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# A computational approach for real-time stochastic recovery of electric power networks during a disaster

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# ABSTRACT

Disasters are occurring with increasing frequency worldwide, causing significant social hardship and economic losses. Critical infrastructures such as electric power networks are prone to failure under such events, and this significantly impacts the daily lives of people in affected areas. It is hence critical that the restoration planning of these power networks be done proactively. Disaster response in power networks is a well-studied problem, especially for pre-and post-event restoration Similar to pre-event, we consider uncertainty associated with the failure paths, and we look into real-time response *while* failures are happening. In this regard, at each time step, we move repair teams towards distribution loads based on their current state, their likelihood to fail, and the impact of the damage in case of node failure. We consider large-scale networks (> 50 nodes and > 20 repair teams) and propose an efficient algorithm to support real-time recovery.

In particular, to address the curse of dimensionality, we design a novel approximate dynamic program that (i) evaluates the future impact of current actions using rollout, (ii) reduces the action space relying on aggregate dynamic programming. The proposed approach is applied to the power distribution network in Aguada municipality, Puerto Rico. Our results show that the proposed rollout approach significantly improves the network service level compared to the base heuristic through prepositioning of the repair crew. Moreover, we find that the performance gap grows larger with the concave restoration function (i.e., a decreasing *Rate of Increase in the Load Service Level* as the recovery progresses) compared to the linear restoration (a constant recovery rate throughout the recovery operation). Finally, the performance gap also grows larger under stronger failure scenarios.

# 1. Introduction

Disasters are causing significant economic losses and social hardship, and, unfortunately, these phenomena are increasingly frequent. In the U.S., the total insured loss was estimated at around 52B\$ for the year 2018 (Anon, 0000a). In response to this situation, government agencies such as FEMA and private utility companies are increasingly engaged in developing the capability to proactively respond to disasters to reduce their impact and long-term effects. FEMA has invested 5.50B\$ in grants on increasing the resiliency of critical infrastructures and communities; out of which 1.3B\$ is specifically granted to protect critical

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infrastructures (Anon, 0000b). Hurricanes are major extreme weather events that occur seasonally in different regions of the country and cause catastrophic damages due to extreme wind and storm surges. Hence, Hurricane planning and response through National Hurricane Program (NHP) initiative is a major component of efforts by FEMA (Anon, 0000c). As part of this initiative, FEMA developed the Hurricane Decision Support Tool (HURREVAC) (Anon, 0000d), a web-based platform that combines real-time forecasts by the National Hurricane Center and insights obtained from evacuation studies to provide evacuation recommendations to local governments. The FEMA Hurricane Liaison Team (HLT) is the operational component of the NHP, which facilitates the information exchange between the National Hurricane Center and the emergency management community. These initiatives are focused on effective evacuation strategies to save lives. However, this research concerns the interesting problem of optimal recovery of critical infrastructures such as power networks. As it will be shown, formalizing the uncertainty of disasters as stochastic scenarios, and using these scenarios to preposition the repair resources in real-time, can significantly increase the network service level.

Within the academic community, disaster response has attracted the attention of many researchers. The problems of interest include, but are not limited to: restoration of transportation infrastructure (Aksu and Ozdamar, 2014), optimal evacuation (Shahparvari et al., 2016; Li et al., 2012), and optimal prepositioning and distribution of relief items (Rezaei-Malek et al., 2016; Torabi et al., 2018; Lu et al., 2016; Caunhye et al., 2016). Specifically, the restoration of power networks as an important critical infrastructure has been the focus of several contributions. Assignment, scheduling, and routing of repair crew under deterministic failures is one focus of the research in the academic community (see Perrier et al. (2013) for a comprehensive review). These contributions do not consider the uncertainty in the failures due to the disaster and look into post-event recovery. Some other researchers take the uncertainty around the disaster into account and propose frameworks composed of several components for the optimal recovery of the power network (the work in Arab et al. (2015a) is an example). However, these frameworks still cannot be leveraged for real-time recovery, which is of interest in this research. In fact, while disaster response in power networks is a well-studied problem, to the best of our knowledge, the current literature does not address the following aspects of the problem jointly: (i) Probabilistic nature of the propagation of the disaster and the resulting failures in the critical infrastructure of interest; (ii) designing a dynamic recovery policy that can consider the state of the network and be applied to support real-time decision making.

The remainder of this paper is organized as follows. In Section 2, relevant literature is critically reviewed, and gaps in the current literature are highlighted, which our research seeks to circumvent. Section 3 details the problem formulation. The methodology is presented in Section 4. The development of a realistic testbed and numerical experiments are discussed in Section 5. We conclude with a summary of results and directions for future research in Section 6.

## 2. Literature review

In this paper, we focus on the real-time restoration of power distribution networks subject to hurricanes. The entity of the loss at the power network level is usually a primary indicator of the magnitude of the disaster phenomenon, and it is a key determinant for how quickly a region will get back to normal operating conditions. Indeed, the power network infrastructure is fundamental for the population welfare and essential to the recovery and humanitarian operations (Abbey et al., 2014). While a broad literature is available in power network analysis and recovery, in the interest of this work, we classify contributions into two main categories: (i) deterministic approaches for post-event power restoration (Section 2.1), and (ii) stochastic approaches for pre-event power network resource management (Section 2.2).

# 2.1. Deterministic approaches for post-event power restoration

One stream of research on disaster response in power distribution networks is focused on the logistics aspects of restoration operations, including crew assignment, scheduling, and routing. Since these optimization problems are large-scale, many contributions in the field propose heuristic or meta-heuristic approaches to solve the problem (Perrier et al., 2013). Several research contributions consider the problem of crew scheduling to recover a power network post-disaster, formulating it as an optimization problem and using different algorithms to solve it (Wu et al., 2009; Arif et al., 2017; Van Hentenryck and Coffrin, 2015; Simon et al., 2012; Van Hentenryck et al., 2011; Coffrin and Van Hentenryck, 2015; Lei et al., 2019). These models do not consider uncertainty and look into static post-disaster recovery.

