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Numerical Solutions for Detecting Contingency in Modern Power Systems

Xiaohang Ma
Department of Mathematics
University of Connecticut
Storrs, CT, USA
ORCID: 0009-0001-4241-141X

Hongjiang Qian
Department of Mathematics
University of Connecticut
Storrs, CT, USA
ORCID: 0000-0002-7139-6407

Le Yi Wang
Department of Electrical & Computer Engineering
Wayne State University
Detroit, MI, USA
ORCID: 0000-0002-1756-4633

Masoud H. Nazari
Department of Electrical & Computer Engineering
Wayne State University
Detroit, MI, USA
ORCID: 0000-0002-3123-5000

George Yin
Department of Mathematics
University of Connecticut
Storrs, CT, USA
ORCID: 0000-0002-2951-0704

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***Abstract*—This paper is devoted to the detection of contingencies in modern power systems. Because the systems we consider are under the framework of cyber-physical systems, it is necessary to take into consideration of the information processing aspect and communication networks. A consequence is that noise and random disturbances are unavoidable. The detection problem then becomes one known as quickest detection. In contrast to running the detection problem in a discrete-time setting leading to a sequence of detection problems, this work focuses on the problem in a continuous-time setup. We treat stochastic differential equation models. One of the distinct features is that the systems are hybrid involving both continuous states and discrete events that coexist and interact. The discrete event process is modeled by a continuous-time Markov chain representing random environments that are not resented by a continuous sample path. The quickest detection then can be written as an optimal stopping problem. This paper is devoted to finding numerical solutions to the underlying problem. We use a Markov chain approximation method to construct the numerical algorithms. Numerical examples are used to demonstrate the performance.**

Key words. power system, contingency, optimal stopping, Markov chain approximation, numerical calculation.

I. Introduction

In the new era, information processing and communication technology have had much impact on a wide variety of applications. The modern power systems are very different from the traditional one. They are highly connected to the cyber-physical systems. They include wired and wireless communications. In this paper, we investigate the problem for detecting sudden changes of systems in modern power systems. Our main work is to construct numerical solutions. The solution techniques are based on Markov chain approximation methods for treating optimal stochastic controls [17].

Timely detection of abrupt changes is vitally important for numerous applications, for example, in fault detection, risk management, and management of modern power systems. Early work on fault detection can be found in [3], [4], [6], [30]. Many works mentioned above dealt with discrete-time problem. The underlying problem was setup as a sequential detection problem. Later, researchers started working on problems in a continuous-time setting. Nowadays, it has been well recognized that estimation of jump time of cyber-physical contingencies (CPC) is a quickest detection problem. The work on quickest detection can be traced back to Liptser and Shiryayev [19], continued by many researchers such as Peskir and Shiryaev; see [34] for an extensive survey of the period of fifty years up to 2010. Denote the jump time by τ with the size of the change as the stopping region. A classical approach for a discrete-time system (e.g., a sequence of random variables) is the approach of sequential detection. For MPS, locating the stopping time and stopping boundary is substantially more difficult due to involvement of stochastic hybrid systems. Detecting the time τ of the contingency can be recast as an optimal stopping problem (see [29], [33] for an earlier introduction). One uses an appropriate cost function so that the problem is converted to choosing τ. Solving a stochastic control problem typically requires deriving a partial differential equation (PDE) and verifying the solution of PDE is indeed the value function (the optimum). To solve this PDE, one needs to identify the auxiliary conditions (boundary conditions). Different from the usual stochastic control problem, the optimal stopping problems have unknown "free boundaries" that need to be identified. In addition, the stopping time itself can be regarded as "control."

