

Search for the decay $B_s^0 \rightarrow \eta' K_S^0$

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We report the results of the first search for the decay $B_s^0 \rightarrow \eta' K_S^0$ using 121.4 fb⁻¹ of data collected at the $\Upsilon(5S)$ resonance with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. We observe no signal and set a 90% confidence-level upper limit of 8.16×10^{-6} on the $B_s^0 \rightarrow \eta' K_S^0$ branching fraction.

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The measurements of rare decays of hadrons containing the heavy b quark provide an indirect way to search for new hypothetical particles (see, e.g., Sec. 17.4 in [1]) and, generally, effects not described by the Standard Model (SM). In this paper we describe the first search for the decay $B_s^0 \rightarrow \eta' K_S^0$, a charmless decay with contributions from gluonic and electroweak penguin amplitudes. On the one hand, processes that include such amplitudes are sensitive to beyond-the-SM physics, which could affect decay rates and CP asymmetries [2]. On the other hand, even the SM-based theoretical predictions [3–7] for the decay $B_s^0 \rightarrow \eta' K_S^0$ vary between 0.72×10^{-6} and 4.5×10^{-6} , which makes measuring the branching fraction for the studied decay valuable in its own right.

The two-body decay searched for in the analysis described in this paper is also interesting because it includes η' , the particle whose anomalous production in inclusive and exclusive B decays, first observed by the CLEO experiment more than two decades ago [8,9], became the catalyst for a large body of dedicated theoretical work [10], followed by a recent experimental study of $B_s^0 \rightarrow \eta' X_{s\bar{s}}$ at Belle using a semi-inclusive method [11]. While the large rate for exclusive decays, such as

$B^\pm \rightarrow \eta' K^\pm$, could be accounted for by SM factorization [12], any process involving η' , such as the decays of B_s^0 mesons, could provide valuable information about the role of this particle in decays of heavy flavors and has been an important part of motivation for the work presented here.

The search for the decay $B_s^0 \rightarrow \eta' K_S^0$ described in this paper is based on a data sample of 121.4 fb⁻¹ collected with the Belle detector at the KEKB asymmetric-energy e^+e^- collider [13] when it operated at the $\Upsilon(5S)$ resonance. The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector, a 50-layer central drift chamber, an array of aerogel threshold Cherenkov counters, a barrel-like arrangement of time-of-flight scintillation counters, and a CsI(Tl) crystal-based electromagnetic calorimeter (ECL) located inside a superconducting solenoid coil that provided a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect K_L^0 mesons and to identify muons. The detailed description of the Belle detector could be found elsewhere [14].

There are three two-body decays of $\Upsilon(5S)$ that serve as sources of B_s^0 mesons: $B_s^{*0} \bar{B}_s^{*0}$, $B_s^{*0} \bar{B}_s^0$, or $B_s^0 \bar{B}_s^{*0}$, and $B_s^0 \bar{B}_s^0$. The first two channels have relative fractions of $f_{B_s^{*0} \bar{B}_s^{*0}} = (87.0 \pm 1.7)\%$ and $f_{B_s^{*0} \bar{B}_s^0} = (7.3 \pm 1.4)\%$ [15]. We reconstruct signal B_s^0 mesons coming directly from $\Upsilon(5S)$ decay or from the radiative decay of the excited vector state B_s^{*0} (the charge-conjugate decay mode is included throughout this paper). The $\Upsilon(5S)$ resonance production cross section is 340 ± 16 pb [15], and f_s , its total branching fraction for decays to $B_s^{(*)0} \bar{B}_s^{(*)0}$, is 0.201 ± 0.031 [16]. Therefore, the Belle data sample is estimated to

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contain $(16.60 \pm 2.68) \times 10^6 B_s^0$ mesons. We obtain the results for the branching fraction $\mathcal{B}(B_s^0 \rightarrow \eta' K_S^0)$ as well as for the product $f_s \times \mathcal{B}(B_s^0 \rightarrow \eta' K_S^0)$.

