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Nambu-Goto Dynamics of Field Theory Cosmic String Loops

Jose J. Blanco-Pillado,[a,b,c] **Daniel Jiménez-Aguilar,**[a,b] **Joanes Lizarraga,**[a,b] **Asier Lopez-Eiguren,**[a,b,d] **Ken D. Olum,**[d] **Ander Urio,**[a,b] **and Jon Urrestilla**[a,b]

[a]Department of Physics, UPV/EHU, 48080, Bilbao, Spain
[b]EHU Quantum Center, University of Basque Country, UPV/EHU
[c]IKERBASQUE, Basque Foundation for Science, 48011, Bilbao, Spain
[d]Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, MA 02155, USA

E-mail: josejuan.blanco@ehu.eus, daniel.jimenez@ehu.eus, joanes.lizarraga@ehu.eus, asier.lopez@ehu.eus, kdo@cosmos.phy.tufts.edu, ander.urio@ehu.eus, jon.urrestilla@ehu.eus

Abstract. We perform a detailed comparison of the dynamics of cosmic string loops obtained in cosmological field theory simulations with their expected motion according to the Nambu-Goto action. We demonstrate that these loops follow the trajectories predicted within the NG effective theory except in regions of high curvature where energy is emitted from the loop in the form of massive radiation. This energy loss continues for all the loops studied in this simulation until they self-intersect or become small enough that they annihilate and disappear well before they complete a single oscillation. We comment on the relevance of this investigation to the interpretation of the results from cosmological field theory simulations as well as their extrapolation to a cosmological context.

Contents

1 Introduction

Many non-linear field theories have in their spectrum solitonic solutions that play a significant role in the dynamics of the theory. These objects have been extensively studied over the past few decades in connection to many different fields of research from condensed matter to cosmology. One important aspect of these studies has been the investigation of the dynamics and interactions of these solitons.

One can explore these issues by performing numerical experiments in a lattice simulation. This lattice does not know anything about the solitonic configuration and simply evolves every point in space as any other one following the equations of motion. One can then recover the information of the evolution of the fields in the lattice and interpret the results in terms of a collection of solitons in motion possibly interacting with each other. This is a useful way to analyse the microphysics of the solitons since one can probe the profiles of the fields at scales smaller than the characteristic soliton size[1]. In particular, one could use this technique to find out whether the solitons remain in their lowest energy state or whether they are typically excited at some point during the simulation.

On the other hand, this procedure is computationally very expensive if one wants to simulate a volume that is large compared to the size of the soliton. This is particularly relevant in simulations that involve many solitons or whenever one is interested in investigating an effect whose typical scale is much larger than the soliton's thickness. In these cases one is forced to look for some effective theory that captures the degrees of freedom that are relevant for the problem without having to simulate every point in a lattice. This drastic reduction in the number of degrees of freedom that one needs to compute to simulate the soliton's dynamics suggests the possibility of another kind of simulation. Such simulations based on effective theories allow for a much larger dynamic range in the simulation, which in turn will help us obtain a better understanding of the large scale dynamics of the problem. However, one needs to make sure that there are no microphysical effects that are missing in

[1] This is always the case since one of the requirements of our lattice spacing should be that it is always smaller than the soliton characteristic scale.

the effective theory that could potentially become relevant for the large scale dynamics that one wants to faithfully reproduce in the simulation.

These two types of simulations are therefore complementary. One can use the lattice simulations to learn the important field theory effects that need to be accounted for in the effective theory of the solitons. Once this is done, one should be able to find some common ground where both these simulations can be compared and where an agreement can be reached on the important dynamics to study. Once this is achieved, an extrapolation to the interesting scales can be safely done using the effective theory.

In this paper we would like to take the first step towards showing this agreement between these two approaches in the context of local cosmic string networks. In this case, we will consider the Abelian-Higgs model as the field theory where local cosmic strings occur as solitons [1] and the Nambu-Goto (NG) action as the effective theory at low energies [2, 3]. Cosmological simulations of both types, lattice field theory [4–11] and Nambu-Goto dynamics [12–21], have been extensively studied in the past. However, there seems to be an important disagreement about the abundance of non-self-intersecting (NSI) loops between these two numerical techniques. As its name suggests, non-self-intersecting loops are loops that in their dynamics do not self-intersect. All the loops found so far in field theory simulations do self intersect, and the loops seem to decay in a short period of time. Nambu-Goto simulations also produce self-intersecting loops, but over the course of their evolution a large number of non-self-intersecting loops appear. These non-self-intersecting loops are important, since their main energy loss mechanism is via gravitational waves, whereas loops that continually intersect lose energy also via massive radiation.

This apparent disagreement has been the source of a debate for a very long time, since some of the first field theory simulations of string networks [4]. One possible explanation could be that field theory simulations have not been lucky enough to produce a large enough NSI loop. This is a reasonable explanation, since field theory simulations have much fewer loops in general, and Nambu-Goto dynamics shows that NSI conditions in a random loop are rare compared to self-intersecting ones.

It is important to remember at this point that the NG action is only an approximation to the actual dynamics of the strings and, as we will describe in detail in this paper, can certainly break down under some special circumstances. However, field theory simulations of individual smooth strings have shown conclusively that these strings follow almost exactly the Nambu-Goto dynamics [22, 23]. This makes this disagreement with field theory simulations more puzzling and has prompted some authors to suggest that another reason for the disagreement may be due to the presence of excitations on the strings in the network simulations [8, 24].

These field theory excitations have been found in many soliton solutions, in particular in cosmic strings [25–28], and have been recently studied in a series of papers in different models in [29–31]. The results of these studies show that many solitonic solutions may store a significant amount of energy in the form of excitations [29, 32]. In particular, the phase transition that creates these solitons could lead to an excited initial state due to the extra energy floating around the bulk as the soliton is formed. Furthermore, these papers show that the time scale for the decay of these excitations could be much larger than the typical time scale of the soliton, the light-crossing time of the width of the soliton [29, 33]. This suggests that the effect of these excitations could play some role in the dynamics of strings in field theory simulations. In particular, the presence of these extra modes localized on the string could modify their equation of state rendering their dynamics quite different from the one predicted by the Nambu-Goto action. However, the existence of this extra energy on cosmic strings has never been conclusively shown in any field theory simulation of the string network[2].

