

CALIBRATION AND UNCERTAINTY ASSESSMENT TOOL WITH UNCERTAINTY MINIMIZATION FOR OPTIMAL HIGH-SPEED CAMERA SETTINGS IN FLAME PROPAGATION

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ABSTRACT

Measurement of rapidly changing velocities for short time durations is challenging because the optimal capture rate and pixel resolution changes with velocity. It is known for velocity measurements that high temporal resolution will greatly increase the velocity uncertainty. This makes selecting the optimal camera settings unintuitive and can result in highly uncertain measurement. For slow velocities (<10 m/s) that require high temporal resolution (because of rapid acceleration) require an exponentially increasing pixel resolution to minimize experimental uncertainty. The optimal spatial and temporal resolution for single pixel motion can be found using a simple calculation where the expected velocity measurement and desired framerate is used to find the optimal pixel resolution (pixel/length) where $\text{Resolution} = \text{FPS}/V$. This ensures the observed motion travels at least one pixel per every time step. However, it is not clear that this is the best method for selecting experimental settings because often measured objects are not a single pixel in size and do not necessarily move parallel to the pixel grid. A practical method is presented here to aid in image calibration as well as assess the postprocessing method through use of an object with known geometry and which progresses with a known velocity. Because this work focuses on schlieren imaging which uses parallel light, effects discussed in other works such as parallax and perspective are not considered. The calibration method will consider a spinning disk with measurable geometry that changes with angular position. Three geometries will be observed: a curve that follows a polar plot of $r=b\theta$ such that the geometry grows linearly or with a constant velocity, a logarithmic growth, and a complex curve which will represent a wide range of velocity data to simulate actual experimental flame propagation. Because the velocity versus time and radius versus time profile using this tool is known, the noise associated with the camera settings, experimental conditions, and post processing can be directly assessed in situ.

Further, uncertainty minimization through post processing techniques such as interpolation between pixels using grayscale values or selective timestep skipping can be assessed. Additionally, timesteps skipping may present a method for capturing high time resolved data with relatively low spatial resolution, however the magnitude of velocity at measured local minimums and maximums should be assessed which can be observed with the non-linear geometry. The proposed calibration tool improves simple static calibration by allowing the researcher to assess both radius and velocity measurements with a single measurement as well as present a platform to improve measurement uncertainty in post processing.

1. INTRODUCTION

Exploration of velocity measurements presented here were initiated to support the measurement of spherically expanding flame [1,2]. Measurement of the early flame expansion is challenging because ignition causes high initial velocity followed by a slow flame propagation [3]. In addition, higher initial pressures further reduce the velocity of the flame [4] making accurate measurement at the high sample rates increasingly challenging. The drive to use this early flame propagation arises from the instabilities that form on the surface of large radius flame at these high pressures [5]. The sample data in Figure 1 shows a combustion event for methane air at 1 atm where the observed kernel radius slowed to just under 2m/s. This event was captured at 15,000 or 180,000 fps with a spatial resolution of 12.6 (px/mm). The high temporal resolution reduced the size of the frame and the overall observed flame size; however, the largest effect was observed in the noise of the measurement. There are many smoothing techniques available to reduce noise, but it is desired to first study the nature of this noise to minimize it experimentally and determine methods best suited to further reduce noise in post processing while accurately maintaining the true shape of the curve. Accuracy of the data

throughout the curve is important for the measurement of Markstein length flame property[6,7] which represents the slope of the spherically propagating flame on a plot of against spherical stretch. One concern is that simply applying smoothing algorithms could affect the data in a way that will increase errors in measurements that depend on the curvature of the data.

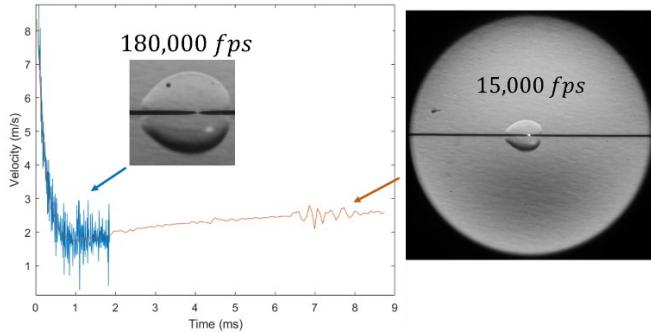


Figure 1: Effect of framerate on velocity measurement

If velocity is calculated using a simple derivative approximation in Equation (1), then the noise observed stems from the fact that velocity is inversely proportional to the time frame used, where the decreased timestep results in a velocity that approaches either infinity or zero. The error from increased time resolution is also discussed in Feng et al.[8]

