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A NEW APPROACH TO NONLINEAR DYNAMIC MODELING AND VIBRATION ANALYSIS OF TENSEGRITY STRUCTURES

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ABSTRACT

Tensegrity structures have experienced continued research and development interests in the past several decades. Revealing dynamic characteristics of a tensegrity structure, for example: vibration analysis, is an important objective in structural design and analysis. Traditional dynamic modeling methods are inaccurate in predicting dynamic responding of a tensegrity structure, due to their neglection of internal displacements of structure members. To solve this issue, a new nonlinear dynamic modeling method for tensegrity structures is proposed in this paper. This method defines position of a structure member as a summation of boundary-induced terms and internal terms in a global coordinate system. A nonlinear dynamic model of a tensegrity structure is derived from Lagrange equation, as a system of ordinary differential equations. This dynamic model can be linearized at an equilibrium configuration for vibration analysis. As shown in simulation results, the proposed method can predict natural frequencies of a tensegrity structure with a better accuracy than the traditional methods. Unlike the traditional methods that can only predict dynamic responses in a low frequency domain, the proposed method can also reveal dynamic responses of a tensegrity structure in a higher frequency domain by only using a small number of internal terms.

Keywords: Tensegrity structure, Nonlinear dynamics, Vibration analysis, Modal analysis, Spatial discretization method,

NOMENCLATURE

$\vec{R}$	position vector in the longitudinal direction of a member
$\tilde{u}$	internal terms of a differential element position
ξ	natural spatial variable
$\hat{u}$	boundary induced terms of a differential element position
N	number of internal terms
q	generalized coordinates representing the internal displacement of a member
$\vec{r}$	unit vector of $\vec{R}$
L	deformed length of a member
u	position of a differential element
$\dot{u}$	velocity of a differential element
$\dot{\tilde{u}}$	internal terms of a differential element velocity
$\dot{\hat{u}}$	boundary induced terms of a differential element velocity
m	mass of a member
T	kinetic energy of a member
u_x, u_y, u_z	x-, y- and z-coordinates of u
dL	deformed length of a differential element
L_0	undeformed length of a member
dL_0	undeformed length of a differential element
ε	strain of a differential element
F_{avg}	average internal axial force applied to a differential element from its undeformed to deformed lengths
P	internal force of a member
E	Young's modulus member material
A	cross section area of a member

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dw_c	work done to a differential element by internal conservative force
V	potential energy of a member
L_L	Lagrangian of a member
Q	generalized coordinates of a member
f_{nc}	generalized force associate with nonconservative loads
M	mass matrix
C	damping matrix
K	stiffness matrix
f_{ext}	generalized force associate with externally applied loads

1. INTRODUCTION

Tensegrity structures, due to their lightweight, foldability, and high stiffness, have experienced continued research and development interests in the past several decades [1-4]. In early work, a tensegrity structure is defined as a pin-jointed structure that is composed of isolated members in compression inside a net of continuous members in tension. Such assembly produces a lightweight, deployable, and yet self-standing structure with high stiffness [5, 6]. A tensegrity structure is designed so that its compressed members do not touch each other and prestressed tensioned members spatially delineate the structure. This definition was later generalized, where a tensegrity structure consists of both bars and cables, with contacts among bar members being allowed [7].

Revealing dynamic characteristics of tensegrity structures, for example: vibration analysis, is an important objective in structural design and analysis. However, research aiming to address this type of problem is seen in very few literatures. Early research was found in work of Motro [8], who obtained dynamic response of a tensegrity structure by both numerical and experimental approaches; and in work of Furuya [9], who performed vibration analysis of a tensegrity mast and revealed a relationship between natural frequencies and level of self-stress of the structure.

For vibration analysis of a tensegrity structure, development of a dynamic model is a key step. One commonly used type of methods for nonlinear dynamic modeling of tensegrity structures is based on generalized coordinates and Lagrange approach, which was seen in work of Sultan [10, 11] and Oppenhem [12]. In this type of methods, a system of generalized coordinates, which is highly coupled with topology and geometric configuration of a specific tensegrity structure, is first defined. All generalized coordinates are required to be independent. By treating bar and cable members of the tensegrity structure as rigid bodies and massless springs, a system of second order nonlinear ordinary differential (ODE) equations that describes the mechanical motion of the structure is obtained by the Lagrange approach. The advantage of this type of approach is that design constraints, such as axial symmetry, of a tensegrity structure are well maintained during a dynamic analysis. But a generalized coordinate system often highly relies on geometric simplicity of a tensegrity structure. Thus, this type of approach is restricted in modeling only regular tensegrity structures with a small number of nodes and members.

