

Bond Bundle Analysis of Ketosteroid Isomerase

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Abstract

Bond bundle analysis is used to investigate enzymatic catalysis in the ketosteroid isomerase (KSI) active site. We identify the unique bonding regions in five KSI systems, including those exposed to applied oriented electric fields and those with amino acid mutations, and calculate the precise redistribution of electron density and other regional properties that accompanies either enhancement or inhibition of KSI catalytic activity. We find that catalytic enhancement results from promoting both inter- and intra-molecular electron density redistribution, between bond bundles and bond wedges within the KSI-docked substrate molecule, in the forward direction of the catalyzed reaction. Though the redistribution applies to both types of perturbed systems, and is thus suggestive of a general catalytic role, we observe that bond properties (*e.g.* volume vs. energy vs. electron count) can respond independently and disproportionately depending on the type of perturbation. We conclude that the resulting catalytic enhancement/inhibition proceeds via different mechanisms, where some bond properties are utilized more by one type of perturbation than the other. Additionally, we find that the correlations between bond wedge properties and catalyzed reaction barrier energies are additive to predict those of bond bundles and atomic basins, providing a rigorous grounding for connecting changes in local charge density to resulting shifts in reaction barrier energy.

Keywords: *bond bundle, bond wedge, gradient bundle analysis, gradient bundle decomposition, KSI, enzymatic catalysis, electron density analysis, QTAIM*

1 Introduction

Enzymes can accelerate chemical reactions by many orders of magnitude. Our understanding of the mechanisms responsible for this process has grown at an increasing rate over the last few decades thanks to two significant advances: i) increasingly accurate structural studies; and ii) improved computational platforms and methods that allows us to predict, among other things, the conformation and energy of folded proteins and the reaction dynamics at enzyme active sites.¹⁻⁵ This wealth of data is useful for identifying new structures and structure-property relationships that may, in turn, be incorporated into the existing chemical formalisms, known collectively as chemical intuition.

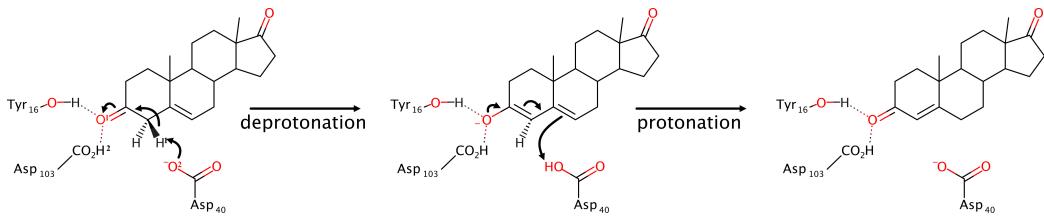
8 Here we investigate the model ketosteroid isomerase (KSI) enzyme, the focus of numerous ex-
9 perimental and computational investigations (see Reference 6 for additional references), specifically
10 to observe the redistribution of electron charge density, $\rho(\mathbf{r})$ caused by amino acid mutations or
11 applied external electric fields (EEFs). These are thought to affect KSI catalytic activity primarily
12 through changes to its electrostatic preorganization,^{7–10} though it remains unconfirmed that these
13 two types of perturbations affect electrostatic preorganization via the same mechanism.

14 Electrostatic preorganization is a strong, non-uniform electric field that augments catalysis
15 through transition state stabilization,⁸ and that results from the arrangement and composition
16 of amino acids about the active site, whose specific arrangement is maintained by the larger protein
17 scaffolding. Using computational methods to predict electrostatic preorganization, researchers are
18 exploring methods for its analysis, comparison, and interpretation.^{6,11,12}

19 Our approach uses bond wedge and bond bundle analysis to reduce a system to a set of chemical
20 bonding regions whose energy, extent, electron count, and many other properties can be accurately
21 calculated.^{13,14} Bond wedges are regions within an atom (atomic basin) where charge density ac-
22 cumulates as a consequence of atomic interactions. The boundaries separating bond wedges are
23 precisely defined zero-flux surfaces whose shape and movement dictate how charge redistribution
24 will affect bond properties via simple geometric constraints on $\rho(\mathbf{r})$.¹⁵

25 Our two-part investigation proceeds with a direct inspection of bond bundle property distribu-
26 tions in the KSI active site in the presence and absence of a uniform EEF known to enhance its
27 catalytic activity. Here we will see that bond bundle electron counts provide an unprecedented
28 level of precision and ease of chemical interpretation. In the second part, we explore a set of KSI
29 systems with varying catalytic activity. The investigation reveals the reactant state bond wedge and
30 bond bundle redistribution that facilitates the forward reaction direction, and locates the active-site
31 regions that most strongly correspond to catalytic enhancement or inhibition, whether caused by
32 applied EEFs or amino acid mutation.

33 We find that regional volume and energy respond in different proportions to applied EEFs than to
34 chlorination of particular amino acids. The disproportionate response of regional properties is found
35 to underlie differences in reaction barrier height. This suggests that regions whose properties have
36 strong correlation with reaction barrier height and that respond proportionately to both types of
37 perturbations play a more central role in catalytic augmentation. Our analysis method allows us to
38 precisely quantify these changes, and because the data are real-space, regional property integrations,



Scheme 1: Steroid isomerization reaction catalyzed by KSI.

it is convenient to visualize and frame our findings within the language and concepts of chemical bonding.¹⁶ This framing provides rigor to what Roald Hoffmann describes as the “fuzzy” application of chemical bonding.¹⁷

2 Background

2.1 Ketosteroid isomerase

The well-studied steroid isomerization reaction that KSI catalyzes involves the repositioning of a double C=C bond in the steroid substrate.^{6,11,18-21} As shown in Scheme 1, this occurs by the removal of a proton from the secondary β -carbon, which is redeposited at the adjacent secondary carbon. Focusing on the first step, deprotonation is typically pictured as the result of a shift of charge through the substrate π system from the β -carbon to the carbonyl oxygen. In KSI this concerted atomic and electronic rearrangement is facilitated by the ideal positioning of Asp₄₀, providing a general base to receive the proton, and by the oxyanion hole that activates the carbonyl and stabilizes the charged enolate intermediate state.

Fuller *et al.* investigated the effects of applied EEFs to this process using the small-scale KSI active site shown in Figure 1, and found that a field applied parallel to the substrate carbonyl bond, pointing from O to C, augmented the electrostatic preorganization, lowering the reaction barrier, while a field in the opposite direction raised the barrier.¹¹ This agrees with classical intuition, that an EEF should push charge opposite the field direction, in this case combining constructively with the KSI oxyanion hole to more readily shift charge to the carbonyl oxygen—further stabilizing the enolate intermediate state—and away from the β carbon, increasing its acidity and thus facilitating deprotonation.

Hennefarth and Alexandrova were then able to show similar reaction barrier effects in KSI vari-

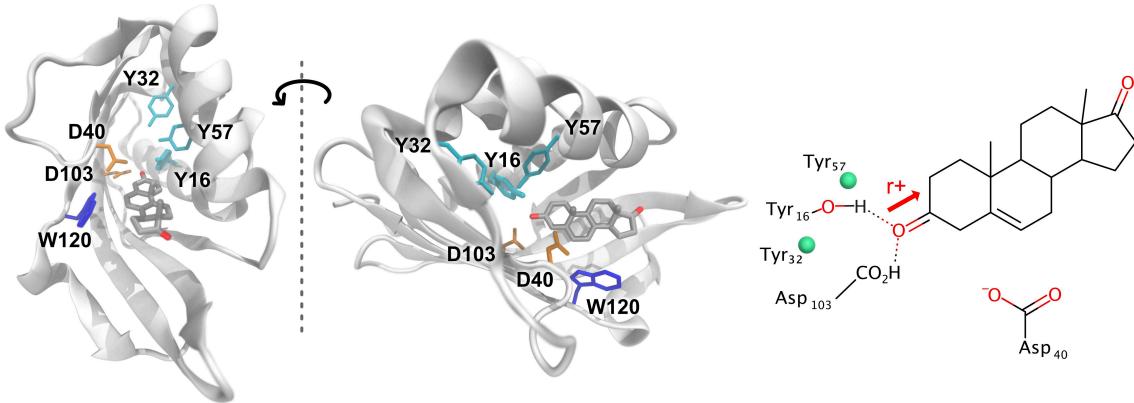


Figure 1: The full KSI protein (PDB code 1O10)²² with docked steroid substrate shown from two angles. The tyrosine (Y or Tyr; cyan), aspartic acid (D or Asp; orange) and tryptophan (W or Trp; blue) residues included in the small scale calculation are shown relative to the substrate (colored by element). The Lewis diagram of the system is shown with the “locations” of the 3-chlorotyrosine for the KSI variant systems (Trp₁₂₀ not shown).