In this paper we will study this problem by carefully analysing the evolution of loops extracted directly from field theory simulations. Specifically, we obtain the position and velocity of some of the largest loops found in the course of a field theory simulation of a network of strings and compare their evolution with the one predicted by the Nambu-Goto action.

The results indicate that these loops follow the same trajectories as their NG counterparts, except in localized regions where the curvature of the strings is large compared with the string core thickness, where the NG approximation is not good by definition. As we will argue in the main part of the text, this shows that there is no significant deviation from the Nambu-Goto dynamics due to a new equation of state for the strings in the parts where the curvature is not high. In other words, it seems that the strings in our simulations do not have a large amount of energy stored in them in the form of localized excitations.

The organization of the paper is the following. In section 2, we comment on the characteristics of the field theory simulations that were used to generate the field theory trajectories of the loops. In section 3, we describe the techniques we use to compare the dynamics of field theory loops and their Nambu-Goto predictions. In section 4, we show our results with a few snapshots of the string trajectories comparing both field theory and Nambu-Goto. Finally, in section 5, we comment on the implications of these results for the cosmological extrapolation of field theory cosmic string networks.

2 Field theory simulations of cosmic string loops

The field theory that we will investigate in this paper is the Abelian-Higgs model, whose Lagrangian density,

$$L = D_\mu\phi D^\mu\phi^* - \frac{\lambda}{4}\left(|\phi|^2 - \eta^2\right)^2 - \frac{1}{4e^2}F^{\mu\nu}F_{\mu\nu} \ , \tag{2.1}$$

describes the dynamics of a complex scalar field, $\phi(x)$, coupled to a vector field, $A_\mu(x)$, through the covariant derivative, $D_\mu\phi = (\partial_\mu - iA_\mu)\phi$. Furthermore, the usual field strength for the vector field is given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We will consider the case $\beta = \lambda/(2e^2) = 1$, which means that the masses of the excitations in the vacuum for the vector and scalar fields are equal: $m = m_s = m_v$.

It is well known that the equations of motion obtained from this theory allow for the existence of solitonic field theory vortices [1]. In $3+1$ dimensions the vortices become strings whose energy is concentrated in a core thickness of the order of $\delta \sim m^{-1}$.

In order to investigate the loops in this theory we follow the prescription detailed in [24], where network loops are created from some random initial conditions in the lattice. We point the interested reader to [24] for details, but we summarize here the necessary basic information. After discretization of the Hamiltonian that corresponds to the Lagrangian (2.1), we obtain the equations of motion and solve them in cubic lattices with periodic boundary conditions.

[2] The closest study of this kind in the literature is the one presented in [32] where the authors perform cosmological simulations of $2+1$ dimensional global vortices. Their results indicate that the amount of energy in this case is rather low.

The initial configuration of the system is chosen to be such that all fields are set to zero except for the scalar field, which is set to be a stationary Gaussian random field with a power spectrum given by

$$P_\phi = Ae^{-kl_\phi} \ , \tag{2.2}$$

where the amplitude A is chosen so that $\left\langle |\phi|^2 \right\rangle = \eta^2$. The free parameter which rules the initial randomness is the correlation length l_ϕ, which can be set initially to different values.

These random initial conditions lead to a considerable excess of energy in the simulation volume. Therefore, a cooling process is applied using a diffusive period of evolution, and once a smooth field distribution is obtained, the network evolves following the true equations of motion in flat space.

It is important to mention that after the diffusive period the network is at rest, which means that any loop at this initial stage will start from a static configuration. We will comment on these primordial loops in Appendix A, but let us for now comment on the fact that they are not the main interest to us since they are not representative of the typical loops in a network simulation. In the course of the evolution of the network, loops of strings will be formed by either self-intersections or intercommutation of long strings and these are the loops that we are mainly interested in. By this stage the string is in motion and so the loops formed are not static. Note that the numerical evolution of the string network was performed in flat spacetime, in other words on a Minkowski lattice spacetime.

In order to localize the position of the strings and the subsequent loops we first identify all the plaquettes with a non-trivial winding in our simulations. In this way the connection between all the centers of these plaquettes will constitute the string. We follow the evolution of the network outputting the windings and confirm the formation of loops coming from intersections by visual inspection.

Since, as mentioned, the simulations are done using periodic boundary conditions, all the strings in the box can be considered to be closed loops; but these loops can be "broken" by the periodic boundary conditions. Thus, if one were to plot the loop directly, some would not appear to be a connected piece of string. In order to avoid this we reconstruct the loops that are broken by the periodic boundary conditions applying spatial translations and assigning new coordinates to the string positions. In this manner, we have a list of connected positions in space for all loops.

In total we have analyzed 7 loops (and their descendants) from the simulations in [24]. These loops were obtained using two different correlation lengths, $l_\phi = 15$ and 25 in η^{-1} units. All of them were produced using lattices of $N = 1024$ points per dimension with a spatial resolution of $\delta x = 0.125$ and temporal resolution of $\delta t = 0.2\delta x$, again in η^{-1} units. We refer the reader to [24] for more specific details on the preparation of these simulations. Moreover, the loops, which are reconstructed following the above prescription, are output at each time step of the evolution so that the field theory information available for the Nambu-Goto prediction is the most accurate possible. Furthermore, as explained in [24] by the time the loops we use here get formed, the large scale dynamics of the string network is consistent with a scaling regime [3].

This data, extracted directly from the field theory simulations, is the starting point of our analysis.

[3]Recall, however, that the simulation here is done in flat space. It would be very interesting to analyze loops from a lattice simulation on an expanding background. Some effort in this direction is already underway.

3 Comparison of Field Theory data with the Nambu-Goto action

As we said above, the effective action that describes the dynamics of local strings is expected to be the Nambu-Goto action [2, 3]. This can be justified by making a judicious choice of coordinate system around the center of the string and integrating the action along the transverse directions of the string [34]: the resulting action can be shown to be of the Nambu-Goto form. This argument rests on several assumptions that we now list in detail.