Recently, Ernst and Mei [7] considered the optimal stopping for a multidimensional linear switching diffusions given by

$$dY^i(t) = (a^i(\alpha(t)) + \sum_{j=1}^{2} b^{ij}(\alpha(t))Y^j(t))dt + \sigma^i(\alpha(t))Y^i(t)dW^i(t),\ i = 1, 2, \quad (1)$$

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where $\alpha(\cdot)$ is a continuous-time Markov chain with a finite state space $\mathcal{M}$, $W^i(\cdot)$ are independent standard Brownian motions. The coefficients $a^i(\cdot), b^{ij}(\cdot), \sigma^i(\cdot)$ are appropriate measurable function such that equation (2) has a unique positive solution. In addition, the couple $(Y(\cdot), \alpha(\cdot))$ is a strong Markov process; see our work on switching diffusions [40, Chapter 2] and our recent work on Kolmogorov systems [22], [23]. In lieu of (2), we work with a more general nonlinear system

$$\begin{aligned} &dY(t) = F(Y(t), \alpha(t))dt + G(Y(t), \alpha(t))dW(t) \\ &Y(0) = y, \ \alpha(0) = i. \end{aligned} \tag{2}$$

where $F : \mathbb{R}^d \times \mathcal{M} \to \mathbb{R}^d$ and $G : \mathbb{R}^d \times \mathcal{M} \to \mathbb{R}^{d\times d}$ are appropriate nonlinear functions satisfying suitable conditions, and W is a standard d-dimensional Brownian motions. To simplify the discussion in what follows, we assume that the Markov chain $\alpha(t)$ can be observed. The case of hidden Markov chain $\alpha(t)$ can be handled by the methods proposed in this paper together with the use of Wonham-type filters. However, for simplicity, we will concentrate on the Markov chain not being hidden in this work. To proceed, define a stopping time of the system by τ and define a cost function

$$J(y, i, \tau) := E_{y,i}\int_0^\tau \exp\big(-\int_0^t \lambda(\alpha(s))ds\big) H(Y(t), \alpha(t))dt, \tag{3}$$

where $E_{y,i}$ is the expectation with initial data (y, i), λ is known as the discount rate, H is the running cost rate, and J is the cost function. The value function is given by

$$V(y, i) = \inf_\tau J(y, i, \tau), \tag{4}$$

where the minimization is taken over $\mathcal{T}_{y,\alpha}$. The collection of all possible stopping times of $(X(\cdot), \alpha(\cdot))$ with respect to the natural filtration $\{\mathcal{F}_t : t > 0\}$ is augmented with all $\mathbb{P}$-null sets (see [7] for formulation). The solution can be represented by a system of Hamilton-Jacobi-Bellman (HJB) equations, and the stopping boundary is a free boundary to be specified. Because of $\alpha(t)$, we deal with a system of partial differential equations rather than a single differential equation. In [7], assuming $\mathcal{M} = \{1, 2\}$ and $d = 2$, the stopping time problem together with stopping boundary was identified as the solution of a system of integral equations. For a finite state space $\mathcal{M}$ with more than two discrete states and $d > 2$, an analytic solution is not available. Even in the case considered in [7], the systems of integral equations cannot be solved in closed form. Thus, numerical approximation becomes important or virtually only possible way for solving the problem. One may seek to approximate the integral equations to get the solution. Unfortunately such an approach does not seem to be feasible for many applications. Thus it is virtually important to find feasible approximation methods.

Rather than solving integral equations, in this paper, we develop an alternative approach. The main idea is the use of Markov chain approximation techniques. Such a technique was initiated by Kushner, continued by Kushner and Dupuis [17], and further extended in our recent work for controlled switching diffusions and games [36], [37].

The rest of this paper is organized as follows. The formulation of the detection or the equivalent the solution of the optimal stopping is given next. Then we briefly discuss the Markov chain approximation techniques, but leave most of the details in related references. In this paper, we will not carry out the convergence analysis. Rather, our attention is devoted to modeling, designing algorithms, as well as carrying out numerical experiments. Section III is devoted to a couple of examples. Finally, some remarks are made in Section IV.

II. Formulation

Our starting point is (3) subject to (2). Note that we only need the uniqueness to be in the sense of unique in distribution. The nonlinear functions $F(\cdot,\cdot)$ and $G(\cdot,\cdot)$ will allow us to cover an even wide class of problems in modern power systems. Sufficient conditions ensuring the existence of solutions may be provided, but it is not the focus of this paper. [For example, even pathwise uniqueness can be enforced by posting a uniform Lipschitz condition on F and G w.r.t. x or the solution can be obtained using a Girsanov transformation from a base process satisfying the condition of uniqueness.]