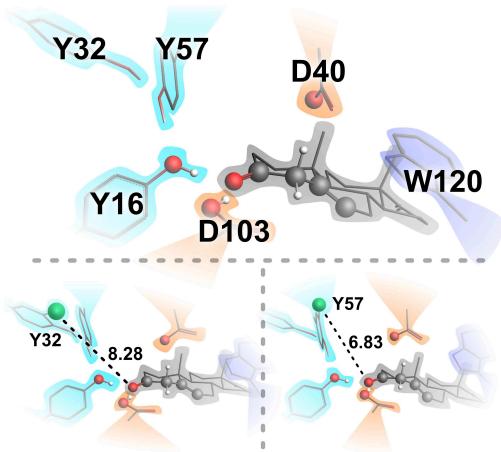


Figure 2: Closer depiction of the arrangement of amino acids forming the “oxyanion hole” about the substrate in KSI (top) and the positioning of the 3-chlorotyrosines in the KSI mutants (bottom) with Cl...O1 distance (Å) indicated.

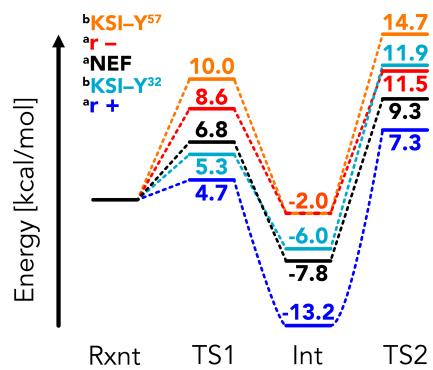


Figure 3: Reaction profiles for the systems in this study. Ordering at top-left is the same as in the TS1 column.
^aData for the NEF and EEF systems taken from Fuller *et al.*¹¹ ^bData for KSI-Y systems taken from Hennefarth and Alexandrova.⁶

ants that had a tyrosine mutated to a 3-chlorotyrosine.⁶ Because the tyrosines of interest are involved
62 in the extended hydrogen bonding network around the oxyanion hole (see Figure 2),²³ this affects
carbonyl activation, altering electrostatic preorganization. They found that a mutated Tyr₃₂ lowered
64 the barrier, and a mutated Tyr₅₇ raised the barrier relative to the wild type enzyme.

The present investigation uses the small-scale KSI active site, EEF directions, and 3-chlorotyrosine
66 KSI variants from References 6 and 11 as the starting point for our calculations, and we explicitly
68 use the same calculated reaction barrier energies. The five systems: wild type (WT) KSI (aka
70 NEF: no electric field), KSI^{r+} KSI^{r-} KSI-Y³² and KSI-Y⁵⁷ provide a minimum example set of KSI
enhancement and inhibition via global and local perturbations; oriented EEFs and chlorination of
amino acids respectively. The regional changes in charge density and energy underlying the reaction
barrier shifts should be accessible using this limited sample.

72 2.2 Assessing the local charge density origins of KSI catalytic enhancement

74 A common approach in the search for *local* relationships between $\rho(\mathbf{r})$ and energy related catalytic
76 properties (reaction barriers) is to seek correlations between the property of interest and charge
density metrics at points, along one-dimensional paths, or within arbitrarily defined volumes. This
approach has been employed to understand the enhancement and inhibition mechanisms of KSI
78 using $\rho(\mathbf{r})$ from *ab-initio* calculations.^{4-6,11}

Fuller *et al.* checked for correlations between KSI-catalyzed reaction barrier shifts due to EEFs
80 and a number of local properties, such as inter-atomic distances and values of $\rho(\mathbf{r})$ at bond critical
82 points (CPs).¹¹ The strongest correlation found was that of the O2-H1 bond length (see Figure 4
84 for atom numbering), which correlated positively with the change in reaction barrier, while the
value of $\rho(\mathbf{r})$ at the corresponding bond CP anticorrelated, indicating that direct facilitation of the
86 deprotonation step affects the barrier energy shift. A weaker positive correlation was found with the
O1-H2 bond length, indicating, as anticipated, that activation of the carbonyl bond—by decreasing
the Asp₄₀–substrate distance—also lowers the reaction barrier.

Given the catalytic role of electrostatics in KSI, Hennefarth and Alexandrova investigated the
88 electric field itself, $\mathbf{E}(\mathbf{r})$, in the KSI active site using both point and regional properties.⁶ Here, the
set of systems included those exposed to oriented EEFs, and also two 3-chlorotyrosine KSI mutants

90 with respectively higher and lower catalytic activity than WT KSI. They found that the electric
91 field magnitude, $|\mathbf{E}(\mathbf{r})|$, at the O2–H1, C2–H1, and C1=O1 bond CPs correlated strongly with the
92 change in reaction barrier, but only for WT KSI structures; KSI-Y³² and KSI-Y⁵⁷ were outliers
93 to this trend. We take these observations to indicate that the mechanism, through $\rho(\mathbf{r})$, by which
94 the KSI mutants enhance/hinder catalysis may be different than that due to EEFs. Furthermore,
95 the correlation at the carbonyl bond CP showed a nearly constant relationship between $|\mathbf{E}(\mathbf{r})|$ and
96 reaction barrier energy.

97 Hennefarth and Alexandrova also conducted a regional electric field curvature analysis within two
98 separate rectilinear volumes, one containing the carbonyl C1=O1 atoms, and the other containing
99 the C2–H1⋯O2 atoms of the reaction site. By evaluating the total curvature along $\mathbf{E}(\mathbf{r})$ streamlines
100 within each volume, they generated histograms that reflect the relative occupations of high and low
101 curvature regions—a regional $\mathbf{E}(\mathbf{r})$ fingerprint. The same relative regions of multiple systems could
102 then be compared by computing their corresponding histogram distances, thus providing a scalar
103 similarity metric also useful for statistical evaluation.

104 Using this similarity metric, Hennefarth and Alexandrova found that the $\mathbf{E}(\mathbf{r})$ curvature about
105 the carbonyl bond had a stronger correlation with reaction barrier than that of the reaction site, with
106 3-chlorotyrosine mutants included in the analysis and which were no longer outliers. This result is
107 counter to the results from point properties (and those of Fuller *et al.*) that the strongest correlations
108 occur in the reaction site rather than within the substrate. The regional results indicate that
109 activation of the carbonyl enhances the reaction rate, where point-based analysis results emphasize
110 changes at the deprotonation site. That is, a regional approach seems to highlight the underlying
111 chemistry at work common to both the EEF and mutant KSI systems, resulting from changes to
112 the electrostatic preorganization in the KSI oxyanion hole.^{10,24}

2.3 The chemical bonding structure of the charge density

114 Here we extend The Quantum Theory of Atoms in Molecules
 (QTAIM), considering only volumes over which energy is well-
 116 defined.^{25,26} In its standard form, QTAIM is used to locate the
 boundaries of the “atoms in molecules,” aka atomic basins, which
 118 possess unambiguous energies, volumes, and charges. That atomic
 basins have a well-defined energy results from their satisfaction of a
 120 zero flux (of the charge density gradient) boundary condition. For
 an arbitrary region in $\rho(\mathbf{r})$, one may calculate the average regional
 122 kinetic energy using the gradient or Laplacian forms of the quan-
 tum mechanical kinetic energy operator, but in general these values
 124 will not match one another. Over a region bounded by zero-flux
 surfaces, however, these values will agree, and hence the regional
 126 energy of an atomic basin is unambiguous.²⁵

128 Bond bundle analysis is an extension of QTAIM that recognizes
 a further partitioning of atomic basins into the smallest regions
 130 bounded by zero-flux surfaces, called differential gradient bundles
 (dGBs).^{27,28} To each dGB there corresponds an unambiguous en-
 ergy, and an atomic basin decomposition into dGBs produces a con-
 132 tinuous and “well-defined energy space.” The topology of this space
 reveals precise boundaries between intra-atomic regions of charge accumulation called bond wedges.
 134 Bond wedges of adjacent atoms then combine to form bond bundles.^{14,29}

