Towards a unified treatment of $\Delta S = 0$ parity violation in low-energy nuclear processes

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We revisit the unified treatment of low-energy hadronic parity violation espoused by Desplanques, Donoghue, and Holstein to the end of an *ab initio* treatment of parity violation in low-energy nuclear processes within the Standard Model. We use our improved effective Hamiltonian and precise non-perturbative assessments of the quark charges of the nucleon within lattice QCD to make new assessments of the parity-violating meson-nucleon coupling constants. Comparing with recent, precise measurements of hadronic parity violation in few-body nuclear reactions, we find improved agreement with these experimental results, though some tensions remain. We thus note the broader problem of comparing low-energy constants from nuclear and few-nucleon systems, considering, too, unresolved theoretical issues in connecting an *ab initio*, effective Hamiltonian approach to chiral effective theories. We note how future experiments and lattice QCD studies could sharpen the emerging picture, promoting the study of hadronic parity violation as a laboratory for testing "end-to-end" theoretical descriptions of weak processes in hadrons and nuclei at low energies.

I. INTRODUCTION

In spite of decades of research, hadronic parity violation in flavor non-changing processes remains poorly understood [1–6]. The pertinent body of experimental work involves the low-energy interactions of hadrons and nuclei, so that we are compelled to address the interplay of the physics of the weak interaction and of nonperturbative strong dynamics. Ultimately we hope that this problem can be largely conquered once the direct computation of two-nucleon matrix elements of a suitable effective Hamiltonian within lattice QCD (LQCD) becomes possible [7], though, as we shall see, there are further issues to address. As an interim step, we revisit the unified treatment of hadronic parity violation by Desplanques, Donoghue, and Holstein (DDH) [1]. There, the description of low-energy hadronic parity violation is framed within an one-meson-exchange model, and DDH show that it is possible to compute the appropriate meson-nucleon coupling constants starting from the Standard Model (SM) Lagrangian. Since that early work, powerful field theoretic treatments exploiting the low-energy symmetries of QCD have been developed and applied to the analysis of hadronic parity violation [4, 6, 8–13]. Yet in these chiral effective field theory treatments, organized in terms of hadronic degrees of freedom, the effective couplings are determined from experiment, and the underlying theoretical connection to QCD and the SM is lost. We note, however, nascent work that would compute the parity-violating pion-nucleon constant in an *ab initio* way [14–16]. Here we assess the current status of this problem by revisiting and updating the treatment of DDH. Namely, we employ our improved effective Hamiltonian [17] to compute the parity-violating meson-nucleon coupling constants, using the factorization approximation (as it is now employed [18]) and LQCD assessments of the quark flavor charges of the nucleon [19]. Our particular purpose is to see how these updated assessments combine to confront the constraints on these parameters from precise experimental measurements of hadronic parity violation in few-body nuclear systems, namely, from the NPDGamma [20] and n3He [21] collaborations that measure the parity-violating asymmetry from neutron-spin reversal in the $\vec{n} + p \rightarrow d + \gamma$ and in $\vec{n} + {}^3\text{He} \rightarrow t + p$ reactions, respectively.

The NPDGamma measurement is particularly sensitive to the parity-violating pion-nucleon coupling, whereas that made by the n3He collaboration also probes four-nucleon contact interactions of isoscalar and isovector character, which we interpret in terms of contributions from vector-meson exchanges between nucleons. Much of the past theoretical effort has concentrated on studying charged pion-nucleon interactions, due to a longstanding notion of its dominance in hadronic-parity-violating observables [1]. However, noting the non-observation of parity violation in ${}^{18}\text{F}$ radiative decay [22–24], and thus finding no clear sign of this dominance, and with the direct theoretical analysis of nucleon-nucleon (NN) amplitudes in pionless effective field theory (EFT) in the large number of colors (N_c) limit showing that isoscalar and isotensor interactions should play driving phenomenological roles [5, 20, 21, 25, 26], we believe the contributions from all isosectors should be computed. Earlier studies of QCD evolution effects have either made calculational approximations [1, 27, 28], or focused on the isovector case [29–31]. We note, for example, that the original estimates of parity-violating meson-nucleon couplings were performed with a low-energy Hamiltonian built using phenomenological K-factors to account for QCD evolution effects on weak processes [1]. In this work, we employ our low-energy effective Hamiltonian [17], which makes a complete renormalization group evolution in leading-order QCD, with matching across heavy-flavor thresholds, to give a unified treatment of all three isosectors in order to compare their contributions to recent experimental measurements.

Powerful searches for physics beyond the SM can be made through low-energy, precision measurements of symmetry-breaking effects in nucleons and nuclei [32, 33]. For example, in the case of searches for permanent electric dipole moments, for neutrinoless double β decay, or for $\mu \to e$ conversion on nuclear targets, the expected SM contribution is either negligibly small with respect to current experimental sensitivities or altogether absent. Thus the discovery of significantly non-zero results in these systems would signal the existence of physics beyond the SM. Here theory is key to assessing the relative sensitivity of different nuclear systems to the effects of interest. Theory is also essential to the interpretation of a non-zero experimental result or limit in terms of the parameters of an underlying new physics model — and, more broadly, to using the experimental limit to estimate a lower bound on the energy scale of new physics, assuming that it lies beyond the weak scale. A theoretical analysis that connects the scale of new physics to that of the pertinent low-energy experiments requires the consideration of multiple physical scales, and "end-to-end" effective-field-theory treatments are being developed to accomplish that [34–36]. In this context, we believe QCD studies of hadronic parity violation have a crucial role to play in the benchmarking of these treatments, because its observables are not only nonzero within the SM but also, given the success of the SM in describing ultra-low energy, parity-violating electron-nucleon interactions [37], new-physics effects presumably play a subdominant role. Thus the comparison of theory and experiment in hadronic parity violation provides a welcome test of the overall theoretical framework, as such tests possess aspects common to new-physics searches as well.

In this paper, we embark on this program by determining the parity-violating meson-nucleon coupling constants at a renormalization scale of 2 GeV, and, as we shall detail, we find improved agreement with the experimental results. Although better agreement speaks to progress, our longer-term goals are to refine our results to higher precision and also to evolve our description to still smaller scales. We note that the parity-violating meson-nucleon couplings, and, generally, the low-energy constants associated with the operators of an effective field theory are not in themselves observables and can be expected to depend on the renormalization scale. In this paper we discuss various assessments of the parity-violating pion-nucleon coupling constant from this perspective, as it is the most precisely determined. Generally, we anticipate difficulties can arise both from the gap between the lowest scale to which we can potentially apply perturbative QCD accurately and the highest scale to which we can employ chiral perturbation theory, as well as from the effects of the massive charm quark. The latter affects the splay of operators that can appear, even if the charm quark is still active [38], as we have developed explicitly in the $\Delta S = 0$ case [17]. Moreover, the truncation error from matching a four to three-flavor theory, at a fixed order of perturbation theory, at charm threshold can be significant, as studied in $K \to \pi\pi$ decay [39, 40], where we refer to Ref. [41] for a broader discussion. In this paper we comment on how some of these effects can impact our results.

We conclude this section with an outline of the rest of the paper. In Sec. II, we recapitulate the outcomes of our effective weak Hamiltonian computation [17] that are pertinent here. In Sec. III we discuss the factorization approximation and its validity. In Sec. IV we employ these results to compute the parity-violating meson-NN coupling constants. In Sec. V we compare our results with the parameters extracted from experiments and discuss the perspectives they offer, and we offer a concluding summary and outlook in Sec. VI.

II. EFFECTIVE HAMILTONIAN AND WILSON COEFFICIENTS

The effective Hamiltonian for hadronic parity violation at a particular energy scale is defined in terms of four-quark operators and corresponding Wilson coefficients. In evolving the theory from one energy scale to another, such as from the W mass to scale μ, the Wilson coefficients follow the relation:

$$\vec{C}(\mu) = \exp\left[\int_{g_s(M_W)}^{g_s(\mu)} dg \frac{\gamma^T(\mu)}{\beta(g_s)}\right] \vec{C}(M_W) \quad \text{with} \quad \beta(g_s) = -\frac{g_s^3}{48\pi^2}(33 - 2n_f), \tag{1}$$

where we work in leading-order (LO) QCD, noting that the anomalous dimension matrix γ arises from the LO QCD mixing of the operators. We refer to Ref. [17] for all details. Allowing the effective theory to flow from the W mass scale to hadronic energy scales, with $\mu = 2\,\text{GeV}$, results in the Hamiltonian:

$$\mathcal{H}_{\text{eff}}^{\text{PV}}(2\,\text{GeV}) = \frac{G_F s_w^2}{3\sqrt{2}} \sum_{i=1}^{12} C_i(2\,\text{GeV})\, \Theta_i\,, \tag{2}$$

where Θ_i are four-quark operators. Twelve such operators form a closed set under LO QCD mixing and thus describe the theory of hadronic parity nonconservation, for all three isosectors, where we refer to App. A1 for a complete list. Moreover, we use the weak-mixing angle θ_W with $\sin^2 \theta_W \equiv s_w^2 = 0.231$ and the Fermi constant $G_F = 1.166 \times 10^{-5}\,\text{GeV}^{-2}$ [42]. Just below the W mass scale, the effects of QCD are negligible, and we can collect the Wilson coefficients by summing the tree-level W and Z^0 exchange contributions in $\Delta S = 0$ quark interactions, giving $\vec{C}(M_W) = (1, 0, 0, 0, -3.49, 0, 0, 0, -13.0\cos^2\theta_c, 0, -13.0\sin^2\theta_c, 0)$,

with the Cabibbo angle given by $\sin\theta_c = 0.2253$. Upon performing the RG flow to 2 GeV using Eq. 1, we have [17]

$$\vec{C}(2\,\text{GeV}) = \begin{pmatrix} 1.09 & [1.17\ldots1.06][1.08\ldots1.04] & [1.07][1.06] \\ 0.018 & [0.014\ldots0.021][0.033\ldots0.006] & [-0.006][-0.006] \\ 0.199 & [0.321\ldots0.133][0.193\ldots0.127] & [0.158][0.153] \\ -0.583 & [-0.990\cdots-0.385][-0.571\cdots-0.374] & [-0.460][-0.456] \\ -4.36 & [-4.99\cdots-4.05][-4.34\cdots-4.03] & [-4.16][-4.14] \\ 1.72 & [2.63\ldots1.19][1.67\ldots1.16] & [1.40][1.36] \\ -0.170 & [-0.288\cdots-0.110][-0.165\cdots-0.105] & [-0.134][-0.129] \\ 0.332 & [0.496\ldots0.235][0.322\ldots0.225] & [0.275][0.268] \\ -16.2 & [-18.6\cdots-15.0][-16.1\cdots-15.0] & [-15.48][-15.4] \\ 6.38 & [9.76\ldots4.44][6.22\ldots4.30] & [5.19][5.05] \\ -16.2 & [-18.6\cdots-15.0][-16.1\cdots-15.0] & [-15.48][-15.4] \\ 6.38 & [9.76\ldots4.44][6.22\ldots4.30] & [5.19][5.05] \end{pmatrix}, \tag{3}$$

where the last four entries should be multiplied by factors of $\cos^2\theta_c$, $\cos^2\theta_c$, $\sin^2\theta_c$, and $\sin^2\theta_c$, respectively. The primary result is given by the leftmost column of numbers. The other columns illustrate the uncertainties in the computation. In the central column, the left set shows the ranges of Wilson coefficients that result in the $N_f = 2+1$ theory for renormalization scales of $\mu = 1-4\,\text{GeV}$ and the right set shows them in the $N_f = 2+1+1$ theory with $\mu = 2-4\,\text{GeV}$. The rightmost column gives Wilson coefficients if the α_s running and matching is computed at NLO (left) and NNLO (right).

