Wilson Loops at Large N and the Quantum M2-Brane

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The Wilson loop operator in the $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ Aharony-Bergman-Jafferis-Maldacena theory at large N and fixed level k has a dual description in terms of a wrapped M2-brane in the M-theory given by the product of four-dimensional anti de Sitter space (AdS_4) and $S^7/\mathbb{Z}_k$. We consider the localization result for the $\frac{1}{2}$-Bogomol'nyi-Prasad-Sommerfield circular Wilson loop expectation value W in this regime and compare it to the prediction of the M2-brane theory. The leading large N exponential factor is matched as expected by the classical action of the M2-brane solution with $\mathrm{AdS}_2 \times S^1$ geometry. We show that the subleading k-dependent prefactor in W is also exactly reproduced by the one-loop term in the partition function of the wrapped M2-brane (with all Kaluza-Klein modes included). This appears to be the first case of an exact matching of the overall numerical prefactor in the Wilson loop expectation value against the dual holographic result. It provides an example of a consistent quantum M2-brane computation, suggesting various generalizations.

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The existence of a consistent quantum supermembrane (or M2-brane) theory remains an enigma (see, e.g., [1,2]). The corresponding 3D world-volume theory is formally nonrenormalizable, apparently requiring a specific definition like a built-in cutoff. Nevertheless, some simple semiclassical computations to one-loop order can still be done in a straightforward way, as one-loop corrections in 3D field theory are free of logarithmic UV divergences, see, e.g., [3–6] or more recent work in [7].

In this Letter, we present a nontrivial example of one-loop calculation in the M2-brane theory, which provides further evidence that the quantization of the supermembrane might be under good control, at least within the semiclassical expansion.

The $\mathrm{AdS}_4/\mathrm{CFT}_3$ duality between the $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ Aharony-Bergman-Jafferis-Maldacena (ABJM) theory [8] and M theory on the direct product of four-dimensional anti de Sitter space (AdS_4) and $S^7/\mathbb{Z}_k$ provides a remarkable opportunity to shed light on the properties of the quantum M2-brane theory by testing its predictions against exact results in 3D superconformal gauge theory. In the large N limit with k fixed, the holographic dual of a Wilson loop in the fundamental representation is expected to be an M2-brane wrapping the M-theory circle direction. Note that this limit is different from the standard large N 't Hooft limit, where N and k are taken to be large with $\lambda = N/k$ fixed, and in which Wilson loops are described by fundamental strings in type IIA string theory in $\mathrm{AdS}_4 \times \mathbb{CP}^3$.

For fixed k, the large N expansion of the Wilson loop operator in the ABJM theory corresponds to the expansion in the large effective M2-brane tension $R^3 T_2 \sim \sqrt{Nk}$, where R is the curvature radius of $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ and $T_2 = (1/(2\pi)^2 \ell_{\mathrm{Pl}}^3)$.

Our starting point will be an analytic expression for the expectation value of $\frac{1}{2}$-supersymmetric circular Wilson loop in the ABJM theory derived using supersymmetric localization in [9] (see also [10–17]),

$$\langle W_{\frac{1}{2}} \rangle = \frac{1}{2\sin(\frac{2\pi}{k})} \frac{\mathrm{Ai}\left[C^{-\frac{1}{3}}\left(N - \frac{k}{24} - \frac{7}{3k}\right)\right]}{\mathrm{Ai}\left[C^{-\frac{1}{3}}\left(N - \frac{k}{24} - \frac{1}{3k}\right)\right]}, \tag{1}$$

where $\mathrm{Ai}(z)$ is the Airy function, and $C = 2/(\pi^2 k)$. This expression resums all of the perturbative $1/N$ corrections at fixed k [19].

In order to compare to the semiclassical expansion in the M2-brane world-volume theory, one is to expand (1) at large N with fixed k, which gives

$$\langle W_{\frac{1}{2}} \rangle = \frac{e^{\pi\sqrt{\frac{2N}{k}}}}{2\sin\frac{2\pi}{k}} \left[1 - \frac{\pi(k^2+32)}{24\sqrt{2}k^{3/2}} \frac{1}{\sqrt{N}} + O\left(\frac{1}{N}\right)\right]. \tag{2}$$

As we will show in the section below, the exponential factor in (2) is reproduced by the classical action of the M2-brane with $\mathrm{AdS}_2 \times S^1$ world volume, while the k-dependent prefactor $(2\sin(2\pi/k))^{-1}$ is matched precisely by the one-loop correction coming from the functional determinants of the quantum fluctuations around this M2-brane solution.