136 Figure 4 illustrates the eight atomic basins and seven bond bundles considered in this study,
 as well as the bond and ring CPs that lie along or interior to their boundaries. See Figure S3 in
 138 the supplemental information for a more realistic, three-dimensional representation of bond bun-
 dles. We will appeal to this sort of abstract representation of bond bundles for the remainder of
 140 this manuscript. Each bond bundle has a set of properties commonly associated with a chemical
 bond, such as an energy and a number of electrons, which, when taken over all bonds in a system,
 recover the system energy and electron count. Like atomic basins, bond bundles have precise, non-
 142 overlapping boundaries that combine to fill all space, giving rise to bond volume—one of a number

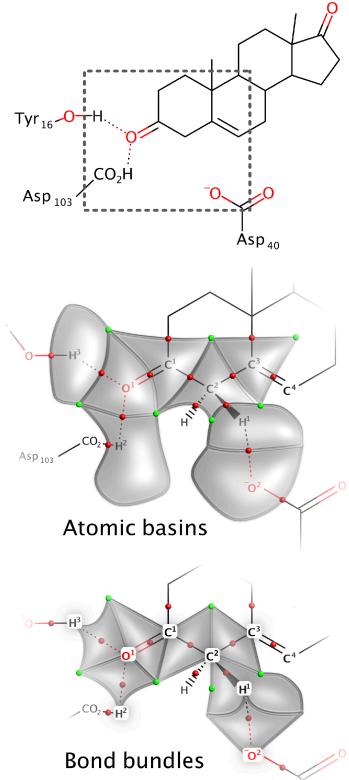


Figure 4: Cartoon depictions of the atomic basins and bond bundles analyzed in this study. Red and green spheres represent bond and ring CPs respectively.

of newly quantifiable bond properties. Of course, the integrated value of electron density within a
144 bond bundle gives its electron count.

Methods

146 All results are from *ab-initio* calculations including the residues shown experimentally and theoretically to control the electrostatic preorganization in KSI.³⁰ The initial placement of these residues
148 is that determined in References 6 and 11, starting from the experimentally determined KSI structure.²² While the KSI active site $|\mathbf{E}(\mathbf{r})|$ can fluctuate between 135 and 150 MV/cm at physiological
150 temperatures,³¹ here, as in References 6 and 11, we are interested in elucidating reactivity using
information from the reactant state, so we exclusively use relaxed, reactant-state conformations
152 where $|\mathbf{E}(\mathbf{r})|$ is closer to 144 ± 6 MV/cm.²⁰ Subsequent bond bundle investigations will focus on the
significance of these fluctuations due to large thermal movements in biological molecules, as well as
154 analogous effects due to molecular vibrations in simple molecular systems.

All *ab-initio* calculations were performed using the ADF package of the Amsterdam Modeling
156 Suite.³²⁻³⁴ Relaxed system geometries were obtained with initial coordinates from References 6 and
11 as mentioned above. Optimization of all five systems was performed using a triple- ζ STO all-
158 electron basis set with one polarization function,³⁵ with the Minnesota'06-2X XC energy density
functional^{36,37} and “good” numerical integration quality. The NEF system relaxation also included
160 implicit COSMO solvation^{38,39} using Allinger solvent radii and a dielectric constant of $\epsilon = 4.0$.
Subsequent single-point calculations were run with the same basis set and functional, with the same
162 COSMO settings now used for all systems. All applied electric fields were of magnitude 10 MV/cm, a
value on the order of that determined for the field produced by the protein beyond the active site by
164 Fuller *et al.* using AMBER charges and classical multipole expansion,¹¹ and an order of magnitude
less than that produced within the KSI active site.²⁰

166 Topological analysis and gradient bundle decomposition was performed on static $\rho(\mathbf{r})$ grids with
the Bondalyzer software suite of the Molecular Theory Group at Colorado School of Mines,⁴⁰ an
168 add-on to the Tecplot360 visualization package.⁴¹ The error associated with these calculations arises
mostly from the use of regular volumetric grid field data, the resolution of which governs the error.
170 Given a “correct” $\rho(\mathbf{r})$, the accuracy of the method increases with the number of differential gradient
bundles used, and with increased discretization of the underlying gradient paths. When a grid spac-

172 ing of 0.025 bohr is used, and each atomic basin is decomposed into at least five-thousand differential
173 gradient bundles, as in this investigation, error in atomic basin electron populations is generally less
174 than 1%, *i.e.* more than 99% of atomic basin charge density is recovered. Additionally, the current
175 method of condensed maximum basin identification results in $\pm 0.2\%$ error in the integrated bond
176 wedge properties.

177 Comparison and statistical analysis of gradient bundle integrated properties was performed in
178 Python, and the MatPlotLib⁴² and SeaBorn⁴³ libraries were used for plotting line/scatter and bar
179 charts respectively.⁴⁴ One-dimensional scatter plots (Figure 7 and Section 5 of the SI) show the
180 KSI-catalyzed reaction barrier height as a function of integrated regional properties, with a linear
181 fit produced with `sklearn.linear_model.LinearRegression` in SciPy.⁴⁵ R^2 , R^2_{adj} , and p-values
182 included with the linear fits (and the other statistical measures included for each fit in the SI)
183 were produced using the ordinary least squares `summary` in StatsModels.⁴⁶ Horizontal bar charts
184 (Figure 8 and Section 4 of the SI) are a compact, graphical depiction of many such scatter plots,
185 showing the sign of the correlation (fit line slope) and the correlation coefficient of the linear fit, but
186 not the slope of the fit line, and were constructed using MatPlotLib. Correlation diagrams (Figure 6
187 and Section 7 of the SI) are typical correlation diagrams showing whether the properties within
188 a particular region correlate with each other (*e.g.* how volume and energy correlate within some
189 particular bond bundle), produced using the SeaBorn `heatmap`. Section 6 of the SI contains other
190 correlation diagrams showing whether particular regional properties correlate across similar regions
191 (*e.g.* how bond bundle volume correlates across all bond bundles).

192 Chemical diagrams were composed in MarvinSketch.⁴⁷ The left two images in Figure 1 were
193 compiled with VMD.⁴⁸ Final figures were compiled in Affinity Designer.⁴⁹

194 3 Results and discussion

3.1 KSI charge density response to a catalyzing EEF

195 To illustrate the concrete nature of gradient bundle properties, we begin by inspecting $\rho(\mathbf{r})$ redis-
196 tribution in the KSI active site due to a catalyzing uniform EEF of magnitude $10^{\text{MV}/\text{cm}}$, which is an
197 order of magnitude less than that produced by the first and second coordination sphere residues,
198 and on the order of the field produced by the protein beyond the active site.¹¹ The KSI active site

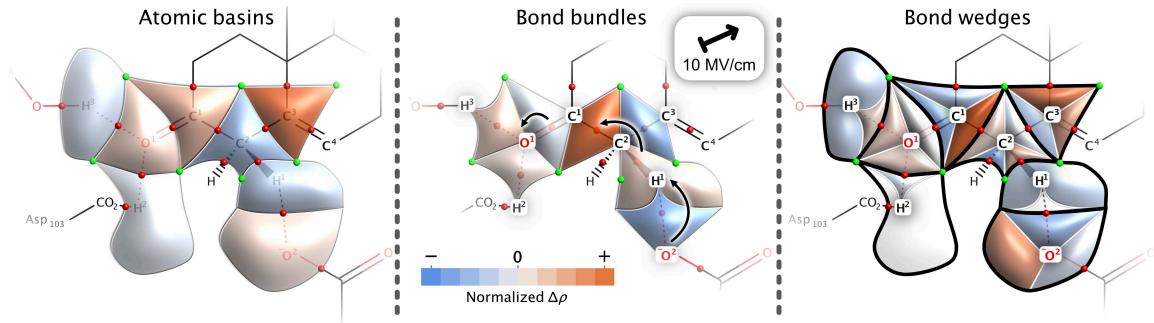


Figure 5: The atomic basins, bond bundles, and bond wedges of KSI (left, middle, and right respectively) shaded according to the changes in their regional electron count due to a 10 MV/cm oriented EEF (the $r+$ direction), which are listed in Table 1. The center image includes the electron-pushing arrows of the deprotonation reaction step.

200 EEF response is captured in the gradient bundle decomposition.

Table 1 contains regional electron counts (ρ) for atomic basins, bond bundles, and bond wedges 202 in the KSI active site, as well as their change due to the catalyzing $r+$ EEF. These regional property 204 changes are graphically depicted in Figure 5, where regions are shaded orange or blue to indicate EEF-induced $\rho(\mathbf{r})$ accumulation or depletion respectively. See Section 3 in the supplemental information for similar tables for all five systems, for ten different regional properties.