First, it assumes that the local curvature of the string is small compared to its thickness. In fact, one can consider the Nambu-Goto action as the lowest order approximation of an infinite expansion in terms of the ratio of δ/R, where δ is the nominal thickness of the string soliton and R is the radius of curvature of the string in space. In a cosmological setting, truncating this series keeping only the first term seems quite reasonable since the separation of scales from the microphysical size of the strings to any cosmologically relevant scale is phenomenally large. Of course, this separation of scales is not so large in a field theory simulation of a string network.

The second assumption, which is somewhat related to the previous one, is that the string does not lose energy by radiation in the course of its evolution. This is of course built in the NG action since, as we discuss below, there is a conservation law for the invariant energy of a loop. However, from the point of view of field theory, one might imagine a situation where solitonic strings lose part of their energy into radiation in the form of propagating modes in the bulk. Of course this cannot happen for a relaxed static string, since by definition this object is the lowest energy configuration with the particular boundary conditions, the winding of the scalar field. Boosting this object can not lead to radiation either. So the only way this string can radiate is due to acceleration. This is easy to achieve in strings since during their evolution they can develop regions of curvature that will induce acceleration. The question is then a quantitative one. How much energy is radiated from the typical acceleration present in the evolution of strings? In order to answer this question we should remind ourselves that all the propagating modes in this model of local strings are massive. This suggests that one should wiggle the string with a frequency at least of the order of this mass, m in our case, if one wants to produce any radiation. Below this frequency, the source for the radiation does not have a large enough frequency to produce propagating modes. This argument has been extensively used in the past and demonstrated explicitly in numerical simulations of strings in [23] and more recently in the analogous situation in domain wall strings in [30][4].

There are of course moments where the string can release part of its energy. The simplest way to visualize this is in the lower dimensional process of vortex-antivortex annihilation. In these events, it is clear that the arguments leading to the conclusions above do not apply since the topological stability of the solitons disappears. Extending this to $3+1$ dimensions, we can classify other instances where similar processes occur. A clear example of these kinds of events is the interaction between two long strings, the so-called intercommutation process by which string loops can detach themselves from long strings [35]. Of course, this process cannot be described by the NG action. Other processes closely related to these are cusp formation and kink-kink collisions. During the formation of the cusp, part of the string annihilates with itself releasing energy in the process [22]. Similarly, kink-kink collisions [36] or in general the appearance of very high curvature regions [23, 30] lead to a similar energy

[4]In fact, the situation is a little more complicated since the thickness of the source (the oscillating string) is typically also of the order of the inverse of the mass of the radiated particle, namely, $\delta \sim m^{-1}$. This means that for higher frequencies the radiation is also cut-off due to interference effects [30].

ejection from the string. All these non-perturbative processes are by now well understood and need to be accounted for separately from the NG evolution.

Finally, another important assumption in the use of the NG action to describe strings is the general expectation that, in their rest frame, the solitonic strings would be well approximated by the static solution of lowest energy. The underlying idea for this expectation is the supposition that excitations on the string will decay in a time scale of the order of the light-crossing time of the thickness of the string. This is a very small time scale to have any relevance for the evolution of a loop even in a field theory simulation. However, this assumption has been recently brought into question [24] due to the existence of localized excitations of the field theory string that can have a long lifetime [29, 32, 33]. The presence of this extra energy can change the equation of state of the strings and so modify the trajectory of strings. These ideas have been recently explored in several papers in lower dimensional models with solitons [29, 30, 32]. The results of these studies seem to indicate that even though some of these solitons could have some extra energy at the moment of formation, it is difficult to see how they can achieve the necessary significance to alter the evolution of the solitons.

Here we set out to investigate the relevance of all these possible effects in the evolution of cosmic string loops from cosmological network simulations. In particular, we will compare the evolution of the string extracted from field theory following the procedure we indicated earlier with the one that one would infer from NG dynamics. In the following section we will explain how to obtain the prediction of the NG dynamics from the field theory data at any moment in time.

3.1 The Nambu-Goto dynamics for a cosmic string loop

The Nambu-Goto action for a relativistic string is given by

$$S_{NG} = -\mu \int d^2\xi \sqrt{-\gamma} \ , \tag{3.1}$$

where μ is the energy per unit length of the string and γ is the induced metric on the worldsheet parametrized by the coordinates $\xi^{1,2}$. The equations of motion for a string propagating in flat spacetime can be obtained from this action and are given by[5]

$$\ddot{\mathbf{x}} - \mathbf{x}'' = 0 \ , \tag{3.2}$$

where $\mathbf{x}(t,\sigma)$ parametrizes the position of the string and dotted and primed quantities denote their differentiation with respect to the two worldsheet parameters, (t,σ). Moreover, we have also imposed the gauge conditions

$$\dot{\mathbf{x}} \cdot \mathbf{x}' = 0 \ , \tag{3.3}$$

so the only physical velocity is perpendicular to the string, and

$$\dot{\mathbf{x}}^2 + \mathbf{x}'^2 = 1 \ , \tag{3.4}$$

which means that the spacelike parameter, σ, is proportional to the energy per unit length along the string. These equations can be integrated, so the most general solution is of the form

$$\mathbf{x}(t,\sigma) = \frac{1}{2}\left(\mathbf{a}(\sigma - t) + \mathbf{b}(\sigma + t)\right) \ , \tag{3.5}$$

[5]See, for example, [37] for an account of all the details of the NG action.

where the constraints impose the conditions

$$|\mathbf{a}'| = 1 \; , \tag{3.6}$$
$$|\mathbf{b}'| = 1 \; . \tag{3.7}$$

This means that all one needs to do in practice to obtain the evolution of a string is to find the form of the two functions $\mathbf{a}(\sigma_-)$ and $\mathbf{b}(\sigma_+)$ with respect to their argument. In the following we will describe an algorithm to obtain these functions from the position of the string at two different time steps. This will allow us to apply this procedure to the data obtained from the field theory simulation.

Note that after obtaining these functions at a particular moment, eq. (3.5) will allow us to find the position of the string at any subsequent time. This can in turn be compared with the position in field theory. If the string moves exactly as the NG predicts, we could obtain the form of these functions at any moment in time and the result would be the same. In the following we will explain that it will be convenient to repeat this procedure at several times.

Another important point about the NG dynamics is the fact that one can find the integral of the parameter σ along the string. This is a constant of motion of any loop of string and so can also be used as a measure of the NG dynamics.