For the present work, it is useful to make the different isosector contributions explicit and separated as

$$\mathcal{H}_{\text{eff}}^{\text{PV}}(2\,\text{GeV}) = \mathcal{H}_{\text{eff}}^{I=1}(2\,\text{GeV}) + \mathcal{H}_{\text{eff}}^{I=0\oplus2}(2\,\text{GeV}). \tag{4}$$

Isovector ($I = 1$) Wilson coefficients at high and low energies are: $\vec{C}^{I=1}(M_W) = (1,0,0,0,3.49,0,3.49,0,-13.0\cos^2\theta_c,0)$ and

$$\vec{C}^{I=1}(2\,\text{GeV}) = \begin{pmatrix} 1.09 & [1.17\ldots1.06][1.08\ldots1.04] & [1.07][1.06] \\ 0.018 & [0.014\ldots0.021][0.033\ldots0.006] & [-0.006][-0.006] \\ 0.199 & [0.321\ldots0.133][0.193\ldots0.127] & [0.158][0.153] \\ -0.583 & [-0.990\cdots-0.385][-0.571\cdots-0.374] & [-0.460][-0.456] \\ 4.36 & [4.99\ldots4.05][4.34\ldots4.03] & [4.16][4.14] \\ -1.72 & [-2.63\cdots-1.19][-1.67\cdots-1.16] & [-1.40][-1.36] \\ 4.36 & [4.99\ldots4.05][4.34\ldots4.03] & [4.16][4.14] \\ -1.72 & [-2.63\cdots-1.19][-1.67\cdots-1.16] & [-1.40][-1.36] \\ -16.2 & [-18.6\cdots-15.0][-16.1\cdots-15.0] & [-15.48][-15.4] \\ 6.38 & [9.76\ldots4.44][6.22\ldots4.30] & [5.19][5.05] \end{pmatrix}, \tag{5}$$

where the last two entries should be multiplied by a factor $\sin^2\theta_c$ and the error estimates are defined as in Eq. (3). Wilson coefficients for the $I = 0 \oplus 2$ sector at high and low energies are: $\vec{C}^{I=0\oplus2}(M_W) = (-1,0,0,0,-3.49,0,0,0,-13.0\cos^2\theta_c,0)$ and

$$\vec{C}^{I=0\oplus2}(2\,\text{GeV}) = \begin{pmatrix} -1.09 & [-1.17\cdots-1.06][-1.08\cdots-1.04] & [-1.07][-1.06] \\ -0.018 & [-0.014\cdots-0.021][-0.033\cdots-0.006] & [0.006][0.006] \\ -0.199 & [-0.321\cdots-0.133][-0.193\cdots-0.127] & [-0.158][-0.153] \\ 0.583 & [0.990\ldots0.385][0.571\ldots0.374] & [0.460][0.456] \\ -4.36 & [-4.99\cdots-4.05][-4.34\cdots-4.03] & [-4.16][-4.14] \\ 1.72 & [2.63\ldots1.19][1.67\ldots1.16] & [1.40][1.36] \\ -0.170 & [-0.288\cdots-0.110][-0.165\cdots-0.105] & [-0.134][-0.129] \\ 0.332 & [0.496\ldots0.235][0.322\ldots0.225] & [0.275][0.268] \\ -16.2 & [-18.6\cdots-15.0][-16.1\cdots-15.0] & [-15.48][-15.4] \\ 6.38 & [9.76\ldots4.44][6.22\ldots4.30] & [5.19][5.05] \end{pmatrix}, \tag{6}$$

where the last two entries should be multiplied by a factor $\cos^2\theta_c$ and the error estimates are defined as in Eq. (3). Although Eqs. (3, 5, 6) appeared in our earlier paper [17], we have included them here to make our presentation self-contained.

III. FACTORIZATION APPROXIMATION

The effective Hamiltonian presented in the previous section can be used in the computation of various parity-violating meson-nucleon coupling constants of isospin I: h_M^I. These parameters are introduced to quantify the observable effects of hadronic

parity violation via the phenomenological Hamiltonian of Ref. [1], $\mathcal{H}_{\text{DDH}}$. By matching the quark-level and hadron-level matrix elements via $\langle MN'|\mathcal{H}^I_{\text{eff}}|N\rangle = \langle MN'|\mathcal{H}_{\text{DDH}}|N\rangle$, these couplings can be estimated. The main challenge in this is determining the quark-level matrix elements $\langle MN'|\mathcal{H}^I_{\text{eff}}|N\rangle$ involving hadrons. This task is significantly simplified within the factorization, or vacuum saturation, approximation, in which the hadronic matrix element of the four-quark operator is computed as the product of the hadron matrix elements of each current. The factorization approximation is heuristic, though its use can be justified *a posteriori* with experimental data, if not *a priori* on theoretical grounds, except in special cases. The difference in the matrix-element computation of the full 4-quark operator and that of its 2-quark pieces is termed a non-factorizable contribution. This difference is not well-known in general, and its outcome depends on the matrix element chosen.

To our knowledge, the factorization approximation was first studied in the context of hadronic parity violation [43, 44]; in particular, the matrix element of a parity-violating four-quark operator to yield a neutral vector meson from a nucleon state is thus written in the form

$$\langle VN'|(\bar{q}_1 q_2)_V (\bar{q}_3 q_4)_A |N\rangle = \langle V|(\bar{q}_1 q_2)_V |0\rangle \langle N'|(\bar{q}_3 q_4)_A |N\rangle \ . \tag{7}$$

Factorization has also been broadly employed in analyses of hadronic weak decays, with the first application being to the computation of so-called tree graphs, arising from partially disconnected intermediate states, and their contribution to the $|\Delta I| = 1/2$ rule in $K \to 2\pi$ and $K \to 3\pi$ decay [45, 46]. With further developments, the factorization approximation has been used to yield predictions for the exclusive decays of charmed mesons [47, 48], compared against experimental data [18, 49], and applied to the B-meson system, in which extensive tests become possible through the rich selection of possible hadronic final states [50–52]. Under certain conditions, factorization has been shown to work extremely well. To that end we consider the specific example of B-meson decays to heavy-light final states, for which factorization has been shown to exist in QCD in leading inverse power in the heavy quark mass [53, 54] — assuming that both b and c quarks are heavy. Tests of these predictions, and of factorization more generally, come from the study of $B \to D^{(*)}_{(s)}(\pi, K)$ decays [55], particularly the comparison of the theoretical decay rates with experiment, yielding excellent agreement. These decays include both vector and pseudoscalar final states and probe the color-suppressed (C) and exchange (E) topologies, in addition to the color-allowed tree (T) contribution. For example, a test derived from $\bar{B}^0_d \to D^+ \rho^-$ and $\bar{B}^0_d \to D^+ \pi^-$ branching ratio data, which is sensitive to both the T and E topologies, probes factorization to a precision of 10%, and the authors note that they could not resolve any nonfactorizable effects within the current experimental precision, which could be as small as 5% in some cases [55]. In contrast, in meson decays to light final states, the energy release is generally much larger, admitting the possibility of rescattering with intermediate-state hadronic resonances and thus yielding contributions beyond the factorization approach. Empirical uncertainties in $B \to \pi\pi, \pi K$ decays are still large enough to preclude such precise tests [56]. In this class of decays, an outstanding problem has been that of understanding the pattern of amplitudes in $K \to \pi\pi$ decay, for which a marked dominance of the $I = 0$ final state amplitude over the $I = 2$ amplitude is observed, with roughly only a factor of 2 of the empirical ratio $\text{Re}A_0/\text{Re}A_2 \simeq 22.5$ in the isospin limit [57] coming from the perturbative Wilson coefficients and a simple factorization of the hadronic matrix elements. Although the problem has long been attributed to an unidentified enhancement of the $I = 0$ amplitude [58, 59], to which a role for the $\sigma(500)$ resonance has been argued [60, 61], LQCD studies have now shown that a numerical resolution of the $|\Delta I| = 1/2$ puzzle [62] includes a significant cancellation of two tree-level operators that contribute to the $I = 2$ amplitude in $K \to \pi\pi$ decay [62–64]. In the factorization treatment the two contributions have the same sign, showing it to be inconsistent. We note that an opposite relative sign also emerged in earlier non-lattice work using chiral perturbation theory and a large N_c analysis [65, 66]. The analysis of $K \to \pi\pi$ decays reveals features that do not occur in our analysis of the parity-violating meson-nucleon couplings. In particular, since QCD dynamics are flavor-blind, we believe that the existing factorization tests in heavy to heavy-light transitions do have bearing on our $N \to NM$ analysis, supporting our results because the kinematics of the process does not support the existence of factorization-violating resonances. We would like to emphasize that we employ the factorization approximation specifically for the computation of the parity-violating meson-nucleon coupling constants. The issue of non-perturbative effects beyond the DDH model, which could be studied within the framework of 2N matrix elements within LQCD remains. Moreover, the theoretical improvements we have made are specific to the computation of the meson-nucleon coupling constants. To put this in context we now turn to the analysis of DDH [1].

The early landmark study of hadronic parity violation by DDH [1] is critical of the use of factorization, and that assessment has appeared to hold sway despite later work suggesting that non-factorizable effects are subdominant [27]. In regards to the comparative study of DDH [1], both the factorization approximation computations and the quark model estimates to which they were compared employed uncontrolled approximations, and poorly known inputs, so that inferring a deficiency in the factorization approximation itself from differences in such predictions is not a reliable conclusion. Moreover, what DDH term "factorization" is not the same procedure as has been employed in the literature since the late 1980's [18]. In their Fig. 1 they present three different quark flow topologies for the parity-violating meson-nucleon couplings and note that "factorization" is associated with the production of a color-singlet meson emerging as the result of Z^0 exchange at tree level exclusively. Moreover, different paths to computing factorized hadronic matrix elements are employed [1]. We, rather, have followed the now standard practice of applying a Fierz transformation to a four-quark operator to expose the quark currents with the flavor content needed to realize a particular hadronic final state, so that we factorize the matrix elements of the four-quark operators into products of the matrix elements of the associated quark-level currents. In so doing the matrix elements of our LO weak Hamiltonian can

generate all the pictorial contributions in Fig. 1 of [1], depending on the meson to be produced. For example, their Fig. 1b can follow from multi-quark (in excess of three) Fock states of the nucleon, as associated with the strange quark axial charge of the nucleon, which is pertinent to the assessment of the vector-meson-nucleon couplings.

In making our assessments, we have employed the recent, precision LQCD computations of the quark-flavor (scalar, axial) charges of the nucleon [19], and we regard that as a great improvement over the poorly controlled flavor-symmetry-based estimates used throughout the literature in the past, as we discuss in Sec. IV. This is key to a sharpened picture of the role of strange quarks, which have been a source of great uncertainty [29, 67]. Thus greatly improved assessments of the factorized matrix elements in the nucleon sector are now possible. Turning to the parity-violating $n \rightarrow p(\pi^\pm, \rho^\pm, \omega)$ transition matrix elements, we note these processes, though now mediated by Z^0 exchange, also contain quark flow topologies of the same forms studied by Ref. [55], and the kinematics of these transition matrix elements is also compatible with that of the heavy-quark/hadron limit they employ. Thus we regard those tests of factorization in hadronic B meson decays, which speak to its success in that context, as also acting in support of our own analysis. In the next section we use the factorization approximation with input from state-of-the-art lattice QCD results to determine the parity-violating meson-nucleon coupling constants.