# 2.2. Stochastic approaches for pre-event power network resource management

In contrast to the above-mentioned deterministic modeling approaches, several studies have considered uncertainty in disaster propagation and the resulting failures in critical infrastructures. Some papers formulated the optimal post-disaster recovery of the power network as stochastic optimization and used heuristic and meta-heuristic algorithms to solve it (Arab et al., 2015b; Xu et al., 2007). While these approaches provide computationally efficient solutions to the power restoration problem and embed the uncertainty deriving from the disaster propagation, they do not consider realistic failure models, and address only post-disaster decisions. Proposing frameworks composed of several components, including failure models, and recovery optimization models is another approach taken in the current literature (Arab et al., 2015a; Ouyang and Duenas-Osorio, 2014). While these frameworks take probabilistic failure scenarios into account to handle pre-hurricane decisions, their optimization approach is static, and once the event starts, they do not allow for updates to the solution based on the current information about the disaster and the network state in real-time. Finally, several contributions formulate the problem of optimal recovery of power networks as dynamic programming and propose approximate methods to solve the problem (Nozhati et al., 2018; Sarkale et al., 2019). However, these models consider only post-disaster decisions.

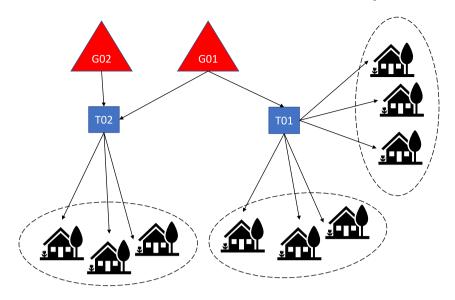


Fig. 1. Example of an abstract power network.

## 2.3. Contributions

The contribution of this paper is two-fold:

- We embed uncertainty in the form of probabilistic propagation of a disaster affecting the power distribution network. Hence, the problem of providing real-time recommendations on prepositioning and assignment of repair resources considering the updated network state and the probabilistic failure scenarios, is formulated as a Stochastic Dynamic Program (SDP).
- In order to tackle the curse of dimensionality resulting from the size of the network, the number of repair teams, and the horizon of the control problem, we propose a new approximate dynamic programming algorithm, which uses the concepts of rollout and aggregate dynamic programming to solve realistic scale problems.

# 3. Problem formulation

When a hurricane starts to unfold, we want to assign and preposition repair crews in a way that minimizes the total future expected cost composed of (i) cost associated with the service loss; (ii) cost associated with moving crews across distribution loads. To estimate such cost, we formalize the future failures as realizations of probabilistic scenarios that are updated as more information on the event becomes available. While the failures primarily affect the distribution network, we will consider "distribution loads" that function as aggregations of the potentially affected areas. Such distribution loads are assigned synthetic demand considering the population of the area.

Fig. 1 depicts an abstract network composed of two generators (triangles), two transformers (squares), and distribution nodes (houses). In this figure, the synthetic loads are formed by aggregation over distribution nodes and shown by dashed ellipsoids. Note that these loads are synthetic entities that are defined to represent the distribution network and model its failures.

# 3.1. Modeling assumptions

The following key assumptions are at the basis of the formulation presented in Section 3.2.

- (1) Several repair teams can be assigned to/prepositioned at a load.
- (2) Assigned/prepositioned teams travel over the shortest *accessible* path between their current location and the location of the target load.
- (3) The demand for electricity at each load, repair time for each load, and travel time between each pair of nodes is known and deterministic.
- (4) The repair teams are identical (in terms of their restoration potential) and operate independently. Thus, when several repair teams are assigned to a load, the rate of increase in load service level will be the sum of the rates of increase in load service due to the recovery operation conducted by each of these teams.
- (5) As repair teams are assigned to a load, the load service level increases gradually, and the increase in the load service level is either linearly or in a non-linear fashion proportionate to the total number of assigned teams (see Section 5.2 for more details).

- (6) The assignments/prepositionings can be modified while the team is on its way to the target load as well as during an ongoing restoration operation.
- (7) The uncertainty around the propagation of the disaster can be formalized as a set of probabilistic failure scenarios, where each scenario lists the probability of failure and failure time for each load.
- (8) While known, the failure scenarios, transportation network, and travel times can be updated as the hurricane propagates, and more recent and accurate information becomes available to decision-makers, and the proposed approach can provide recommendations on assignment and prepositioning in real-time using these updated inputs.

# 3.2. Dynamic programming formulation

We formulate the problem of real-time recovery of the power distribution network described above, as a *stochastic dynamic program* (SDP) where the evolution of the network states is a result of the controllable repair actions and the uncontrollable weather conditions. In our SDP model, the *system state* is fully captured by the condition of the network loads, location of the repair teams, and their current assignment. We consider a discrete-time model, where a *stage* is to be interpreted as time and time progresses according to a fixed time increment. The state of the *i*th load of the network at stage *t* is  $q_{it}$ , the failure magnitude for *i*th load at stage *t*. The state of the *j*th repair team at stage *t* is  $R_{jt} = (l_{jt}, n_{jt}, e_{jt})$ , where  $l_{jt}$  is the team's geographical location at the beginning of period *t*,  $n_{jt}$  is the load *j*th team is assigned to, and  $e_{jt}$  is the number of time periods since arriving at the current geographical location (the start of the repair in case of a busy team). At each stage *t*, the assignment and prepositioning decisions are formulated as follows:

$$x_{ijt} = \begin{cases} 1 & \text{if } j \text{th available repair team is assigned to restore the } i \text{th failed load at stage } t \\ 0 & \text{otherwise.} \end{cases}$$
 
$$y_{ijt} = \begin{cases} 1 & \text{if } j \text{th idle repair team is moved towards } i \text{th load at risk at stage } t \\ 0 & \text{otherwise.} \end{cases}$$