3.2 Obtaining the NG dynamics from field theory data

As we mentioned earlier, we extract the information about individual loops from the lattice field theory simulation of the network by identifying their position at any moment in time. Here we will explain in detail how to transform that into a prediction of the NG evolution.

We start with the position of the string through the lattice as a list of spatial positions of plaquette centers, $\mathbf{p}_n$, $n = 1, 2, \ldots$. The first thing we do is to smooth out these position vectors since otherwise we will have big jumps related to the discrete nature of the lattice. One reasonable possibility is to smooth this data by a Gaussian window function whose width is given by a few lattice spacings, $\mathrm{M}\delta x$. In order to justify this choice, let us first remember that in order to have a faithful simulation of the relevant dynamics the thickness of the string is somewhat larger than the lattice spacing. Therefore, one should not expect to know the position of the center of the strings with a precision much larger than this width[6].

Using this smoothed data, we obtain the list of vectors that describe the normalized tangent vector of the string by computing

$$\hat{\mathbf{p}}'_{\mathbf{n}} = \frac{\mathbf{p}_{n+1} - \mathbf{p}_n}{|\mathbf{p}_{n+1} - \mathbf{p}_n|} \; . \tag{3.8}$$

Now we need to compute the velocity vectors for each point of the string. In order to do that, we will assume the NG evolution and consider that the string moves in the direction perpendicular to its tangent vector. This allows us to compute this velocity using the following algorithm.

First, we find the point of the string at a later time described by $t + \Delta T$ that is the intersection of the plane perpendicular to the tangent vector at the original position of the string (at time t) with the string at $t + \Delta T$. Let's denote this point by $\tilde{\mathbf{p}}_n(\Delta T)$. Then we estimate the velocity vector of this segment of the string in the NG approximation by taking

$$\dot{\mathbf{x}}_n = (\tilde{\mathbf{p}}_n(\Delta T) - \mathbf{p}_n)/\Delta T \; . \tag{3.9}$$

[6] We use $\mathrm{M} = 2$ for the loops presented in this paper. We have checked that our results do not change significantly using $\mathrm{M} = 5$.

Here we should comment on an important point. In the previous algorithm we have not specified what is the relation between the time interval between the two sets of strings positions we have in the original data and the computing time interval in the numerical field theory simulation, δt. In our case, we have experimented with different values and finally settled on $\Delta T = 8\,\delta t$. The reason to take these two snapshots of the string separated in time larger than the minimal possible time separation, δt, is to try to smooth out possible errors in the estimate of the velocity[7].

Using this velocity and the normalized tangent vectors we can now compute the tangent vector correctly parametrized according to the usual NG gauge. This means that we can define the new tangent vectors as

$$\mathbf{x}'_n = \left(\sqrt{1 - |\dot{\mathbf{x}}_n|^2}\right)\hat{\mathbf{p}}'_{\mathbf{n}} \ . \tag{3.10}$$

Using the velocity and the tangent vectors we can in turn compute the functions $\mathbf{a}'$ and $\mathbf{b}'$ using

$$\mathbf{a}'_n = \mathbf{x}'_n - \dot{\mathbf{x}}_n \,, \tag{3.11}$$

$$\mathbf{b}'_n = \mathbf{x}'_n + \dot{\mathbf{x}}_n \,, \tag{3.12}$$

and from here it is easy to find the position of the string at any moment in time following the prescription of the NG solution in eq. (3.5).

Finally, using this data we can easily compute the local Lorentz factor associated with each segment, namely,

$$\Gamma_n = \frac{1}{\sqrt{1 - |\dot{\mathbf{x}}_n|^2}} \ , \tag{3.13}$$

as well as the amount of σ parameter in each segment of the string by computing

$$\Delta\sigma_n = \Gamma_n |\mathbf{x}_{n+1} - \mathbf{x}_n| \ . \tag{3.14}$$

Integrating this over our list of elements of the string we obtain the invariant energy of the loop, $E_{NG} = \mu \sum_n \Delta\sigma_n$.

As we mentioned earlier, in the course of the reconstruction there are points that lead to an estimation of the velocity from $|\dot{\mathbf{x}}_n|$ very close to the speed of light or even above it. In order to suppress the pathological behaviour that these points could have on the total energy we have decided to put an artificial cap to the velocity of each individual point. We replace any estimate of $|\dot{\mathbf{x}}_n| > 0.9$ by $v_{\max} = 0.9$. We have tried other regularization procedures and checked that the total energy of the loop is not significantly affected by the different procedures.

4 Results

Using the techniques we have outlined above, we can compare the evolution of field theory loops from our simulations with the motion predicted from the NG dynamics. Using the data from two steps in the field theory simulation separated by ΔT since the formation of the loop,

[7]Using a smaller value of ΔT induces the presence of points of super-luminal motion. This is of course an error induced in regions of high velocity of the string. Note also that regions of self-annihilation would give rise to these problems. However, it is clear that this is to be expected since in those regions the string does not behave as NG, so this algorithm should definitely fail.

we build the $\mathbf{a}(\sigma)$ and $\mathbf{b}(\sigma)$ functions. This allows us to plot the predicted string NG position for all times. This comparison shows that the field theory string follows the NG solution very accurately for the large majority of the string length. This seems to suggest that these loops extracted directly from the simulation are not endowed with a significant amount of extra energy as conjectured in [24], or at least, not enough to change the trajectory of the string perceptibly. A large amount of energy in bound states would change the equation of state of the string and its local velocity would be changed with respect to the one obtained in NG. This is not observed for most parts of the string length.

There are, however, regions where we see a departure of the field theory string position when compared to NG. Many of the places where we see this departure are regions where the NG dynamics predicts the existence of a high curvature section on the string. As we mentioned earlier it is therefore not a surprise that the field theory string does not follow the NG prediction in those regions.

An example of such local departure from NG dynamics is shown in figure 1. We present several snapshots of this loop's evolution in field theory (in blue) and in the NG dynamics (in red) obtained using the algorithm described in the previous section. In order to represent the field theory string, we give it a width of the order of δ (the thickness of the solitonic object). The predicted position of the NG action is hidden inside of the blue tube describing the position of the field theory string for most of the string. The two curves only deviate from one another in a small section of the whole string. In that region, the NG string curves itself at the scale of the order of the string thickness, but the field theory string does not do that and finds a shortcut.