Another approach to nonlinear dynamic modeling of tensegrity structures is based on finite element methods, which were seen in work of Ali [13], Faroughi [14], Ashwear [15] and Feng [16]. This type of methods starts by obtaining mass and stiffness matrices of a single member in local coordinate system. Then, mass and stiffness matrices of a whole tensegrity structure are established by transforming from local to global coordinates through a co-rotational approach. This type of methods provides a simple approach in obtaining a linear dynamic model for vibration analysis, and offers capability of modeling irregular tensegrity structures that usually lack geometric symmetry. However, it is inefficient when applying to tensegrity structures subjected to large deformations.

A common issue in the above-mentioned nonlinear dynamic modeling methods of tensegrity structure is that structure members are over simplified. This is mainly due to the neglection of internal displacements of structure members in the longitudinal directions. In these traditional methods, bar members were modeled either as rigid bodies with no internal displacement at all (seen in Lagrange and rigid-body based approaches), or as elastic elements whose internal displacements are uniformly distributed along their longitudinal directions (see the shape function used in finite-element-analysis based approaches). Similar issues were also seen in cable member modeling by traditional methods, where cable members were modeled either as massless springs or as elastic elements with uniformly distributed longitudinal displacements. Such oversimplification of structure members will inevitably prevent the nonlinear dynamic model from predicting dynamic responses with a high accuracy, especially for responses in a higher frequency domain.

To resolve the issues in traditional methods, a new nonlinear dynamic modeling method for vibration analysis of tensegrity structures is proposed. This method defines position of a structure member as a summation of boundary-induced terms and internal terms in a global coordinate system. A nonlinear dynamic model of a tensegrity structure is then derived from Lagrange equation, as a system of ordinary differential equations. This dynamic model can be linearized at an equilibrium configuration of the tensegrity structure for vibration analysis. The proposed method is new in that internal deformations of both bar and cable members are considered as depend variables in the dynamic model of a tensegrity structure so developed. Thus, over simplification of a tensegrity structure, which was often seen in traditional dynamic modeling methods, is successfully avoided. The proposed method can predict natural frequencies of a tensegrity structure with a better accuracy than the traditional methods. Unlike the traditional methods that can only predict dynamic responses in a low frequency domain, the proposed method can also reveal dynamic responses of a tensegrity structure in a higher frequency domain by only using a small number of internal terms.

2. NEW METHOD FOR NONLINEAR DYNAMIC MODELING OF TENSEGRITY STRUCTURES

In this section, a new method for nonlinear dynamic modeling and vibration analysis of tensegrity structures shall be presented. Consider a member that connects two nodes of a tensegrity structure in a three-dimensional global coordinate system, see Fig. 1. Coordinates of the two nodes are given as (x_0, y_0, z_0) and (x_1, y_1, z_1), respectively. The longitudinal direction of the member can be expressed by a position vector as

$$\vec{R} = \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{bmatrix} \tag{1}$$

An independent natural spatial variable ξ is defined to represent the internal positions of the member. 0 and 1 are boundary locations for the natural spatial variable, which means that $\xi = 0$ and $\xi = 1$ represent the locations of (x_0, y_0, z_0) and (x_1, y_1, z_1), respectively. According to the spatial discretization method [17], the position u of a differential element of the member at position ξ can be expressed as a summation of internal terms and boundary-induced terms:

$$u(\xi,t) = \tilde{u}(\xi,t) + \hat{u}(\xi,t) \tag{2}$$

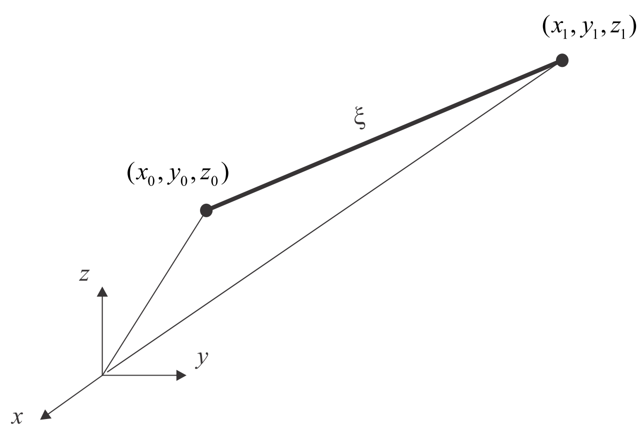

FIGURE 1: MOTION OF A MEMBER IN THE GLOBAL COORDINATE SYSTEM

The internal terms and boundary-induced terms of position u are represented as vector forms in the three-dimensional global coordinate system as