We use Monte Carlo (MC) generator EVTGEN [17] to simulate the production and decay processes, and GEANT toolkit [18] to model detector response. To validate our analysis methods and to calibrate parameters of signal probability density function (PDF), we use a control sample of two-body decays $B^0 \rightarrow \eta' K_S^0$ reconstructed in 711 fb^{-1} of $\Upsilon(4S)$ data collected by Belle at $\Upsilon(4S)$.

To search for $B_s^0 \rightarrow \eta' K_S^0$ decay, we first reconstruct $K_S^0 \rightarrow \pi^+ \pi^-$ and $\eta' \rightarrow \eta \pi^+ \pi^-$ followed by the decay $\eta \rightarrow \gamma \gamma$. For charged pions from η' decay we require the distance of closest approach to the interaction point to be less than 4 cm along the z axis and less than 0.3 cm in the direction perpendicular to it, where the z axis is opposite to the direction of the e^+ beam. Transverse momenta of these charged pion candidates are required to exceed $100 \text{ MeV}/c$. To distinguish between charged pions and other particles, we apply requirements on the likelihood ratio, $R_{h/\pi} = L_\pi / (L_\pi + L_h)$, which is based on particle identification (PID, see Chap. 5 in [1]) measurements, where L_π is the likelihood for the track according to pion hypothesis, and L_h is for kaon ($h = K$) or electron ($h = e$) hypotheses. By requiring $R_{K/\pi} \leq 0.6$ and $R_{e/\pi} \leq 0.95$ for charged pion candidates we reject 25.8% of background events while the signal efficiency loss is 7.6%. Photons are reconstructed as ECL energy clusters not matched to any charged tracks. We require the reconstructed laboratory-frame energy of photon candidates in the barrel (endcap) region of ECL to exceed 50 (100) MeV. Barrel region of ECL covers polar angle θ between 32.2° and 128.7° , where the angle θ is measured with respect to the z axis in the laboratory frame. θ coverage of forward and backward endcaps is between 12.0° and 31.4° , and 131.5° and 157.2° , respectively. The η candidates are reconstructed using the decay channel $\eta \rightarrow \gamma \gamma$, with the reconstructed invariant mass of each candidate required to be between $0.515 \text{ GeV}/c^2$ and $0.580 \text{ GeV}/c^2$, which corresponds, approximately, to a $\pm 3\sigma$ Gaussian resolution window around the nominal η mass [16]. To suppress combinatorial background arising due to low-energy photons, the magnitude of the cosine of the helicity angle ($\cos \theta_{\text{hel}}$) is required to be less than 0.97, where θ_{hel} is the angle in the rest frame of the η candidate between the directions of its Lorentz boost from the laboratory frame and one of the photons. This requirement rejects 11.4% of background events while the efficiency loss for signal events is 3.0%. We perform kinematic fits constraining the reconstructed masses of the η candidates to the nominal η mass [16]. Then η' candidates are reconstructed using η candidates and pairs of oppositely charged tracks identified as pions within a wide window of the reconstructed invariant mass $M(\pi^+ \pi^- \eta)$ between $0.920 \text{ GeV}/c^2$ and $0.980 \text{ GeV}/c^2$, which corresponds, approximately, to the range $[-10, +6]\sigma$ of the

Gaussian resolution and includes a wide sideband, so $M(\pi^+ \pi^- \eta)$ could be used to extract the signal, as described later in this paper. To identify the K_S^0 candidates, we use a neural network technique [19] to search for secondary vertices associated with pairs of oppositely charged tracks treated as pions [20]. To improve mass resolution, a kinematic fit is performed to the vertex. To reconstruct a B_s^0 candidate, we combine K_S^0 and η' candidates after constraining the reconstructed mass of the η' to the nominal η' mass [16]. We further select B_s^0 candidates using three kinematic variables: beam-energy-constrained B_s^0 mass $M_{\text{bc}} = \sqrt{E_{\text{beam}}^2 - p_{B_s^0}^2}$, the energy difference $\Delta E = E_{B_s^0} - E_{\text{beam}}$, and $M(\pi^+ \pi^- \eta)$, where E_{beam} is the beam energy, and $E_{B_s^0}$ and $p_{B_s^0}$ are the reconstructed energy and momentum of the B_s^0 candidate, respectively, calculated in the $e^+ e^-$ center-of-mass frame. Signal B_s^0 candidates are required to satisfy $5.300 \text{ GeV}/c^2 < M_{\text{bc}} < 5.440 \text{ GeV}/c^2$ and $-0.400 \text{ GeV} < \Delta E < 0.300 \text{ GeV}$.