IV. ESTIMATES OF THE PARITY-VIOLATING MESON-NUCLEON COUPLING CONSTANTS

In this section, firstly we flesh out the calculation of h_π^1 in Ref. [17], particularly emphasizing and discussing the different input choices made in arriving at this result. Then, we turn to estimating the remaining meson-nucleon couplings. Phenomenologically, the pion contribution to hadronic parity violation with coupling h_π^1 is

$$\mathcal{H}_{\mathrm{DDH}}^\pi = i h_\pi^1 (\pi^+ \bar{p} n - \pi^- \bar{n} p) \,. \tag{8}$$

Matching the quark and hadron-level matrix elements we have

$$- i h_\pi^1 \bar{u}_n u_p = \langle n\pi^+ | \mathcal{H}_{\mathrm{eff}}^{I=1} | p \rangle \,, \tag{9}$$

where u_N with $N \in p, n$ is a Dirac spinor. Employing the Fierz identities, where we note the useful compilation of Ref. [68], $\Theta_i^{I=1}$ operators within the Hamiltonian are rearranged to yield scalar-pseudoscalar contributions. Using the definition $\langle 0 | (\bar{d}u)_A(0) | \pi^+(p) \rangle = i p^\mu f_\pi$ and the result

$$\langle \pi^+ | (\bar{u}\gamma_5 d) | 0 \rangle = \frac{m_\pi^2 f_\pi}{i(m_u + m_d)} \,, \tag{10}$$

we obtain the equation connecting the pion-nucleon coupling to the Wilson coefficients in $\mathcal{H}_{\mathrm{eff}}^{I=1}$:

$$h_\pi^1 \bar{u}_n u_p = \frac{2 G_F s_w^2}{3\sqrt{2}} \left(\frac{C_1^{I=1}}{3} + C_2^{I=1} - \frac{C_3^{I=1}}{3} - C_4^{I=1} \right) \frac{m_\pi^2 f_\pi}{(m_u + m_d)} \langle n | \bar{d}u | p \rangle \,. \tag{11}$$

In its numerical evaluation, we use the isovector scalar charge g_s^{u-d} computed within lattice QCD (LQCD) [19], where $\langle n | \bar{d}u | p \rangle \equiv g_s^{u-d} \bar{u}_n u_p$. Modern LQCD calculations are "unquenched" so that the effects of the light sea quarks are allowed to appear, noting that these are characterized by N_f, the number of dynamical quark flavors in the simulation. As per Ref. [19], we suppose simulations with $N_f = 2 + 1 + 1$ are more realistic but that $N_f = 2 + 1$ simulations are typically more precise. The evaluation of Eq. (11) is sensitive to the precise value of $m_\pi^2/(m_u + m_d)$, where the light quark masses are evaluated in LQCD. This ratio gives a large enhancement, and its assessment should be made with care. Here $m_\pi = 135\,\mathrm{MeV}$, because the LQCD simulations used do not include electromagnetism, and the charged-pion decay constant $f_\pi = 130\,\mathrm{MeV}$. As for the light quark masses, it is appropriate to use the renormalization-group-invariant (RGI) mass $(m_u + m_d) = 2(4.695(56)_m(54)_\Lambda\,\mathrm{MeV})$ for $N_f = 2 + 1$ [19], an appealing choice because it is scale and scheme independent, thus avoiding extreme sensitivity to the choice of scale. In this case, combining errors in quadrature implies $m_\pi^2/(m_u + m_d) = 1941(32)\,\mathrm{MeV}$, whereas using the result from a $N_f = 2 + 1$ simulation in the $\overline{\mathrm{MS}}$ scheme at a scale of $2\,\mathrm{GeV}$, $(m_u + m_d) = 2(3.381(40)\,\mathrm{MeV})$ [19] we find $2695(32)\,\mathrm{MeV}$ for this ratio. (We note in this scheme at this scale that the PDG compilation recommends $(m_u + m_d) = 2(3.45^{+0.55}_{-0.15}\,\mathrm{MeV})$ [42]; we note, too, $(m_u + m_d) = 2(3.75(0.45)\,\mathrm{MeV})$ using scalar sum rules and chiral perturbation theory [69].) We can also assess it through the use of the Gell-Mann–Oakes–Renner (GOR) relation [70–72]. The GOR relation captures the pion mass with a correction of within a few percent [73–76], where the concomitant quark condensate $B \equiv |\Sigma|/F^2$, with $\Sigma = |\langle 0|\bar{u}u|0\rangle|$ and F the pion decay constant in the chiral limit, can all be computed in LQCD. Using Ref. [19] to compute B from Σ and F, in the SU(2) chiral limit and $N_f = 2 + 1$ we have, assuming the errors are uncorrelated, $2560(240)\,\mathrm{MeV}$, whereas in the SU(3) chiral limit we have $2280(280)\,\mathrm{MeV}$, a difference reflecting the role of the strange sea quarks in its numerical evaluation. The result with the RGI quark mass has been employed in what follows. Turning to the isovector quark scalar charge of the nucleon, $N_f = 2 + 1$

result: we use $g_s^{u-d} = 1.06(10)(06)_{sys}$ [77], noting that this compares favorably with the result $g_s^{u-d} = 1.02(11)$ determined from strong-isospin breaking in the nucleon mass from LQCD [78], whereas the SU(3) estimate in Ref. [30] yields 0.6. Finally,

$$h_\pi^1 = (3.06 \pm 0.34 + \left(^{+1.29}_{-0.64}\right) + 0.42 + (1.00)) \times 10^{-7}, \tag{12}$$

where the error estimates come, respectively, from the LQCD inputs employed, the change in the Wilson coefficients over (i) a scale variation of $1 - 4\,\text{GeV}$ and (ii) higher-order corrections in α_s as per Eq. (5), and, finally, the estimates of the accuracy of Eq. (11) through the contribution to it from $O(1/N_c)$ terms, which are noted in parentheses.

We now turn to the assessment of other meson-nucleon coupling constants, starting with the remaining $I = 1$ couplings. For the ρ^0 meson, e.g., $\langle \rho^0 N | \mathcal{H}_{\text{eff}}^{I=1} | N \rangle = h_\rho^1 \epsilon_\rho^{*\mu} (\bar{u}_N u_N)_A$. With $\langle \rho^0 | (\bar{u}u)_V - (\bar{d}d)_V | 0 \rangle \equiv \sqrt{2}\epsilon_\rho^{*\mu} f_\rho m_\rho$, $m_\rho = 775.4\,\text{MeV}$ [42], and $f_\rho = 210\,\text{MeV}$ [50] and using the quark axial charges of the nucleon from LQCD [19]

$$\begin{aligned}
\langle p | (\bar{u}u)_A | p \rangle &= g_A^u (\bar{u}_p u_p)_A \,; & g_A^u &= 0.777(25)(30)\ [0.847(18)(32)]\,, \\
\langle p | (\bar{d}d)_A | p \rangle &= g_A^d (\bar{u}_p u_p)_A \,; & g_A^d &= -0.438(18)(30)\ [-0.407(16)(18)]\,, \\
\langle p | (\bar{s}s)_A | p \rangle &= g_A^s (\bar{u}_p u_p)_A \,; & g_A^s &= -0.053(8)\ [-0.035(6)(7)]\,,
\end{aligned} \tag{13}$$

in the $\overline{\text{MS}}$ scheme at $\mu = 2\,\text{GeV}$ from $N_f = 2 + 1 + 1$ [79] [$N_f = 2 + 1$ [80]] flavor simulations, we have

$$h_\rho^1 = \frac{G_F s_w^2}{3} f_\rho m_\rho \left(\left(C_3^{I=1} + \frac{C_4^{I=1}}{3} \right)(g_A^u + g_A^d) + \left(C_3^{I=1} + \frac{C_4^{I=1}}{3} + C_7^{I=1} + \frac{C_8^{I=1}}{3} + C_9^{I=1} + \frac{C_{10}^{I=1}}{3} \right) g_A^s \right), \tag{14}$$

and with Eq. (13) this yields

$$h_\rho^1 = -0.294 \pm 0.045 + \left(^{0.014}_{-0.036}\right) + 0.009 + (0.026)) \times 10^{-7}, \tag{15}$$

For the ω meson, $\langle \omega N | \mathcal{H}_{\text{eff}}^{I=1} | N \rangle = h_{\omega N}^1 \epsilon_\omega^{*\mu} (\bar{u}_N u_N)_A$. With $\langle \omega | (\bar{u}u)_V + (\bar{d}d)_V | 0 \rangle \equiv \sqrt{2}\epsilon_\omega^{*\mu} f_\omega m_\omega$, $m_\omega = 782.65\,\text{MeV}$ [42], and $f_\omega = 195\,\text{MeV}$ [50], we have

$$h_{\omega N}^1 = \frac{G_F s_w^2}{3} f_\omega m_\omega \left(\left(C_1^{I=1} + \frac{C_2^{I=1}}{3} \right) \eta_N (g_A^u - g_A^d) + \left(C_9^{I=1} + \frac{C_{10}^{I=1}}{3} \right) g_A^s \right), \tag{16}$$

where $\eta = \pm 1$ for a proton or neutron state, respectively. With Eqs. (13)

$$h_{\omega p}^1 = +1.825 \pm 0.111 + \left(^{-0.047}_{0.125}\right) - 0.040 + (-0.020)) \times 10^{-7} \,; \, h_{\omega n}^1 = -1.828 \pm 0.112 + \left(^{0.053}_{-0.134}\right) + 0.043 + (0.000)) \times 10^{-7}, \tag{17}$$

where the difference in their magnitudes speaks to the role of charged-current effects. Similarly we can make use of $H_{\text{eff}}^{I=0\oplus2}$ to determine $\langle \omega N | \mathcal{H}^{I=0\oplus2} | N \rangle = h_\omega^0 \epsilon_\omega^{*\mu} (\bar{u}_N u_N)_A$. Thus

$$h_\omega^0 = \frac{G_F s_w^2}{3} f_\omega m_\omega \left(\left(C_7^{0+2} + \frac{C_8^{0+2}}{3} + C_9^{0+2} + \frac{C_{10}^{0+2}}{3} \right)(g_A^u + g_A^d) + \left(C_1^{0+2} + \frac{C_2^{0+2}}{3} + C_7^{0+2} + \frac{C_8^{0+2}}{3} \right) g_A^s \right), \tag{18}$$

and with Eqs. (13) this gives

$$h_\omega^0 = +0.270 \pm 0.015 + \left(^{-0.32}_{0.55}\right) - 0.202 + (1.148)) \times 10^{-7} \tag{19}$$

To determine the isocalar and isotensor ρ couplings from $\mathcal{H}_{\text{eff}}^{I=0\oplus2}$ we note from $\mathcal{H}_{\text{DDH}}$ [1] that

$$h_\rho^0 + \frac{1}{\sqrt{6}} h_\rho^2 = h_{\rho^0}^{0\oplus2} \,; \quad \sqrt{2} h_\rho^0 - \frac{1}{\sqrt{12}} h_\rho^2 = h_{\rho^-}^{0\oplus2}. \tag{20}$$