$$V_i = \frac{x_{i+1} - x_i}{t_{i+1} - t_i} \quad (1)$$

If a constant velocity motion is considered, which is measured over a camera sensor with a resolution in (px/mm) we can expect the error in our velocity measurement to decrease to a minimum where the spatial time step is some integer of the temporal step. If the pixel resolution is poorly scaled, this motion can never be properly observed, illustrated in Figure 2. This of course assumes that the detection pixel is either on or off with the actual motion shown in red and the observable motion in blue.

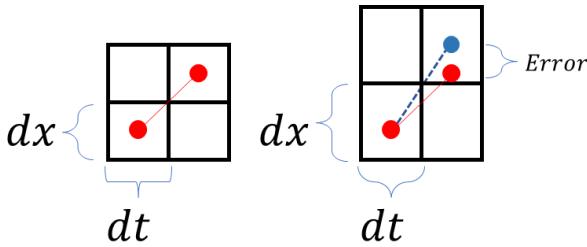


Figure 2: Velocity error from improper pixel vs time scale

This then results in velocity measurements which can only be an integer of these resolutions shown in Equation (2).

$$V_{measurable} = n * \frac{FPS}{Scale} \mid_{n=1,2,3,\dots} \quad (2)$$

where FPS is the sample rate of the camera in Hz and the Scale is the spatial resolution in Pixels per mm. Equation (2) effectively gives the resolution or scale for the velocity measurement where $V_{measurable}$ represents all possible velocity measurements for a given camera and optical setup. It should be noted here that considering a central difference will effectively double the measurement number of measurable values or divide the velocity scale in half. This can be seen in Raffel et al. [9] where the uncertainty of the differentiation scheme for central difference is half that of the forward difference.

For the flame data presented at 180,000 fps the smallest observable change in velocity will be ~ 7 m/s (central difference) and is the source of the noise. This 7m/s step size is not immediately observable in the data because the measurement is a summation of many pixel measurements in 2 dimensions and several of the edge detection routines may increase the effective spatial resolution. These effects create artificial resolution in the velocity domain, but, if a single pixel was used to locate the radius; data would only be present at 0 m/s and 7 m/s. The exploration here only considers a single pixel radius measurement, and this will clearly show the resolution of a velocity measurement without extraneous experimental factors affecting the result. This will show fundamentally what is occurring as we measure and provide an understanding of the affect post processing truly has on the data.

A simple simulation can be created to show how the error of a constant velocity measurement is decreased as the resolution increases seen in Figure 3. In this simulation the time division where held constant as spatial resolution was increased. For the condition where the observed motion is slower than one pixel per second the error in the measurement tends to infinity while greater than 1px/s has maximum error when the motion occurs where the resolutions are out of phase with each other. Clearly from this result the minimum criteria for velocity measurement should be a resolution which is at least scaled to the first local minimum to prevent unbounded inaccuracy. It is also clear that for velocity measurement which changes with time should prioritize a high spatial vs temporal resolution to minimize errors as the measurement crosses between optimum measurement locations as this will be unavoidable.

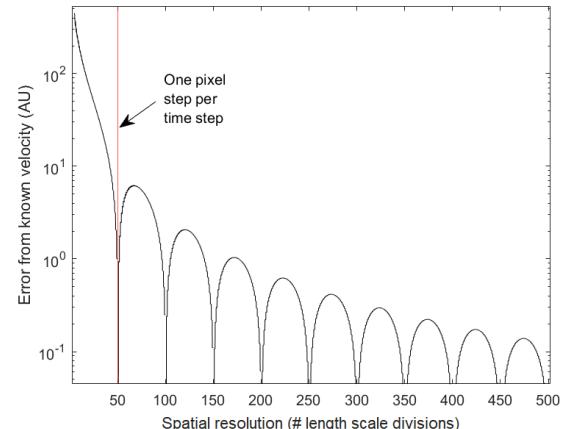


Figure 3: Error reduction as spatial resolution is increased

The idea of defining the optimum experimental conditions can be considered now where a curve of the first local minimum can be plotted on axes of framerate, resolution, and measured velocity. This provides a guide for experimental minimum resolutions to capture velocity at a given framerate.