these result in the binary assignment and prepositioning matrices,  $X_t$  and  $Y_t$ , and the optimal actions will minimize the following cost function:

$$V_{t}(Q_{t}, R_{t}, \xi_{t}) = \min_{(X_{t}, Y_{t}) \in A_{t}} C(Q_{t}, R_{t}, X_{t}, Y_{t}) + \mathbb{E}_{\xi_{t}} [V_{t+1}(Q_{t+1}, R_{t+1}, \xi_{t+1})]$$

$$\tag{1}$$

where  $\xi_t$  is a vector of independent Bernoulli random variables (one for each load) whose values indicate whether each load failed at time period t. The parameter p of these Bernoulli distributions are failure probabilities and vary according to the considered failure scenario. Given the repair schedule and the network state, the instantaneous cost in Eq. (1) is:

$$C(Q_t, R_t, X_t, Y_t) = \sum_{i=1}^{N} q_{it}c_i + \sum_{i=1}^{N} \sum_{j=1}^{M} (x_{ijt} + y_{ijt})u_j$$
(2)

where  $q_{it}$  is the failure magnitude for the *i*-th load at stage t,  $c_i$  is the hourly outage cost for the *i*-th load,  $u_j$  is the unit transportation cost for the *j*-th repair team, N is the number of loads, and M is the number of repair teams.

After deriving the optimal assignment and prepositioning decisions, the repair teams are moved towards the selected node within the transportation network, which is located on the shortest path between the current location of the repair team and the location of the temporarily assigned load. To update the state, at each time step, we define the *transition function*, which takes as input the current system state, including the realized failure(s) and, accounting for the actions, returns the system state at the next stage. For the power network recovery problem, we defined separate transition functions for network loads and repair teams. For each load, the state transition function can be written as:

$$q_{i(t+1)} = \begin{cases} 1 & \text{If } f_{it} = 1\\ q_{it} - \frac{a_{it}}{r_i} & \text{If } f_{it} = 0 \end{cases}$$
 (3)

where  $f_{it}$  is the realized value of the failure Bernoulli random variable,  $\xi_{it}$ , and  $a_{it}$  is the number of teams involved in the recovery operation, both for the ith load at ith time period. Note that, in Eq. (3), the first case refers to the functional loads that have failed at stage i, while the second case refers to all other loads. For each repair team, the state transition function can be formalized as:

$$(l_{j(t+1)}, n_{j(t+1)}, e_{j(t+1)}) = \begin{cases} (b_{l_{jt}}, l_i, 0) & \text{If } \exists \ i \in N | x_{ij} = 1 \\ (b_{l_{jt}}, l_i, \emptyset, 0) & \text{If } \exists \ i \in N | y_{ij} = 1 \\ (l_{jt}, n_{jt}, e_{jt} + 1) & \text{If } l_{jt} = l_{n_{jt}} \\ (b_{l_{jt}}, l_{n_{jt}}, n_{jt}, 0) & \text{If } l_{jt} \neq l_{n_{jt}}, n_{jt} \neq \emptyset \\ (l_{jt}, \emptyset, e_{jt} + 1) & \text{If } n_{jt} = \emptyset \end{cases}$$

$$(4)$$

where  $b_{g,h}$  is the next location, with respect to the gth node of the transportation network, located on the shortest path to the hth node of the transportation network, and  $l_i$  is the location of the ith load. Note that, in Eq. (4), the first case refers to repair teams that are assigned to a new failed load. The second case refers to repair teams that are moving due to the prepositioning action taken. The third case refers to repair teams that are engaged in an ongoing restoration operation and are not assigned to a different failed load. The fourth case refers to repair teams that are assigned to a failed load, but are still on their way to the target load. Finally, the fifth case refers to free repair teams that are neither assigned nor prepositioned.

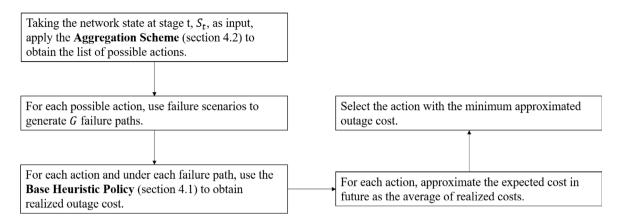


Fig. 2. Steps of the proposed algorithm.

# 4. Methodology

The dynamic programming optimization problem formulated in Section 3.1, can be solved exactly using the stochastic dynamic programming algorithm, originally introduced by Bellman (1957). However, to evaluate the expected cost in the future, this algorithm needs to explore all the possible states that the system can transition to, which makes the problem computationally prohibitive in realistic sizes. In this paper, we use the rollout method to solve our problem (Bertsekas and Tsitsiklis, 1996; Bertsekas, 2019, 2020).

In fact, the rollout heuristic can be used to approximate the expected cost controlling the computational effort. Furthermore, an aggregation scheme in the action space is proposed to reduce the number of possible actions. The steps of the proposed rollout approach are shown in Fig. 2. It should be noted that rollout is an established heuristic method for solving large-scale dynamic programming problems by approximating the expected cost in the future. The contribution of this research is proposing a computationally efficient base heuristic to approximate the expected cost in the future (Section 4.1) and an optimization-based aggregation scheme in action space to reduce the number of possible actions, both within the rollout heuristic approach (Section 4.2).

The rollout method was first introduced to solve discrete optimization problems in the paper by Bertsekas et al. (see Bertsekas et al. (1997)), and the book by Bertsekas and Tsitsiklis (see Bertsekas and Tsitsiklis (1996)). Rollout aims to find a solution of the general problem of minimizing a function F(a) over all  $a = (a_1, \ldots, a_T)$  satisfying certain constraints. The components  $a_1, \ldots, a_T$  can take a finite number of values, so this is a difficult discrete optimization. Rollout aims to find a sub-optimal solution that improves over the solution produced by a given algorithm, called the base heuristic (in our case, the base heuristic is presented in Section 4.1). This is accomplished by a sequence of minimizations of cost functions that are defined by F(a) and by partial solutions produced by the base heuristic.