We still use a cost function of the form (3) and with the value function given by (4). In practice, the process will be confined to some compact set $\mathcal{O}$ for numerical purpose. That is, the process must stop by the time $\tau' = \inf\{t : Y(t) \notin \mathcal{O}\}$ if it has not stopped earlier. We wish to find the stopping time $\tau \le \tau'$ which minimize the cost (3). In other words, we consider $V(y, i) = \inf_{\tau\le\tau'} J(y, i, \tau)$. We define $\mathcal{L}$ as an operator for a twice continuously differentiable function $\psi(\cdot, i)$ (differentiable w.r.t. y)

$$\begin{aligned} \mathcal{L}\psi(y, i) &= [\nabla\psi(y, i)]' F(y, i) \\ &+ \frac{1}{2}\mathrm{tr}[\nabla^2\psi(y, i)G(y, i)G'(y, i)] + Q\psi(y, \cdot)(i), \end{aligned} \tag{5}$$

where $Q\psi(y, \cdot)(i) = \sum_{j=1}^m q^{ij}\psi(y, j)$, and $\nabla\psi(y, i)$ and $\nabla^2\psi(y, i)$ denote the gradient and Hessian of $\psi(y, i)$, respectively.

Under the conditions and notation above, the Hamilton-Jacobi-Bellman equation satisfied by the value function is:

$$\min\{\mathcal{L}V(y, i) - \lambda(i)V(y, i) + H(y, i), -V(y, i)\} = 0, \tag{6}$$

in [7], it was argued this equation for the switching system (linear in the continuous state y) holds if the running cost function $H(y, i)$ satisfies the Lipschitz and linear growth condition [20].

Given the absence of the linear growth condition, a truncation method can be introduced to provide that the HJB equation holds for the value function in [7]. For fixed but otherwise arbitrary integer N, define

$$H^N(y, i) = \min(H(y, i), N). \tag{7}$$

We can then obtain the truncated problem,

$$\min\{\mathcal{L}V^N(y, i) - \lambda(i)V^N(y, i) + H^N(y, i), -V^N(y, i)\} = 0. \tag{8}$$

When the value function of the truncated problem $V^N(y, i)$ is finite $\forall (y, i) \in \mathbb{R}^d \times \mathcal{M}$ and the associated optimal stopping time is finite almost surely, then it will converge to $V(y,i)$ as $n \to \infty$; see [7, Proposition 3.1, Lemma 3.3]. Nevertheless, we do not need such a truncation in the problem considered in this paper because we will work with a compact region. In fact, our point of start is: We assume that the optimal stopping problem has a unique solution.

Note that in this paper, our main objective is to construct efficient numerical schemes, so we will begin with assuming that there is an optimal solution of the quickest detection problem. To approximate the solution to (4), we assume that the cost rate $H(\cdot,i)$ is confined in a compact region $y \in K_0$. Let $h > 0$ be a small discretization step size and $S_h = \{y : y = \sum_{j=1}^{d} n_j e_j h, j = 1,\ldots,d,\ n_j = 0,\pm1,\pm2,\ldots\} \cap K_0$, where e_j is the standard unit vector in the j-th coordinate direction. We construct a discrete-time two-component Markov chain $\{(\xi_k^h,\alpha_k^h) : k < \infty\}$ on the discrete state space $S_h \times \mathcal{M}$ with $p^h((y,\iota),(z,\ell))$ being the transition probabilities from a state $(y,\iota) \in S_h \times \mathcal{M}$ to state $(z,\ell) \in S_h \times \mathcal{M}$, which was originated for switching diffusions in our work [36], [37]. The idea is: ξ_k^h approximates Y and α_k^h approximates α. We approximate the stopping time τ by using a stopping time N^h and write the approximate cost function

$$J^h(y,i,N^h) := E_{y,i} \sum_{k=0}^{N^h-1} \exp(-\sum_{j=0}^{k} \lambda(\alpha_j^h)) H(\xi_k^h,\alpha_k^h)\Delta t_k^h. \tag{9}$$