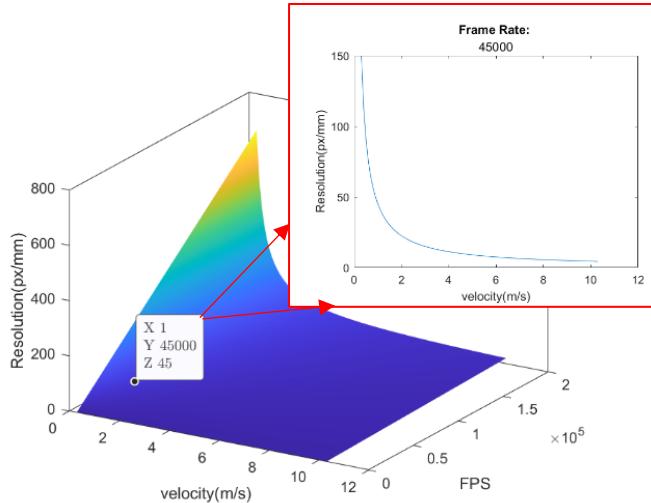


Figure 4: Required spatial resolution to achieve single pixel motion per timestep over entire experimental domain

Ideally any experimental resolution will be many times larger than the curve presented in Figure 4, however this may not be possible depending on the velocities observed. For the extreme flame conditions the velocity may slow down to nearly zero velocity when close to the minimum ignition radius, and with acceleration, poor camera settings may be required which is why addition study of post processing methods is done.

The fundamental velocity measurement is studied to present a true characterization of what these settings mean for uncertainty and after considering the fundamental measurement the data processing method can be easily observed.

2. MATERIALS AND METHODS

Disks with well-defined equations encoded on the edge are used to simulate radial expansion and are illustrated in Figure 5. By rotating the disk with constant angular velocity, the growth over a single pixel column will be described the following equations. Three disks are considered and shown below. Where the measured path starts and ends at a radius of 12.2 and 25.4 mm respectively to cover the available visible region.

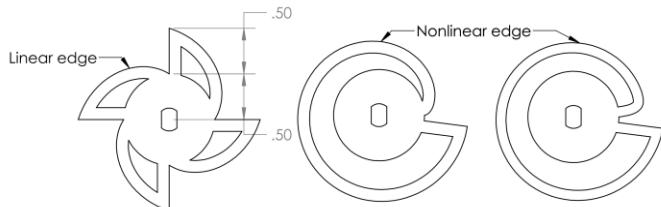


Figure 5: Calibration disk geometries

The equations were described for the edge using parametric equations but presented here in Equations (3) through (5) in the Cartesian form where y is in inches and x is in radians.

$$y = \frac{x}{\pi} + 0.5 \text{ for } x = 0 \rightarrow 0.5\pi \quad (3)$$

$$y = \frac{\ln(x+1)^{0.6}}{\pi} + 0.5 \text{ for } x = 0 \rightarrow 1.96\pi \quad (4)$$

$$y = 10.2 \left(\sqrt{x^2 + 1000^2} - 1000 \right) - 0.8 \quad (5)$$

$$-0.3 \left(e^{-\frac{x}{0.07}} \right) \text{ for } x = 0 \rightarrow 1.96\pi$$

The disks were cut into 1.3 mm thick stainless steel sheet metal using a water jet and were spun during testing using a 5V dc motor. The edge was imaged using Photron SA-Z camera at 45, 35 and 17.5 kfps with an image size of 640 x 640 px. A linear Schlieren system was used as a light source with the disk placed at the center of a parallel 625 nm light path. This setup, however, is technically not schlieren since no knife edge is used and the object observed is solid. An example of the imaged edge is shown in Figure 6 where the vertical column used to detect the edge is highlighted and the edge for this image is marked with a star.

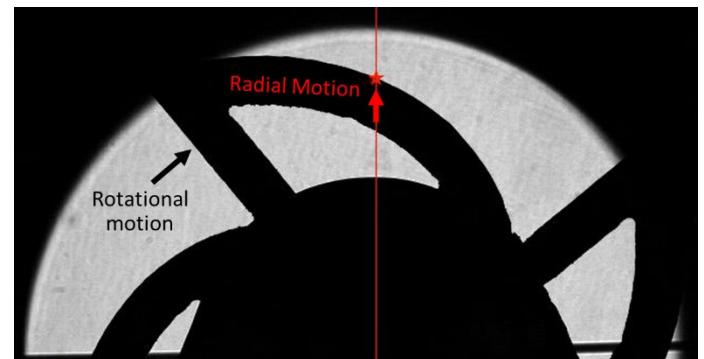


Figure 6: Illustration of motion and measurement location for linear disk

The edge is observed with a spatial resolution of 13.5 pixels/mm at various framerates. This resolution represents a poor camera setup for observation of the velocities expected however the goal will be to improve this measurement with the post processing methods. Three methods are considered (1) interpelation, (2) artificial timestep reduction and (3) mean average smoothing. The first two method address and improve spatial and time resolution while the third will simply filter the variation in the data. The pixel interpolation which will effectively increase the spatial resolution should provide the most improvement while there is concern that the data smoothing algorithm, if applied too heavily, will no longer represent the original data.