Our assumption is that, given any partial solution  $(\tilde{a}_1, \dots, \tilde{a}_{t-1})$ , and any feasible value  $a_t$ , the base heuristic produces a feasible complete solution of the form

$$(\tilde{a}_1, \dots, \tilde{a}_{t-1}, a_t, \hat{a}_{t+1}, \dots, \hat{a}_T),$$
 (5)

by constructing the complementary sequence  $(\hat{a}_{t+1}, \dots, \hat{a}_T)$  based on the knowledge of  $(\tilde{a}_1, \dots, \tilde{a}_{t-1}, a_t)$ . We also assume that the base heuristic can be applied to generate a feasible solution

$$(\hat{a}_1, \dots, \hat{a}_T),$$
 (6)

starting from the empty partial sequence. Initially, the rollout algorithm uses the base heuristic to generate, for each feasible value  $a_1$ , the complementary sequence  $(\hat{a}_2, \dots, \hat{a}_T)$ , and computes the initial solution component  $\tilde{a}_1$  as

$$\tilde{a}_1 \in \arg\min_{a_1 \in A_1} F(a_1, \hat{a}_2, \dots, \hat{a}_T).$$

Where  $A_1$  is the set of feasible solutions at the first iteration. Then, sequentially for every iteration  $t \ge 2$ , given the partial solution  $(\tilde{a}_1, \dots, \tilde{a}_{t-1})$ , the rollout algorithm, considers all feasible values of  $a_t$  and applies the base heuristic to generate the complete solution

$$(\tilde{a}_1, \dots, \tilde{a}_{t-1}, u_t, \hat{u}_{t+1}, \dots, \hat{u}_T).$$

It then computes the value of  $a_t$  that minimizes the cost function over all these complete solutions:

$$\tilde{a}_t \in \arg\min_{a_t} F(\tilde{a}_1, \dots, \tilde{a}_{t-1}, a_t, \hat{a}_{t+1}, \dots, \hat{a}_T),$$

and fixes  $a_t$  at the computed value  $\tilde{a}_t$ . It then repeats with  $(\tilde{a}_1,\ldots,\tilde{a}_{t-1})$  replaced by  $(\tilde{a}_1,\ldots,\tilde{a}_t)$ . After T steps the rollout algorithm produces the complete sequence

$$\tilde{a} = (\tilde{a}_1, \dots, \tilde{a}_T)$$

which is called the rollout solution. The fundamental result underlying the rollout algorithm is that under certain assumptions, we have cost improvement, i.e.,

$$F(\tilde{a}_1, \dots, \tilde{a}_T) \le F(\hat{a}_1, \dots, \hat{a}_T),\tag{7}$$

where  $(\hat{a}_1, \dots, \hat{a}_T)$  is the solution produced by the base heuristic starting with the empty partial solution (cf. Eq. (7)). Even when the assumptions needed for cost improvement are not satisfied, a simple modification, the so-called fortified rollout algorithm, produces a modified sequence that satisfies the cost improvement property in Eq. (7). Algorithm 1 summarizes the approach applied to the sequential recovery problem.

# Algorithm 1: Stochastic Rollout for Optimal Recovery

```
Result: \left\{ a_t = \left[ \left( (x_{ijt})_{i=1}^N \right)_{j=1}^M, \left( (y_{ijt})_{i=1}^N \right)_{j=1}^M \right] \right\}, t = 1, ..., T
Input: Feasible actions \mathbf{a} \in \mathbf{A}, horizon length T, number of failure paths G;
Initialize: load state q_{i0}, i = 1..., N, teams state r_{i0} = (l_{i0}, n_{i0}, e_{i0}), j = 1..., M t \leftarrow 1;
while t \le T do
     for a_t \in \mathcal{U}_t do
           Obtain instantaneous cost C_t(a_t) using Eq. (2);
           for g = 1 to G do
                 Generate failure path, \Xi^g;
                 for k = t + 1, ..., T do
                     Use a_{k-1}^g to derive \left(q_{i,k}^g, r_{j,k}^g\right), i=1,\ldots,N; j=1,\ldots,M with Eqn. (3)–(4); Use the <u>base heuristic</u> to derive a_k^g;
                     Use a_k^g to calculate h_k^g = C_k(a_k^g) using Eq. (2);
                 end
           end
           Approximate the expected cost as E\left(V_{t+1}\right) \approx H\left(a_{t}\right) = \frac{1}{G(T-t)} \sum_{k=t}^{T} \sum_{g=1}^{G} h_{k}^{g};
      Set a_t \in \arg\min (C_t(a_t) + H(a_t));
     t \leftarrow t + 1;
end
```

# 4.1. Base heuristic design

The design of a good heuristic policy is crucial to the performance of the rollout algorithm (Bertsekas, 1995). However, heuristic policies are greedy problem-dependent decision rules that try to minimize the cost locally.

In our case, the basic idea is to solve an optimal assignment problem for the one-step restoration maximization. Considering the set of failed loads and the available repair teams according to the current network state, this optimization model can be used to determine the number of repair teams to be assigned to each of the failed loads. Decision variables, objective function, and the constraints of this optimization model are given below.