The main source of background to our signal is hadronic continuum, i.e., quark-pair production in $e^+ e^-$ annihilation. To suppress continuum background, we take advantage of the difference between signal and background event topologies by utilizing a set of 17 modified Fox-Wolfgram moments [21]. By optimizing Fisher discriminant [22] coefficients evaluated using these moments, we calculate a likelihood ratio ($\mathcal{R}_{s/b}$) according to signal and background hypotheses. To suppress background, we require $\mathcal{R}_{s/b} > 0.6$. This 80.5%-efficient requirement removes 90.0% of continuum background. The details of our continuum suppression algorithm are provided elsewhere [23].

After applying the described selection criteria, 16% of signal MC events have more than one candidate. We select the best candidate with the smallest value of $\chi^2 = \chi_\eta^2 + \chi_{\pi^+ \pi^-}^2 + \chi_{\eta': \pi^+ \pi^-}^2$, where $\chi_\eta^2 = (\frac{M_{\eta\gamma} - m_\eta}{\sigma_{\eta\gamma}})^2$ is from the kinematic fit for the η candidate, $\chi_{\pi^+ \pi^-}^2$ is from the vertex fit for pion candidates from the K_S^0 decay, and $\chi_{\eta': \pi^+ \pi^-}^2$ is from fitting charged pion tracks from η' decay to a common vertex. This method chooses the correct B_s^0 candidate 91% of the time in signal MC events. The overall reconstruction efficiency in this analysis is 26.8%.

To extract the signal yield, we perform a three-dimensional (3D) unbinned extended maximum likelihood (ML) fit to M_{bc} , ΔE , and $M(\pi^+ \pi^- \eta)$. The likelihood function is defined as

$$\mathcal{L} = \frac{e^{-\sum_j N_j}}{N!} \prod_{i=1}^N \left(\sum_j^{b,s} N_j \mathcal{P}_j[M_{\text{bc}}^i, \Delta E^i, M(\pi^+ \pi^- \eta)^i] \right), \quad (1)$$

where N is the total number of events in the sample, N_j are the fit parameters for the number of signal ($j = s$) and background events ($j = b$), $\mathcal{P}_j$ are the PDFs for the signal and background components of our fitting model.

The background PDF is further represented by the sum of two 3D PDFs which describe a peaking $M^i(\pi^+\pi^-\eta)$ component with real η' mesons and a nonpeaking component of combinatorial origin. Since the correlations among these three fitting variables are small, each of the three 3D PDFs describing the signal contribution, and the peaking and nonpeaking backgrounds, is assumed to factorize as

$$\mathcal{P}_j[M_{bc}, \Delta E, M(\pi^+\pi^-\eta)] = \mathcal{P}_j[M_{bc}] \times \mathcal{P}_j[\Delta E] \times \mathcal{P}_j[M(\pi^+\pi^-\eta)]. \quad (2)$$

The signal component is further described by the sum of contributions from three signal sources $B_s^{*0}\bar{B}_s^{*0}$, $B_s^{*0}\bar{B}_s^0$ or $B_s^0\bar{B}_s^{*0}$, and $B_s^0\bar{B}_s^0$, with relative fractions for the two former contributions according to their branching fractions [15].

The signal M_{bc} distribution is modeled with a Gaussian, and that of ΔE by the sum of a Gaussian and Crystal Ball function [24] (with different means). A sum of two Gaussians with the same mean is used to describe the reconstructed invariant mass of η' candidates in signal events.

To model the background M_{bc} distribution, an ARGUS [25] function with a fixed endpoint at $5.433 \text{ GeV}/c^2$ is used. We use a second-order Chebyshev polynomial to describe the background ΔE distribution. To account for the presence of real η' mesons in background events, we use the signal $M(\pi^+\pi^-\eta)$ PDF to model the peaking part and a first-order Chebyshev polynomial to model the nonpeaking component.