The field activates the carbonyl bond, which is assumed to occur by redistributing electron charge 206 density to the carbonyl O atom, and is evidenced here by the accumulation of charge in the O1 atomic basin (Figure 5; left). However, the charge redistribution *within* the atomic basin (*i.e.* between 208 bond wedges), recovers a more detailed description of the activation. The charge accumulation in the O1 \cdots H2 and O1 \cdots H3 bond bundles (Figure 5; center) is a consequence of charge depletion in 210 the O1=C1 bond bundle. More pronounced is the charge redistribution occurring at the other end 212 of the conjugated system, interior to the substrate. The C1=C2 bond bundle, which is believed to 214 increase its bond order from single to double in the deprotonation step, accumulates nearly 0.09 electrons in response to the field, offset by decreases in the carbonyl bond and more so in the C2=C3 bond.

At the reaction site, the C2–H1 bond, which is broken in the deprotonation step, accumulates 216 charge due to the EEF, while the incipient O2 \cdots H1 bond loses charge, seemingly the reverse of the 218 anticipated electron motion for a forming bond. Inspection of the corresponding bond wedge values (Figure 5; right), however, reveals the expected behavior. The increase in the C2–H1 bond bundle 220 electron count is due to the contribution of the C2 atom. The H1 bond wedge component of the C2–

Table 1: Regional electron counts in the KSI active site with (EEF) and without (NEF) an applied external electric field of $10^{\text{MV/cm}}$ pointing from the O nuclear position to the C; the r+ direction. Unnumbered atoms were not included in the study. All regions are truncated at the $\rho = 0.001$ isosurface. Complete gradient bundle integration tables are available in the SI, Section 3.

Atomic basin decomposition	Electron count (ρ) [e]			
	NEF	EEF	Δ	% Δ
C1	5.151	5.185	0.035	0.67
C2	6.207	6.187	-0.021	-0.33
C3	6.182	6.255	0.073	1.18
H1	0.862	0.846	-0.017	-1.92
H2	0.370	0.369	-0.001	-0.22
H3	0.380	0.367	-0.013	-3.46
O1	9.245	9.258	0.013	0.14
O2	9.324	9.329	0.005	0.05
Total	37.721	37.795	0.074	0.20
<hr/>				
Bond bundle decomposition	NEF	EEF	Δ	% Δ
C1 — C bond wedge	2.049	2.010	-0.040	-1.93
C1 — C2 bond bundle	3.435	3.524	0.089	2.58
\hookrightarrow C1 bond wedge	2.073	2.157	0.084	4.07
\hookrightarrow C2 bond wedge	1.363	1.367	0.004	0.31
C1 — O1 bond bundle	3.362	3.353	-0.009	-0.28
\hookrightarrow C1 bond wedge	1.029	1.019	-0.010	-0.99
\hookrightarrow O1 bond wedge	2.334	2.334	0.001	0.04
C2 — H bond wedge	1.625	1.574	-0.051	-3.16
C2 — C3 bond bundle	3.214	3.171	-0.042	-1.31
\hookrightarrow C2 bond wedge	1.567	1.563	-0.004	-0.25
\hookrightarrow C3 bond wedge	1.647	1.609	-0.038	-2.32
C2 — H1 bond bundle	2.465	2.478	0.013	0.52
\hookrightarrow C2 bond wedge	1.652	1.683	0.030	1.84
\hookrightarrow H1 bond wedge	0.813	0.795	-0.018	-2.17
C3 — C bond wedge	1.769	1.842	0.073	4.15
C3 — C bond wedge	2.766	2.804	0.038	1.37
H1 — O2 bond bundle	3.662	3.617	-0.046	-1.25
\hookrightarrow H1 bond wedge	0.049	0.050	0.001	2.15
\hookrightarrow O2 bond wedge	3.613	3.566	-0.047	-1.30
H2 — Asp ₁₀₃ bond wedge	0.335	0.335	0.000	0.12
H2 — O1 bond bundle	3.539	3.543	0.005	0.13
\hookrightarrow H2 bond wedge	0.035	0.034	-0.001	-3.46
\hookrightarrow O1 bond wedge	3.503	3.509	0.006	0.17
H3 — Tyr ₁₆ bond wedge	0.321	0.300	-0.022	-6.77
H3 — O1 bond bundle	3.467	3.482	0.014	0.42
\hookrightarrow H3 bond wedge	0.059	0.067	0.009	14.66
\hookrightarrow O1 bond wedge	3.408	3.414	0.006	0.17
O2 — Asp ₄₀ bond wedge	2.114	2.109	-0.005	-0.24
O2 lone pair wedge	3.597	3.654	0.057	1.58
Total	37.721	37.795	0.074	0.20

H1 bond decreases by 0.018 electrons. Because the H1 atom's other bond wedge, corresponding to the
222 O2 · · · H1 bond, only increases by 0.001 electrons, we conclude that the remaining 0.017 electrons—
the amount lost by the H1 atomic basin—is transferred to the C2 atom rather than redistributed
224 within the H1 atom. That is, shared charge density in the C2–H1 bond was lost by the H1 atom,
responsible for at least half of the charge density gained in the corresponding C2 bond wedge; a
226 precursor to its subsequent gains when the bond is broken. Furthermore, given the much lower
electron counts of the O-bonded H atoms—around $0.37e$ in this case—we posit that the H1 atom,
228 at $0.86e$, loses roughly half an electron during its abstraction to Asp₄₀. The EEF-induced decrease
of $0.017e$ constitutes around 4% of the necessary H1 atomic charge depletion as dictated by the
230 reaction, so here too the $\rho(\mathbf{r})$ response to the EEF appears to facilitate deprotonation. It's tempting
to discuss this in terms of electron flow across boundaries. However, a more correct approach is to
232 recognize that the boundaries themselves move.

3.2 Correlated gradient bundle property redistribution

The specification of real-space bonding regions manifests a number of new (or newly quantified)
234 bond properties, some of which are geometric. To assess the contribution of boundary motion
to regional property responses, we can compare gradient bundle field properties against gradient
236 bundle geometric properties. We will include in our analysis two field properties— ρ and kinetic
238 energy (T)¹—and two geometric properties—volume (V) and bond wedge solid angle (α). The
former represent fundamental chemical bonding concepts—bond electron count and bond energy
240 respectively—that are quantified and generalized via their counterpart gradient bundle properties.
The latter represent two of a number of regional geometric charge density descriptors, which are
242 necessary components of a real-space chemical bond.¹⁵ Using this approach, we will see that gradient
bundle properties can vary independently in response to applied EEFs or enzyme mutation, and that
244 these two types of perturbations affect gradient bundles in qualitatively different ways.

Although these geometric descriptors are newly computed $\rho(\mathbf{r})$ properties, they too, like the
246 field properties, can have straightforward chemical interpretations. In addition to bond wedge V ,
we can describe a bond wedge by α . Conceptually, one can think of α as the percent of near-nuclear
248 (“core”) $\rho(\mathbf{r})$ contained within a bond wedge. In a methane molecule, each bond wedge on the sp³

¹Due to the virial theorem, gradient bundle kinetic energy is equal to minus the total energy,^{14,50,51} and so relative changes to T give us insight into changes in total bond energy.

250 C atom would have α of $\frac{1}{4}$. In a benzene molecule, each sp^2 C atom is found to have a C–H bond wedge and two C–C bond wedges with respective α of 0.30, 0.35, and 0.35. For the sp^2 C3 atom from this study, which, in the WT KSI system, has α of 0.28, 0.30, and 0.43 (from Section 3.1 in the supplemental information). Bond wedge V and α both describe atomic partitioning and depend 252 on bond wedge bounding surfaces, though α specifically describes the surfaces close to the nucleus.

254 As a system changes, these surfaces move, and this motion primarily determines redistribution of properties between bond wedges.^{16,52} Numerically, this means that changes to geometric bond 256 wedge properties (e.g. V and α) often predict changes to other bond wedge properties. However, bond wedge surface motion close to the nucleus can be different from the motion in the inter- 258 atomic (“valence”) region. For example, a decrease in α accompanied by an increase in V would indicate that the bond wedge surfaces contracted in the near-nuclear region and expanded in the 260 inter-atomic region. Moreover, bond wedge property responses to a perturbation are not equal, and can be compared to assess whether a bond wedge merely expanded into more space with the same 262 average property distribution (e.g. V , T , and ρ increase proportionately), or if the property content of the region actually changed (e.g. V decreases while T and ρ increase). Rather than inspect relative 264 gradient bundle geometric and field property redistribution resulting from the r+ EEF directly, we will simultaneously compare the redistribution resulting from both types of perturbations.