Computing $h_{\rho^0}^{0\oplus2}$, with $\langle \rho^0 N | \mathcal{H}_{\text{eff}}^{I=0\oplus2} | N \rangle = h_{\rho^0}^{0\oplus2} \eta_N \epsilon_\rho^{*\mu} (\bar{u}_N u_N)_A$,

$$h_{\rho^0}^{0\oplus2} = \frac{G_F s_w^2}{3} f_\rho m_\rho \left(C_5^{I=0+2} + \frac{C_6^{I=0+2}}{3} - \frac{C_9^{I=0+2}}{6} - \frac{C_{10}^{I=0+2}}{2} \right)(g_A^u - g_A^d), \tag{21}$$

which, with Eqs. (13), implies

$$h_{\rho^0}^{0\oplus2} = -7.55 \pm 0.46 + \left(^{1.54}_{-2.76}\right) + 1.00 + (-5.57)) \times 10^{-7}. \tag{22}$$

Computing $h_{\rho^-}^{0\oplus2}$, with $\langle\rho^- p|\mathcal{H}_{\text{eff}}^{I=0\oplus2}|n\rangle = h_{\rho^-}^{0\oplus2}\epsilon_\rho^{*\mu}(\bar{u}_N u_N)_A$, noting $\langle\rho^-|(\bar{d}u)_v|0\rangle = \epsilon_\rho^{*\mu}f_\rho m_\rho$, and using the quark isovector axial charge in LQCD in $\overline{\text{MS}}$ at 2 GeV from a $N_f = 2+1$ [77] [$N_f = 2+1+1$ [81]] flavor simulation, namely,

$$\langle p|(\bar{u}d)_A|n\rangle = g_A^{u-d}(\bar{u}_p u_n)_A; \quad g_A^{u-d} = 1.31(06)(05)_{\text{sys}}\,[1.218(25)(30)_{\text{sys}}]\,, \tag{23}$$

we have

$$h_{\rho^-}^{0\oplus2} = \frac{G_F s_w^2}{3\sqrt{2}}f_\rho m_\rho\left(\frac{-C_5^{I=0+2}}{3} - C_6^{I=0+2} + \frac{C_7^{I=0+2}}{3} + C_8^{I=0+2} + C_9^{I=0+2} + \frac{C_{10}^{I=0+2}}{3}\right)g_A^{u-d}\,. \tag{24}$$

With Eqs.(23), this implies

$$h_{\rho^-}^{0+2} = -18.10 \pm 1.1 + \binom{1.2}{-2.4} + 0.72 + (-4.63))\times10^{-7}\,. \tag{25}$$

Solving Eq. (20) we find

$$h_\rho^0 = -11.05 \pm 0.672 + \binom{1.079}{-2.051} + 0.673 + (-4.039))\times10^{-7}\,; \quad h_\rho^2 = +8.57 \pm 0.519 + \binom{1.129}{-1.736} + 0.802 + (-3.749))\times10^{-7}\,. \tag{26}$$

Although our determinations have been made at a scale of 2 GeV, we follow the spirit of DDH [1] and compare our results with the constraints on the coupling constants that emerge from experiments at much lower energies. In this way we hope to discern the driving theoretical limitations in our approach.

V. PERSPECTIVES FROM COMPARISONS WITH EXPERIMENT

In what follows we consider how the results of Sec. IV compare with the outcomes of hadronic parity violation experiments with nucleons and nuclei. We anticipate that our results may be most closely suited to studies of hadronic parity violation in few-body systems, though we also consider more complex nuclear systems, comparing, in particular, our h_π^1 result to a precise limit extracted from a search for parity violation in the radiative decay of excited-state ^{18}F [24]. Finally, following earlier work [3, 5], we use the DDH potential [1], based on one-meson exchange, to evaluate the Danilov parameters and compare them with the outcomes of low-energy experiments, particularly those from parity-violating proton-proton scattering. We regard these computations as rough estimates, to be checked against the predictions of a large N_c analysis and that may serve as guidance in determining the limitations of the DDH potential.

Comparing with the constraints on the parity-violating vector-meson-nucleon coupling constants that emerge from the combined analysis of the $\vec{n}p \to d\gamma$ [20] and $\vec{n}\,^3\text{He} \to p\,^3\text{H}$ [21] experiments, within the theoretical framework of Ref. [82], we have $h_\pi^1 = (2.6 \pm 1.2_{\text{stat}} \pm 0.2_{\text{sys}})\times10^{-7}$ [20], and $h_{\rho-\omega} \equiv h_\rho^0 + 0.605h_\omega^0 - 0.605h_\rho^1 - 1.316h_\omega^1 + 0.026h_\rho^2 = (-17.0 \pm 6.56)\times10^{-7}$ [21], for which we compute

$$h_{\rho-\omega} = -12.9 \pm 0.52 + \binom{0.97}{-1.9} + 0.62 + (-3.4))\times10^{-7}\,, \tag{27}$$

so that both this and our h_π^1, Eq. (12), are within $\pm1\sigma$ of the experimentally determined parameters. We note, moreover, that analyzing the result of the $\vec{n}p \to d\gamma$ [20] experiment within chiral perturbation theory yields $h_\pi^1 = (2.7\pm1.8)\times10^{-7}$ [6, 13]. Using our results, we evaluate the asymmetry in $\vec{n}\,^3\text{He} \to p\,^3\text{H}$ as -0.69×10^{-8} in the framework of Ref. [82] but as 1.6×10^{-8} in the framework of Ref. [83], as per Eqs.(8,9) of Ref. [21], to compare with the experimental result $(1.55\pm0.97_{\text{stat}}\pm0.24_{\text{sys}})\times10^{-8}$ [21]. Evidently the value of the asymmetry is sensitive to a partial cancellation of the various contributions [21]. The h_π^1 determination from the $\vec{n}p \to d\gamma$ experiment, $h_\pi^1 = (2.6 \pm 1.2_{\text{stat}} \pm 0.2_{\text{sys}})\times10^{-7}$ [20] is in slight tension with the value determined by the non-observation of the photon circular polarization in ^{18}F radiative decay from the 1.081 MeV $J^P T = 0^-0$ state, reflecting an absence of mixing with the nearby 1.042 MeV 0^+1 state, yielding the bound $|h_\pi^1| < 1.3\times10^{-7}$ at 68% CL [3]. The ^{18}F system is special in that the theoretical uncertainties can be largely controlled through the experimental assessment of the pertinent nuclear matrix element, after an isospin rotation, from a well-measured β^+-decay transition in ^{18}Ne [2, 22, 23]. Thus the error in each h_π^1 assessment is thought to be statistics dominated. Other reliably calculated, parity-violating observables that depend on the couplings probed in the few-body reactions include the longitudinal asymmetry in elastic $\vec{p}-\alpha$ scattering at 46 MeV, $A_L[\vec{p}\alpha]$, and the gamma asymmetry in ^{19}F decay, $A_\gamma[^{19}\text{F}]$. Using the expressions in Ref. [3] we find -2.6×10^{-7}, to compare with $A_L[\vec{p}\alpha]_{\text{expt}} = -(3.3 \pm 0.9)\times10^{-7}$[84, 85], and -6.7×10^{-5}, to compare with $A_\gamma[^{19}\text{F}]_{\text{expt}} = -(7.4 \pm 1.9)\times10^{-5}$[23, 86]. Therefore only the ^{18}F study is precise enough to challenge the determination of h_π^1 in few-body systems, and we show these results in Fig. 1, along with the value of h_π^1 determined from the parity-violating gamma asymmetry in $\vec{n}p \to d\gamma$ [20] using chiral perturbation theory [6, 13], as well as our own determination of that and of $h_{\rho-\omega}$. Our assessment of these couplings
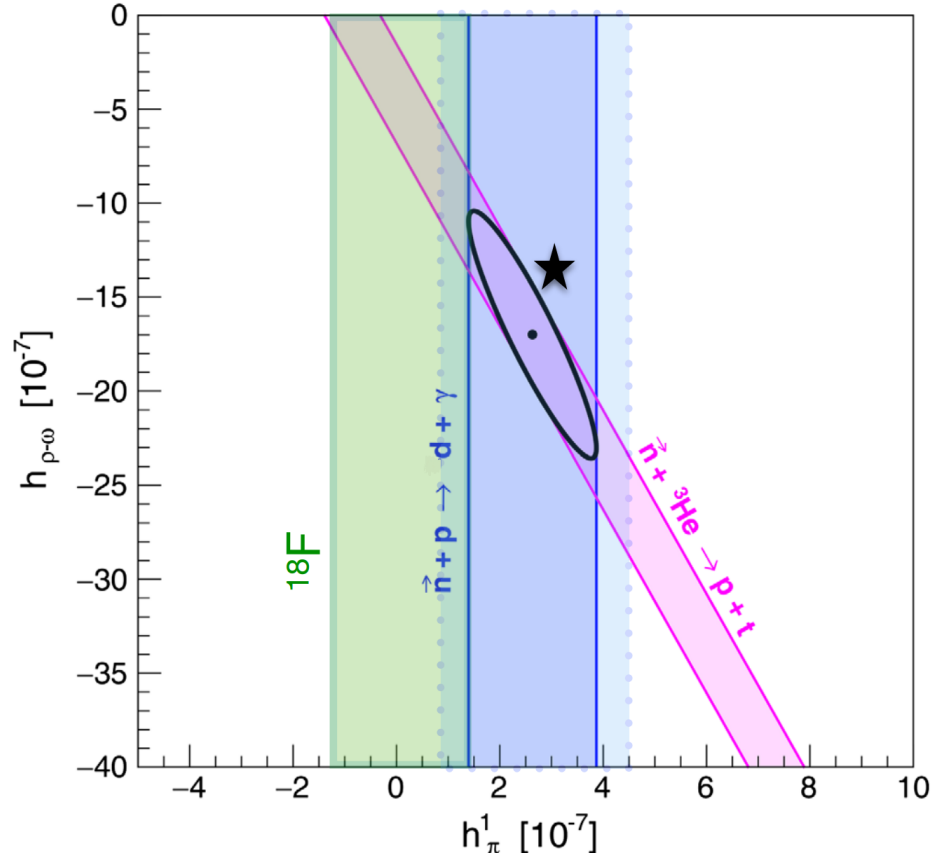
FIG. 1. Constraints on the parity-violating coupling constants $h_{\rho-\omega}$ and h_π^1, after Ref. [21]. The couplings are not direct physical observables and thus can be sensitive to the energy scale of the system under consideration, see the text for further discussion. Combining statistical and systematic errors in quadrature and working at 68% CL, we show the value $h_\pi^1 = (2.6 \pm 1.2) \times 10^{-7}$ from the measured parity-violating asymmetry in $\vec{n} + p \rightarrow d + \gamma$ [20] as the vertical band bounded by a solid line, and its determination $h_\pi^1 = (2.7 \pm 1.8) \times 10^{-7}$ in chiral perturbation theory as the vertical band bounded by a dotted line [6, 13], and the diagonal constraint from the measured parity-violating asymmetry in $\vec{n} +^3$ He $\rightarrow p + t$ [21], with the combined fit of the two experiments yielding the ellipse shown. The analysis of ^{18}F radiative decay from its 1.081 MeV excited state yields the bound $|h_\pi^1| < 1.3 \times 10^{-7}$ [3], shown as the leftmost vertical band. Our *ab initio* result at a scale of 2 GeV is represented by the star with the associated error from its inputs roughly by its size. The tension with the ^{18}F result at a nominal scale of less than 100 MeV, may also be reflective of an extraction in a different physical setting.