The corresponding value function of the approximating Markov chain is

$$V^h(y,i) = \inf_{N^h} J^h(y,i,N^h).$$

The associated dynamic programming equation becomes

$$\begin{aligned} &V^h(y,i)\\ &= \min\Big\{e^{-\lambda(i)\Delta t^h(y,i)} \sum_{(z,\ell)} p^h((y,i),(z,\ell))V^h(z,\ell)\\ &\quad + H(y,i)\Delta t^h(y,i)),0\Big\}. \end{aligned} \tag{10}$$

To establish the convergence, we use continuous-time interpolations. Suppose that we have an interpolation interval $\Delta t^h(\cdot,\cdot) > 0$ on $S_h \times \mathcal{M}$, and denote $\Delta t_k^h = \Delta t^h(\xi_k^h,\alpha_k^h)$. Define the interpolated time $t_k^h = \sum_{j=0}^{k-1} \Delta t_j^h(\xi_j^h,\alpha_j^h)$. To ensure that the approximation is in line with (2), we need to make sure that the construction is locally consistent. That is, we construct the discrete-time Markov chain (ξ_k^h,α_k^h) so that the conditional mean and covariance "match" that of switching diffusion (2) with an error tending to 0 as $h \to 0$. We omit the details but refer to our work [36, Definition 1]. We will show that the constructed Markov chain leads to the correct limit. Define the interpolated processes as

$$\begin{aligned} &\xi^h(t) = \xi_k^h,\ \alpha^h(t) = \alpha_k^h,\ z^h(t) = k \ \text{ for } \ t \in [t_k^h, t_{k+1}^h),\\ &\tau^h = t_{N^h}^h,\\ &J^h(y,i,\tau^h) = E_{y,i}\int_0^{\tau^h} \exp\big(-\int_0^t \lambda(\alpha^h(s))ds\big)\\ &\qquad\qquad \times H(\xi^h(t),\alpha^h(t))dt,\\ &V^h(y,i) = \inf_{\tau^h} J^h(y,i,\tau^h). \end{aligned}$$

Denote by $\mathcal{F}_t^h$ the σ-algebra generated by $\{\xi^h(s),\alpha^h(s),z^h(s) : s \le t\}$. Then τ^h is an $\mathcal{F}_t^h$ stopping time. With proper definition of transition probabilities, it is expected that as $h \to 0$,

(a) the constructed approximating Markov chain is locally consistent;
(b) the sequence $(\xi^h(\cdot),\alpha^h(\cdot),\tau^h)$ converges weakly to $(X(\cdot),\alpha(\cdot),\tau)$;
(c) $J^h(y,i,\tau^h) \to J(y,i,\tau)$ and $V^h(y,i) \to V(y,i)$;
(d) using techniques similar to our work [28], we can proceed to find the continuation region and stopping region.

We show that the limit of $(\xi^h(\cdot),\alpha^h(\cdot))$ is a solution of the martingale problem with operator $\mathcal{L}$, which allows us to conclude that $(\xi^h(\cdot),\alpha^h(\cdot))$ converges weakly to $(Y(\cdot),\alpha(\cdot))$.

To carry out the analysis in the aforementioned steps, there is a crucial point that needs to be addressed. In [17, pp. 278-279], it is recognized that in approximating the stopping times of a diffusion system, a "tangency" problem may arise. The problem can be described as the interpolated processes of the trajectories that will approximate the diffusion. Although τ^h converges, the limit is not the true τ. In a nutshell, the problem is concerned with continuity of value functions, which was considered by many researchers, for example, [1], [2], [5], [8]. The tangency problem can also occur for switching diffusions. In [35], we identified certain conditions under which there will be no tangency taking place for the Markov chain approximation with a stopping time.