266 Figure 6 shows gradient bundle property correlation matrices for a selection of atomic basins, bond bundles, and bond wedges, generated using the data from all five systems: wild type KSI (a.k.a. 268 NEF), EEF-enhanced KSI^{r+} EEF-inhibited KSI^{r-} enhanced mutant KSI-Y,³² and inhibited mutant KSI-Y.⁵⁷ With both types of catalytically enhancing and inhibiting perturbations considered, we 270 see that the correlation among gradient bundle property responses varies from region to region. Within the H1 atomic basin at the reaction site (top-left), we see that V correlates weakly with ρ 272 and T , indicating that atomic basin boundary motion occurs primarily in space with little charge or energy content. That is, the boundary motion responsible for this V change occurs in regions 274 that contribute minimally to ρ and T . Contrast to the H2 atomic basin, part of the oxyanion hole, in which V , T , and ρ all strongly correlate. Similar contrast exists between the C1 and C3 atomic 276 basins (left column, rows three and four), demonstrating that property correlation varies from atom to atom regardless of species. Note that α is always equal to 1 for an atomic basin, so it provides 278 no information here.

Bond bundle property correlation is more dynamic. In the substrate carbonyl bond bundle (C1–

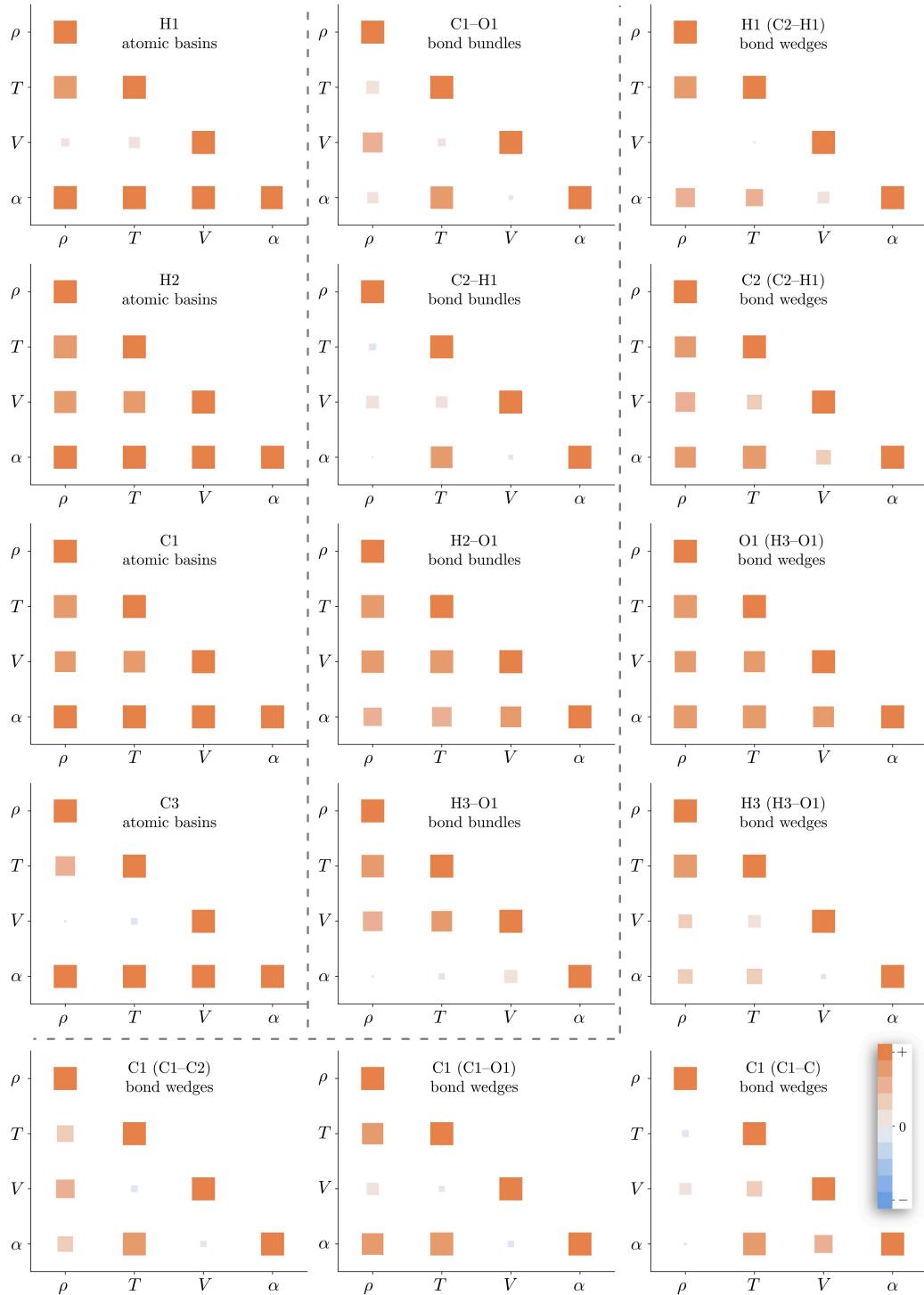


Figure 6: Correlation matrices showing property correlations within a selection of gradient bundles. Electron density (ρ), kinetic energy (T), volume (V), and solid angle (α) are included. Full sets of correlation matrices are provided in the SI, Section 7.

Table 2: Regional property changes for each system relative to NEF KSI for the C1 bond wedge corresponding to the C1 bond with the unnumbered C atom.

C1 (C1–C) bond wedges	ρ (NEF : 2.049)		T (NEF : 13.899)		V (NEF : 21.068)		α (NEF : 0.3688)	
	Δ	% Δ	Δ	% Δ	Δ	% Δ	Δ	% Δ
r+	-0.040	-1.93	-0.306	-2.21	-0.183	-0.87	-0.0065	-1.75
r-	-0.030	-1.45	-0.204	-1.47	-0.360	-1.71	-0.0047	-1.28
Y ³²	-0.001	-0.05	-0.339	-2.44	5.608	26.62	-0.0091	-2.48
Y ⁵⁷	-0.014	-0.67	-0.382	-2.75	5.433	25.79	-0.0101	-2.73

280 O1; top-center of Figure 6), α and V weakly anticorrelate, suggesting that inter-atomic region bond
 281 bundle surface motion is independent (and somewhat opposite) from near-nuclear region surface
 282 motion. Also in the C1–O1 bond bundle, ρ and T weakly correlate with each other, but strongly
 283 correlate with V and α respectively, indicating that changes in T result primarily from near-nuclear
 284 region bond bundle surface motion (where electronic kinetic energy is greatest), while inter-atomic
 285 surface motion is responsible for changing ρ . The ability of T and ρ to change independently is
 286 most stark in the C2–H1 bond bundle at the reaction site (center column, second row), where they
 287 anticorrelate, and again (as with the C1–O1 bond bundle) this results from differences in bond bun-
 288 dle surface motion in the near-nuclear versus inter-atomic region, as evidenced by anticorrelation
 289 between V and α . Not all bond bundles present such dynamic property correlation, however. Con-
 290 sidering the carbonyl O atom’s lone pair coordinated hydrogen bonds (center column, rows three and
 291 four), within the H2–O1 bond bundle, all four properties strongly correlate. Contrast to the other
 292 O atom lone pair hydrogen bond, H3–O1, where α more weakly correlates with V and anticorrelates
 293 with both T and ρ , so again the bond bundle surface motion near the nuclear CP is quite different
 294 from that far from the nuclear CP.

295 Varying bond bundle property correlations are partially explained as the combination of their
 296 constituent bond wedges. For example, the H3–O1 bond bundle property correlation appears to
 297 combine that of the O1 bond wedge component (right column, third row), in which all four properties
 298 correlate, and that of the H3 bond wedge component (right column, fourth row), which seems
 299 responsible for the dynamic correlation of the resulting bond bundle.