The initial edge detection considers the a pixel to contain the edge at some threshold value which means each pixel only represents 2 values on or off. By considering the intensity of the pixel before and after this edge pixel, a location within this pixel

can be found through interpolation. Since the pixel has a 12-bit resolution the pixel resolution is greatly increased.

Artificial time step reduction treats the data as multiple sets of lower framerate data where frames are skipped to produce several lower time resolution velocity curves. These lower frame rate data set are then merged back together into one data set to preserve quantity of data with the quality of the lower framerate. This is similar to a processing done in Feng et al [8].

The conditions observed are shown in Table 1 where the linear disk is captured at several framerates to show that the timestep reduction is identical to capturing lower framerate data at the higher sample rate. Ie. 35000 fps data with the framerate reduction applied has the same precision as the 17500 data except with additional data points.

Table 1: Experimental conditions

	Frame rate	Disk Speed (Hz)	Edge Velocity (m/s)	Velocity resolution (m/s)
Linear Disk	45000	116.88	5.94	1.66
	35000	111.46	5.66	1.29
	17500	112.20	5.70	0.65
Non-Linear Disk 1	45000	76.01	N/A	1.66
Non-Linear Disk 2	45000	84.59	N/A	1.66

The expected edge velocity is shown and the expected velocity resolution (change in velocity between data points) is also given for the camera setup. The non-linear conditions only use the higher frame rate with the interpolation of pixel location in order to best explore the smoothing routines.

3. RESULTS AND DISCUSSION

The linear disk radius and velocity measurement can be seen in Figure 7. As discussed previously the velocity resolution is clearly visible and marked where the measurable values are integers of the resolution described in Equation (2). The measured values must alternate between velocity levels above and below the actual edge velocity shown with a dashed line.

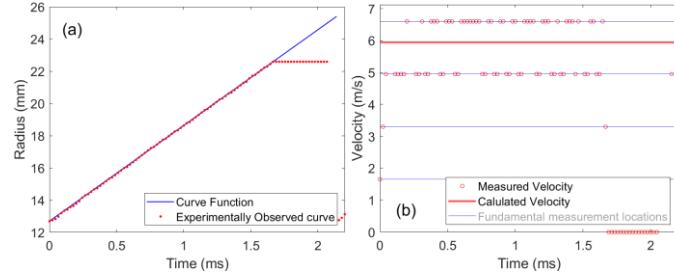


Figure 7: Measured linear radius (a) and velocity (b) where all possible velocity measurements are marked with horizontal black lines

By setting the framerate lower in Figure 8 the resolution can be improved but at the cost of total data. This is shown to illustrate

that the resolution and level of noise in velocity measurement is described by the resolution in Equation (2) and that it behaves in a predictable way. When the high time resolution data has the artificial reduction in time resolution where every other data point is skipped. The resolution is effectively doubled without loosing any data points

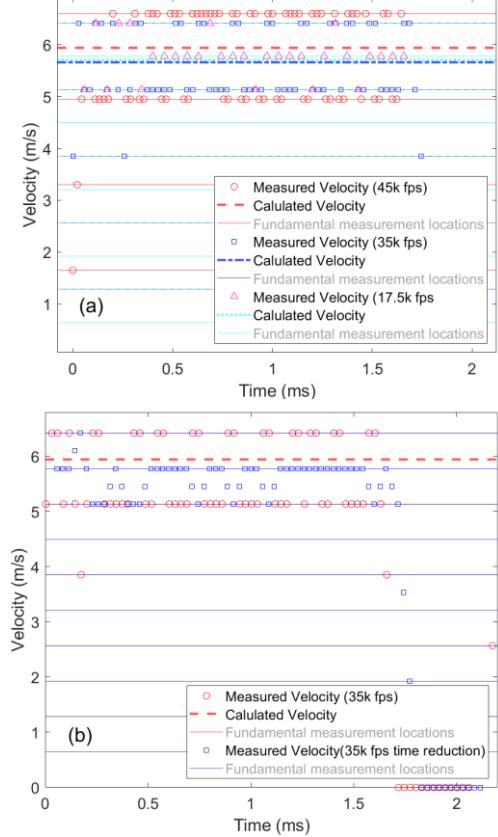


Figure 8: (a) effect of frame rate reduction and (b) timestep reduction.

