single mass, although in a clearer form. If $\alpha = 1$, the mean of the residual energy is small but the variance is large. Setting $\alpha = 0$ increases the mean compared to the case of $\alpha = 1$ but the variance is much smaller.

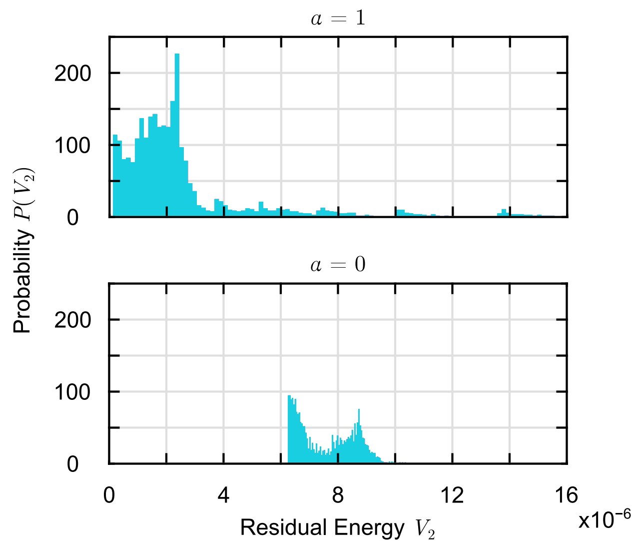

Fig. 12. Histogram of the residual energy of the floating oscillator system over uncertain spring stiffness and damping constant when a global sensitivity based controller design is applied for choosing $\alpha = 1$ and $\alpha = 0$.

4. CONCLUSION

In this paper a global sensitivity based controller design is proposed. The newly developed method is compared to the established robust time delay filter technique. Both methods are studied on a single spring mass system and a floating oscillator. The main idea is to reduce the residual energy of the system at the final time when uncertainties in the spring stiffness and the damping coefficient exist. To provide a fair comparison, the maneuver time for the global sensitivity based design is identical to the robust time delay filter. The cost function for the optimization process depends on the mean of the residual energy and its derivatives with respect to the spring stiffness and the damping coefficient. A weighing parameter embedded in the cost function assigns the focus on minimizing the mean of the residual energy or on desensitizing the residual energy with respect to the model uncertainties.

It should be noted that in both tested models, the uncertainty in the damping coefficient doesn't play a significant role compared to the spring stiffness. On average the global sensitivity based design outperforms the established time delay filter technique except around the nominal stage. However, it should be mentioned that the character of sensitivity based controller design lies on the desensitization to uncertainties, meaning that a lower variance in the residual energy comes automatically with a higher mean.

The current formulation of the optimization problem results in a nonlinear programming problem which is burdened with all the challenges associated with solving nonconvex problems. Convex problem formulation and demonstrating it experimentally are planned for the near future.

ACKNOWLEDGEMENTS

The authors acknowledge the support of this work by the US National Science Foundation through CMMI Award number 2021710.

REFERENCES

Borgonovo, E. (2007). A new uncertainty importance measure. *Reliability Engineering & System Safety*, 92(6), 771–784. doi:10.1016/j.ress.2006.04.015.

Gill, P.E., Murray, W., and Saunders, M.A. (2005). Snopt: An sqp algorithm for large-scale constrained optimization. *SIAM Review*, 47(1), 99–131. doi:10.1137/S0036144504446096.

Kucherenko, S. and Song, S. (2016). Derivative-based global sensitivity measures and their link with sobol sensitivity indices. *(None)*, 163(1), 455–469. doi:10.1007/978-3-319-33507-0{\textunderscore}23. URL http://arxiv.org/pdf/1605.07830v1.

Liu, S.W. and Singh, T. (1997). Robust time-optimal control of nonlinear structures with parameter uncertainties. *ASME J. of Dynamic Systems, Measurement and Control*, 119(4), 743–748.

Nandi, S. and Singh, T. (2021). Global sensitivity analysis measures based on statistical distances. *International Journal for Uncertainty Quantification*, 11(6), 1–30. doi:10.1615/Int.J.UncertaintyQuantification.2021035424.

Saltelli, A. (2008). *Global sensitivity analysis: The primer / Andrea Saltelli ... [et al.]*. John Wiley & Sons, Chichester. URL http://www.loc.gov/catdir/enhancements/fy0805/2007045551-d.html.

Singer, N.C. and Seering, W.P. (1990). Preshaping command inputs to reduce system vibration. *Journal of Dynamic Systems, Measurement, and Control*, 112(1), 76–82. doi:10.1115/1.2894142.

Singh, T. (2002). Minimax design of robust controllers for flexible systems. In *Proceedings of the 2002 American Control Conference (IEEE Cat. No.CH37301)*, 2510–2515 vol.3. IEEE. doi:10.1109/ACC.2002.1024021.

Singh, T. and Singhose, W. (2002). Input shaping/time delay control of maneuvering flexible structures. In *Proceedings of the 2002 American Control Conference (IEEE Cat. No.CH37301)*, 1717–1731 vol.3. IEEE. doi:10.1109/ACC.2002.1023813.

Singh, T. (2010). *Optimal reference shaping for dynamical systems: Theory and applications / Tarunraj Singh*. CRC Press, Boca Raton.

Singh, T. and Vyhlídal, T. (2020). Recent results in reference prefiltering for precision motion control. *IFAC-PapersOnLine*, 53(2), 8656–8667. doi:10.1016/j.ifacol.2020.12.315.

Singhose, W., Derezinski, S., and Singer, N. (1996). Extra-insensitive input shapers for controlling flexible spacecraft. *Journal of Guidance, Control, and Dynamics*, 19(2), 385–391. doi:10.2514/3.21630.

Sobol', I.M. (1990). On sensitivity estimation for nonlinear mathematical models. *Matematicheskoe modelirovanie*, 2(1), 112–118.