at a renormalization scale of $\mu = 2$ GeV is compatible with the determinations from the few-body results, but both it and the experiment values are in tension with the ^{18}F result. Of course it is possible that the disagreement between the experiments could be experimental in origin, though the procedures used in the NPDGamma experiment have been validated through the experimental study of parity-violating $\vec{n}$ capture on ^{35}Cl [87], or be the result of an underestimated theoretical systematic error, yet we emphasize that these couplings are not directly observable. Thus they can be expected to vary with the renormalization scale of the system in which they are determined, which is typically bounded from above by the cutoff scale that determines the active degrees of freedom in a particular EFT. In the current context we contrast chiral perturbation theory, a NN EFT with active pion degrees of freedom and a cutoff scale of about 1 GeV [72, 74], with chiral effective theory, an EFT in which pion degrees of freedom are absent and thus with a cutoff scale of about 100 MeV. In settings where the scale variation is set by perturbative physics, such as in the case of the running of $\sin^2 \theta_W$ in the SM, noting Fig. 5 of Ref. [37], in which the natural scale choice is the typical momentum transfer Q of the experiment, the computed variations are numerically very small, a few percent at most. However, in low-energy QCD, the scale variation is no longer controlled by weakly-coupled effects, and it need not be very small. To illustrate, we turn to a NN effective theory without pions, so-called pionless effective theory [88–90]. The large S-wave scattering lengths a_0^J, with $J = 0, 1$, associated with the low-energy NN system reflect the possibility of nearly or weakly bound states, and to address the incompatibility of that large length scale in an effective theory with a break-down scale of $\Lambda_{\not\pi}$ [91–94], where $a_0^J \gg 1/\Lambda_{\not\pi}$, a power-divergence subtraction (PDS) scheme can be employed at a subtraction point of $\mu \approx Q$ [92]. In this scheme the LECs that result vary with μ as a ratio of simple polynomials, and we note that the μ variation in ratios of LECs can vary by a factor of a few over scales μ ranging from 80 to 180 MeV [95]. Although the PDS scheme enlarges the range of momenta for which the EFT is valid, other, long-standing approaches to the systematic organization of a chiral EFT

continue to be followed [96]. We note Ref. [97] for a detailed comparative study of the PDS renormalization and the Wilsonian renormalization group schemes in an analytically solvable NN EFT; here we consider the implications of their conjecture that fitting LECs to a data set implicitly selects a renormalization scheme. To us, this means the particular parity-violating couplings shown in Fig. 1 can intrinsically depend on the physical momentum scale of the studies in which they are extracted. Here we note that a cutoff scale of the EFT that would describe the radiative decay of an excited state of ^{18}F, which is pertinent even if the existing extraction is regarded as semi-empirical [2, 22, 23], is much lower than the one associated with chiral perturbation theory for $\vec{n} + p \to d + \gamma$. The extracted couplings could be discernibly different in the two settings, and we consider probes of this possibility in what follows.

Recent analyses have suggested that matrix elements of a quark-based effective Hamiltonian can be matched to chiral perturbation theory at a renormalization scale of $\mu = 2\,\text{GeV}$ [34, 36]. Conventionally, however, the cutoff scale of chiral perturbation theory is taken to be 1 GeV [71, 74], or the ρ mass [19]. If we were to try to evolve our description to still lower scales, we expect to encounter the charm quark scale at $\mu = m_c$ [98]. For $\mu \gg m_c \approx 1.3\,\text{GeV}$, the effects of the charm-quark mass are negligible, allowing u-like quark penguin contributions from the charged-current contributions in the weak effective Hamiltonian to cancel. However, at scales for which $\mu \gtrsim m_c$, this cancellation is no longer efficient, and if $\mu \leq m_c$, it no longer operates. Thus for $\mu < 2\,\text{GeV}$ the effects of these additional operators, all of $I = 0$ character, can exist [17], along with the possibility of non-perturbative matching [39] that we have already noted. These effects are presumably small with respect to the precision of the h_π^1 extraction from chiral perturbation theory [6, 13], nominally at a scale of $\mu = 1\,\text{GeV}$, shown in Fig. 1. Nevertheless, to begin to assess the possible numerical implications of these effects, we use the coupling constants we have computed as they stand to estimate the LECs of very-low-energy, parity-violating observables in the NN system, which are essentially the Danilov parameters [8], to compare more broadly with existing experiments. Working within the context of the DDH potential, with parameters $g_{\pi NN}^2/4\pi = 14.4$, $g_\rho^2/4\pi = 0.62$, $g_\omega^2/4\pi = 9g_\rho^2/4\pi$, $\chi_\rho = 3.70$, and $\chi_\omega = -0.12$, we compute the Danilov parameters to find

$$
\begin{aligned}
\Lambda_0^{^1S_0-^3P_0} &= -g_\rho(2 + \chi_\rho)h_\rho^0 - g_\omega(2 + \chi_\omega)h_\omega^0 \to 176\,[210] \\
\Lambda_0^{^3S_1-^1P_1} &= -3g_\rho\chi_\rho h_\rho^0 + g_\omega\chi_\omega h_\omega^0 \to 343\,[360] \\
\Lambda_1^{^1S_0-^3P_0} &= -g_\rho(2 + \chi_\rho)h_\rho^1 - g_\omega(2 + \chi_\omega)h_\omega^1 \to 4.67\,[21] \\
\Lambda_1^{^3S_1-^3P_1} &= \frac{g_{\pi NN}}{\sqrt{2}}\left(\frac{m_\rho}{m_\pi}\right)^2 h_\pi^1 + g_\rho(h_\rho^1 - h_\rho^{1'}) - g_\omega h_\omega^1 \to 859\,[1340] \\
\Lambda_2^{^1S_0-^3P_0} &= -g_\rho(2 + \chi_\rho)h_\rho^2 \to -137\,[160]\,,
\end{aligned}
\tag{28}
$$

where we neglect $h_\rho^{1'}$ [3, 99] and provide our numerical values, with the DDH "best values [1]" given in brackets — and all in units of 10^{-7}. Following the large N_c analysis of Ref. [5], we compute

$$
\Lambda_0^+ \equiv \frac{1}{4}\Lambda_0^{^1S_0-^3P_0} + \frac{3}{4}\Lambda_0^{^3S_1-^1P_1} \to 301 \;;\; \Lambda_0^- \equiv \frac{1}{4}\Lambda_0^{^3S_1-^1P_1} - \frac{3}{4}\Lambda_0^{^1S_0-^3P_0} \to -46\,,
\tag{29}
$$

and recall the scaling predictions $\Lambda_0^+ \sim N_c$, $\Lambda_2^{^1S_0-^3P_0} \sim N_c \sin^2\theta_w$, $\Lambda_0^- \sim 1/N_c$, $\Lambda_1^{^1S_0-^3P_0} \sim \sin^2\theta_w$, $\Lambda_1^{^3S_1-^3P_1} \sim \sin^2\theta_w$ [5, 25, 26]. Certainly the value of h_π^1 we compute yields a value of $\Lambda_1^{^3S_1-^3P_1}$ at odds with the large N_c expectation, though $\Lambda_1^{^3S_1-^3P_1}|_{h_\pi^1=0} = -31$. We note, too, that in this we have ignored the possibility of scale dependence entirely, though an explicit study [95] in the parity-conserving case shows that only certain ranges of μ are compatible with large N_c expectations for partial waves beyond the S-wave channels.

We now turn to other observables, starting with the parity-violating longitudinal asymmetry in low-energy $\vec{p}p$ scattering, $A_L(\vec{p}p)$, for which the Danilov parameters associated with $S - P$ interference should suffice. Fixed target $\vec{p}p$ experiments at beam energies of 13.6 MeV, 15 MeV, and 45 MeV can be analyzed within a DDH framework [100] to yield [3]

$$
\frac{2}{5}\Lambda_0^+ + \frac{1}{\sqrt{6}}\Lambda_2^{^1S_0-^3P_0} + \left[\Lambda_1^{^1S_0-^3P_0} - \frac{6}{5}\Lambda_0^-\right] = 419 \pm 43\,,
\tag{30}
$$

which we evaluate as $120 - 56 + 60 = 124$. Thus our results in this case do not compare favorably. For context, we note that an analysis of this observable in chiral effective theory shows that correlated two-pion exchange (TPE) also plays an important role [11, 12, 83], bringing in an interaction largely controlled by h_π^1 as well, although TPE is not present in the DDH framework. As for the other observables we have considered, the value of h_π^1 plays an important numerical role, with the subleading contributions, which are largely isovector, and the leading ones, which are isoscalar, playing comparable numerical roles. Thus although our original assessment of the Danilov parameters, with the exception of the one in which h_π^1 appears, are crudely consistent with large N_c scaling, it appears that the large N_c relationships are not effective in predicting the aggregate size of the various contributions. In this the parameter h_π^1 drives this conclusion, making its computation within LQCD [15, 16], noting the pioneering work of Ref. [14], or an improved experimental assessment of it, possibly through a next-generation $\vec{n}p \to d\gamma$ experiment,

extremely welcome. Another interesting possibility would be a neutron spin rotation experiment in liquid ^{4}He; the existing limit is consistent with zero but is statistics limited [101], and a new experiment with a planned factor of 10 improvement in sensitivity is being developed [102]. With our Danilov parameter estimates that experiment should be able to measure a non-zero result. As for our suggestion that the extraction of h_π^1, and possibly other couplings, could vary with the cutoff scale of the physical description, we hope that further studies of hadronic parity violation in complex systems could be made and be of sufficient precision to reveal this effect in other isosectors as well. Since we have noted that additional penguin contributions, of purely isoscalar character, emerge once the charm quark is no longer an active degree of freedom, we think that precision experimental studies of hadronic parity violation in the isoscalar sector, as detailed in Ref. [5], both in few-body and complex nuclei, would be needed to assess the quantitative importance of these long-neglected effects. A particularly appealing example would be the measurement of the parity-violating asymmetry in $\vec{n} + d \rightarrow t + \gamma$, because the asymmetry is expected to be somewhat larger than those of other measured reactions, with little sensitivity to the isotensor sector — and it would be interesting to compare that outcome to the measured γ-ray asymmetry in ^{19}F decay [3, 5] and even more so if the precision of the latter experiment could be improved.

VI. SUMMARY

We have used the LO QCD effective weak Hamiltonian for parity-violating, $\Delta S = 0$ hadronic processes to determine the parity-violating meson-nucleon coupling constants, h_π^1, $h_\rho^{0,1,2}$, $h_\omega^{0,1}$, familiar from the DDH framework. We have achieved this by employing the factorization *Ansatz* and assessments of the pertinent quark charges of the nucleon in lattice QCD at the 2 GeV scale. Working further, we have found that our assessment of h_π^1 and $h_{\rho-\omega}$ agree within 1σ of their experimental determinations in few-body nuclear systems [20, 21], though both our h_π^1 result and the size of the asymmetry in $\vec{n}p \rightarrow d\gamma$ [20] are in slight tension with the null result from the study of $P_\gamma[^{18}$F] [2, 22, 23], and we have noted the possibility that the extracted coupling could depend on the cutoff scale of the EFT description that would describe it.

Turning to the study of the parity-violating asymmetries in low-energy $\vec{p}p$ scattering, which is sensitive to the $I = 2$ Davilov parameter $\Lambda_2^{^1S_0-^3P_0}$ as well, we do not find agreement with experiment. The analysis of this process within chiral effective theory, however, suggests that TPE, an effect not included in the DDH potential, plays an important role [83], and this can also modify the $I = 1$ Danilov parameters, though it may be that our factorization assessment of h_ρ^2, or of neglected higher order effects in α_s, and thus of $\Lambda_2^{^1S_0-^3P_0}$ that is to blame. We note that the parameter $h_{\rho-\omega}$ depends only very weakly on the $I = 2$ sector.