These episodes of high curvature act as a source of energy loss from the string. Some of these events resemble the cusp annihilation simulated several years ago in [22]. Others just correspond to the interaction of wiggles on the string that produce high curvature regions. In some cases, these interactions lead to the formation of tiny daughter loops that immediately annihilate in the field theory side.

The subsequent evolution of the field theory string does not follow the NG prediction after those episodes. The reason for this is also clear. The NG action conserves energy, and therefore it does not account for this energy loss mechanism. This means that the evolution from field theory would start being different in the region where energy is radiated. As time passes, this departure from FT and NG spreads over the rest of the string. If one waits long enough, the difference becomes quite visible, and if one were to continue the comparison forward, the shapes of the loops would grow more different. However, this is not a real measure of the different local dynamics.

In order to do a better job of identifying the reason for the different evolution we follow a procedure also used in the past in [22]. After we have identified one of these high curvature events on the string, we reconstruct the NG data again. This yields different $\mathbf{a}$ and $\mathbf{b}$ functions that should be valid for the subsequent evolution. The interesting point is that the field theory after these events have passed is again accurately described by the new NG data. We show in figure 2 a comparison of the position of the string obtained from field theory to the NG reconstruction after the first episode of high curvature radiation. We notice that there is no visible departure of the field theory evolution from the prediction of NG.

This behaviour continues for a while until a new episode occurs. This is shown in figure 3, where we clearly see another region where the NG prediction deviates from the field theory result in a localized region. It is clear that the string in field theory does not want to curve itself so much as the NG predicts and takes a shortcut. This process radiates again

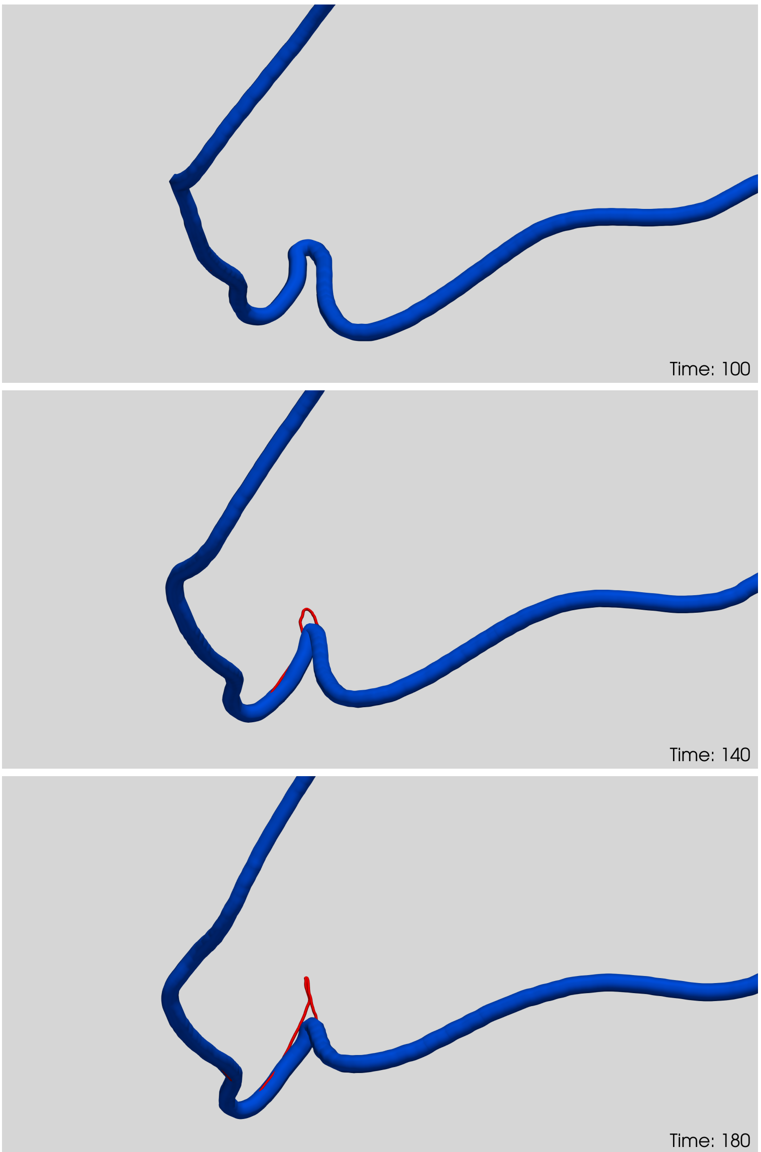

Figure 1. Several snapshots of the evolution of one of the field theory loops. We show in blue the position of the string obtained directly from the lattice simulation. In red is the predicted position obtained from the reconstruction of the NG data at the initial conditions and evolved using the NG dynamics until the time shown. The agreement between these two descriptions is very good for most of the loop's evolution. We have zoomed in on a region of the string at a particular moment where there is a visible departure between them.