# Decision Variables:

 $x_i$  = Number of repair teams assigned to the *i*th load;

# Objective and Constraints:

$$\max Z = \sum_{i=1}^{N} o_i x_i$$
Subject to:

$$\sum_{i=1}^{N} x_i \le M \tag{9}$$

$$x_i \le r_i q_i \ \forall i \in \{1, 2, \dots, N\}.$$
 (10)

In the objective function,  $o_i$  is the increase in load service level for ith load if only one repair team is assigned to that load. Thus, the base heuristic aims at maximizing the increase in the network service level at the next stage. Constraint (9) ensures that the total number of assigned teams does not exceed the number of available teams. Constraints (10) ensure that the number of assigned repair teams to each load does not exceed the total required workforce to recover that load fully. This base heuristic can alleviate the curse of dimensionality by approximating the expected cost in the future. However, for realistic problems with a large number of possible actions, this approximation is computationally prohibitive. Specifically, starting from a fully functional distribution network with N

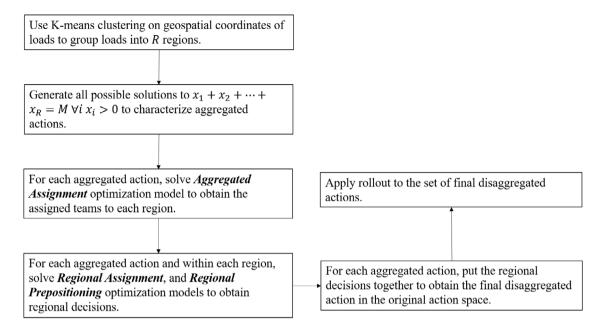


Fig. 3. Steps of the proposed aggregation scheme in action space.

loads and assuming M available repair teams, the number of possible actions would be  $N^M$ , which is enormous even for reasonably small values of N and M. In the following sub-sections, we use aggregation to reduce the size of this mathematical programming model dramatically.

# 4.2. Aggregation in action space

The aggregation in the action space can effectively reduce the overall size of the problem in (8), by drastically reducing the set of feasible actions. To this end, first, considering the loads' geospatial coordinates, we use k-means clustering algorithm (Lloyd, 1982) to group the N loads into R regions. With R regions, the aggregated actions are characterized as the number of repair teams assigned to each region. Assuming that at least one repair team is assigned to each region, the set of  $\binom{R-1}{M-1}$  feasible aggregated actions satisfies:

$$x_1 + x_2 + \dots + x_R = M : x_k > 0 \ \forall k.$$

These aggregated actions cannot dictate the assignment and prepositioning decisions in the original action space directly. Thus, we transform each action in the aggregated space into a corresponding action in the original space by taking the following three steps:

- 1. For each aggregated action, taking the number of teams assigned to each region as input, solve the *Aggregated Assignment Model* to obtain the set of teams assigned to each region;
- 2. For each aggregated action, and within each region, solve the *Regional Assignment Model*, and *Regional Prepositioning Model* to obtain the prepositioning and assignment decisions within each region (regional decisions).
- 3. For each aggregated action, integrate regional decisions together to obtain the corresponding action in the original space.

The above steps are summarized in Fig. 3.

Aggregated assignment model. Taking the number of assigned teams to each region as input, the solution to this optimization model determines the assignment of each repair team to a specific region. The decision variables, objective function, and constraints of this optimization model are given below.

# Decision Variables:

$$u_{jk} = \begin{cases} 1 & \text{if } thej \text{th repair team is assigned to the } k \text{th region} \\ 0 & \text{otherwise.} \end{cases}$$

# Objective and Constraints:

$$\min Z = \sum_{j=1}^{M} \sum_{k=1}^{R} u_{jk} \bar{t}_{jk} \tag{11}$$

$$\sum_{k=1}^{R} u_{jk} = 1 \,\forall j \in \{1, 2, \dots, M\}. \tag{12}$$

$$\sum_{j=1}^{M} u_{jk} = a_k \forall k \in \{1, 2, \dots, R\}.$$
(13)

where  $\bar{t}_{ik}$  is the weighted average of travel time between the current location of the jth repair team and the loads in the kth region with weights being the demand share multiplied by the probability of failure for each load, and  $a_k$  is the number of repair teams assigned to the kth region according to the aggregated action. The objective function minimizes the expected total loss of service during the time the repair teams are on their way to the target loads.

Constraints (12) ensure that each repair team is assigned to only one region, while constraints (13) ensure that the number of assigned repair teams to each region is equal to what is dictated by the given aggregated action.

Regional assignment model. Considering the loads and the repair teams within each specific region, the solution to this optimization model gives the optimal regional assignment decision. Note that for each region, a separate optimization model should be formulated and solved. The decision variables, objective function, and constraints of this optimization model are given below.

 $v_{ij} = \begin{cases} 1 & \text{if the } j \text{th repair team is assigned to the } i \text{th load within the } k \text{th region} \\ 0 & \text{otherwise.} \end{cases}$ 

# Objective and Constraints:

$$\max Z = \sum_{i \in M_k} \sum_{j \in N_k} v_{ij} (o_i - \epsilon t_{l_i, l_{j0}})$$
(14)

$$\sum_{i \in N_k} v_{ij} \le 1 \ \forall j \in M_k. \tag{15}$$

$$\sum_{i \in M_k} v_{ij} \le r_i q_i \ \forall i \in N_k. \tag{16}$$

where  $N_k$  is the set of loads and  $M_k$  is the set of repair teams both within the kth region,  $o_i$  is the rate of increase in service level for the *i*th load, and  $t_{l_i,l_{j0}}$  is the travel time between the location of the *i*th load and the location of the *j*th team at the initial stage. The objective function maximizes the increase in the network service level at the next stage. But, the increase in the network service level is adjusted by a small factor of total travel time so that the model prefers solutions with smaller travel time when two or more solutions lead to the same increase in the network service level. Constraints (15) ensure that each team is assigned to at most one of the region loads, while constraints (16) ensure that the number of assigned teams to each load does not exceed the required workforce to recover that load.

Regional prepositioning model. For each region, if after solving the Regional Assignment Model presented above, any idle repair team is left, the Regional Prepositioning Model is solved to obtain the optimal regional prepositioning decisions. Note that, for each region, one optimization model should be formulated and solved. The decision variables, objective function, and constraints of this model are given below.