To obtain PDF shape parameters for signal, we first use the $\Upsilon(5S)$ signal MC sample and determine the peak positions for M_{bc} and ΔE . Then we use $\Upsilon(4S)$ data for the decay $B^0 \rightarrow \eta' K_S^0$ to determine all the other PDF parameters. To obtain background PDF shapes, we use $\Upsilon(5S)$ sideband data collected outside of the signal region defined as $5.401 \text{ GeV}/c^2 < M_{bc} < 5.423 \text{ GeV}/c^2$ and $-0.200 \text{ GeV} < \Delta E < 0.100 \text{ GeV}$, and $0.940 \text{ GeV}/c^2 < M(\pi^+\pi^-\eta) < 0.970 \text{ GeV}/c^2$.

To validate our $\Upsilon(5S)$ analysis, we use the full Belle data sample collected at $\Upsilon(4S)$ energy to analyze the decay $B^0 \rightarrow \eta' K_S^0$. The results of the fit to $\Upsilon(4S)$ data are shown in Fig. 1, where each fit projection is plotted after additional selection criteria are applied to the other two variables, $0.948 \text{ GeV}/c^2 < M(\pi^+\pi^-\eta) < 0.966 \text{ GeV}/c^2$, $5.274 \text{ GeV}/c^2 < M_{bc} < 5.286 \text{ GeV}/c^2$, and $-0.100 \text{ GeV} < \Delta E < 0.060 \text{ GeV}$. We estimate the branching fraction, $\mathcal{B}(B^0 \rightarrow \eta' K^0) = (52.3 \pm 2.1) \times 10^{-6}$ (where only the statistical uncertainty is shown), which is consistent with our previous result [26] within the estimated systematic uncertainties.

To extract the signal yield at $\Upsilon(5S)$, we fix all PDF shape parameters to the values obtained from our MC-assisted data-based studies, except for the fraction of background containing real η' mesons, which remains a free parameter in our final fit. To obtain our nominal result, we fit the data with the following three floating parameters in the fit: the number of signal events N_s , the number of background events N_b , and the fraction of background with real η' mesons.

By performing a 3D fit to $\Upsilon(5S)$ data, we obtain -3.21 ± 1.85 signal and 801 ± 28 background events. The results of the fit are plotted in Fig. 2. To emphasize the dominant signal source, $B_s^{*0}\bar{B}_s^{*0}$, each fit projection in this figure is plotted after additional selection criteria are applied to the other two variables, $0.948 \text{ GeV}/c^2 < M(\pi^+\pi^-\eta) < 0.966 \text{ GeV}/c^2$, $5.400 \text{ GeV}/c^2 < M_{bc} < 5.440 \text{ GeV}/c^2$, and $-0.100 \text{ GeV} < \Delta E < 0.060 \text{ GeV}$. We observe no signal and estimate the upper limits for the branching fraction and its product with f_s .

Sources of systematic uncertainties and their contributions are summarized in Table I. The relative uncertainties for f_s and $\sigma(\Upsilon(5S))$ are 15.4% [16] and 4.7% [15], respectively. Systematic uncertainty due to $f_{B_s^{(*)0}\bar{B}_s^{(*)0}}$, i.e., relative contributions of the three signal sources, is 1.87%, estimated by varying the relative fractions of the three contributions to signal PDF. For daughter branching