Higher-order $1/(\sqrt{N})^n$ terms in (2) are expected to represent higher-loop corrections in the semiclassical expansion of the partition function of the quantum M2-brane theory, and checking this is a very interesting but challenging future problem.

$\mathrm{AdS}_2 \times S^1$ M2-brane in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$.—The $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ metric is given by ($\varphi \equiv \varphi + 2\pi$) [20],

$$ds^2 = \frac{R^2}{4} ds^2_{\mathrm{AdS}_4} + R^2 ds^2_{S^7/\mathbb{Z}_k}, \tag{3}$$

$$ds^2_{\mathrm{AdS}_4} = \frac{1}{z^2}(-dt^2 + dz^2 + dx_1^2 + dx_2^2), \tag{4}$$

$$ds^2_{S^7/\mathbb{Z}_k} = ds^2_{CP^3} + \frac{1}{k^2}(d\varphi + kA)^2. \tag{5}$$

The 11D supergravity background also includes the 4-form field strength

$$F_4 = dC_3 = -\frac{3}{8}\frac{R^3}{z^4} dt \wedge dx_1 \wedge dx_2 \wedge dz. \tag{6}$$

The radius R in units of the 11D Planck length ℓ_{Pl} is related to the parameters N and k of the dual ABJM gauge theory by

$$\left(\frac{R}{\ell_{\mathrm{Pl}}}\right)^6 = 2^5 \pi^2 N k. \tag{7}$$

The world-volume action for a probe M2-brane in this background is given by [21–23]

$$S_{\mathrm{M2}} = T_2 \int d^3\sigma \sqrt{-\det g} + T_2 \int C_3 + \text{fermionic terms}, \tag{8}$$

where the M2-brane tension is

$$T_2 = \frac{1}{(2\pi)^2} \frac{1}{\ell^3_{\mathrm{Pl}}}. \tag{9}$$

Classical M2-brane solution.—The action (8) admits a simple classical solution given by the M2-brane wrapping the M-theory circle direction [the φ angle in (3)] and occupying the AdS_2 subspace of AdS_4 spanned by the coordinates t, z in (4). The resulting membrane has the $\mathrm{AdS}_2 \times S^1$ world-volume geometry and is dual to the $\frac{1}{2}$-BPS Wilson loop along the t direction at the boundary of AdS_4. By an appropriate Wick rotation and coordinate transformation, one may obtain in the same way the solution dual to the circular Wilson loop, for which the AdS_2 factor is just the Euclidean hyperbolic disk with circular boundary.

The value of the classical action (8) for this $\mathrm{AdS}_2 \times S^1$ solution is simply given by

$$S^{\mathrm{cl}}_{\mathrm{M2}} = T_2 R^3 \frac{1}{4} \mathrm{vol}(\mathrm{AdS}_2) \frac{2\pi}{k}, \tag{10}$$

where $(1/4)$ comes from the AdS_4 radius in (3), and the $2\pi/k$ is the length of the M-theory φ circle in (5). Using (7) and (9), and the well-known value of the regularized volume of the unit-radius hyperbolic disk $\mathrm{vol}(\mathrm{AdS}_2) = -2\pi$, this gives (we always assume $k > 0$)

$$S^{\mathrm{cl}}_{\mathrm{M2}} = -\pi\sqrt{\frac{2N}{k}}. \tag{11}$$

Thus, $e^{-S^{\mathrm{cl}}_{\mathrm{M2}}}$ precisely matches the exponential in the localization prediction (2) [25].

In the next section, we will also compute the one-loop correction to the M2-brane partition function due to the quantum fluctuations about this classical solution and will reproduce precisely the prefactor in (2).