300 While uncorrelated behavior between T and ρ can be accompanied by a lack of correlation
 301 between α and V , this is not always the case, as in the H3 component of the H3–O1 bond bundle
 302 where ρ and T strongly correlate *despite* anticorrelation between V and α . In this case it appears
 303 that the H3 bond wedge surface motion differs in the near-nuclear versus inter-atomic regions, but

304 ρ and T occupy both regions in the same proportions, such that boundary motion does not affect
305 one more than the other. The same behavior is evident in the C1 bond wedge components of the
306 C1–O1 and C1–C2 bond bundles (bottom row, left two columns). More interesting is the opposite
307 case, where α and V do correlate, while T and ρ do not, as in the C1 atom's third bond wedge
308 to another (unnumbered) C atom (bottom row, right column). Here α and V correlate, indicating
309 similar motion of near-nuclear and inter-atomic bond wedge surfaces, but regardless, ρ and T weakly
310 anticorrelate. More ambiguous than the previous cases, we turn to the tabulated data in Table 2
311 to find that this behavior stems from disproportionate changes to bond wedge properties resulting
312 from applied EEFs compared to enzyme mutation. Specifically, T , ρ , and α are lowered in response
313 to all four perturbations, though the response of T is proportionate with respect to both types, but
314 not the response of ρ , which is much less responsive to chorination of amino acids than to applied
315 EEFs. Meanwhile, bond wedge V dramatically increases in response to atomic substitution, while
316 slightly decreasing in response to applied EEFs. When the C1 atomic basin is considered as a whole
317 (Figure 6; left column, third row) this uncorrelated behavior vanishes. We conclude that the lack
318 of correlation between ρ and the other three properties in the C1–C bond wedge in Figure 6 results
319 from the two mutant systems for which bond wedge ρ barely changes relative to the other properties,
320 in marked contrast to the effects of applied EEFs.

321 These observations show that gradient bundle properties respond differently to chorination of
322 amino acids than to applied EEFs. Although T , V , and α are all functionals of ρ , they can respond
323 independently of ρ , as in Table 2 where enzyme mutations and applied EEFs had the same relative
324 effect on T , but dramatically different effects on ρ and V .

3.3 KSI catalytic enhancement and inhibition

325 Having demonstrated that enzyme mutation affects gradient bundle properties differently than applied
326 EEFs, we now investigate the relationship between this redistribution and changes to catalytic
327 activity. As in References 6 and 11, we now check for linear correlations between reaction barrier
328 energy and regional properties across the five systems: wild type KSI (a.k.a. NEF), EEF-enhanced
329 KSI^{r+} EEF-inhibited KSI^{r-} enhanced mutant KSI-Y³² and inhibited mutant KSI-Y⁵⁷. We start by
330 inspecting some particularly relevant individual correlations, then move to a method of viewing cor-
331 relations of all regions and properties simultaneously. We will find that the bond bundle and bond
332 wedge $\rho(\mathbf{r})$ redistribution accompanying the r+ EEF is well represented in the reaction barrier energy

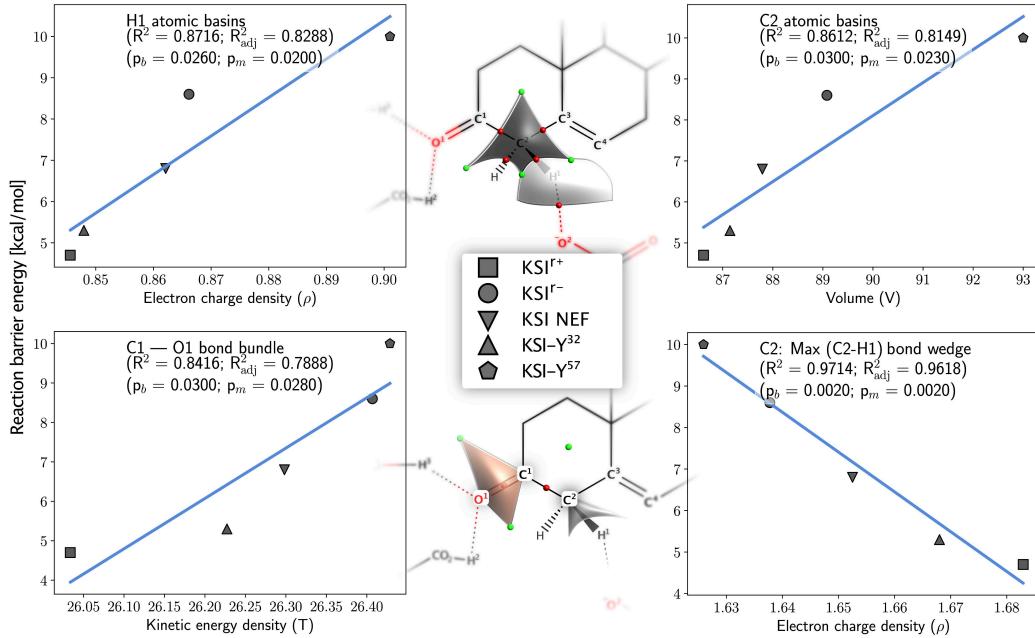


Figure 7: Sampling of atomic basin and bond bundle properties correlated against KSI-catalyzed deprotonation reaction barrier energy. Center: Schematic representation of the atomic basins (top) and bond bundles (bottom) shown. Sides: Plots of regional properties vs barrier energy. Full sets of plots used to generate correlations presented in this manuscript are available in the SI, Section 5.

correlations, but that this is not the case for atomic basins, for which there are notable exceptions. By considering correlations between system energy and gradient bundle properties, we will see that the mechanism by which amino acid mutation affects catalytic proficiency appears different from that of applied EEFs, and that this difference stems from the ability of gradient bundle properties to independently vary.

Beginning with inspection of individual correlations, Figure 7 shows a selection of some of the fits of reaction barrier height as a function of different gradient bundle condensed properties (see Section 5 in the supplemental information for the complete set of plots for all systems, and section SI-3 for the corresponding tabulated gradient bundle integration data).² Atomic basin correlations were strong for the C2 and H1 atoms. Specifically, the H1 atomic ρ correlates positively with reaction

²To aid in the statistical interpretation of the fits, in addition to the correlation coefficient (R^2) we have included the adjusted correlation coefficient (R^2_{adj}) and p-values for the constant and linear fit coefficients (p_b and p_m respectively). R^2_{adj} is always $\leq R^2$, and a large difference between the two indicates that there are unhelpful independent variables in the regression model. For example, in the two top plots in Figure 7, the points are skewed to the left, that is, the left half of the x-axis is more thoroughly sampled than is the right half, resulting in R^2_{adj} being about 0.054 less than R^2 . Meanwhile, the C2 bond wedge fit (bottom-right) has more even sampling of the x-axis, and less of a resulting penalty in its R^2_{adj} . P-values indicate the likelihood that a given fit can be explained by the null hypothesis; that it is explained by chance. A p-value of less than 0.05 conventionally indicates that the null hypothesis can be rejected and that the fit is statistically significant. Section 5 of the supplementary information contains additional measures of statistical significance, independence, and separation.

344 barrier, as does the C2 atomic V . Intra-atomic property redistribution additionally indicates that
 345 property shifts within the conjugated substrate carbon system—*e.g.* carbon atom rehybridization—
 346 play an important role. In this case, positive correlation of O1=C1 bond T and negative correlation
 347 of the C2-to-H1 bond wedge ρ together indicate that, to lower the reaction barrier, charge should
 348 increase in the C2 bond wedge (gained from the heterolytic cleavage of the C2–H1 bond), and kinetic
 349 energy should decrease in the O1=C1 bond (thus increasing total energy, weakening the carbonyl
 350 bond). This is in agreement with results in the previous section, where the catalyzing EEF did in
 351 fact produce these property shifts.

Moving on to the simultaneous inspection of correlations of all regions and properties, it is helpful to simplify and plot multiple correlation coefficients at once to see how *e.g.* volume and energy each correlate, and to access a more immediate chemical interpretation. Inspired graphically by Reference 53, we have included correlations of regional ρ , T , V , and α as bar charts in Figure 8. Regions are sorted according to the reaction barrier correlations of ρ , which is also used to graphically shade representative regions as in Figure 5. Note that the shading has opposite meaning between the two figures, in regards to energetic significance. In Figure 8 a blue-shaded region anticorrelates with reaction barrier energy, so an increase in regional properties should lower the reaction barrier. Unlike in Figure 5, where an orange region was one in which ρ increased in response to a catalyzing EEF. Regions and properties that resulted in low correlation coefficients are still included in the bar charts in order to contrast the full set of regions. Redistribution of $\rho(r)$ and other gradient bundle integrated properties is more correlated to reaction barrier energy for some regions/properties than others.