$$\begin{aligned} \tilde{u}(\xi,t) &= \sum_{i=1}^{N} q_i \sin(i\pi\xi)\vec{r} \\ \hat{u}(\xi,t) &= (1-\xi)\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + \xi \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \end{aligned} \tag{3}$$

where N is the number of internal terms, and q_i is generalized coordinates that describes internal displacement of the member in the longitudinal direction. $q_i = 0$ for all i means the longitudinal displacement is uniformly distributed along the axial direction of the member. If the two boundary nodes are fixed, the member is at equilibrium when all q_i are to zero. $\vec{r}$ is a unit vector that represents the longitudinal direction of the member, given as

$$\vec{r} = \frac{\vec{R}}{L} \tag{4}$$

where L is deformed length of the member under a level of self-stress, calculated as

$$L = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2} \tag{5}$$

It should also be emphasized that the boundary induced term in Eq. (3) coincides with a shape function in elastic rod modeling in the finite element method [18]. So, the finite element method for dynamic modeling of bar members of a tensegrity structure can also be viewed as a simplified version of the proposed method without internal terms.

2.1 Kinetic energy

Velocity of a differential element at location ξ on the member can be obtained by taking the time derivative to Eq. (2):

$$\dot{u}(\xi,t) = \dot{\tilde{u}}(\xi,t) + \dot{\hat{u}}(\xi,t) \tag{6}$$

The time derivatives of the internal terms and the boundary-induced terms are given as

$$\dot{\tilde{u}}(\xi,t) = \sum_{i=1}^{N} \left[\dot{q}_i \sin(i\pi\xi)\vec{r} + \left(\frac{q_i}{L}\right)\sin(i\pi\xi)\dot{\vec{R}} - \left(\frac{q_i}{L}\right)\sin(i\pi\xi)\frac{\dot{L}}{L}\vec{R} \right] \tag{7}$$

$$\dot{\hat{u}}(\xi,t) = (1-\xi)\begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{z}_0 \end{bmatrix} + \xi \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{z}_1 \end{bmatrix} \tag{8}$$

Since the internal displacement, described by the general coordinate q_i, is usually significantly small than the deformed length of the member ($q_i \ll L$), it can be assumed that

$$\frac{q_i}{L} \approx 0 \tag{9}$$

By substituting Eq. (9) into Eq. (6), the second and third terms of Eq. (7) vanish. The internal term of velocity becomes

$$\dot{\vec{u}}(\xi,t) = \sum_{i=1}^{N} \dot{q}_i \sin(i\pi\xi)\vec{r} \tag{10}$$

Substituting Eqs. (10) and (8) into Eq. (6) yields

$$\dot{u}(\xi,t) = \sum_{i=1}^{N} \dot{q}_i \sin(i\pi\xi)\vec{r} + (1-\xi)\begin{bmatrix}\dot{x}_0\\ \dot{y}_0\\ \dot{z}_0\end{bmatrix} + \xi\begin{bmatrix}\dot{x}_1\\ \dot{y}_1\\ \dot{z}_1\end{bmatrix} \tag{11}$$