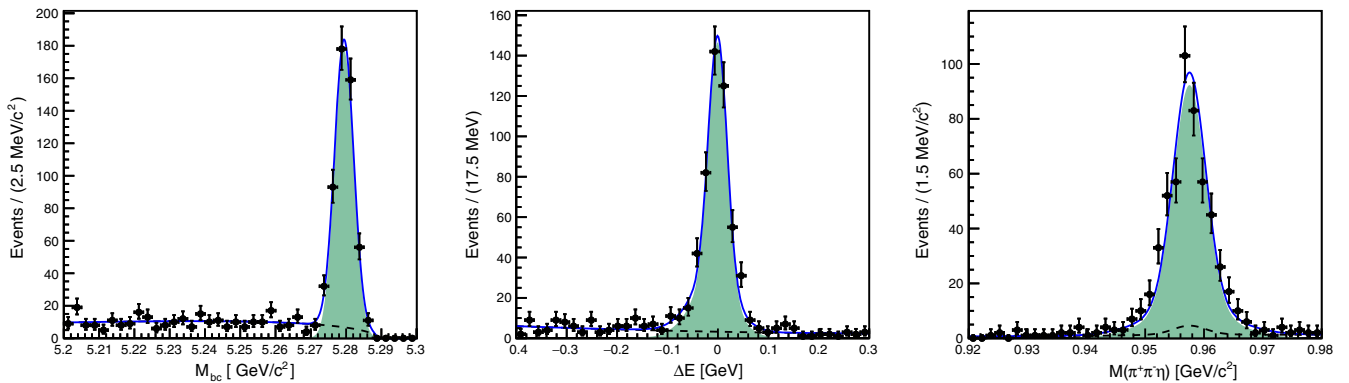


FIG. 1. Signal region fit projections onto M_{bc} , ΔE and $M(\pi^+\pi^-\eta)$ for $B^0 \rightarrow \eta' K_S^0$ event candidates in $\Upsilon(4S)$ data after additional selection criteria are applied, as described in the text. Points with the error bars show the binned data. Blue solid lines show the results of the fit, filled area and black dashed line show the signal and background fit components, respectively.

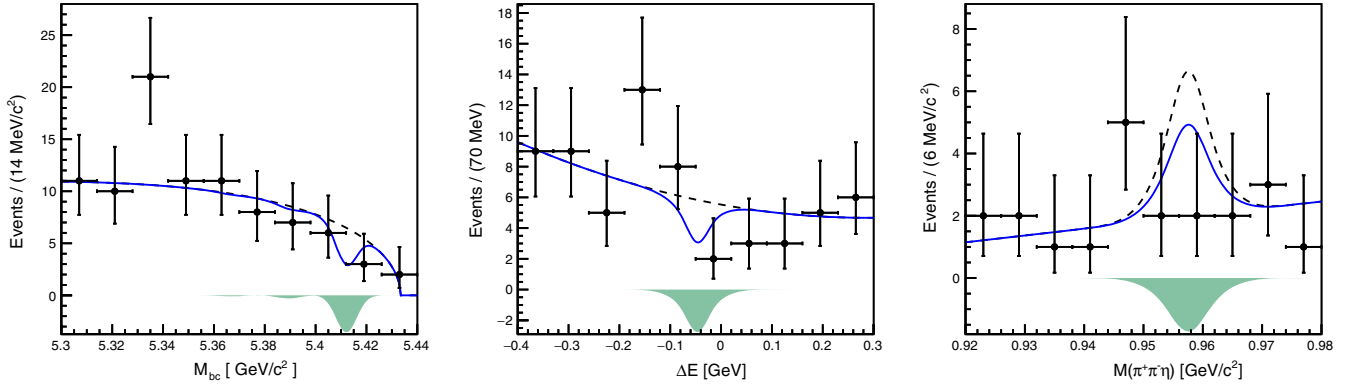


FIG. 2. Signal region fit projections onto M_{bc} , ΔE and $M(\pi^+\pi^-\eta)$ for $B_s^0 \rightarrow \eta' K_S^0$ event candidates in $\Upsilon(5S)$ data after additional selection criteria are applied, as described in the text. Points with the error bars show the binned data. Blue solid lines show the results of the fit, filled area and black dashed line show the signal and background fit components, respectively.

fractions, the uncertainties for $\eta \rightarrow \gamma\gamma$, $\eta' \rightarrow \eta\pi^+\pi^-$, and $K_S^0 \rightarrow \pi^+\pi^-$ are 0.2%, 0.7%, and 0.05%, respectively [16]. Statistical uncertainty due to MC statistics is 0.11% via $\sqrt{\epsilon \times (1 - \epsilon)/N}$, where N is the total number of signal MC events, and ϵ is the overall reconstruction efficiency. The uncertainties in π , η , and K_S^0 reconstruction efficiencies are 1.4% (0.35% per track [27]), 4.1% [28], and 1.4% [29] per particle, respectively. The uncertainty due to PDF parametrization is 11.9%, estimated from the change in signal yield while varying fixed PDF shape parameters one at a time by one unit of their Gaussian uncertainties as measured from the control data sample for signal and from data sideband for background.