Looking at ρ redistribution between bond bundles, the C1–C2 bond most anticorrelates with reaction barrier energy, indicating that promoting (or hindering) its transition from single to double bond generally lowers (or raises) the reaction barrier. The O1=C1 and O1···H2 bond bundles respectively correlate and anticorrelate with barrier height, indicating that activation of the carbonyl bond—by increasing the O1 atom lone pair density and decreasing the carbonyl bond density—lowers the barrier. At the deprotonation site, the C2–H1 bond properties anticorrelate with barrier height, paradoxically suggesting that the electron count (and other properties, especially T) of the breaking bond should increase rather than decrease, which is again resolved by inspecting the bond's constituent bond wedges. Altogether, the bond bundle property correlations seem to indicate the same underlying catalytic charge density shifts as those resulting from an applied EEF, including

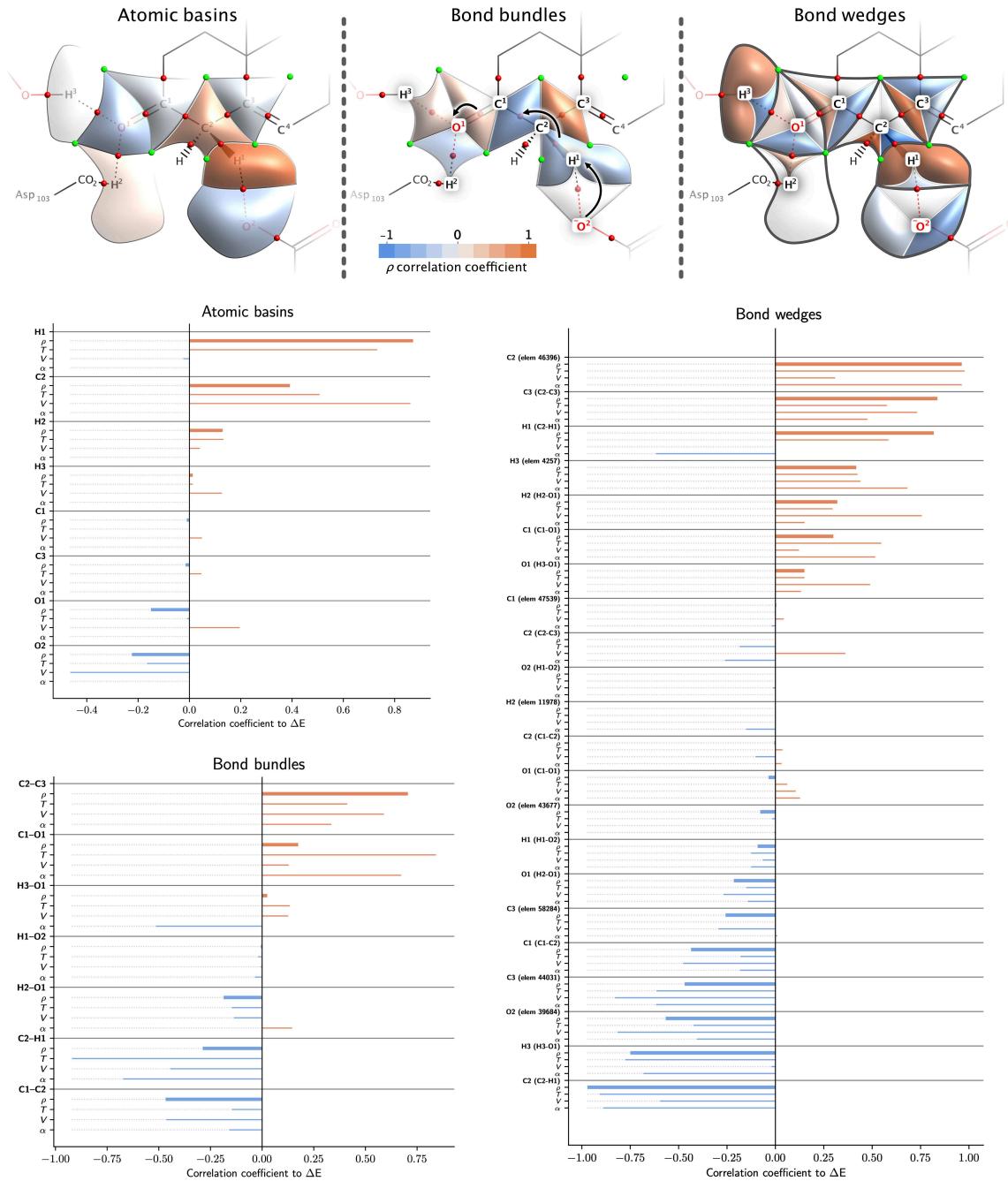


Figure 8: Atomic basin, bond bundle, and bond wedge (top; left to right) property correlations with reaction barrier energy, signed as positive or negative to indicate correlation and anti-correlation. Above are cartoon, schematic depictions of the overlaid on the Lewis representation of the active site, with electron pushing arrows in the center pane denoting the deprotonation reaction step from Scheme 1. Regions are shaded above, and sorted in the plots below, according to the sign and magnitude of their electron population (ρ) correlations with reaction barrier. The regional kinetic energy (T), volume (V), and normalized solid angle (α) are plotted as well. Complete multi-variable correlation bar charts are available in the SI, Section 4.

376 the reversed behavior at the deprotonation site. Here the results indicate a structure property
376 relationship between reactant state substrate bond bundle properties and the barrier height of the
376 KSI-catalyzed reaction, regardless of the perturbation causing the bond bundle properties to change.

378 Considering the correlations of atomic basin properties, the C2 and H1 atoms at the deproto-
378 nization site correlate with barrier height, indicating their combined atomic electron count should be
380 lowered in order to facilitate deprotonation. The O1 atom weakly anticorrelates, suggesting that it
380 should be activated via accumulating charge density. Both of these match the redistribution that
382 resulted from the applied r^+ field.

384 However, while the conjugated carbon system experienced the greatest amount of inter-atomic
384 ρ redistribution due to the EEF (Figure 5), those (C1 and C3) atomic properties show almost no
384 correlation with reaction barrier. Instead, the H1 and C2 atoms most strongly correlate. This
386 suggests that accumulation of $\rho(r)$ in the C1 and C3 atomic basin due to the r^+ EEF is not a
386 general requirement for catalytic enhancement. On the other hand, the r^+ EEF-induced bond
388 bundle redistribution does seem indicative of the general behavior shown in Figure 8.

390 Indeed, the (anti)similarity between bond bundle correlations and EEF-induced bond bundle
390 property shifts is stark. With very few exceptions, regions of $\rho(r)$ accumulation due to the r^+ field
392 (Figure 5) are those that anticorrelate with reaction barrier (Figure 8). Likewise, regions of $\rho(r)$
392 depletion due to the field correlate positively with reaction barrier. For example, the C1–C2 bond
394 bundle, which experienced the greatest increase in $\rho(r)$ due to the EEF, anticorrelates with barrier
394 height. Given that the r^+ system was included in the regional correlations, and that it is the most
396 rate-enhancing system in the (small) sample, it is no surprise that it should be represented in the
396 resulting correlations. However, if KSI^{r^+} was simply dominating the correlations, one would expect
398 all regions to match, but as noted above, atomic basin regional correlations share less similarity with
398 their r^+ field-induced redistribution in KSI. This interplay can only be recovered via bond wedge
400 and bond bundle analysis.