III. Numerical Examples

In power system analysis, we encounter nonlinear state space models [15], [27] for micrigrids (MG). To demonstrate briefly how a nonlinear system is derived, we concentrate on real power management in frequency regulation problems. Let the voltage of the ith bus be denoted by the phasor $\vec{V}_i = V_i\angle\delta_i$. For a given transmission line (a link) $(i,j) \in \mathcal{E}$ between two buses (see Fig. 1), suppose that the transmission line between Bus i and Bus j has impedance $X_{ij}\angle\theta_{ij}$. Then the transmitted real power at Bus i is

$$P_{ij} = \frac{V_i^2}{X_{ij}}\cos(\theta_{ij}) - \frac{V_iV_j}{X_{ij}}\cos(\theta_{ij}+\delta_{ij}). \tag{11}$$

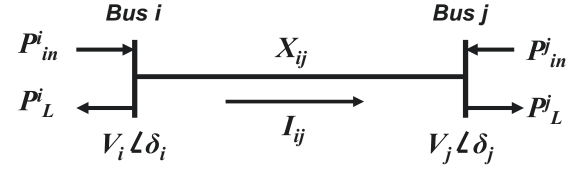

Fig. 1. A link in microgrids

To derive state space models for MG dynamics, it is important to distinguish dynamic and non-dynamic buses. If the ith bus is dynamic, then it entails a local state space model

$$\dot{z}_i = f(z_i, z_i^-, v_i, e_i)$$

where z_i is the local state, z_i^- is the neighboring states of Bus i, v_i is the local control input, e_i is the local load viewed as a disturbance. The collection of the state variables of the dynamic buses is denoted by z_d. On the other hand, if the ith bus is non-dynamic, then it entails an implicit algebraic relationship between their variables z_{nd} and z_d, expressed as a nonlinear function $z_{nd} = \Gamma(z_d, v, e)$. After combining these two types of bus models and eliminate z_{nd}, we can arrive at the nonlinear state space model $\dot{z}_d = F^0(z_d, \Gamma(z_d, v, e), v, e) = F(z_d, v, e)$. In power system control problems, it is common to linearize the nonlinear dynamics above near nominal operating points [9], [14]. Given the steady-state loads e and steady-state input real powers $\overline{v}$, we can compute the steady-state $\overline{z}_d$ (the equilibrium point). Then, by defining the perturbation variables from their nominal values as $x = z_d - \overline{z}_d$ as state, $u = v - \overline{v}$ as control, and $d = e - \overline{e}$ as noise, the linearized system is $\dot{x} = Ax + B_1 u + B_2 d$. Finally, after we apply a common power system feedback controller, such as droop control plus secondary frequency regulator, expressed as $u = -Kx$, we reach the closed-loop system $\dot{x} = Ax - B_1 K x + B_2 d = A_c x + B_2 d$, where $A_c = A - B_1 K$ is stable. If the noise is modeled as a Brownian motion, this model can be expressed in the form of a stochastic differential equation (SDE)

$$dx(t) = A_c x(t)dt + \Sigma dW, \tag{12}$$

where W is the standard Brownian motion. Consider a two-bus system shown in Fig. 1 with $i = 1$ and $j = 2$. Bus 1 is a generator bus and Bus 2 is a load bus with load P_L^2 and no input power $P_{in}^2 = 0$. Denote $\theta = \theta_{12}$, $X = X_{12}$, $\beta_1 = \frac{V_1^2}{X}\cos(\theta)$, $\beta_2 = \frac{V_2^2}{X}\cos(\theta)$, $\beta = V_1 V_2 / X$, and $\delta = \delta_1 - \delta_2$. $\omega_1 = \dot{\delta}_1$ is the frequency on Bus 1. Define the state variable $z_d = \begin{bmatrix} \delta_1 \\ \omega_1 \end{bmatrix}$, and assume that $g_1(\omega_1) = b_1\omega_1$, $b_1 > 0$. Then,

$$f_1(z_d, \delta_2) = \begin{bmatrix} \omega_1 \\ -\frac{b_1\omega_1}{M_1} - \frac{1}{M_1}(\beta_1 - \beta\cos(\theta + \delta_1 - \delta_2)) \end{bmatrix}.$$