By an assumption of uniform mass distribution along the axial direction of the member, mass of the differential element can be expressed $md\xi$, where m is the mass of the whole member. The kinetic energy of the differential element is given as

$$dT = \frac{1}{2} m \|\dot{u}\|^2 \, d\xi \tag{12}$$

Thus, the kinetic energy of the member can be obtained by the taking the integration with respect to ξ in the domain of the entire member length ([0,1]) as

$$T = \int_{\xi=0}^{\xi=1} dT \tag{13}$$

2.2 Potential energy

A differential element of a structure member, that starts at location ξ and ends at location ($\xi + d\xi$), of a member is shown in Fig. 2. The global coordinates of the starting and the ending points of the differential element are given as (u_x, u_y, u_z) and $(u_x + \frac{\partial u_x}{\partial \xi} d\xi, u_y + \frac{\partial u_y}{\partial \xi} d\xi, u_z + \frac{\partial u_z}{\partial \xi} d\xi)$, respectively, where u_x, u_y and u_z are the x-, y- and z-coordinates of $u(\xi, t)$ obtained from Eq. (2). $\frac{\partial u_x}{\partial \xi}$, $\frac{\partial u_y}{\partial \xi}$ and $\frac{\partial u_z}{\partial \xi}$ are calculated as

$$\begin{aligned}
\frac{\partial u_x}{\partial \xi} &= (x_1 - x_0)\left[1 + \frac{1}{L_b}\sum_{i=1}^{N} q_i i\pi \cos(i\pi\xi)\right] \\
\frac{\partial u_y}{\partial \xi} &= (y_1 - y_0)\left[1 + \frac{1}{L_b}\sum_{i=1}^{N} q_i i\pi \cos(i\pi\xi)\right] \\
\frac{\partial u_z}{\partial \xi} &= (z_1 - z_0)\left[1 + \frac{1}{L_b}\sum_{i=1}^{N} q_i i\pi \cos(i\pi\xi)\right]
\end{aligned} \tag{14}$$

The deformed length dL of the differential element can be calculated by the corresponding geometry information shown in Fig. 2 as

$$dL = \sqrt{\left(\frac{\partial u_x}{\partial \xi}\right)^2 + \left(\frac{\partial u_y}{\partial \xi}\right)^2 + \left(\frac{\partial u_z}{\partial \xi}\right)^2} \, d\xi \tag{15}$$

Substituting Eq. (14) and Eq. (5) into Eq. (15) yields an explicit expression of dL:

$$dL = \left[L + \sum_{i=1}^{N} q_i i\pi \cos(i\pi\xi)\right] d\xi \tag{16}$$

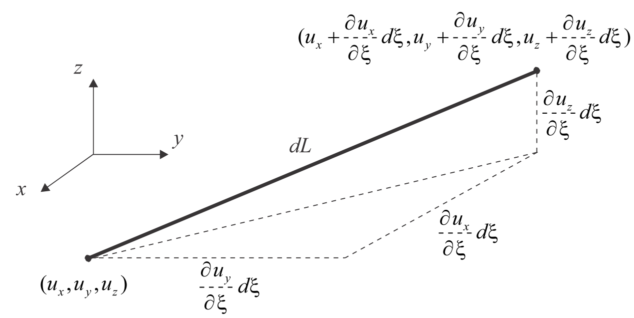

FIGURE 2: KINETIC DIAGRAM OF A DIFFERENTIAL ELEMENT OF A STRUCTURE MEMBER

Let the undeformed length of the member be L_0, and the undeformed length of the differential element be

$$dL_0 = L_0 d\xi \tag{17}$$

Then, the strain ε of the differential element of the member at location ξ can be obtained as

$$\varepsilon = \frac{dL - dL_0}{dL_0} \tag{18}$$

Based on linear elasticity assumption, the average internal axial force F_{avg} applied to the differential element from undeformed length dL_0 to deformed length dL is $P/2$ (internal force increases linearly, from zero to P), where the internal force P is calculated as

$$P = EA\varepsilon \quad (19)$$

where E and A are the Young's modulus and cross section area of the member, respectively. The work dw_c done to the differential element by the internal conservative force F_{avg} is given as

$$dw_c = F_{avg}(dL - dL_0) \quad (20)$$

The potential energy V of the member is then obtained taking the integration of dw_c with respect to ξ in domain of the entire member length ([0,1]).

$$V = \int_{\xi=0}^{\xi=1} dw_c \quad (21)$$

Finally, let the Lagrangian be $L_L = T - V$, a system of nonlinear equations of motion of a member of a tensegrity structure can be obtained by the Lagrange's Equation as

$$\frac{d}{dt}\left(\frac{\partial L_L}{\partial \dot{Q}}\right) - \frac{\partial L_L}{\partial Q} = f_{nc} \quad (22)$$

where Q is the generalized coordinate, defined as

$$Q = \begin{bmatrix} x_0 & y_0 & z_0 & x_1 & y_1 & z_1 & q_1 & \cdots & q_N \end{bmatrix} \quad (23)$$

and f_{nc} is a vector generalized force associate with nonconservative loads, obtained by the principle of virtual work.