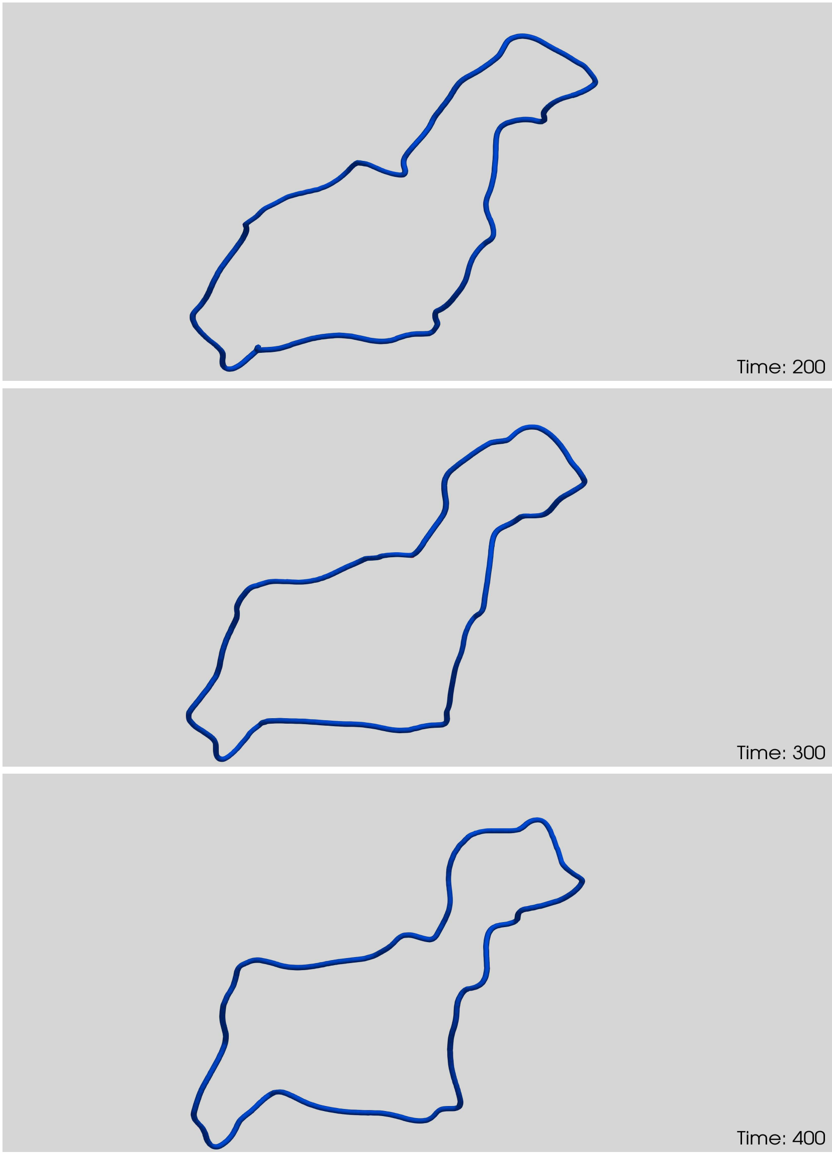

Figure 2. The evolution of the field theory string seems to follow the NG prediction obtained from the reconstruction of the string after the high curvature event. There is no visible departure between the NG and the FT descriptions.

some portion of the energy of the string.

We have seen a similar behaviour in all our loops. Some of the examples are clear, but some other ones are harder to visualize since more than one of these high curvature events happen to have some non-trivial overlap in time [8]. In fact, this already happens in our example loop. We show in figure 4 a third event situated quite far away in space from the previous one but that overlaps in time with the event represented in figure 3.

We also look at the energy computed from the local reconstruction of the NG string

[8]However, we would like to emphasize that the results we present here, with this particular loop, are indeed a good representation of what we obtain in our analysis of all the other 6 loops from [24].

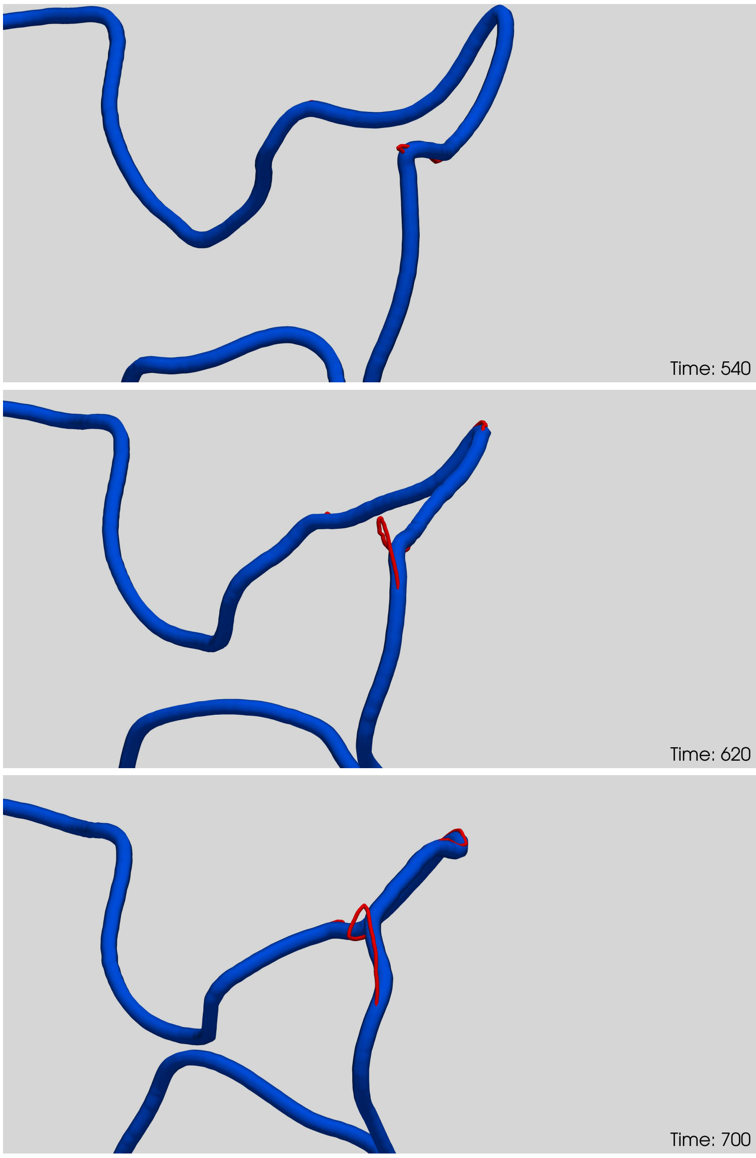

Figure 3. Another high curvature event on the same loop. The NG description (in red) is the one obtained after the first event in figure 1.

obtained from the field theory data at any moment in time. In a purely NG dynamics, this quantity (the total amount of σ of the loop) should be a constant of motion. We plot in figure 5 this energy for the same loop that we discussed before. We observe that the energy overall tends to go down. There are some episodic events where the decrease in energy is sharper, and some of those can be linked with the high-curvature events. We mark in figure 5 the three different episodes that we have been discussing above by shading in light purple the ranges of times displayed in figures 1, 3, and 4. Looking at the energy, there seems to be a connection between these events and the regions where the energy starts to decrease.