From the equation on Bus 2, $P_L^2 = -\beta_2 + \beta\cos(\theta - \delta)$. We solve for $\delta = \theta - \arccos((\beta_2 + P_L^2)/\beta)$. It follows that

$$\widetilde{f}_1(z_d) = \begin{bmatrix} \omega_1 \\ -\frac{b_1\omega_1}{M_1} - \frac{1}{M_1}\left[\beta_1 - \beta\cos(2\theta - \arccos(\beta_2 + P_L^2)/\beta)\right] \end{bmatrix}.$$

The nonlinear state equation for this grid is

$$\begin{aligned} \dot{z}_d &= \widetilde{f}_1(z_d) + \begin{bmatrix} 0 \\ 1/M_1 \end{bmatrix} v_1 + \begin{bmatrix} 0 \\ -1/M_1 \end{bmatrix} e_1 \\ &= \widetilde{f}_1(z_d) + B_1 v_1 + B_2 e_1. \end{aligned}$$

For numerical values, assume $M_1 = 1$, $b_1 = 0.2$, $\beta = 200$, we can derive the linearized system with $A = \begin{bmatrix} 0 & 1 \\ -197.7372 & -0.2 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$. When a contingency occurs at an unknown time t_s, it could be modeled as the SDE obtaining an known extra drift term μ after the system parameters A_c. If the randomness of the environment disturbance also includes the switching of μ. This can be represented by a continuous Markov chain $\alpha(t) \in \{1, \dots, m\}$. After adding sensor noises and the contingency we have

$$dz_d = (Az_d + B_1 v_1 + B_2 e_1 + I(t \ge t_s)\mu(\alpha(t)))dt + \Sigma dW. \tag{13}$$

For simplicity, we denote $\tilde{z}_d(t) = z_d(t) - \int_0^t (Az_d(s) + B_1 v_1 + B_2 e_1)ds$. Then the contingency can be regarded as the drift term of the processes $\tilde{z}_d(t)$ changes from zero to a non-zero drift term.

Due to extremely stringent requirements on power system quality management, it is critically important to detect t_s accurately and quickly so that the correcting actions can be implemented. Thus the problem is to find a stopping time τ with respect to the natural filtration $\mathcal{F}_t^{\tilde{z}_d, \alpha} = \sigma\{\tilde{z}_d(s), \alpha(s), \alpha(0) = \alpha; 0 \le s \le t\}$ that is "as close as possible" to the unknown contingency time t_s. Introducing the following cost functional

$$\widetilde{J}(\varphi, \alpha, \tau) = \mathbb{P}_{\varphi,\alpha}(\tau < t_s) + c\mathbb{E}_{\varphi,\alpha}\Big[F(\tau - \theta)I(\tau > t_s)\Big]$$

where $\varphi = \tilde{z}_d(0)$, $F(x) := e^{\gamma t} - 1$ for a given $\gamma > 0$. Now the quickest detection problem is to find the solution of the minimization problem $\widetilde{V}(\varphi, \alpha) = \inf_\tau \widetilde{J}(\varphi, \alpha, \tau)$. A suitable measure change recast the process in (13) or (12) to another process governed by switching diffusion on a new probability space. The corresponding minimization problem can be reformulated as the optimal stopping problem with cost functional defined in (3). The state of the new process could be recorded by the observation of z_d. Moreover, given the value function and the stopping region of the measure transformed problem, the optimal detection policy of (13) can be obtained; see [7]. For numerical demonstration and simplicity, we start with measure transformed switching diffusions and focus on the optimal stopping problems.

Example 1: Consider a 1-dimensional detection of contingency in a modern power system. It is recast as minimizing