Five independent parameters characterize low-energy hadronic parity violation, and the use of pionless effective theory in the large N_c limit gives insight into the relative size of the contributions [5, 25, 26, 103]. Yet these are scaling relationships, rather than numerical predictions, and we have noted that our numerical assessments in Eq. (28), save for the $I = 1$ parameter containing h_π^1, compare favorably with those expectations. Thus the overall success of the large N_c predictions very much depends on the precise value of h_π^1, with future input from either LQCD or experiment important to a definitive test. Despite this, the application of our results, within the DDH framework, to parity-violating observables in $A > 3$ systems suggest that it is not effective, because the subleading pieces are not only quite large, but they are also needed for theoretical compatibility with the observed effects. This outcome is nevertheless suggestive that the systematic study of hadronic parity violation in $A > 3$ systems, for which studies in molecular systems [104] also show great promise [105, 106], is within reach. Precision experimental studies, particularly in the isoscalar sector, can illuminate the additional theoretical effects we have noted, providing an important opportunity to bench-mark end-to-end EFT descriptions of low-energy weak observables in nuclei, which play a broad role in searches for physics beyond the SM.

The hints of success in our work of updating DDH come from comparing our estimations of meson-nucleon couplings with the outcomes of recent experiments, as discussed at length in the current paper. We are careful not to claim any such comparisons with our crude estimations of the Danilov parameters. Yet, we are of the opinion that such rough estimations can be checked against the predictions of large N_c analysis and may serve as a supplement in assessing the limitations of our approach. We would like to emphasize that our work neither discounts the possibility of TPE nor of the importance of additional non-perturbative effects in a complete picture of hadronic parity violation at low energies. But, in striving to refine the benchmark expectations of the parity-violating meson-nucleon couplings, we have updated the work of DDH [1] via the introduction of renormalization-group methods, a modern definition of factorization, and lattice QCD inputs and thus in so doing overcome many challenges in such theoretical computations starting in the 1980s. Future theoretical work that would aspire to confront low-energy experiments more directly would surely benefit from the realization of a LQCD program for the computation of 2N matrix elements for hadronic parity violation, which is under development [107–109], though there are ongoing challenges [7].

ACKNOWLEDGMENTS

Appendix A: Four-quark Operators

The operators of the complete theory ($\mathcal{H}_{\text{eff}}^{\text{PV}}$) with all three isosectors are:

$$
\begin{aligned}
\Theta_1 &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta} \\
\Theta_2 &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\alpha} \\
\Theta_3 &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha}[(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\beta\beta} \\
\Theta_4 &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\beta\alpha} \\
\Theta_5 &= [(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta} \\
\Theta_6 &= [(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\alpha} \\
\Theta_7 &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha}[(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\beta\beta} \\
\Theta_8 &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\beta\alpha} \\
\Theta_9 &= (\bar{u}d)_V^{\alpha\alpha}(\bar{d}u)_A^{\beta\beta} + (\bar{d}u)_V^{\alpha\alpha}(\bar{u}d)_A^{\beta\beta} \\
\Theta_{10} &= (\bar{u}d)_V^{\alpha\beta}(\bar{d}u)_A^{\beta\alpha} + (\bar{d}u)_V^{\alpha\beta}(\bar{u}d)_A^{\beta\alpha} \\
\Theta_{11} &= (\bar{u}s)_V^{\alpha\alpha}(\bar{s}u)_A^{\beta\beta} + (\bar{s}u)_V^{\alpha\alpha}(\bar{u}s)_A^{\beta\beta} \\
\Theta_{12} &= (\bar{u}s)_V^{\alpha\beta}(\bar{s}u)_A^{\beta\alpha} + (\bar{s}u)_V^{\alpha\beta}(\bar{u}s)_A^{\beta\alpha} \, .
\end{aligned}
\tag{A1}
$$

Operators for isovector sector ($\mathcal{H}_{\text{eff}}^{\text{I=1}}$) are:

$$
\begin{aligned}
\Theta_1^{I=1} &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\beta} \\
\Theta_2^{I=1} &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\alpha} \\
\Theta_3^{I=1} &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha}[(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\beta} \\
\Theta_4^{I=1} &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\alpha} \\
\Theta_5^{I=1} &= (\bar{s}s)_V^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\beta} \\
\Theta_6^{I=1} &= (\bar{s}s)_V^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\alpha} \\
\Theta_7^{I=1} &= (\bar{s}s)_A^{\alpha\alpha}[(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\beta} \\
\Theta_8^{I=1} &= (\bar{s}s)_A^{\alpha\beta}[(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\alpha} \\
\Theta_9^{I=1} &= (\bar{u}s)_V^{\alpha\alpha}(\bar{s}u)_A^{\beta\beta} + (\bar{s}u)_V^{\alpha\alpha}(\bar{u}s)_A^{\beta\beta} \\
\Theta_{10}^{I=1} &= (\bar{u}s)_V^{\alpha\beta}(\bar{s}u)_A^{\beta\alpha} + (\bar{s}u)_V^{\alpha\beta}(\bar{u}s)_A^{\beta\alpha}
\end{aligned}
\tag{A2}
$$

and the operators for $I = 0 \oplus 2$ sector ($\mathcal{H}_{\text{eff}}^{I=0\oplus2}$) are:

$$
\begin{aligned}
\Theta_1{}^{I=0\oplus2} &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{s}s)_A]^{\beta\beta} \\
\Theta_2{}^{I=0\oplus2} &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\beta}[(\bar{s}s)_A]^{\beta\alpha} \\
\Theta_3{}^{I=0\oplus2} &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha}[(\bar{s}s)_V]^{\beta\beta} \\
\Theta_4{}^{I=0\oplus2} &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{s}s)_V]^{\beta\alpha} \\
\Theta_5{}^{I=0\oplus2} &= [(\bar{u}u)_V - (\bar{d}d)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\beta} + (\bar{s}s)_V^{\alpha\alpha}(\bar{s}s)_A^{\beta\beta} \\
\Theta_6{}^{I=0\oplus2} &= [(\bar{u}u)_V - (\bar{d}d)_V]^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\alpha} + (\bar{s}s)_V^{\alpha\beta}(\bar{s}s)_A^{\beta\alpha} \\
\Theta_7{}^{I=0\oplus2} &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\beta\beta} \\
\Theta_8{}^{I=0\oplus2} &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\beta\alpha} \\
\Theta_9{}^{I=0\oplus2} &= (\bar{u}d)_V^{\alpha\alpha}(\bar{d}u)_A^{\beta\beta} + (\bar{d}u)_V^{\alpha\alpha}(\bar{u}d)_A^{\beta\beta} \\
\Theta_{10}{}^{I=0\oplus2} &= (\bar{u}d)_V^{\alpha\beta}(\bar{d}u)_A^{\beta\alpha} + (\bar{d}u)_V^{\alpha\beta}(\bar{u}d)_A^{\beta\alpha}
\end{aligned}
\tag{A3}
$$

[1] B. Desplanques, J. F. Donoghue, B. R. Holstein, Unified Treatment of the Parity Violating Nuclear Force, Annals Phys. 124 (1980) 449. `doi:10.1016/0003-4916(80)90217-1`.

[2] E. G. Adelberger, W. C. Haxton, Parity violation in the nucleon-nucleon Interaction, Ann. Rev. Nucl. Part. Sci. 35 (1985) 501–558. `doi:10.1146/annurev.ns.35.120185.002441`.

[3] W. Haxton, B. Holstein, Hadronic parity violation, Prog. Part. Nucl. Phys. 71 (2013) 185–203. `arXiv:1303.4132`, `doi:10.1016/j.ppnp.2013.03.009`.

[4] M. Schindler, R. Springer, The theory of parity violation in few-nucleon systems, Prog. Part. Nucl. Phys. 72 (2013) 1–43. `arXiv:1305.4190`, `doi:10.1016/j.ppnp.2013.05.002`.

[5] S. Gardner, W. C. Haxton, B. R. Holstein, A New Paradigm for Hadronic Parity Nonconservation and its Experimental Implications, Ann. Rev. Nucl. Part. Sci. 67 (2017) 69–95. `arXiv:1704.02617`, `doi:10.1146/annurev-nucl-041917-033231`.

[6] J. de Vries, E. Epelbaum, L. Girlanda, A. Gnech, E. Mereghetti, M. Viviani, Parity- and time-reversal-violating nuclear forces, Front. in Phys. 8 (2020) 218. `arXiv:2001.09050`, `doi:10.3389/fphy.2020.00218`.

[7] A. Nicholson, et al., Toward a resolution of the NN controversy, in: 38th International Symposium on Lattice Field Theory, 2021. `arXiv:2112.04569`.

[8] S.-L. Zhu, C. Maekawa, B. Holstein, M. Ramsey-Musolf, U. van Kolck, Nuclear parity-violation in effective field theory, Nucl. Phys. A748 (2005) 435–498. `arXiv:nucl-th/0407087`, `doi:10.1016/j.nuclphysa.2004.10.032`.

[9] D. R. Phillips, M. R. Schindler, R. P. Springer, An effective-field-theory analysis of low-energy parity-violation in nucleon-nucleon scattering, Nucl. Phys. A822 (2009) 1–19. `arXiv:0812.2073`, `doi:10.1016/j.nuclphysa.2009.02.011`.

[10] M. R. Schindler, R. P. Springer, Two parity violating asymmetries from $np \rightarrow d\gamma$ in pionless effective field theories, Nucl. Phys. A846 (2010) 51–62. `arXiv:0907.5358`, `doi:10.1016/j.nuclphysa.2010.06.002`.

[11] J. de Vries, U.-G. Meißner, E. Epelbaum, N. Kaiser, Parity violation in proton-proton scattering from chiral effective field theory, Eur. Phys. J. A 49 (2013) 149. `arXiv:1309.4711`, `doi:10.1140/epja/i2013-13149-9`.

[12] J. de Vries, N. Li, U.-G. Meißner, N. Kaiser, X. H. Liu, S. L. Zhu, A study of the parity-odd nucleon-nucleon potential, Eur. Phys. J. A 50 (2014) 108. `arXiv:1404.1576`, `doi:10.1140/epja/i2014-14108-8`.

[13] J. de Vries, N. Li, U.-G. Meißner, A. Nogga, E. Epelbaum, N. Kaiser, Parity violation in neutron capture on the proton: Determining the weak pion–nucleon coupling, Phys. Lett. B 747 (2015) 299–304. `arXiv:1501.01832`, `doi:10.1016/j.physletb.2015.05.074`.

[14] J. Wasem, Lattice QCD Calculation of Nuclear Parity Violation, Phys. Rev. C 85 (2012) 022501. `arXiv:1108.1151`, `doi:10.1103/PhysRevC.85.022501`.

[15] X. Feng, F.-K. Guo, C.-Y. Seng, Novel Soft-Pion Theorem for Long-Range Nuclear Parity Violation, Phys. Rev. Lett. 120 (18) (2018) 181801. `arXiv:1711.09342`, `doi:10.1103/PhysRevLett.120.181801`.

[16] A. Sen, M. Petschlies, N. Schlage, C. Urbach, Hadronic Parity Violation from 4-quark Interactions, in: 38th International Symposium on Lattice Field Theory, 2021. `arXiv:2111.09025`.

[17] S. Gardner, G. Muralidhara, QCD analysis of $\Delta S = 0$ hadronic parity violation, Phys. Lett. B 833 (2022) 137372. `arXiv:2203.00033v2`, `doi:10.1016/j.physletb.2022.137372`.

[18] M. Bauer, B. Stech, M. Wirbel, Exclusive Nonleptonic Decays of D, D(s), and B Mesons, Z. Phys. C 34 (1987) 103. `doi:10.1007/BF01561122`.

[19] Y. Aoki, et al., FLAG Review 2021 (11 2021). `arXiv:2111.09849`.