2.3 Linearized equations of motion for vibration analysis

The system of nonlinear equations of motion of a member can be linearized at an equilibrium configuration of a tensegrity structure for vibration analysis. Denote global nodal coordinates of the two ends of the member at the equilibrium state as (x_0^e, y_0^e, z_0^e) and (x_1^e, y_1^e, z_1^e). Values of other generalized coordinates q_i associate with internal longitudinal displacements are zero at the equilibrium state. The linearized equations of motion for the member are given in the second order form as

$$M\ddot{Q} + C\dot{Q} + KQ = f_{ext} \quad (24)$$

where M, C and K are mass, damping and stiffness matrices, and f_{ext} is a vector of generalized force associate with externally applied loads.

A system of equations of motion for the entire tensegrity structure can be obtained by directly assembling Eq. (24) for each member without any local-to-global coordinate transformation. The linearized dynamic model for the whole tensegrity structure is also useful for various design and analysis tasks, such as model analysis, control system design and structure health monitoring.

3. SIMULATION RESULTS

For demonstration of the proposed nonlinear dynamic modeling method for vibration analysis of tensegrity structures, a planar Snelson's X tensegrity structure with four nodes, two bars and four cables, is investigated. Topology and dimensions of the structure are shown in Fig. 3. Materials of bar and cable members are assumed to be carbon fiber and steel, respectively. The dimensional and material parameters of the bar and cable members are given in Table 1. The structure is self-stressed, with member internal force being $141.42N$ in compression for the two bar members, and $100N$ in tension for the four cable members. To eliminate the three modes of rigid-body motion of the structure, displacements of node one in the x- and the y-directions, and displacement of node two in the y-direction are restricted.

TABLE 1: DIMENSIONAL AND MATERIAL PARAMETERS OF CABLE AND BAR MEMBERS OF THE SNELSON'S X TENSEGRITY STRUCTURE

Parameter	Value
Young's modulus of bar member	$183GPa$
Young's modulus of cable member	$200GPa$
Radius of bar member	$5mm$
Radius of cable member	$1mm$
Material density of bar member	$1750kg/m^3$
Material density of cable member	$7850kg/m^3$

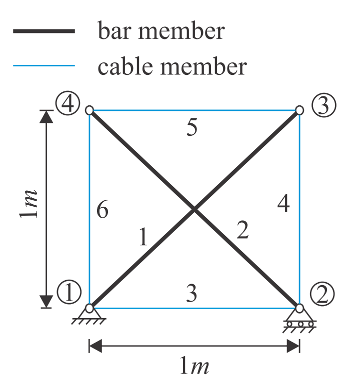

FIGURE 3: A PLANAR SNELSON'S X TENSEGRITY STRUCTURE

The proposed method is compared with the finite element analysis (FEA) method given in Ref. [18], which assumed uniform distribution of displacements of material particles on structure members. In other words, the FEA method for comparison does not include internal displacements of structure members.

3.1 Model analysis

The five natural frequencies associate with nodal motions are investigated for a direct comparison. This is because the FEA method can only reveal the modes associate with nodal motions. Values of the five natural frequencies are presented in Tables 2, with the corresponding mode shapes shown in Fig. 4.