Let us note that, in the case of the $\frac{1}{2}$-BPS Wilson loop along the infinite straight line, one should use in (10) the regularized volume of AdS_2 in Poincaré coordinates, which is zero, thus getting $S^{\mathrm{cl}}_{\mathrm{M2}} = 0$, consistent with $\langle W_{\frac{1}{2}} \rangle = 1$ in this case. All quantum corrections also vanish here since the AdS_2 space is homogeneous and hence the quantum M2-brane free energy is proportional to $\mathrm{vol}(\mathrm{AdS}_2)$ to all orders [27].

One-loop correction.—Starting with the action (8), one may expand it near a classical solution to quadratic order fixing a 3D reparametrization and κ-symmetry gauge to get an action for $8+8$ physical 3D fluctuation fields. The resulting spectrum of the quantum fluctuations around the above $\mathrm{AdS}_2 \times S^1$ solution was obtained in Ref. [30], which we follow below (see also [31]).

It is natural to chose a static gauge identifying two membrane coordinates σ_1, σ_2 in (8) with the AdS_2 directions and the third σ_3 with the S^1 angle φ. After a Kaluza-Klein (Fourier) expansion of the 3D fields in the periodic coordinate σ_3, one obtains a tower of bosonic and fermionic fluctuations that can be viewed as 2D fields propagating on the (unit-radius) AdS_2 background. Thus, one gets an equivalent 2D theory with an infinite number of fields.

The bosonic fluctuations in the two transverse directions within AdS_4 give a tower of complex scalar fields η_n (two real scalars for each n) with masses

$$m^2_{\eta_n} = \frac{1}{4}(kn-2)(kn-4), \qquad n \in \mathbb{Z}, \tag{12}$$

while from the fluctuations in the six $\mathbb{CP}^3$ directions one finds a tower of three complex fields ζ^s_n ($s = 1, 2, 3$) with masses

$$m^2_{\zeta^s_n} = \frac{1}{4}kn(kn+2), \qquad n \in \mathbb{Z}. \tag{13}$$

For the fermionic fluctuations, the Kaluza-Klein (KK) reduction leads to a tower of eight two-component spinors ϑ^A_n ($A = 1, \ldots, 8$) for each value of the KK mode number n, with masses given by ($n \in \mathbb{Z}$) [32]

$$m_{\vartheta^a_n} = \frac{kn}{2} \pm 1 \ (3+3 \text{ modes}), \qquad m_{\vartheta^i_n} = \frac{kn}{2} \ (2 \text{ modes}). \tag{14}$$

The above masses explicitly depend on the integer k, which is the inverse radius of the φ circle in (5). Thus, in the type IIA string limit $k \to \infty$, all KK modes with $n \neq 0$ become infinitely heavy.

For $n = 0$, this spectrum coincides (as expected upon double-dimensional reduction [33]) with the spectrum of bosonic and fermionic fluctuations around the corresponding AdS_2 string solution in the type IIA superstring theory on $AdS_4 \times \mathbb{CP}^3$ [18,34]: we get two scalars of $m^2 = 2$, six scalars of $m^2 = 0$, $3+3$ fermions of $m = \pm 1$, and 2 fermions of $m = 0$.

One can also check that the full spectrum is consistent with 2D supersymmetry. The bosonic and fermionic masses in a $\mathcal{N} = 1$ supermultiplet in AdS_2 containing one real scalar and a Majorana fermion are related as (see, e.g., [35])

$$m_B^2 = m_F(m_F - 1). \tag{15}$$

Indeed, the bosonic and fermionic modes listed above can be grouped so that their masses satisfy this relation.

A stronger consistency test of the spectrum is obtained by checking the vanishing of the vacuum energy in Lorentzian AdS_2 in global coordinates (as that happens also in the simple case of the flat toroidal M2-brane [3,36]). The vacuum energies for massive bosons and fermions in AdS_2 are given by (see, e.g., [28])

$$E_B(m_B) = -\frac{1}{4}\left(m_B^2 + \frac{1}{6}\right), \qquad E_F(m_F) = \frac{1}{4}\left(m_F^2 - \frac{1}{12}\right).$$