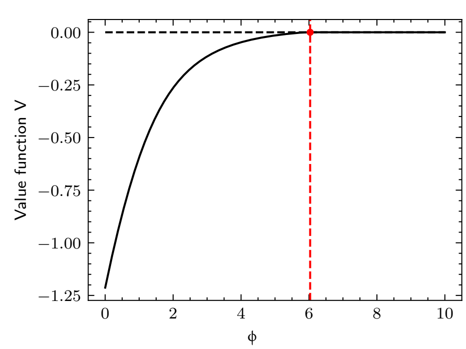

Fig. 2. Value function of Example 1

$$J(\varphi, \tau) = \mathbb{E}_\varphi \int_0^\tau e^{-\lambda t}(Y(t) - \frac{\lambda}{c})dt \quad \text{subject to}$$

$$dY(t) = \lambda(1 + Y(t))dt + \mu Y(t)dW, \; Y(0) = \varphi. \tag{14}$$

Denote the value function by $V(\varphi) = \inf_\tau J(\varphi, \tau)$. For the assumption of the detection problem before measure transformation see [32]. In our numerical experiment, use parameters $\lambda = 3.0, \mu = 0.1, c = 0.5$. In addition, we set the boundaries $\mathcal{O}$ to be $[0, 10]$ for numerical purpose and take the step size $h = 0.001$. Fig. 2 displays the value vs. φ. It demonstrated that the stopping region is $\widetilde{S} = \{\varphi : V(\varphi) = 0\} = \{\varphi : \varphi \geq 6.034\}$ and the continuation region $\widetilde{C} = \{\varphi : V(\varphi) < 0\} = \{\varphi : \varphi < 6.034\}$. To get the stopping region of $\tilde{z}_d$, we use the transformation from $\tilde{z}_d$ to $Y(t)$,

$$Y(t) = \lambda e^{\lambda - \frac{1}{2}\mu^2 t + \mu \tilde{z}_d(t)} \int_0^t e^{-(\lambda - \frac{1}{2}\mu^2)s - \mu \tilde{z}_d(s)} ds. \quad (15)$$

The optimal stopping time is the first entrance time to $\widetilde{S}$ of $Y(t)$.

Example 2: Similar to [7], detecting contingency in a modern power 2-dimensional system can be solved by minimizing

$$J(\varphi, \ell, \tau) = \mathbb{E}_{\varphi,\ell} \int_0^\tau e^{-\lambda t} \Big(p_1 Y^1(t) + p_2 Y^2(t) - \frac{\lambda}{c\gamma} \Big) dt$$

subject to a system of stochastic differential equations with Markov switching

$$dY^i(t) = [\lambda + (\lambda + \gamma) Y^i(t)]dt + \mu(\alpha(t)) Y^i(t) dW^i(t), \ i = 1, 2 \quad (16)$$

where the initial data are $Y(0) = (\varphi^1, \varphi^2)$, $\alpha(0) = \ell$ and $W^i(t)$ are independent Brownian motions. Take $\lambda = 0.6, \gamma = 0.5, p_1 = 0.3, p_2 = 0.7$, $\mu(1) = 10, \mu(2) = 1$ and $c = 1$ for numerical experiment. Furthermore, we set the boundaries to be $\mathcal{O} = [0, 5] \times [0, 5]$ and the step size $h = 0.0125$. The Markov chain has two states $\mathcal{M} = \{1, 2\}$ with generator given by $Q = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$. Denote $V(\varphi, \ell) = \inf_\tau J(\varphi, \ell, \tau)$. The negative of the value function is presented in Fig. 3. The continuation region (blue) and the stopping region (red) are plotted in Fig. 4. The black lines in Fig. 4 is the optimal stopping boundaries. Using $\tilde{z}_d$, the processes $Y^i(t)$ for $i = 1, 2$ are given by

$$L^i(t) = \exp\{\int_0^t \mu(\alpha(s)) d\tilde{z}_d^i - \frac{1}{2}\int_0^t \mu^2(\alpha(s)) ds\},$$
$$Y^i(t) = \lambda e^{(\lambda+\gamma)t} L^i(t) \int_0^t \frac{1}{e^{(\lambda+\gamma)s} L^i(s)} ds.$$

Starting from initial data (φ, ℓ), we stop the process when it hits the boundaries yielding the optimality.