[20] D. Blyth, et al., First Observation of P-odd γ Asymmetry in Polarized Neutron Capture on Hydrogen, Phys. Rev. Lett. 121 (24) (2018) 242002. `arXiv:1807.10192`, `doi:10.1103/PhysRevLett.121.242002`.

[21] M. T. Gericke, et al., First Precision Measurement of the Parity Violating Asymmetry in Cold Neutron Capture on ^{3}He, Phys. Rev. Lett. 125 (13) (2020) 131803. `arXiv:2004.11535`, `doi:10.1103/PhysRevLett.125.131803`.

[22] W. Haxton, Parity nonconservation in ^{18}F and meson exchange contributions to the axial charge operator, Phys. Rev. Lett. 46 (1981) 698. `doi:10.1103/PhysRevLett.46.698`.

[23] E. G. Adelberger, M. M. Hindi, C. D. Hoyle, H. E. Swanson, R. D. Von Lintig, W. C. Haxton, Beta decays of Ne-18 and Ne-19 and their relation to parity mixing in F-18 and F-19, Phys. Rev. C27 (1983) 2833–2856. doi:10.1103/PhysRevC.27.2833.

[24] S. A. Page, et al., Weak Pion - Nucleon Coupling Strength: New Constraint From Parity Mixing in ^{18}F, Phys. Rev. C 35 (1987) 1119–1131. doi:10.1103/PhysRevC.35.1119.

[25] D. R. Phillips, D. Samart, C. Schat, Parity-Violating Nucleon-Nucleon Force in the $1/N_c$ Expansion, Phys. Rev. Lett. 114 (6) (2015) 062301. arXiv:1410.1157, doi:10.1103/PhysRevLett.114.062301.

[26] M. R. Schindler, R. P. Springer, J. Vanasse, Large-N_c limit reduces the number of independent few-body parity-violating low-energy constants in pionless effective field theory, Phys. Rev. C 93 (2) (2016) 025502, [Erratum: Phys.Rev.C 97, 059901 (2018)]. arXiv:1510.07598, doi:10.1103/PhysRevC.93.025502.

[27] V. M. Dubovik, S. V. Zenkin, Formation of parity nonconserving nuclear forces in the standard model SU(2)$_L$ X U(1) X SU(3)$_c$, Annals Phys. 172 (1986) 100–135. doi:10.1016/0003-4916(86)90021-7.

[28] B. Tiburzi, Hadronic parity violation at next-to-leading order, Phys. Rev. D85 (2012) 054020. arXiv:1201.4852, doi:10.1103/PhysRevD.85.054020.

[29] J. Dai, M. J. Savage, J. Liu, R. P. Springer, Low-energy effective Hamiltonian for Delta I = 1 nuclear parity violation and nucleonic strangeness, Phys. Lett. B 271 (1991) 403–409. doi:10.1016/0370-2693(91)90108-3.

[30] D. B. Kaplan, M. J. Savage, An analysis of parity-violating pion-nucleon couplings, Nuclear Physics A 556 (4) (1993) 653–671.

[31] B. Tiburzi, Isotensor hadronic parity violation, Phys. Rev. D86 (2012) 097501. arXiv:1207.4996, doi:10.1103/PhysRevD.86.097501.

[32] V. Cirigliano, M. J. Ramsey-Musolf, Low Energy Probes of Physics Beyond the Standard Model, Prog. Part. Nucl. Phys. 71 (2013) 2–20. arXiv:1304.0017, doi:10.1016/j.ppnp.2013.03.002.

[33] N. Brambilla, et al., QCD and Strongly Coupled Gauge Theories: Challenges and Perspectives, Eur. Phys. J. C 74 (10) (2014) 2981. arXiv:1404.3723, doi:10.1140/epjc/s10052-014-2981-5.

[34] V. Cirigliano, W. Dekens, J. de Vries, M. L. Graesser, E. Mereghetti, A neutrinoless double beta decay master formula from effective field theory, JHEP 12 (2018) 097. arXiv:1806.02780, doi:10.1007/JHEP12(2018)097.

[35] W. C. Haxton, E. Rule, K. McElvain, M. J. Ramsey-Musolf, Nuclear-level Effective Theory of Muon-to-Electron Conversion (8 2022). arXiv:2208.07945.

[36] V. Cirigliano, et al., Towards Precise and Accurate Calculations of Neutrinoless Double-Beta Decay: Project Scoping Workshop Report, 2022. arXiv:2207.01085.

[37] R. D. Carlini, W. T. H. van Oers, M. L. Pitt, G. R. Smith, Determination of the Proton's Weak Charge and Its Constraints on the Standard Model, Ann. Rev. Nucl. Part. Sci. 69 (2019) 191–217. doi:10.1146/annurev-nucl-101918-023633.

[38] G. Buchalla, A. J. Buras, M. E. Lautenbacher, Weak decays beyond leading logarithms, Rev. Mod. Phys. 68 (1996) 1125–1144. arXiv:hep-ph/9512380, doi:10.1103/RevModPhys.68.1125.

[39] M. Tomii, Non-perturbative matching of three/four-flavor Wilson coefficients with a position-space procedure, PoS LATTICE2019 (2020) 174. doi:10.22323/1.363.0174.

[40] M. Tomii, T. Blum, D. Hoying, T. Izubuchi, L. Jin, C. Jung, A. Soni, $K \to \pi\pi$ decay matrix elements at the physical point with periodic boundary conditions, PoS LATTICE2021 (2022) 394. arXiv:2201.02320, doi:10.22323/1.396.0394.

[41] T. Blum, et al., Discovering new physics in rare kaon decays, in: 2022 Snowmass Summer Study, 2022. arXiv:2203.10998.

[42] P. A. Zyla, et al., Review of Particle Physics, PTEP 2020 (8) (2020) 083C01. doi:10.1093/ptep/ptaa104.

[43] F. C. Michel, Parity Nonconservation in Nuclei, Phys. Rev. 133 (1964) B329–B349. doi:10.1103/PhysRev.133.B329.

[44] E. Fischbach, D. Tadic, K. Trabert, Schwinger terms, field algebra, the parity-violating internucleon potential, and models of the weak hamiltonian, Phys. Rev. 186 (1969) 1688–1691. doi:10.1103/PhysRev.186.1688.

[45] R. P. Feynman, in: A. Zichichi (Ed.), Symmetries in Elementary Particle Physics, Academic Press, 1965, p. 167.

[46] O. Haan, B. Stech, Violation of the Delta-I=1/2 rule in non-leptonic decays, Nucl. Phys. B 22 (1970) 448–460. doi:10.1016/0550-3213(70)90420-7.

[47] D. Fakirov, B. Stech, F and D Decays, Nucl. Phys. B 133 (1978) 315–326. doi:10.1016/0550-3213(78)90306-1.

[48] N. Cabibbo, L. Maiani, Two-Body Decays of Charmed Mesons, Phys. Lett. B 73 (1978) 418, [Erratum: Phys.Lett.B 76, 663 (1978)]. doi:10.1016/0370-2693(78)90754-2.

[49] M. Bauer, B. Stech, Exclusive D Decays, Phys. Lett. B 152 (1985) 380–384. doi:10.1016/0370-2693(85)90515-5.

[50] A. Ali, G. Kramer, C.-D. Lu, Experimental tests of factorization in charmless nonleptonic two-body B decays, Phys. Rev. D 58 (1998) 094009. arXiv:hep-ph/9804363, doi:10.1103/PhysRevD.58.094009.

[51] H.-Y. Cheng, B. Tseng, Nonfactorizable effects in spectator and penguin amplitudes of hadronic charmless B decays, Phys. Rev. D 58 (9) (1998) 094005. arXiv:hep-ph/9803457, doi:10.1103/PhysRevD.58.094005.

[52] M. Diehl, G. Hiller, New ways to explore factorization in b decays, Journal of High Energy Physics 2001 (6) (2001) 067. arXiv:hep-ph/0105194, doi:10.1088/1126-6708/2001/06/067.

[53] M. Beneke, G. Buchalla, M. Neubert, C. T. Sachrajda, QCD factorization for exclusive, nonleptonic B meson decays: General arguments and the case of heavy light final states, Nucl. Phys. B 591 (2000) 313–418. arXiv:hep-ph/0006124, doi:10.1016/S0550-3213(00)00559-9.

[54] C. W. Bauer, D. Pirjol, I. W. Stewart, A Proof of factorization for $B \to D\pi$, Phys. Rev. Lett. 87 (2001) 201806. arXiv:hep-ph/0107002, doi:10.1103/PhysRevLett.87.201806.

[55] R. Fleischer, N. Serra, N. Tuning, Tests of Factorization and SU(3) Relations in B Decays into Heavy-Light Final States, Phys. Rev. D 83 (2011) 014017. arXiv:1012.2784, doi:10.1103/PhysRevD.83.014017.

[56] R. Fleischer, R. Jaarsma, E. Malami, K. K. Vos, Exploring $B \to \pi\pi, \pi K$ decays at the high-precision frontier, Eur. Phys. J. C 78 (11) (2018) 943. arXiv:1806.08783, doi:10.1140/epjc/s10052-018-6397-5.

[57] S. Gardner, G. Valencia, The Impact of $|\Delta I| = 5/2$ transitions in $K \to \pi\pi$ decays, Phys. Rev. D 62 (2000) 094024. arXiv:hep-ph/

0006240, doi:10.1103/PhysRevD.62.094024.

[58] G. Buchalla, Kaon and charm physics: Theory, in: Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 2000): Flavor Physics for the Millennium, 2001, pp. 143–205. arXiv:hep-ph/0103166, doi:10.1142/9789812811509_0004.

[59] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portoles, Kaon Decays in the Standard Model, Rev. Mod. Phys. 84 (2012) 399. arXiv:1107.6001, doi:10.1103/RevModPhys.84.399.

[60] E. P. Shabalin, $K_S \to 2\pi$ Decays in a Theory With the Effective Chiral Lagrangian, Sov. J. Nucl. Phys. 48 (1988) 172.

[61] T. Morozumi, C. S. Lim, A. I. Sanda, Chiral Weak Dynamics, Phys. Rev. Lett. 65 (1990) 404–407. doi:10.1103/PhysRevLett.65.404.

[62] R. Abbott, et al., Direct CP violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decay from the standard model, Phys. Rev. D 102 (5) (2020) 054509. arXiv:2004.09440, doi:10.1103/PhysRevD.102.054509.

[63] P. Boyle, N. Christ, N. Garron, E. Goode, T. Janowski, C. Lehner, Q. Liu, A. Lytle, C. Sachrajda, A. Soni, et al., Emerging understanding of the δ i= 1/2 rule from lattice qcd, Physical review letters 110 (15) (2013) 152001.

[64] T. Blum, et al., $K \to \pi\pi \Delta I = 3/2$ decay amplitude in the continuum limit, Phys. Rev. D 91 (7) (2015) 074502. arXiv:1502.00263, doi:10.1103/PhysRevD.91.074502.

[65] W. A. Bardeen, A. J. Buras, J. M. Gerard, A Consistent Analysis of the Delta I = 1/2 Rule for K Decays, Phys. Lett. B 192 (1987) 138–144. doi:10.1016/0370-2693(87)91156-7.

[66] A. Pich, E. de Rafael, Weak K amplitudes in the chiral and 1/n(c) expansions, Phys. Lett. B 374 (1996) 186–192. arXiv:hep-ph/9511465, doi:10.1016/0370-2693(96)00171-2.