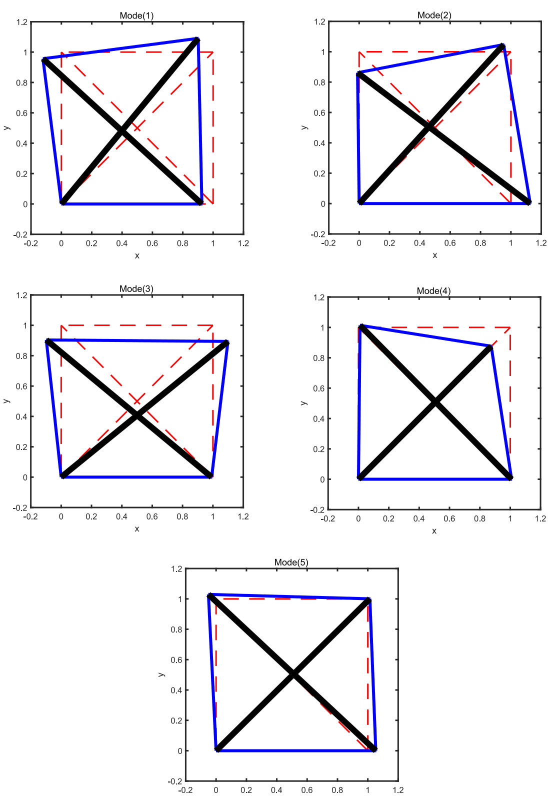

FIGURE 4: FIRST FIVE MODE SHAPES ASSOCIATE WITH NODAL MOTIONS

TABLE 2: THE FIVE NATURAL FREQUENCIES (NF) ASSOCIATE WITH NODAL MOTIONS (*HZ*)

	NF1	NF2	NF3	NF4	NF5
FEM	259.36	450.22	538.73	1840.94	2675.81
$N = 1$	259.17	448.27	538.32	1692.53	2274.68
$N = 2$	259.17	448.24	538.25	1687.20	2261.56
$N = 3$	259.17	448.21	538.25	1686.24	2261.16

As observed from the results, accuracy of natural frequencies associate with modes of nodal motions can be significantly improved by the proposed method. According to Table 2, improvement of 9.17% and 18.34% in accuracies of the fourth and fifth natural frequencies are achieved by only using three internal terms of member positions ($N = 3$).

3.2 Frequency response

Let a point-wise sinusoidal force $F_f = F_0 \sin(2\pi f t)$ be applied at node two of the tensegrity structure in the *x*-direction, with $F_0 = 1000N$. The frequency response of node four in the *y*-direction of the structure can be determined by the FEA method and the proposed method, respectively. As seen in Fig. 5, the FEA method and the proposed method are in good agreement under 1000 Hz. In the range of 1500-3000Hz, which is near the fourth and fifth natural frequencies, significant improvement of accuracy is achieved by the proposed method.

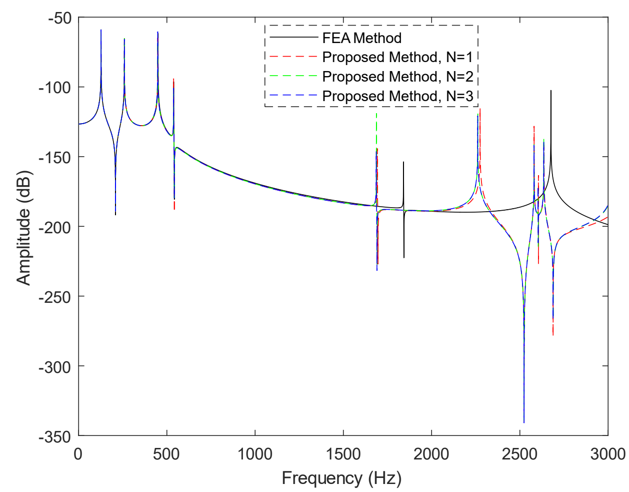

FIGURE 5: FREQUENCY RESPONSE IN 0-3000HZ OF NODE FOUR IN THE Y-DIRECTION

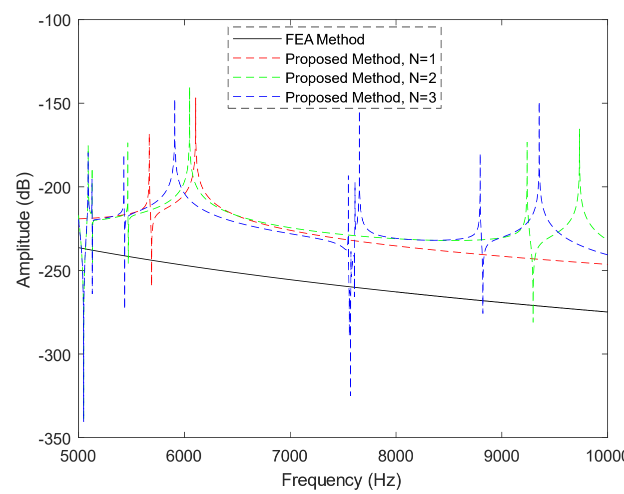

FIGURE 6: FREQUENCY RESPONSE IN 5000-10000HZ OF NODE FOUR IN THE Y-DIRECTION

Frequency responses of the tensegrity structure in the range of 5000-10000Hz are shown in Fig. 6. As observed, the proposed method shows a significant advantage over the FEA method, as the FEA method failed to reveal the natural frequencies in this range. The proposed method can reveal more peaks in the frequency response as the value of N increases.