IV. Conclusion

In this paper, under the consideration of information processing and communication networks, numerical solutions for detecting sudden changes for a modern power systems are developed for switching diffusion processes. Different from the discrete-time setup [3], [6], we used stochastic differential equations. The switching process is used to model the random environmental changes that are not representable using a diffusion process alone. We adopted the Markov chain approximation methods initiated by Kushner and developed further by Kushner and colleagues, and presented the problem formulation, the numerical methods, and numerical computational results. In this paper, the random switching process is assumed to be available. Switching diffusions in which the switching process is a hidden Markov switching process can be treated. In such a case, necessarily we need to add another component, namely, Wonham filtering. We refer an interested reader to our work [39]. This is the first effort in treating detection of contingency for power systems as a quickest detection problem. Only simple examples are considered, but more complex systems can be treated.

Although power systems are usually large and complex, using time-scale separation and singular perturbation, we can normally reduce a large-scale and complex system to a much simplified, smaller in dimension, and reduced order system. Thus in what follows, we will demonstrate numerics for the quickest detection using relatively simplified models.

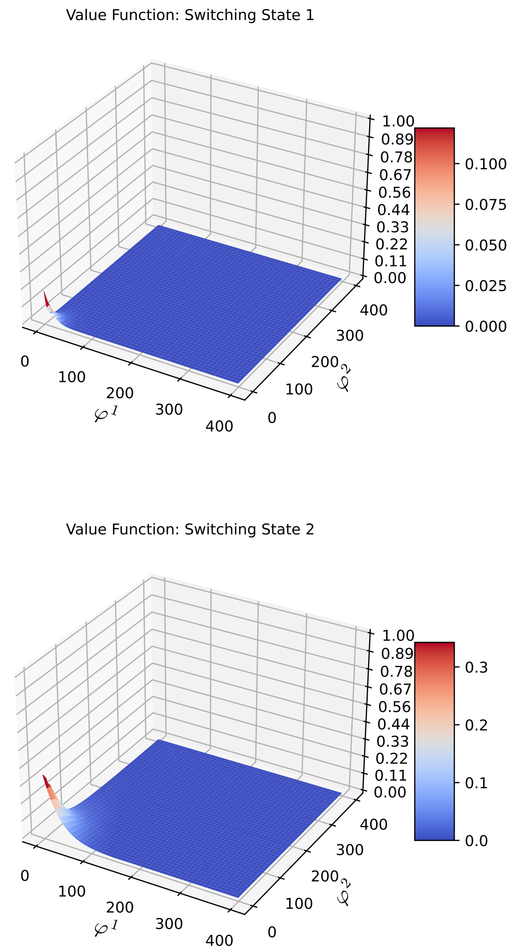

Fig. 3. Negative of value function for Example 2

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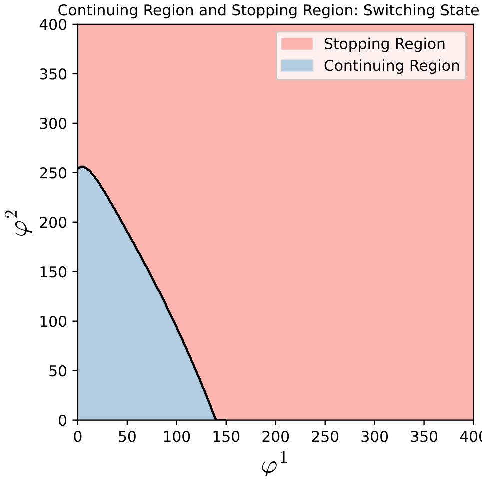

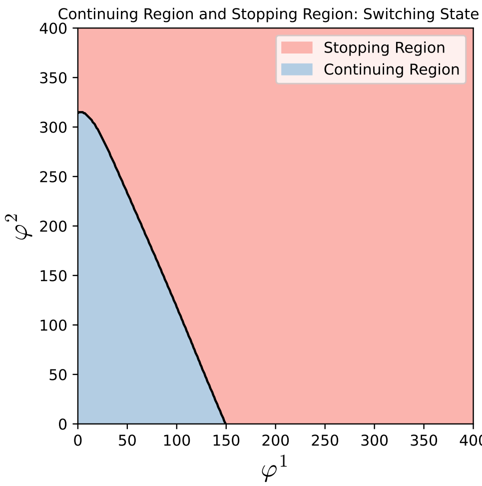

Fig. 4. Optimal stopping boundaries for Example 2

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