[67] U. G. Meissner, H. Weigel, The Parity violating pion nucleon coupling constant from a realistic three flavor Skyrme model, Phys. Lett. B447 (1999) 1–7. arXiv:nucl-th/9807038, doi:10.1016/S0370-2693(98)01569-X.

[68] J. F. Nieves, P. B. Pal, Generalized Fierz identities, Am. J. Phys. 72 (2004) 1100–1108. arXiv:hep-ph/0306087, doi:10.1119/1.1757445.

[69] M. Jamin, J. A. Oller, A. Pich, Scalar K pi form factor and light quark masses, Phys. Rev. D 74 (2006) 074009. arXiv:hep-ph/0605095, doi:10.1103/PhysRevD.74.074009.

[70] M. Gell-Mann, R. J. Oakes, B. Renner, Behavior of current divergences under SU(3) x SU(3), Phys. Rev. 175 (1968) 2195–2199. doi:10.1103/PhysRev.175.2195.

[71] J. Gasser, H. Leutwyler, Chiral Perturbation Theory to One Loop, Annals Phys. 158 (1984) 142. doi:10.1016/0003-4916(84)90242-2.

[72] J. Gasser, H. Leutwyler, Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark, Nucl. Phys. B 250 (1985) 465–516. doi:10.1016/0550-3213(85)90492-4.

[73] M. Jamin, Flavor symmetry breaking of the quark condensate and chiral corrections to the Gell-Mann-Oakes-Renner relation, Phys. Lett. B 538 (2002) 71–76. arXiv:hep-ph/0201174, doi:10.1016/S0370-2693(02)01951-2.

[74] V. Bernard, U.-G. Meissner, Chiral perturbation theory, Ann. Rev. Nucl. Part. Sci. 57 (2007) 33–60. arXiv:hep-ph/0611231, doi:10.1146/annurev.nucl.56.080805.140449.

[75] J. Bordes, C. A. Dominguez, P. Moodley, J. Penarrocha, K. Schilcher, Chiral corrections to the $SU(2) \times SU(2)$ Gell-Mann-Oakes-Renner relation, JHEP 05 (2010) 064. arXiv:1003.3358, doi:10.1007/JHEP05(2010)064.

[76] C. McNeile, A. Bazavov, C. T. H. Davies, R. J. Dowdall, K. Hornbostel, G. P. Lepage, H. D. Trottier, Direct determination of the strange and light quark condensates from full lattice QCD, Phys. Rev. D 87 (3) (2013) 034503. arXiv:1211.6577, doi:10.1103/PhysRevD.87.034503.

[77] S. Park, R. Gupta, B. Yoon, S. Mondal, T. Bhattacharya, Y.-C. Jang, B. Joó, F. Winter, Precision nucleon charges and form factors using 2+ 1-flavor lattice qcd, arXiv preprint arXiv:2103.05599 (2021).

[78] M. González-Alonso, J. Martin Camalich, Isospin breaking in the nucleon mass and the sensitivity of β decays to new physics, Phys. Rev. Lett. 112 (4) (2014) 042501. arXiv:1309.4434, doi:10.1103/PhysRevLett.112.042501.

[79] H.-W. Lin, R. Gupta, B. Yoon, Y.-C. Jang, T. Bhattacharya, Quark contribution to the proton spin from 2+1+1-flavor lattice QCD, Phys. Rev. D 98 (9) (2018) 094512. arXiv:1806.10604, doi:10.1103/PhysRevD.98.094512.

[80] J. Liang, A. Alexandru, Y.-J. Bi, T. Draper, K.-F. Liu, Y.-B. Yang, Detecting flavor content of the vacuum using the Dirac operator spectrum (2 2021). arXiv:2102.05380.

[81] R. Gupta, Y.-C. Jang, B. Yoon, H.-W. Lin, V. Cirigliano, T. Bhattacharya, Isovector Charges of the Nucleon from 2+1+1-flavor Lattice QCD, Phys. Rev. D 98 (2018) 034503. arXiv:1806.09006, doi:10.1103/PhysRevD.98.034503.

[82] M. Viviani, R. Schiavilla, L. Girlanda, A. Kievsky, L. E. Marcucci, The Parity-violating asymmetry in the ^{3}He(n, p)^{3}H reaction, Phys. Rev. C 82 (2010) 044001. arXiv:1007.2052, doi:10.1103/PhysRevC.82.044001.

[83] M. Viviani, A. Baroni, L. Girlanda, A. Kievsky, L. E. Marcucci, R. Schiavilla, Chiral effective field theory analysis of hadronic parity violation in few-nucleon systems, Phys. Rev. C 89 (6) (2014) 064004. arXiv:1403.2267, doi:10.1103/PhysRevC.89.064004.

[84] J. Lang, T. Maier, R. Müller, F. Nessi-Tedaldi, T. Roser, M. Simonius, J. Sromicki, W. Haeberli, Parity nonconservation in elastic $p\alpha$ scattering and the determination of the weak meson-nucleon coupling constants, Phys. Rev. Lett. 54 (1985) 170–173. doi:10.1103/PhysRevLett.54.170.

[85] R. Henneck, C. Jacquemart, J. Lang, R. Müller, T. Roser, M. Simonius, F. Tedaldi, W. Haeberli, S. Jaccard, Study of parity nonconservation in $p\alpha$ scattering, Phys. Rev. Lett. 48 (1982) 725–728. doi:10.1103/PhysRevLett.48.725.

[86] K. Elsener, W. Grüebler, V. König, P. A. Schmelzbach, J. Ulbricht, D. Singy, C. Forstner, W. Z. Zhang, B. Vuaridel, Constraints on weak meson-nucleon coupling from parity nonconservation in ^{19}F, Phys. Rev. Lett. 52 (1984) 1476–1479. doi:10.1103/PhysRevLett.52.1476.

[87] N. Fomin, et al., Measurement of the Parity-Odd Angular Distribution of Gamma Rays From Polarized Neutron Capture on ^{35}Cl (7 2022). arXiv:2207.11295, doi:10.1103/PhysRevC.106.015504.

[88] J.-W. Chen, G. Rupak, M. J. Savage, Nucleon-nucleon effective field theory without pions, Nucl. Phys. A 653 (1999) 386–412. arXiv:

nucl-th/9902056, doi:10.1016/S0375-9474(99)00298-5.

[89] U. van Kolck, Effective field theory of nuclear forces, Prog. Part. Nucl. Phys. 43 (1999) 337–418. arXiv:nucl-th/9902015, doi:10.1016/S0146-6410(99)00097-6.

[90] S. R. Beane, P. F. Bedaque, W. C. Haxton, D. R. Phillips, M. J. Savage, From hadrons to nuclei: Crossing the border (2000) 133–269arXiv:nucl-th/0008064, doi:10.1142/9789812810458_0011.

[91] D. B. Kaplan, M. J. Savage, M. B. Wise, Nucleon - nucleon scattering from effective field theory, Nucl. Phys. B 478 (1996) 629–659. arXiv:nucl-th/9605002, doi:10.1016/0550-3213(96)00357-4.

[92] D. B. Kaplan, M. J. Savage, M. B. Wise, A New expansion for nucleon-nucleon interactions, Phys. Lett. B 424 (1998) 390–396. arXiv:nucl-th/9801034, doi:10.1016/S0370-2693(98)00210-X.

[93] D. B. Kaplan, M. J. Savage, M. B. Wise, Two nucleon systems from effective field theory, Nucl. Phys. B 534 (1998) 329–355. arXiv:nucl-th/9802075, doi:10.1016/S0550-3213(98)00440-4.

[94] U. van Kolck, Effective field theory of short range forces, Nucl. Phys. A 645 (1999) 273–302. arXiv:nucl-th/9808007, doi:10.1016/S0375-9474(98)00612-5.

[95] M. R. Schindler, H. Singh, R. P. Springer, Large-N_c Relationships Among Two-Derivative Pionless Effective Field Theory Couplings, Phys. Rev. C 98 (4) (2018) 044001. arXiv:1805.06056, doi:10.1103/PhysRevC.98.044001.

[96] S. Weinberg, Nuclear forces from chiral Lagrangians, Phys. Lett. B 251 (1990) 288–292. doi:10.1016/0370-2693(90)90938-3.

[97] E. Epelbaum, J. Gegelia, U.-G. Meißner, Wilsonian renormalization group versus subtractive renormalization in effective field theories for nucleon–nucleon scattering, Nucl. Phys. B 925 (2017) 161–185. arXiv:1705.02524, doi:10.1016/j.nuclphysb.2017.10.008.

[98] W. J. Marciano, Flavor Thresholds and Lambda in the Modified Minimal Subtraction Prescription, Phys. Rev. D 29 (1984) 580. doi:10.1103/PhysRevD.29.580.

[99] B. R. Holstein, Nuclear parity-violation parameter $h_\rho^{1'}$, Phys. Rev. D 23 (1981) 1618–1622. doi:10.1103/PhysRevD.23.1618.

[100] J. Carlson, R. Schiavilla, V. Brown, B. Gibson, Parity violating interaction effects 1: The longitudinal asymmetry in pp elastic scattering, Phys. Rev. C65 (2002) 035502. arXiv:nucl-th/0109084, doi:10.1103/PhysRevC.65.035502.

[101] H. E. Swanson, et al., Experimental upper bound and theoretical expectations for parity-violating neutron spin rotation in ^{4}He, Phys. Rev. C 100 (1) (2019) 015204. arXiv:1902.08026, doi:10.1103/PhysRevC.100.015204.

[102] M. Sarsour, et al., Neutron spin rotation measurements, EPJ Web Conf. 219 (2019) 06002. doi:10.1051/epjconf/201921906002.

[103] S.-L. Zhu, Large N_c expansion and the parity-violating π, n, Δ couplings, Phys. Rev. D 79 (2009) 116002. doi:10.1103/PhysRevD.79.116002.

[104] A. Borschevsky, M. Ilias, V. A. Dzuba, K. Beloy, V. V. Flambaum, P. Schwerdtfeger, Nuclear-spin dependent parity violation in diatomic molecular ions, Phys. Rev. A 86 (2012) 050501. arXiv:1209.4282, doi:10.1103/PhysRevA.86.050501.

[105] E. Altuntas, J. Ammon, S. B. Cahn, D. DeMille, Demonstration of a Sensitive Method to Measure Nuclear Spin-Dependent Parity Violation, Phys. Rev. Lett. 120 (14) (2018) 142501. arXiv:1801.05316, doi:10.1103/PhysRevLett.120.142501.

[106] J. Karthein, Precision spectroscopy of single molecular ions in a penning trap (2022).
URL https://archive.int.washington.edu/talks/WorkShops/int_19R_76/People/Karthein_J/Karthein.pdf

[107] T. Kurth, E. Berkowitz, E. Rinaldi, P. Vranas, A. Nicholson, M. Strother, A. Walker-Loud, E. Rinaldi, Nuclear Parity Violation from Lattice QCD, PoS LATTICE2015 (2016) 329. arXiv:1511.02260, doi:10.22323/1.251.0329.

[108] B. Hörz, et al., Two-nucleon S-wave interactions at the $SU(3)$ flavor-symmetric point with $m_{ud} \simeq m_s^{\text{phys}}$: A first lattice QCD calculation with the stochastic Laplacian Heaviside method, Phys. Rev. C 103 (1) (2021) 014003. arXiv:2009.11825, doi:10.1103/PhysRevC.103.014003.

[109] A. Walker-Loud, Lattice qcd calculations of nn interactions and prospects for parity violating matrix elements (2022).
URL https://archive.int.washington.edu/talks/WorkShops/int_19R_76/People/Walker-Loud_A/Walker-Loud.pdf