We find that the total vacuum energy in the present case is zero separately for each KK level n,

$$E^{\text{tot}} = \sum_{n=-\infty}^{\infty} E_n^{\text{tot}}, \tag{16}$$

$$E_n^{\text{tot}} = -\frac{1}{4}\left[\frac{2}{4}(kn-2)(kn-4) + \frac{6}{4}kn(kn+2) - 3\left(\frac{kn}{2}+1\right)^2 - 3\left(\frac{kn}{2}-1\right)^2 - 2\left(\frac{kn}{2}\right)^2 + 2\right] = 0. \tag{17}$$

Using the above spectrum, we can derive the one-loop correction to the partition function of the M2-brane theory expanded around the Euclidean $AdS_2 \times S^1$ solution with circular boundary. The semiclassical partition function is given by

$$Z_{\text{M2}} = Z_1 e^{-S^{\text{cl}}_{\text{M2}}}\left[1 + O\left(\frac{1}{R^3 T_2}\right)\right], \tag{18}$$

where the one-loop term Z_1 is the ratio of the determinants of the corresponding fluctuation operators

$$\begin{aligned} Z_1 &= \prod_{n\in\mathbb{Z}} \frac{\mathcal{Z}_{n,F}}{\mathcal{Z}_{n,B}}, \\ \mathcal{Z}_{n,F} &= \left[\det\left(-\nabla^2 + \frac{R^{(2)}}{4} + \left(\frac{kn}{2}+1\right)^2\right)\right]^{\frac{3}{2}}\left[\det\left(-\nabla^2 + \frac{R^{(2)}}{4} + \left(\frac{kn}{2}-1\right)^2\right)\right]^{\frac{3}{2}} \det\left(-\nabla^2 + \frac{R^{(2)}}{4} + \frac{k^2n^2}{4}\right)^2, \\ \mathcal{Z}_{n,B} &= \det\left(-\nabla^2 + \frac{(kn-2)(kn-4)}{4}\right)\left[\det\left(-\nabla^2 + \frac{kn(kn+2)}{4}\right)\right]^3. \end{aligned} \tag{19}$$

Here $R^{(2)} = -2$ is the curvature of AdS_2 [37]. The $n = 0$ factor in (19) is of course the same as the one-loop partition function [18,34] for the fluctuations near the corresponding type IIA AdS_2 string world sheet ending on a circle at the boundary of $AdS_4 \times \mathbb{CP}^3$.

The functional determinants in (19) may be computed by the standard AdS_d spectral zeta-function techniques (as was done in the similar AdS_2 string case in, e.g., [18,28,29]). For a massive boson, one has

$$\begin{aligned} \Gamma_{1_B} &= \frac{1}{2}\log\det(-\nabla^2 + m_B^2) \\ &= -\frac{1}{2}\zeta(0; m_B^2)\log(\Lambda^2) - \frac{1}{2}\zeta'(0; m_B^2), \end{aligned} \tag{20}$$

where Λ is a 2D UV cutoff, and

$$\zeta_B(0;m_B^2) = \frac{m_B^2}{2} + \frac{1}{6},$$
$$\zeta_B'(0;m_B^2) = -\frac{1}{12} - \frac{\log 2}{12} + \log \mathrm{A} - \int_0^{m_B^2+\frac{1}{4}} dx \psi\left(\sqrt{x} + \frac{1}{2}\right). \tag{21}$$

Here A is the Glaisher constant and $\psi(x) = \Gamma'(x)/\Gamma(x)$. Similarly, for a massive fermion,

$$\Gamma_{1_F} = -\frac{1}{2}\log\det\left(-\nabla^2 + \frac{R^{(2)}}{4} + m_F^2\right)$$
$$= -\frac{1}{2}\zeta_F(0;m_F)\log(\Lambda^2) - \frac{1}{2}\zeta_F'(0;m_F), \tag{22}$$

$$\zeta_F(0;m_F) = -\frac{m_F^2}{2} + \frac{1}{12},$$
$$\zeta_F'(0;m_F) = -\frac{1}{6} + 2\log \mathrm{A} + |m_F| + \int_0^{m_F^2} dx \psi(\sqrt{x}). \tag{23}$$