The uncertainty from PID selection is 2.4% [27]. By comparing the $\mathcal{R}_{s/b}$ distributions for $B_s^0 \rightarrow K_S^0 \eta'$ events in $\Upsilon(4S)$ data and signal MC events, we estimate the uncertainty in the efficiency of likelihood ratio requirement

TABLE I. Summary of relative systematic uncertainties for $B(B_s^0 \rightarrow \eta' K_S^0)$ and $f_s \times B(B_s^0 \rightarrow \eta' K_S^0)$.

Source	Uncertainty (%)
$\sigma(\Upsilon(5S))$	4.7
$f_{B_s^{(*)0} \bar{B}_s^{(*)0}}$	1.87
$B(\eta \rightarrow \gamma\gamma)$	0.2
$B(\eta' \rightarrow \eta\pi^+\pi^-)$	0.7
$B(K_S^0 \rightarrow \pi^+\pi^-)$	0.05
MC statistics	0.11
π reconstruction	1.4
η reconstruction	4.1
K_S^0 reconstruction	1.4
PDF parametrization	11.9
PID selection	2.4
Background suppression	4.4
Subtotal (without f_s)	17.6
f_s	15.4
Total	23.4

to be 4.4%. We estimate the total multiplicative uncertainties to be 17.6% for $f_s \times B(B_s^0 \rightarrow K_S^0 \eta')$ and 23.4% for $B(B_s^0 \rightarrow K_S^0 \eta')$.

To estimate the upper limit using the frequentist approach [30], an 80% confidence-level (C.L.) belt (including systematic uncertainties) is prepared. To prepare this belt, we generate MC pseudoexperiments according to signal and background PDFs described previously. For each experiment, we generate 800 background events, which is, approximately, the number of background events obtained from fitting $\Upsilon(5S)$ data. We generate toy MC samples with the number of signal events in the range between 0 and 15. For each number of signal MC events we generate 2000 pseudoexperiments, obtain the number of signal events from a 3D fit and smear the resulting distributions of signal yields using the Gaussian σ of the total systematic uncertainty. The overall uncertainty is obtained by combining the uncertainty in the yield, σ , with the multiplicative uncertainty δ using the following formula [31]:

$$(N \pm \sigma)(1 \pm \delta) = N \pm (\sigma \oplus N\delta \oplus \sigma\delta), \quad (3)$$

where $\oplus$ denotes addition in quadrature. We use the results of our pseudoexperiments to prepare an 80% classical confidence belt (without ordering), for which the lower and upper ends of respective confidence intervals correspond to the values for which 10% of fitting results lie below and above the boundary of the contour. We use this 80% confidence belt and its lower 10% sideband to estimate a 90% C.L. upper limit on the number of signal events in data to be 2.1, corresponding to a 90% C.L. upper limit on the branching fraction $B(B_s^0 \rightarrow \eta' K_S^0) < 8.16 \times 10^{-6}$. We also estimate a 90% C.L. upper limit on the product $f_s \times B(B_s^0 \rightarrow \eta' K_S^0) < 1.64 \times 10^{-6}$. The confidence intervals prepared using this statistical method are known to slightly “overcover” for the number of signal events [32], therefore resulting in a conservative upper limit.

In summary, we search for the charmless rare decay $B_s^0 \rightarrow \eta' K_S^0$ using the full data sample collected by the Belle experiment at $\Upsilon(5S)$ resonance. We find no statistically significant signal and set 90% C.L. upper limits $\mathcal{B}(B_s^0 \rightarrow \eta' K_S^0) < 8.16 \times 10^{-6}$ and $f_s \times \mathcal{B}(B_s^0 \rightarrow \eta' K_S^0) < 1.64 \times 10^{-6}$. Our results are the only experimental information currently available for this decay channel, and the reported 90% C.L. upper limit on the branching fraction is several times larger than the current theoretical predictions based on QCDF, SCET and flavor SU(3) symmetry [3–7]. This decay should be further searched for by the Belle II experiment [33] at the next-generation B -factory SuperKEKB, where its discovery would require $\Upsilon(5S)$ statistics of the order of 2 ab^{-1} .

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