# Decision Variables:

 $w_{ij} = \begin{cases} 1 & \text{if } the \ j-\text{th repair team is prepositioned at the } i-\text{th load within the } k-\text{th region} \\ 0 & \text{otherwise.} \end{cases}$ 

# Objective and Constraints:

$$\max Z = \sum_{i \in N_k^f} \sum_{j \in M_k^a} w_{ij} (o_i p_i - \epsilon t_{l_i, l_{j0}})$$

$$\tag{17}$$

Subject to: 
$$\sum_{i \in N_k^f} w_{ij} \le 1 \ \forall j \in M_k^a. \tag{18}$$

$$\sum_{j \in M_a^n} w_{ij} \le r_i \ \forall i \in N_k^f. \tag{19}$$

where  $N_k^f$  is the set of functional loads within the kth region,  $M_k^a$  is the set of available repair teams within the kth region that are not assigned to any failed load,  $o_i$  is the rate of increase in service level for the *i*th load,  $p_i$  is the probability of failure for the *i*th load, and  $t_{l_i,l_m}$  is the travel time between the location of the *i*th load and the location of the *j*th team at the initial stage. The objective function maximizes the total expected outage cost that can be avoided, but it is adjusted by a small factor of transportation time such that when two or more solutions have equal scores, the model picks the solution with minimum total travel time. Constraints (18) ensure that each team is prepositioned at only one of the region loads if any, while constraints (19) ensure that the number of repair teams prepositioned at each load does not exceed the required workforce to recover that load.

# 4.3. Computational complexity

In previous sections, the proposed rollout approach is claimed to be able to approximate the expected cost in the future efficiently. Also, we mentioned that the proposed aggregation scheme in action space reduces the computational time of the proposed rollout approach. In this section, the computational complexity of the exact SDP algorithm, rollout algorithm, and rollout with aggregation in action space is analyzed to quantify and show these computational savings.

# 4.3.1. Stochastic dynamic programming

To calculate the exact value of expected cost in the future, at each stage, and starting from each possible state, all possible actions have to be considered and for all of these actions, the resulting states at the next stages should be obtained and this is repeated at all subsequent stages. It can be shown that an upper bound on the number of possible states at the last stage is  $N^{MT}$  where N is the number of loads, M is the number of repair teams, and T is the number of stages. As a result, an upper bound on the number of operations required to solve the problem at hand by the exact SDP algorithm will be  $TN^{MT}$ . Therefore, the worst-case computational complexity of the exact SDP algorithm will be  $O(TN^{MT})$ .

# 4.3.2. Rollout

Rollout approximates the expected cost function for each possible action at the initial state. The number of such actions is  $N^M$ . To approximate the cost function for each action, the base heuristic should be adopted at each stage (T times). To this end, the loads can be sorted in the decreasing order of their rate of increase in the load service level and then starting from the first load in the list, the repair teams can be greedily until no repair team is left. The worst-case performance of such an algorithm will be  $O(N^2)$ . Thus, the computational complexity of rollout will be  $O(TN^M)$ . As it can be observed, the rollout can exponentially decrease the computational time compared to the exact SDP and this allows for solving the realistic size problems approximately.

# 4.3.3. Rollout with aggregation in action space

With aggregation in action space, the cost function will be approximated for only  $\binom{R-1}{M-1}$  solutions. Assuming only 2 regions, the number of solutions will be M-1. Thus the computational complexity of the algorithm will be  $O(MTN^2)$ . Again compared to the original rollout, aggregation in action space can exponentially decrease the computational complexity and effectively scale the proposed approach.

# 5. A case study in Puerto Rico

In this section, based on the solution methodology discussed in Section 4, the aim is to develop a realistic testbed for the optimal recovery of the power distribution network in Aguada municipality, Puerto Rico, subject to synthetic failure scenarios.

# 5.1. Model inputs estimation

In this subsection, the estimation process for model input parameters, i.e., demand, restoration workforce, and travel time, is detailed. Without loss of generality, we focus our study on the Aguada municipality because the population in this municipality, is in 60th percentile of municipality populations in Puerto Rico, and as a result, it is a representative municipality in terms of population.

Focusing the analysis on an individual municipality is based on the assumption that the recovery operations at each municipality are handled independently, which is not necessarily realistic. However, to apply the proposed approach in practice, the level of granularity for the region can be set such that this assumption holds true.

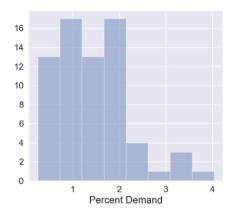
The power authority in Puerto Rico (PREPA) (Anon, 0000e) shared with our research team, the data on repair crew deployment to restore the power distribution network after hurricane Maria (Tormos-Aponte et al., 0000). According to these data, during hurricane Maria, the repair teams were deployed in 69 different geographical locations in Aguada municipality visualized on the map in Fig. 4(a). We used the information on repair locations to create a synthetic position for each load as an input to our restoration algorithm. We highlight that this is an approximation under the assumption that *all* main loads failed during the Maria event.

Given the map, we established the travel time matrix. In particular, we used the *Haversine distance* between each pair of loads as a proxy of the travel distance. We then transformed this into the team travel time, assuming that the average travel speed for repair teams is 10 Mph. In this regard, a low average speed was considered because the actual distance is usually larger due to the road network, and the road conditions are particularly critical during disasters.

Concerning the demand, we assume that the demand for electricity at each load is proportionate to the population within the region surrounding the load. Such regions were determined by covering Aguada by a partition of simplexes of equal size, each centered in one of the 69 locations. With these assumptions, we used the *Facebook population density data* (Anon, 0000f), and estimated the demand. The demand share (in percentages) for different loads is shown in Fig. 4(b).



(a) Location of the assumed loads.



(b) Electricity demand share for different loads.

Fig. 4. Examples of input data for testing.