Using these expressions we can first verify the cancellation of the logarithmically divergent part of the one-loop free energy $\Gamma_1 = -\log Z_1$ in (19). Indeed, from the above calculation of the vacuum energy, one can see that the sum over the bosonic and fermionic masses at each KK level n satisfies $\sum(m_B^2 - m_F^2) = -2$. Then the total coefficient of the logarithmic divergence in the sum of the corresponding terms in (20), (21) and (22), (23) over the spectrum is

$$\zeta_{\text{tot}}(0) = \frac{1}{2}\sum_{n\in\mathbb{Z}}(-2+4) = \sum_{n\in\mathbb{Z}} 1 = 1 + 2\zeta_R(0) = 0, \tag{24}$$

where we have used the Riemann zeta-function regularization to evaluate the (linearly divergent) sum. Note that the contribution of all massive KK modes at nonzero n levels cancels 1 coming from the $n = 0$ modes, i.e., cancels the logarithmic UV divergence that was present in the similar computation in the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ superstring regime [18].

The vanishing of the logarithmic divergence in the free energy was actually expected, as the M2-brane theory we started with is three dimensional, and there are no logarithmic divergences in the corresponding functional determinants in 3D. The reduction to 2D with all KK modes included cannot produce logarithmic divergences that were not present in the 3D formulation [38].

The one-loop free energy is thus finite and is given by

$$\Gamma_1 = -\log Z_1 = -\frac{1}{2}\zeta_{\text{tot}}'(0), \tag{25}$$

where according to (19),

$$\zeta_{\text{tot}}'(0) = \sum_{n\in\mathbb{Z}} \zeta_{\text{tot}}'(0;n),$$
$$\zeta_{\text{tot}}'(0;n) = 2\zeta_B'\left(0;\frac{(kn-2)(kn-4)}{4}\right) + 6\zeta_B'\left(0;\frac{kn(kn+2)}{4}\right) + 3\zeta_F'\left(0;\frac{kn}{2}+1\right) + 3\zeta_F'\left(0;\frac{kn}{2}-1\right) + 2\zeta_F'\left(0;\frac{kn}{2}\right). \tag{26}$$

Summing up the bosonic and fermionic contributions, some remarkable simplifications occur. Combining the contributions of the positive and negative modes (so that below $n \geq 0$), we find the following result [39]:

$$\zeta_{\text{tot}}'(0;n) + \zeta_{\text{tot}}'(0;-n) = \begin{cases} -2\log\left(\frac{k^2n^2}{4} - 1\right), & kn > 2, \\ \log\pi^2, & kn = 2, \\ -\log\frac{9}{4}, & kn = 1, \\ 0, & n = 0. \end{cases}$$

If we assume that $k > 2$, only the $n = 0$ and $kn > 2$ cases in (27) occur, and the complete one-loop free energy is given by the following simple result:

$$\Gamma_1 = \sum_{n=1}^{\infty}\log\left(\frac{k^2n^2}{4} - 1\right)$$
$$= 2\sum_{n=1}^{\infty}\log\frac{kn}{2} + \sum_{n=1}^{\infty}\log\left(1 - \frac{4}{k^2n^2}\right). \tag{27}$$

Using again the Riemann zeta-function regularization $[\zeta_R(0) = -\frac{1}{2}, \zeta_R'(0) = -\frac{1}{2}\log(2\pi)]$, we get

$$2\sum_{n=1}^{\infty}\log\frac{kn}{2} = 2\zeta_R(0)\log\frac{k}{2} - 2\zeta_R'(0) = -\log\frac{k}{4\pi}. \tag{28}$$

The second sum in (27) is finite and given by

$$\sum_{n=1}^{\infty}\log\left(1 - \frac{4}{k^2n^2}\right) = \log\prod_{n=1}^{\infty}\left(1 - \frac{4}{k^2n^2}\right)$$
$$= \log\left[\frac{k}{2\pi}\sin\left(\frac{2\pi}{k}\right)\right]. \tag{29}$$

Here we used Euler's expression for the sine as a product of its zeros, $\sin(\pi x) = \pi x \prod_{n=1}^{\infty}(1 - (x^2/n^2))$.