3.3 Transient response

Let a point-wise sinusoidal force $F_f = F_0\sin(2\pi ft)$ be applied at node two in the x-direction, with $F_0 = 10N$. The displacements of node three of the tensegrity structure in the x-direction are plotted in Figs. 7 and 8, at excitation frequencies of 2000Hz and 15000Hz, respectively.

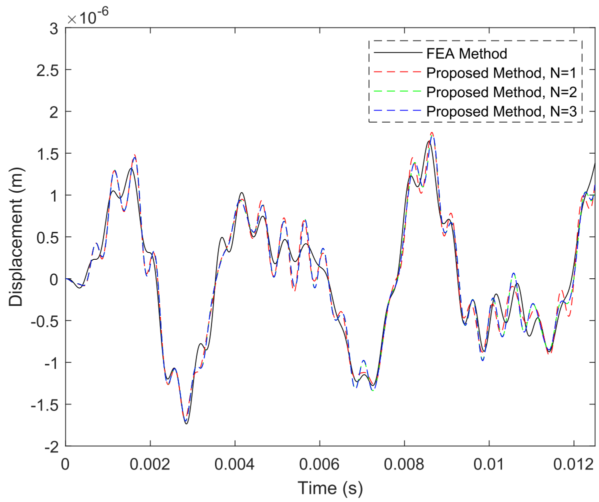

FIGURE 7: DISPLACEMENT OF NODE THREE IN THE X-DIRECTION AT EXCITATION FREQUENCY OF 2000HZ

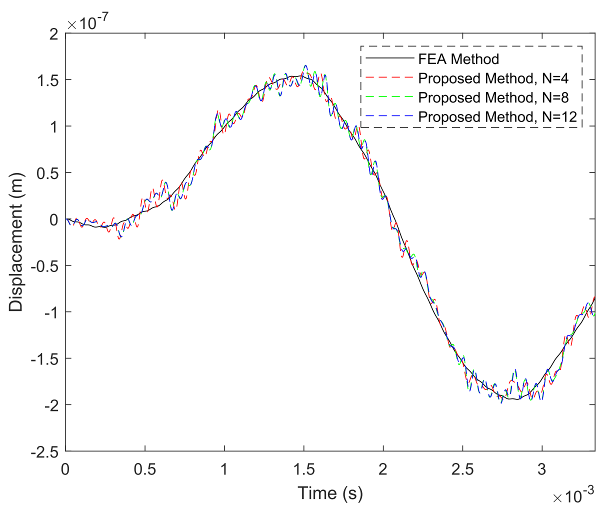

FIGURE 8: DISPLACEMENT OF NODE THREE IN THE X-DIRECTION AT EXCITATION FREQUENCY OF 15000HZ

In both cases, the results obtained by the FEA method are inaccurate, as it fails to reveal dynamic responses with higher frequencies. As observed from Fig. 7, the proposed method can accurately predict the transient response for the 2000Hz excitation frequency with only a small number of internal terms of member positions. The results obtained by the proposed method for $N = 1{\sim}3$ are in good agreement. As observed from Figs. 8, only the proposed method can reveal the high-frequency response for the 15000Hz excitation frequency. The results obtained by the proposed method for $N = 8$ and 12 are in good agreement. However, the results do not match those for $N = 4$. So, it can be concluded that more internal terms of member positions are needed for the proposed method to predict tensegrity structure dynamic response with higher frequencies.

4. CONCLUSION

A new nonlinear dynamic modeling method for vibration analysis of tensegrity structures is proposed. In this method, position of a structure member is defined of as a summation of boundary-induced terms and internal terms in a global coordinate system. A system of nonlinear equations of motion is derived from Lagrange equation. The nonlinear equations of motion can be linearized at an equilibrium configuration of a tensegrity structure for vibration analysis. Simulation results show that the proposed method can accurately predict natural frequencies by only using a small number of internal terms, which validates the numerical efficiency of the proposed method. It is also proved that the proposed method can predict dynamic response of tensegrity structures in a higher frequency domain than the finite element method.

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