Response to "Comments on the paper by B.E. Grossman-Ponemon, L.M. Keer, and A.J. Lew 'A method to compute mixed-mode stress intensity factors for nonplanar cracks in three dimensions' (Int. J. Numer. Methods Eng., 2020)"

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C.A. Duarte commented on our paper Grossman-Ponemon et al. [1], arguing that our work misrepresents and leads to incorrect conclusions about his manuscript, Gupta et al. [2]. The source of the argument is the use of the name "Gupta, Duarte, and Dhankar's displacement correlation method" or "GDD-DCM" to refer to the particular version of the Displacement Correlation Method (DCM) that we implemented in order to have a reference of performance against the method introduced by us. Specifically, the argument states that the DCM is a standard method in the literature, and that in adopting the name "GDD-DCM" to denote the results in our work we are leading the reader to the incorrect conclusion that the method with G/XFEM in [2] does not provide convergent values for the stress intensity factors. This is incorrect for the following reasons:

1. DCM is a label to denote a family of methods based on a common idea, but methods in that family present differences among them. In our work, we implemented a particular version of DCM introduced by C.A. Duarte and collaborators in [2]. A novel aspect of this version is an averaging scheme introduced and described in §3.5.1 of their work which does not appear in any other earlier proposed DCM method. As such, it is an original contribution by C.A. Duarte and collaborators. In labeling our implementation "GDD-DCM" we are specifically referring to the method introduced in such reference. For concreteness, the computation of stress intensity factors with Eq. (32) in [2] corresponds to Eq. (3.10) in [1], and it is a unique feature of the DCM introduced by C.A. Duarte and collaborators in [2].

2. In the letter authored by C.A. Duarte, the following statement is incorrect: *"The DCM formulation adopted by Gupta et al. [2] is the same one used much earlier by several authors, as reviewed in [2]."* An exhaustive review of all the references cited in [2] (and beyond) shows that the version of DCM introduced in [2], namely the averaging scheme, is novel to that manuscript. C.A. Duarte and collaborators implicitly acknowledge this in the introduction of [2], in which they state "*Strategies to handle non-planar crack surfaces with DCM and to improve the accuracy of the method are also presented*," which they do in §3.5, whose title is *"Strategies to improve the Displacement Correlation Method,"* and which contains the aforementioned averaging scheme.

3. At no point in [1] do we make any statement about the convergence or lack thereof of the stress intensity factors computed with the DCM in combination with displacement fields computed with G/XFEM. In other words, we never state that the method that results from the combination of GDD-DCM and G/XFEM by C.A Duarte and collaborators in [2] does not converge[1].

In view of these facts, it is evident that referring to the DCM we implemented solely as "DCM" would have been imprecise, and that there was no misrepresentation of the results in [2].

[1]This fact contradicts the assertion in C.A. Duarte's letter that says: *"Grossman-Ponemon et al. state (at the end of page 14) the following about GDD-DCM results for the semi-infinite straight crack: 'Referring to Figure 8 and in contrast to GM-II, we did not observe convergence of the GDD-DCM in any of error metrics and for any of the mesh families'. This is an incorrect statement about the work by Gupta et al."* Also, we disagree with the statement in the letter by C.A. Duarte that states that it is incorrect to separate the DCM and G/XFEM in [2]: *"It is incorrect to ignore this recommendation while calling the DCM as the "Gupta, Duarte, and Dhankhar's displacement correlation method (GDD-DCM)" and then state that the method is not convergent."*

Arguing, as the letter by C.A. Duarte does, that the label "GDD-DCM" leads the reader to incorrect conclusions about [2] fails to acknowledge the clear distinction we make of the DCM and G/XFEM methods in [2] in our manuscript. In the introduction of our manuscript we state, *"More recently, Gupta et al. applied the Displacement Correlation Method to numerical solutions found using the Generalized/Extended Finite Element Method (XFEM). Here, the enrichment of the FEM function spaces with the analytical asymptotic behavior of the displacement field yields more accurate displacements in the vicinity of the crack front."*, acknowledging that DCM and G/XFEM are separate methods. Later in §3.1 we refer to GDD-DCM as *"the displacement correlation method of Gupta, Duarte, and Dhankhar,"* specifically acknowledging the contribution to the DCM family in [2]. We believe that a reader of our manuscript is able to distinguish between the DCM and the G/XFEM method in [2].

Finally, and for the record, prior to the submission of our manuscript, on January 9, 2019 we shared with C.A. Duarte the numerical results that comprise Fig. 6 in [1]; our correspondence was concluded on January 13, 2019, well before the manuscript was submitted for review on August 14, 2019. In our communication, we clearly explained that we were using the DCM in [2] with FEM. Further, we explicitly asked whether the results we obtained were reasonable in view of his experience with GDD-DCM as a way to certify that no mistake was made on our side with respect to our DCM calculations.

References

[1] Benjamin E. Grossman-Ponemon, Leon M. Keer, and Adrian J. Lew. A method to compute mixed-mode stress intensity factors for nonplanar cracks in three dimensions. *International Journal for Numerical Methods in Engineering*, 121(19):4292–4328, 2020.

[2] P. Gupta, C.A. Duarte, and A. Dhankhar. Accuracy and robustness of stress intensity factor extraction methods for the generalized/extended finite element method. *Engineering Fracture Mechanics*, 179:120–153, 2017.