400 We now return to the observation that the H1 and C2 atomic basins most strongly correlate with
400 reaction barrier energy, and how it relates to the underlying mechanism of KSI catalysis. In general,
402 the catalytic effect of an oriented EEF depends on the magnitude of the field, its angle relative to the
402 dipole of the reaction coordinate (*e.g.* parallel to a bond that is broken or formed in the reaction), and
404 the magnitude of that dipole.^{54–66} The local electrostatic preorganization in KSI most underlying its
404 catalytic proficiency is that within the oxyanion hole, which is aligned along the substrate carbonyl

406 bond (the $r+$ direction).^{20,66} One might ask how nature determined that the best way to achieve
407 this atomic basin redistribution is via a carbonyl-oriented field, rather than a field oriented along
408 the reaction coordinate (C2–H1 bond). We previously observed that a catalyzing EEF, applied to a
409 simple Diels-Alder reaction, primarily shifted electron density between bonds bundles *within* rather
410 than *between* reactant molecules, even though the field was applied *along the reaction coordinate*.¹⁶
411 That is, the field-aligned bond was not the most affected. Here too we observe, in Figure 5, that
412 the $r+$ field causes bond bundle redistribution *within* the substrate molecule through its conjugated
413 carbon system, and that this results in the same H1 and C2 atomic basin electron count changes
414 that most correlate with reaction barrier energy. Additionally, the carbonyl is a better candidate for
415 electric field-induced transition state stabilization because its dipole is strong and does not reorient
416 during the reaction, while the C–H (reaction coordinate) dipole is weak and does reorient. Thus
417 the carbonyl provides a more consistent EEF “grip” on the molecule (dipole orientation), and more
418 “leverage” (dipole magnitude) for inducing intra-molecular bond bundle redistribution of charge and
419 other properties.

420 A final and somewhat unexpected similarity to the results of the previous section is the additivity
421 of bond wedge property correlations to predict those of bond bundles and atomic basins. For
422 example, the two H3 bond wedges strongly correlate and anticorrelate respectively, and the H3
423 atomic basin has nearly zero correlation, as if its bond wedges combined to give the whole. The
424 same behavior is apparent in the C1 and C3 atoms. Bond wedge property correlations with reaction
425 barrier height combine to predict those of bond bundles in a similar way, and in this case without
426 exception; the correlation of each bond bundle appears to be the sum of its bond wedge correlations.
427 While gradient bundle properties are definitionally additive, it cannot be said that gradient bundle
428 energetic significance is additive, in this case in relation to the catalyzed reaction barrier. Regardless,
429 bond wedge regional energetic significance does seem to combine to predict bond bundle and atomic
430 basin significance. How far this energetic additivity extends has yet to be investigated, but it is
431 a welcome exception to the lack of such additivity that has been noted as a major obstacle in
432 enzymology.¹⁹

433 Regarding the nearly uniform agreement between the correlations of different regional properties
434 with reaction barrier energy, we observed in Figure 6 that regional V , T , and α often correlate
435 strongly with ρ , as we have shown previously for organic systems.¹⁴ Hence, it is unsurprising that
436 these properties should yield similar correlations with respect to reaction barrier height, as in KSI.

However, there are exceptions. For example, within the C1–O1 bond bundle in Figure 8, T and α correlate more strongly with reaction barrier ($R^2 \approx 0.7$) than do ρ and V ($R^2 \approx 0.2$). Disagreements of this type typically indicate weak correlation between the properties themselves, in this case indicating that within the C1–O1 bond bundle, T and α do not correlate with ρ and V , just as we observed in Figure 6. The connection between regional property correlation and reaction barrier height correlation is observable for almost all regions in the study. The H1 atomic basin, for example, has strong ρ and T barrier correlation ($R^2 \approx 0.8$) but weak V anti-correlation ($R^2 \approx 0.02$), and we saw in Figure 6 that ρ and T correlate strongly with each other and weakly with V . The C2–H1 bond wedge is particularly relevant because it yields some of the strongest overall correlations to reaction barrier height, though V correlates more weakly ($R^2 \approx 0.55$) than ρ , T , or α ($R^2 \approx 0.89$ to 0.98), and in Figure 6, ρ , T , and α correlate strongly with each other and more weakly with V .

Because the correlation among regional properties appears to be closely tied to their relative correlations with reaction barrier height, it appears that different types of system changes (*e.g.* an applied EEF vs. amino acid mutation) enhance catalysis through different mechanisms that affect and utilize some gradient bundle properties more than others. We conclude that weak correlation between a region’s properties indicates the region is affected by different perturbations in different ways. Conversely, strong correlation between a region’s properties, *together with strong correlation to reaction barrier height* indicates that the region has similar energetic significance and catalytic functionality regardless of the type of system change, *i.e.* that it more fundamentally underlies the catalysis. In future investigations involving larger samples, *e.g.* many mutant and EEF-exposed systems, we can further test this conclusion by comparing results of similar analyses on EEF-only, mutant-only, and mixed subsamples. Additionally, by including the local electric field in the analysis, we can directly address the pivotal role of electrostatic preorganization, *e.g.* the strong observed link between $|\mathbf{E}(\mathbf{r})|$ at the substrate carbonyl and KSI catalytic activity (see Figure 4d of Reference 66).

Overall, in regards to the mechanism of KSI catalytic augmentation, this approach leads us to chemically similar conclusions to those of direct inspection of EEF-induced bond bundle property redistribution from the previous section. Activation of the substrate carbonyl bond, and charge accumulation in the C1–C2 bond bundle, as result from the catalyzing r+ EEF, here suggest a more general structure–property relationship between KSI active site gradient bundle properties and reaction barrier height. Correlations of reaction barrier energy to bond bundle regional properties recover a picture of property redistribution that resembles the expected qualitative electron redistribution of

468 the catalyzed chemical reaction, as implied with electron-pushing formalisms, but in unambiguous,
469 quantitative terms, absent the presuppositions of organic chemistry. Surprisingly, we also found
470 that bond wedge correlations to reaction barrier energy appear to combine to predict those of bond
471 bundles and atomic basins. That is, typically the energetic relevance of bond wedges appears to be
472 additive, contrary to the conventional notion of lacking energetic additivity in enzymes, such that
473 the sum of local energies cannot be used to deduce resulting changes to reactivity.¹⁹ We also expect
474 that this approach can be used to investigate reactant state destabilization in addition to transition
475 state stabilization. Lastly, by extending this analysis to the many computable properties of gradient
476 bundles, we saw that not all properties correlate with reaction barrier to the same degree, and that
477 these differences typically stem from weak correlation between the regional properties themselves.

478 4 Conclusion

480 Here we have presented a method of computing the energies and energy-mediated properties of real-
481 space bonding regions in any chemical system, applied to the specific problem of KSI catalysis. We
482 inspected the redistribution of $\rho(\mathbf{r})$ due to a catalyzing EEF in the KSI active site, and directly
483 observed, without the use of chemical presuppositions, that the field catalyzes by shifting charge
484 in the forward reaction direction. Extending the analysis to include multiple perturbed KSI sys-
485 tems, we recovered a similar overall picture of property redistribution, generally descriptive of KSI
486 augmentation via either chlorination of amino acids or applied EEFs. Moreover, even though KSI
487 can be enhanced or inhibited by either type of perturbation, they affect gradient bundle properties
488 differently, and hence augment KSI through different mechanisms.

489 The surfaces separating bond wedges, along with concepts like bond wedge solid angle that
490 describe their relative atomic occupation, are necessary physical components of a real-space chemical
491 bond. In the same way, the ability of a bond wedge to change one integrated property independent
492 from or in different proportion to its other properties (Figure 6) is something expected of a well-
493 defined, real-space chemical system. These concepts are put to abundant qualitative and quantitative
494 use in other chemical systems, *e.g.* via equations of state. Gradient bundle analysis provides the
495 same spatial and energetic grounding at the atomic scale, where regional additivity can be assumed
496 and leveraged. The gradient bundle properties presented here, together with the larger integration
497 tables contained in the supplementary information, are but a small sample of what can currently

be calculated. We have only begun to investigate the statistical and conceptual significance of this
498 extensive set of properties.

Constructing a complete, descriptive network of enzyme $\rho(\mathbf{r})$ structure-property relationships,
500 such that local changes in $\rho(\mathbf{r})$ could be used to accurately predict catalytic rate enhancement, will
be a formidable task owing to the lack of underlying energetic additivity between the independent
502 properties of enzymes.¹⁹ Bond bundles do possess spatial and energetic additivity, are uniquely
defined in any chemical system, and their analysis appears to naturally leverage our hard-won un-
504 derstanding of chemical bonding. Furthermore, in this case the correlation of bond wedge properties
with rate enhancement does, in fact, appear to be additive, so there is a readily quantifiable sense
506 in which catalyzing features may have underlying energetic additivity.

Supporting information

508 Supporting information is provided, containing:

- 3D depictions of some of the regions discussed in the manuscript
- 510 • Coordinates and energies of *ab-initio* calculations
- Atomic basin, bond bundle, and bond wedge tabulated integration values *with many more*
512 *properties than are included in this manuscript*
- Multi-variable bar charts of simplified regional property correlations
- 514 • Plots of fits of each region and property with reaction barrier energy
- Correlations among regional properties

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