To approximate the required workforce to recover each load, we used the **repair crew data** shared by PREPA. While this manuscript does not make direct use of these data, we used the information on the number of teams and deployment dates to infer the total required workforce to restore each of the 69 loads in Aguada. Next, considering these values, we computed the ratio between the synthetic demand share data and the required workforce values inferred from the PREPA repair data. Such ratio is intuitively an estimate of the *rate of increase in load service level* for each load during hurricane Maria. We refer to the average of these *rate of increase in load service level* values as the *reference rate of increase in the load service level*. Subsequently, we used such reference recovery rate to transform the demand share data into *required workforce levels*, defined as the ratio between the demand share and the reference *rate of increase in the load service level*. Moving from demand share to the required workforce through the reference recovery rate allowed us to create a more homogeneous distribution of the required workforce and reduce the dependency on the original repair crew deployment data set. Nonetheless, we highlight that users of our platform may have accurate repair and power demand data.

As a result of our approach, all the loads are characterized by the same *rate of increase in the load service level* under linear restoration. Equivalently, considering the required workforce, a specified number of repair teams leads to the same increase in the service level regardless of the load to which these teams are assigned if a linear restoration function is assumed (see Section 5.2 for more details).

# 5.2. Modeling alternative restoration functions

The restoration function characterizes the relationship between the assigned workforce and the increase in the load service level. Particularly, this function takes the ratio of the assigned workforce to the total required workforce as input, and outputs the load service level. Herein, we assume two different restoration functions: *linear restoration*, and *concave restoration*. The mathematical form and the underlying assumptions of these restoration functions follow.

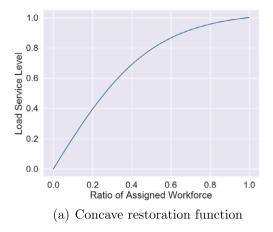
*Linear restoration function.* With this restoration function, the load service level is assumed to be linearly proportionate to the total assigned workforce. Under linear restoration function, the relationship between the load service level and the ratio of the assigned workforce can be written as:

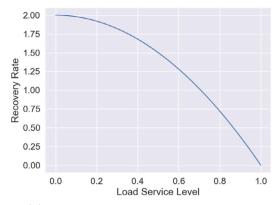
$$s_{it} = \frac{a_{it}}{r_i} \tag{20}$$

where  $s_{it}$  is the service level for the *i*th load at time period t,  $a_{it}$  is the total number of team-hours deployed at the *i*th load until time period t, and  $r_i$  is the required workforce to recover the *i*th load. The rate of increase in load service level, is the derivative of the restoration function with respect to the total number of assigned team-hours. For the linear restoration function we have:

$$\frac{\partial s_{it}}{\partial a_{it}} = \frac{1}{r_i} \tag{21}$$

In practice, this implies that with linear restoration function, the rate of increase in the load service level is constant throughout the recovery period.





(b) Rate of increase in the load service level

Fig. 5. Service level and rate of increase in service level for the concave restoration function.

Concave restoration function. A more realistic restoration function should consider the fact that the recovery operation is done at a faster rate initially, and the *the rate of increase in the load service level* decreases as the restoration operation progresses. To this end, we need a concave function that takes  $a_{ii} \in [0,1]$  as input, and outputs  $s_{ii} \in [0,1]$ . Starting from *Sigmoid* function, herein we use the following concave function to serve this purpose. This adjustment changes the range of *Sigmoid* function from  $(-\infty, \infty)$  to [0,1].

The function curve is shown in Fig. 5(a)

$$s_{it} = \frac{2}{1 + e^{\left(\frac{-4a_{it}}{r_i}\right)}} - 1 \tag{22}$$

Similar to linear case, *the rate of increase in load service level* would be the derivative of restoration function with respect to the total number of assigned teams-hours,  $a_{ii}$ , which can be calculated as:

$$\frac{ds_{it}}{\partial a_{it}} = \frac{8}{\left(e^{\frac{-4a_{it}}{r_i}}\right)r_i} \tag{23}$$

While the *rate of increase in load service level* is a function of a total number of assigned team-hours,  $a_{it}$ , with system state, we only keep track of load service level,  $s_{it}$ . The inverse of the restoration function can be used to obtain  $a_{it}$  taking  $s_{it}$  as input. The recovery rate can be rewritten as a function of the load service level as follows:

$$\frac{\partial s_{it}}{\partial a_{it}} = 2\left(1 - s_{it}\right)\left(1 + s_{it}\right) \tag{24}$$

The resulting rate of the increase in the load service level curve is shown in Fig. 5(b). From this figure, it can be observed that with the concave restoration function, as the recovery operation progresses, the rate of increase in the load service level decreases.

It is worth noting that the linear restoration function is based on the assumption that the *rate of increase in the load service level* of a load does not change over the course of recovery operation and the concave restoration function on the other hand assumes decreasing rate of increase in service level. Neither of these underlying assumptions and as a result, none of these restoration functions, are necessary realistic. Also, validating any restoration function, needs highly granular data on network recovery which was not at hand. Nonetheless, these restoration functions are used in the current literature (Zhang et al., 2017; Cimellaro et al., 2010).

# 5.3. Numerical experiments

This section evaluates the proposed rollout approach by comparing its performance to that of the base heuristic through conducting numerical experiments using the Aguada testbed developed in Section 5.1. Three failure scenarios, three travel time scenarios, and three different demand scenarios were considered. Under each scenario, the failure probabilities, travel time between loads, and demand for power are sampled from a uniform distribution but with different parameters as shown in Table 1. At each level of these factors, 25 different sample problems were generated and to control the variance, the same random seed was used at each level of factors. For each sample problem, the base heuristic policy (Section 4.2) and the rollout policy were adopted assuming 25 repair teams and 100 time periods (hours), and the network service level trajectory obtained. The experiments were conducted with both *linear* and *concave* restoration functions introduced in Section 5.2. The code to conduct the experiments was implemented in Python 3.7 (Van Rossum and Drake Jr., 1995), Gurobi 8.1 Python API (Gurobi Optimization, 2019) was used to solve the optimization problems, and K-means clustering algorithm (MacQueen et al., 1967) implemented in Python Sklearn

Table 1
Distribution scenarios for travel time, demand for power, and failure probability.