There are other instances (between $t \sim 200$ and $t \sim 400$) where the energy seems to be constant, represented in grey in figure 5. These correspond to the times where the NG

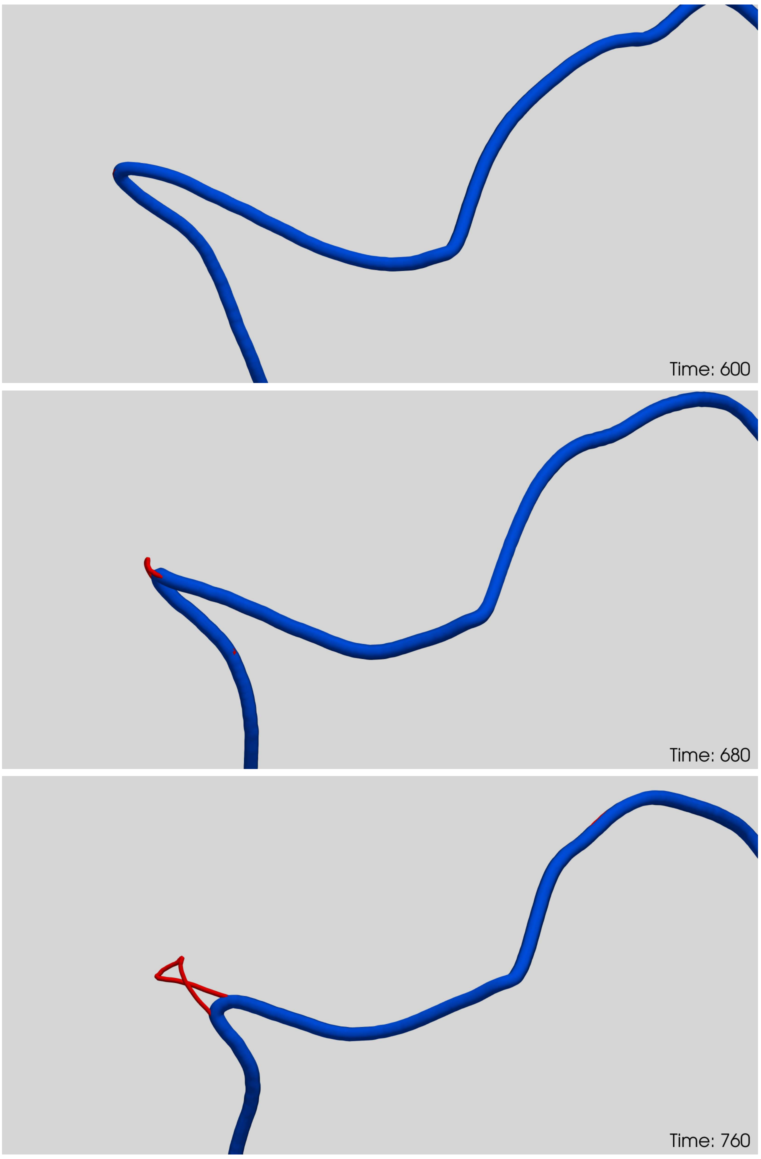

Figure 4. Another high curvature event on the same loop. The NG description (in red) is the one obtained after the first event in figure 1. The evolution between these episodes is well represented by the NG dynamics.

reconstruction was made after the first episode, as shown in figure 2, where there is a very good agreement with the FT data.

There are, however, regions where the energy slowly decreases that are not so obviously associated with any of these individual events (see figure 6, which corresponds to times 450, 500 and 550). The reasons for these are not so clear. The curvature does not seem to be very high, and could also be understood as small scale structure in the loop that leads to energy loss by radiation. In other words, this could be due to events similar to the ones already mentioned that are not so clearly visible in our procedure. Looking at figure 6 one indeed sees several regions of slight departure between NG and FT dynamics, especially at

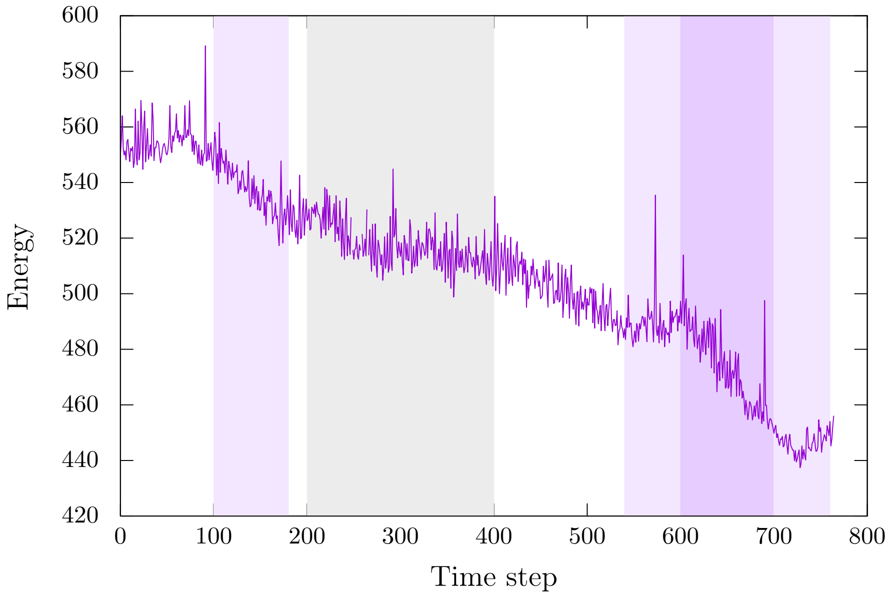

Figure 5. Total amount of invariant energy (total amount of σ) for the NG reconstruction of the field theory data of the loop analyzed in other figures. The purple regions correspond to the high curvature events presented in figures 1,3 and 4. The grey region corresponds to figure 2, where the energy remains roughly constant. The white region (between $t \sim 400$ and 550) corresponds to figure 6, where there is a small deviation from the NG dynamics.

$t = 550$. Note that identifying small regions of high curvature from the NG reconstruction is sometimes quite difficult due to numerical error. In the future we will design new methods to quantify this effect and understand the reason for this energy loss in these regions as well. It is remarkable, and somewhat puzzling, that though the energy goes down roughly by the same percentage as in a high-curvature event, the visual inspection of the loop dynamics does show a rather small deviation from the NG trajectory. This could just be due to the combined effect of several smaller regions where the deviation is small instead of a single large event like in the other cases.