Combining (28) and (29), we get the final result for the one-loop partition function for $k > 2$,

$$Z_1 = e^{-\Gamma_1} = \frac{1}{2\sin(\frac{2\pi}{k})}, \tag{30}$$

which is thus in precise agreement with the localization result in (2).

Let us now discuss the special cases of $k=1$, 2 which require a separate treatment [40]. For $k=1$, all of the cases listed in (27) occur in the sum over the KK modes, i.e.,

$$\Gamma_1^{k=1}=\frac{1}{2}\log\frac{9}{4}-\frac{1}{2}\log\pi^2+\sum_{n=3}^{\infty}\log\left(\frac{n^2}{4}-1\right). \qquad (31)$$

The infinite sum here can be evaluated in a similar way: $\sum_{n=3}^{\infty}\log((n^2/4)-1)=2\sum_{n=3}^{\infty}\log(n/2)+\sum_{n=3}^{\infty}\times\log(1-(4/n^2))=\log(16\pi)-\log 6$, and from (31) we get $\Gamma_1^{k=1}=\log 4$, i.e.,

$$Z_1^{k=1}=\frac{1}{4}. \qquad (32)$$

Similarly, $\Gamma_1^{k=2}=-\frac{1}{2}\log(\pi^2)+\sum_{n=2}^{\infty}\log(n^2-1)=0$,

$$Z_1^{k=2}=1. \qquad (33)$$

These results cannot be directly compared to localization, as the result (1) of [9] is singular for $k=1$, 2 [44]. It might be that the derivation of (1) in [9] is to be reconsidered specifically for $k=1$, 2. The matching in these special cases thus remains an open problem.

Concluding remarks.—Extending the above computation to higher loops in the semiclassical expansion of the partition function (18) would allow one to compare the quantum M2-brane prediction with the subleading terms in the expansion of the localization expression at large N. For instance, the term of order $1/\sqrt{N}$ in (2) should come from a two-loop calculation in the M2-brane theory (recall that $(1/R^3T_2)\sim(1/\sqrt{N})$). One issue with this computation is whether the two-loop correction will be UV finite.

The cancellation of logarithmic divergences (despite apparent nonrenormalizability) may happen due to the large amount of supersymmetry of the supermembrane theory (cf. [48]). An example of a cancellation of two-loop UV divergences in a formally nonrenormalizable theory is provided by the successful computation of the subleading $(1/\sqrt{\lambda})$ correction to the cusp anomalous dimension $f(\lambda)=a_0\sqrt{\lambda}+a_1+(a_2/\sqrt{\lambda})+\cdots$ in the $AdS_5\times S^5$ superstring theory [49–51], which matched the corresponding term in the strong-coupling expansion of $f(\lambda)$ derived on the $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) side using integrability [52] (the analogous two-loop computation in the case of the $AdS_4\times\mathbb{CP}^3$ string was done in [53]).

An alternative possibility could be that the M2-brane theory has a built-in UV cutoff $\Lambda\sim\ell_{\rm Pl}{}^{-1}\sim T_2^{-1/3}$. However, then a logarithmically divergent term would scale as $\log(R\Lambda)=(1/6)\log(Nk)+\cdots$ [see Eq. (7)], but there is no such $\log N$ term in the localization expansion of the Wilson loop in (2) [54]. This suggests that the logarithmic divergences may cancel at higher loops in the M2-brane theory, at least for such a $\frac{1}{2}$-BPS observable.

Let us now comment on the 10D type IIA string theory limit, which corresponds to k and N both taken to be large, with the 't Hooft coupling $\lambda=N/k$ kept fixed. In this regime, the 11D background (3) reduces to $AdS_4\times\mathbb{CP}^3$, and a Wilson loop operator is dual to an open string ending on a loop at the boundary of AdS_4. The corresponding type IIA string coupling constant g_s and the effective string tension $T=(1/2\pi)(R_s^2/\ell_s^2)$ are then [8]

$$g_s=\frac{\sqrt{\pi}(2\lambda)^{5/4}}{N},\qquad T=\frac{\sqrt{2\lambda}}{2},\qquad \lambda=\frac{N}{k}. \qquad (34)$$

Note that, to be in the $g_s\ll 1$ and $\lambda\gg 1$ regime, we need to assume that $k\ll N\ll k^5$.