If the pixel interpolation is considered instead in Figure 9a, the velocity resolution increases by orders of magnitude and the data becomes centered around the true velocity and appears to be bound by the fundamental measurement band. Presumably noise in the pixel intensity from sources such as the light intensity fluctuations, dust in the air and disk edge quality will determine the spread of this interpolated data. Reduction of noise on the pixel level might improve the precision of interpolation and reduce noise.

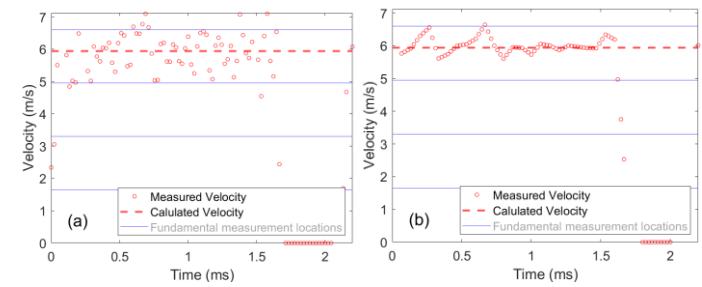


Figure 9: (a) Effect of inter-pixel interpolation and (b) the inclusion of both methods

By applying both interpolation and time step reduction in Figure 9b the deviation of the velocity measurement is greatly improved, however, for the constant velocity condition, application of a data smoothing method could be easily used to produce similar results.

It is clear that for small velocity values where high temporal resolution is needed the interpolation method is necessary to increase the possible spatial measurement location (increasing the resolution of velocity) further, the timestep reduction appears to improve the quality of data with little reduction in data quality. Since the interpolation method can only be applied to the data once but the time step and smoothing methods can be applied with various intensities. These two smoothing methods will be considered against each other using the interpolated data for the non linear cases.

The detected edge and the respective curve equations are plotted in Figure 10 a and b with the fundamental and interpolated velocity measurements in Figure 10 c and d. As seen previously the use of interpolation is absolutely necessary for these slow velocities where the velocity measurements are now bounded by the fundamental resolution. If the known velocity profile is subtracted from measured velocity the velocity measurement is found to be evenly distributed within this first fundamental band. This makes sense and shows that these velocity steps are essentially the precision available at the camera settings.

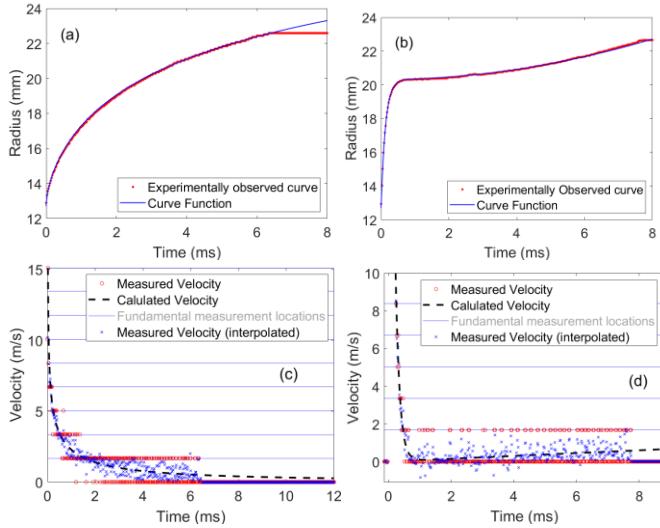


Figure 10: measurement (a and b) of non-linear disks showing velocity(c and d) with and without interpolation applied.

Next, a single application of the timestep reduction and smoothing algorithm are applied to the data. The two methods appear to be similar. We know from the linear condition that the timestep reduction effectively reduces the measurement deviation by half of the previous velocity measurement step. The strength of the smoothing method appears to match this result. In Figure 12 the smoothing routine is applied 8 times to the data and the time step skipping takes the derivative for every 8th data point. While these methods are not equivalent in nature for direct

comparison, the traditional smoothing method is shown to deviate from the rapidly changing portion off the non-linear case showing it no longer represents the original curve. Granted, if a larger number of timestep skips were used the result would eventually not reflect the correct curve either.