Parameter	Number of levels	Levels
Travel time	3	U[4, 6], U[3, 7], U[1, 9]
Demand for power	3	U[4, 6], U[3, 7], U[1, 9]
Failure probability	3	U[0.4, 0.6], U[0.6, 0.8], U[0.8, 1]

package (Pedregosa et al., 2011) was used to form the regions taking the geospatial coordinates of the loads as an input. Finally, the experiments were run on Arizona State University Research Computing HPC Cluster.

To demonstrate the experimental results, we defined *Percent increase in network service level with rollout*,  $\Delta_n^r$ , as follows:

$$\Delta_{gt}^{r} = \frac{s_{gt}^{r} - s_{gt}^{b}}{s_{gt}^{b}} \times 100\%$$
 (25)

where  $s_{gt}^b$  is the network service level under the base heuristic policy, and  $s_{gt}^r$  is the network service level under the rollout policy, both under the gth failure path and at tth time period.

To calculate the network service level, with linear (concave) restoration function, Eq. (22) (Eq. (24)) used to obtain the load service level for each load. Then the network service level is calculated as the average of these load service levels weighted by normalized demand for power at each load.

It is worth noting that rollout is guaranteed to beat the base heuristic. Nonetheless, many researchers use the base heuristic as baseline (Bertsekas et al., 1997; Sarkale et al., 2018; Secomandi, 2003; Ciavotta et al., 2016), as due to the curse of dimensionality, performance comparison with the exact SDP algorithm (which is guaranteed to outperform rollout) is not possible.

Fig. 6(a) shows the boxplot of the Percent increase in the network service level with rollout with different average failure probabilities and restoration functions. From this figure, the higher the average failure probability, the larger the performance gap between the base heuristic, and rollout. This holds true with both linear and concave restoration functions, but the gap grows even larger with the concave restoration function. This implies that under strong disasters, prepositioning of the repair crew becomes more critical, and can lead to a more significant improvement in the network service level, especially with the concave restoration function. Fig. 6(b) shows the boxplot of the performance metric at different levels of demand variance and for different restoration functions. From this figure, the demand variability does not significantly change the distribution of the performance metric, for both linear and concave restoration functions. Thus, it can be concluded that the proposed solution approach can perform satisfactorily and lead to significant increases in the network service levels under different levels of variability in demand for power. Fig. 6(c) shows the boxplot of the performance metric, for different levels of travel time variance. From this figure, the variability of travel time does not change the distribution of the performance metric significantly, and this is valid for both linear and concave restoration functions. Therefore, the proposed solution is also robust against changes in the variability of travel time.

# 6. Conclusions and future research

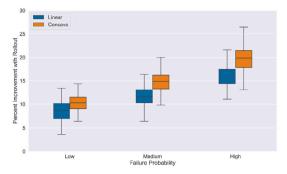
In this paper, we formulated the problem of proactive recovery of power networks as dynamic programming under uncertainty. The proposed model can provide prepositioning and assignment decisions in a dynamic, real-time fashion. However, due to the curse of dimensionality, it cannot be solved exactly for realistic problem sizes.

To deal with the curse of dimensionality, we proposed an efficient rollout policy, and an aggregation scheme in action space to make the proposed approach computationally more efficient. A realistic testbed for the power distribution network in Aguada municipality, Puerto Rico, subject to synthetic failure scenarios, was developed to demonstrate the applicability of the proposed approach.

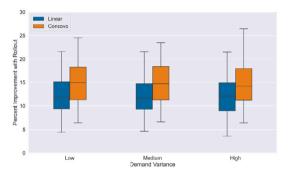
Numerical experiments showed that with both linear and concave restoration functions, the proposed rollout policy offers a significant improvement in network service level over the base heuristic policy through prepositioning. However, the performance gap grows larger with stronger failure scenarios. This implies that under more intense failure scenarios, modeling the failure uncertainty in the form of probabilistic failure scenarios and using them to preposition the repair teams can lead to a more significant improvement in the power distribution network service level. Also, the performance gap increases with the concave restoration function.

In this paper, the power distribution network was modeled with a set of synthetic loads; thus, a natural extension is to develop high-fidelity simulation models for the power distribution network and use them to evaluate the proposed approach. Also, in this research, Facebook Population Density data (Anon, 0000f) was used to estimate the demand at each load. Developing more accurate methods to estimate the demand for electricity at each load is another extension of this research.

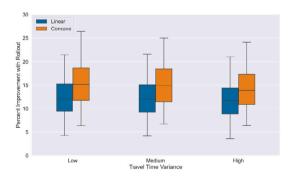
Moreover, the travel time between loads were calculated assuming a fixed average speed and using the Haversine distance between the centroid of those loads. However, in reality, the travel time is a function of the available road network, free-flow speed, and traffic congestion which in turn is a function of the date, and the time of the day. Thus, more realistic models for travel time is another interesting extension of this research. While our dynamic model can take the updated travel time and repair time values as input whenever it is deployed to return the prepositioning and assignment decisions, it still assumes known and deterministic travel time and repair time. Therefore, probabilistic modeling of uncertainty in these input parameters is another promising direction for future research.



(a) Boxplot of the performance metric for different levels of average failure probability



(b) Boxplot of the performance metric for different levels of demand variance



(c) Boxplot of the performance metric for different levels of travel time variance

Fig. 6. Service level and rate of increase in service level for the concave restoration function.

To conduct the numerical experiments, we used synthetic failure scenarios. Thus, developing more realistic failure scenarios by training predictive or simulation models on historical weather as well as power outage data is another interesting avenue for future research. Moreover, while our proposed approach was focused on maximizing the network service level, the same amount of power loss could dictate different levels of social hardship in different regions. This calls for quantifying and considering social hardship to formulate and solve the problem of optimal recovery of power distribution network as a multi-objective optimization as another valuable extension to this paper.

Finally, to develop the proposed model we assumed identical repair teams which could be an unrealistic assumption. Thus, extending the modeling approach to account for several types of teams with different *rate of increase in the load service level* values is another interesting direction for future research.

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# Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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