As was pointed out in [18], the string partition function computed near the AdS_2 minimal surface representing the $\frac{1}{2}$-BPS circular Wilson loop in both type IIB $AdS_5\times S^5$ and type IIA $AdS_4\times\mathbb{CP}^3$ theories has an expansion in small g_s and then in large tension T of the following universal form [56]:

$$\langle W_{\frac{1}{2}}\rangle=e^{2\pi T}\frac{\sqrt{T}}{g_s}\left\{c_0[1+O(T^{-1})]+c_1\frac{g_s^2}{T}[1+O(T^{-1})]+c_2\left(\frac{g_s^2}{T}\right)^2[1+O(T^{-1})]+\cdots\right\}. \qquad (35)$$

In our present case, we can see that this is consistent with the structure of the corresponding large N, large k expansion of (1), according to which one should get $c_0=(1/\sqrt{2\pi})$, $c_1=(\pi/12)c_0$, etc. The presence of the overall $\sqrt{T}$ factor was shown in [18] to originate from the leading one-loop string sigma model correction on the disk [it is related to the $n=0$ contribution in (24)]. The precise value of the one-loop coefficient $c_0=(1/\sqrt{2\pi})$ was not so far derived directly on the string side (that appears to require a careful normalization of the measure in the superstring path integral). Remarkably, the M2-brane one-loop computation described above effectively determines this coefficient and, moreover, the coefficients of all of the leading large tension terms at higher genus (disk with handles). Indeed, comparing (35) to the corresponding large N, large k expansion of (1), the leading large tension terms in (35) can be seen [57] to arise from a resummed expression,

$$\langle W_{\frac{1}{2}}\rangle=\frac{1}{2\sin\left[\sqrt{\frac{\pi}{2}}\frac{g_s}{\sqrt{T}}\right]}e^{2\pi T}[1+O(T^{-1})], \qquad (36)$$

where $\sqrt{(\pi/2)}(g_s/\sqrt{T})=2\pi(\lambda/N)=(2\pi/k)$. Here the sine factor is just the same as the one found in (2) and (30) [the exponential factor is also the same as in (18) and (11)].

Thus, the one-loop M2-brane correction happens to describe the leading large tension terms at all orders in the genus expansion in the type IIA string theory [58].

One natural generalization of our calculation is to consider the $\frac{1}{6}$-BPS Wilson loop [10,61]. In this case, the localization result derived in [9], expanded in the large N, fixed k limit, gives

$$\langle W_{\frac{1}{6}} \rangle = \frac{i}{2\sin(\frac{2\pi}{k})}\sqrt{\frac{2N}{k}}e^{\pi\sqrt{\frac{2N}{k}}}(1+\cdots). \tag{37}$$

Note that there is an extra factor $i\sqrt{2N/k}$ compared to the $\frac{1}{2}$-BPS case. The origin of this factor should be similar to what was discussed in the corresponding string case, where it was argued [10] that the string solution should be smeared over a $\mathbb{CP}^1$ in $\mathbb{CP}^3$, leading to two zero modes and hence an overall factor $(\sqrt{T})^2 \sim \sqrt{\lambda}$ in the partition function [13,18]. For the M2-brane, we similarly expect that the solution relevant to the $(1/6)$-BPS case should be smeared over a $\mathbb{CP}^1$, leading again to an extra tension-dependent prefactor $\sim\sqrt{N}$. It would be interesting to study the fluctuation spectrum of the corresponding M2-brane in detail and reproduce from a one-loop calculation the remaining normalization factor in (37).

Another interesting extension would be to explore the defect conformal field theory (CFT) defined by the $\frac{1}{2}$-BPS Wilson loop in the large N, fixed k limit. The corresponding problem in the type IIA string regime was studied in [62]. In particular, in that case, one finds that the $8+8$ fluctuation modes about the AdS_2 string solution form a short supermultiplet containing the displacement operator. The same multiplet appears for the M2-brane as the $n=0$ mode in the KK reduction. It would be interesting to understand the interpretation of the higher KK modes from the defect CFT point of view and compute their boundary correlation functions [63].

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