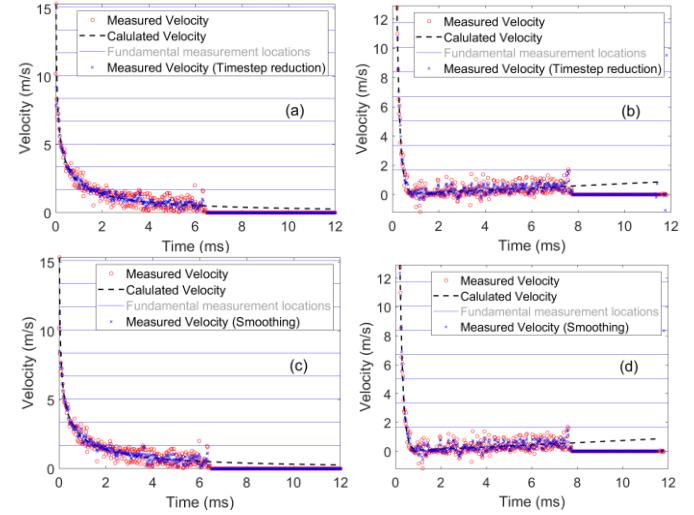


Figure 11: Timestep reduction (a and b) vs smoothing (c and d)

However, the benefit of the Timestep method is the affect on the data is better known. The data is effectively getting squeezed into a smaller range. By taking the derivative for every 8 step the resolution would be the equivalent to a 5600 fps capture which would have a fundamental velocity measurement scale of ~ 0.2 m/s. This resolution improvement still only considers the original data whereas smoothing algorithms use relations unrelated to the physical phenomena observed and therefore have a higher chance the data will be negatively altered

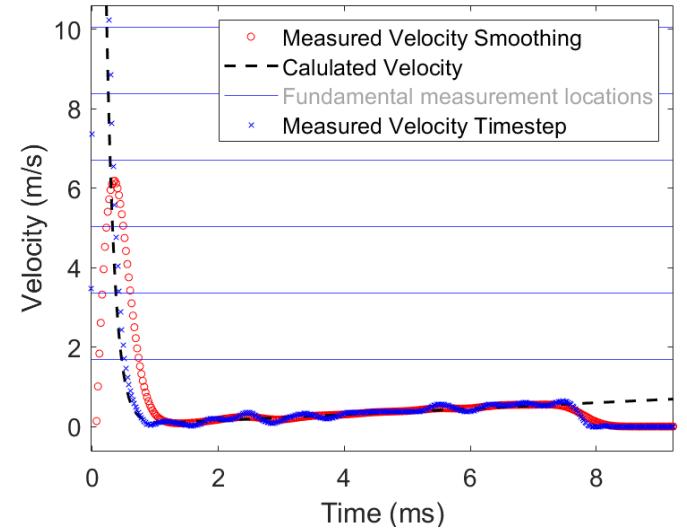


Figure 12: Increased smoothing intensity (circle) vs skipping 9 data points in the time step reduction method (cross)

4. CONCLUSION

The velocity measurement for an expanding radius has been reduced to its fundamental form. Inadequate camera settings were used to capture velocity measurements at slow and rapidly changing values. This condition allows for the experimentation of various post processing methods and the affect it has on the measurement results. The resolution of velocity is defined as the framerate divided by the spatial scale and represents the upper range of the measurement uncertainty. In this case only the pixel resolution is considered. The interpolation between pixels was found to be necessary for these high framerate measurements where the measurement noise at sub pixel resolution was contained by the base pixel resolution velocity of the camera settings. The bounds on this noise can be improved by reducing the effective framerate through frame skipping.

Applying smoothing algorithms is possible, but for fast acceleration the results begin to deviate from the true value. For conditions that approach these experimental conditions it is best to test the post processing methods using a calibration device to ensure the data accurately reflects the true data and information is not being lost. Such as important curvature and peaks through aggressive post processing.

An additional benefit to using a non-static calibration tool is the ability to fine tune the scaling factor to the measured curve. For example, if an initial (px/mm) scaling factor is established from a single measurement. A higher precision scaling factor can be found by matching the slope of the rotary tool measurement which uses a much larger dataset.

Further study of the interpolation routine, to assess the noise present on the pixel level, is desired to determine if further noise improvements can be made without the need for the secondary smoothing methods.

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