5 Conclusions

In this paper we have compared the evolution of field theory loops obtained in the course of a cosmological lattice simulation with their expected dynamics in the NG approximation. Understanding the discrepancy between these two approaches is of paramount importance in order to make an accurate prediction of the observational signatures of strings. In particular, it is crucial in the estimate of the gravitational wave signature from strings in current and future gravitational wave observatories (see [38] and references therein).

Our investigations show that loops in field theory seem to behave according to the NG action in regions where the curvature is not high. The visual comparison with the NG motion does not support the need for any departure in the equation of state of field theory loops, also, in regions where the curvature is not high. The strings move with the local trajectories dictated by NG. However, we have found that the strings lose part of their energy in the course of their evolution. Some of these energy loss regions correspond to high curvature sections

Figure 6. The evolution of the field theory string in times where the energy decreases. One can not identify any single event of high curvature associated with this decline in energy. Instead we see several regions of high curvature where the deviation from NG is clearly visible but not as striking as in the previous examples.

where some portion of the string annihilates with an adjacent part. These high curvature events are indeed predicted by the NG evolution obtained from the original reconstruction of the field theory string. Of course, the NG dynamics cannot account accurately for the subsequent evolution of the field theory string since part of the energy of the string is lost in these events. This can be bypassed by reconstructing the NG data again after one of these incidents. The result we obtain by following this prescription seems to show that the evolution of the strings is again described by NG with this new data. There are other instances where the string loses energy, which cannot be so clearly pinpointed as regions of high curvature, at least following our visual inspection. Remarkably, though, the trajectory of the string does not seem to be altered perceptibly in these events. They deserve further investigation.

The picture that emerges from our detailed comparison of both descriptions of the string motion is the following. Most of the time, loops behave as NG predicts, but there are instances where the NG action breaks down and one needs to interrupt this comparison for a while until the NG behaviour resumes again. The study of the conserved NG energy backs up that there are instances where the energy drops that correspond to high curvature events.

This localized energy loss mechanism makes the loops shrink and sometimes self-intersect before they have a chance to oscillate for a full period. So at the end the resultant loops are too small to expect them to behave as NG and they finally disappear. This could explain why we do not get at the end any non-self-intersecting loop from our simulation even though the dynamics of loops is well explained by NG for most of their evolution.

The question arises then: if the loops behave almost everywhere like NG, then, would one expect to get NSI loops also in FT, and thus a big chunk of the energy of the network be released as GW? The direct obvious way of answering this question might be to keep simulating loops of this kind until a NSI loop is found in field theory. This is not a good strategy, because, as indicated previously, NSI conditions are not so easy to come by. Many large NSI loops are found in NG simulations [20], because one simulates a much, much larger volume with many more loops [20]. Unfortunately, we do not have the dynamic range in field theory to do such simulations. Of course, we may be lucky and find one such loop in our simulations after a large number of them.

Another idea would be to start with a different set of loops. For example, we could get loops from field theory simulations in the radiation or matter era. These loops should be smoother and have a greater chance to become non-self-intersecting.

One would also be tempted to look for larger initial loops. However, even though it is important to have large loops, so their size is large compared to their thickness, what we have seen in these simulations is that this is not the most important fact. One can have a very large loop with wiggles that lead to high curvature regions, which would thus lose energy by this mechanism.

The best scenario would be to start with a large enough loop that radiates most of this energy in the high curvature regions in its first few moments leaving behind a smoother loop that now should behave mostly as NG (except maybe for the presence of cusps). This expectation is based on the results obtained in field theory simulations of the collisions of wiggles that lead to high curvature regions [23, 30]. That is the analogous situation to what we are seeing here in these loops, but in long, infinite strings. The results there ndicate that the radiation from these events decreases quite fast after their first encounter. The wiggles become milder, and their subsequent interaction is not so violent. This argues for a period of smoothing of the loops of the order of one oscillation time after which the loops would become quite close to NG in all their evolution.

If this is the correct view, it means that loops created in a real cosmological network have a transient period where they emit massive radiation from these highly curved regions. After this initial stage (of the order of the period of the loop) this effect should smooth out the loop and loops would behave as the NG action predicts. Furthermore, as the universe evolves, if the above picture is right, the sizes of structures on loops would increase proportionally to the horizon distance, while the string core size remains fixed. Thus, over cosmological time, the curvature radii seen on loops would become many orders of magnitude larger than the string thickness, the radiative processes we see here would disappear, and the loop motion would be accurately given by NG.

Nevertheless, the study reported here is quite preliminary. We have analyzed only a few loops, and we do not understand their dynamics completely. So it is possible that more is going on than the simple description above. In that case many alternative scenarios [4, 7, 8, 24] may be possible.

In summary, we believe we have made a major step forward in our understanding of the dynamics of loops from field theory simulations. It is clear that loops appear to move as NG for most of their evolution. One clear situation in which this is not happening is, not surprisingly, in events of high-curvature. There are other instances in which the energy of the loops does not seem to follow a NG prediction, and yet the behaviour of the loop does not present big departures from the NG trajectory. The details of this picture need to be confirmed by further study. In the future we will perform new numerical experiments and use different field theory simulations that could corroborate this picture. Some of this work is already underway.

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A Primordial loops

As we explained in the main part of the text, we create our string network with an initial period of diffusion. One can of course look at the initial evolution of some of the loops created at this time as well. However, we should note that they are quite different from the ones we have analyzed in the rest of the paper. First of all, they are much smoother due to the period of diffusion and furthermore they are all created at rest, meaning all the segments of the string start their evolution without any initial velocity. These properties make these loops rather special from the point of view of their Nambu-Goto evolution. It is easy to show that an initially static loop will overlap with itself along the entirety of its physical length in a half of its period if it moves according to the NG description [39]. It is therefore clear that we cannot use these loops to illustrate the typical behaviour of a loop in a realistic cosmological setting. However, as we will describe in the following, we can use these primordial loops to check the validity of our results and our conclusions.

We show in figure 7 the comparison of the evolution of these primordial loops with their predicted NG dynamics. The pictures demonstrate that the evolution is pretty much

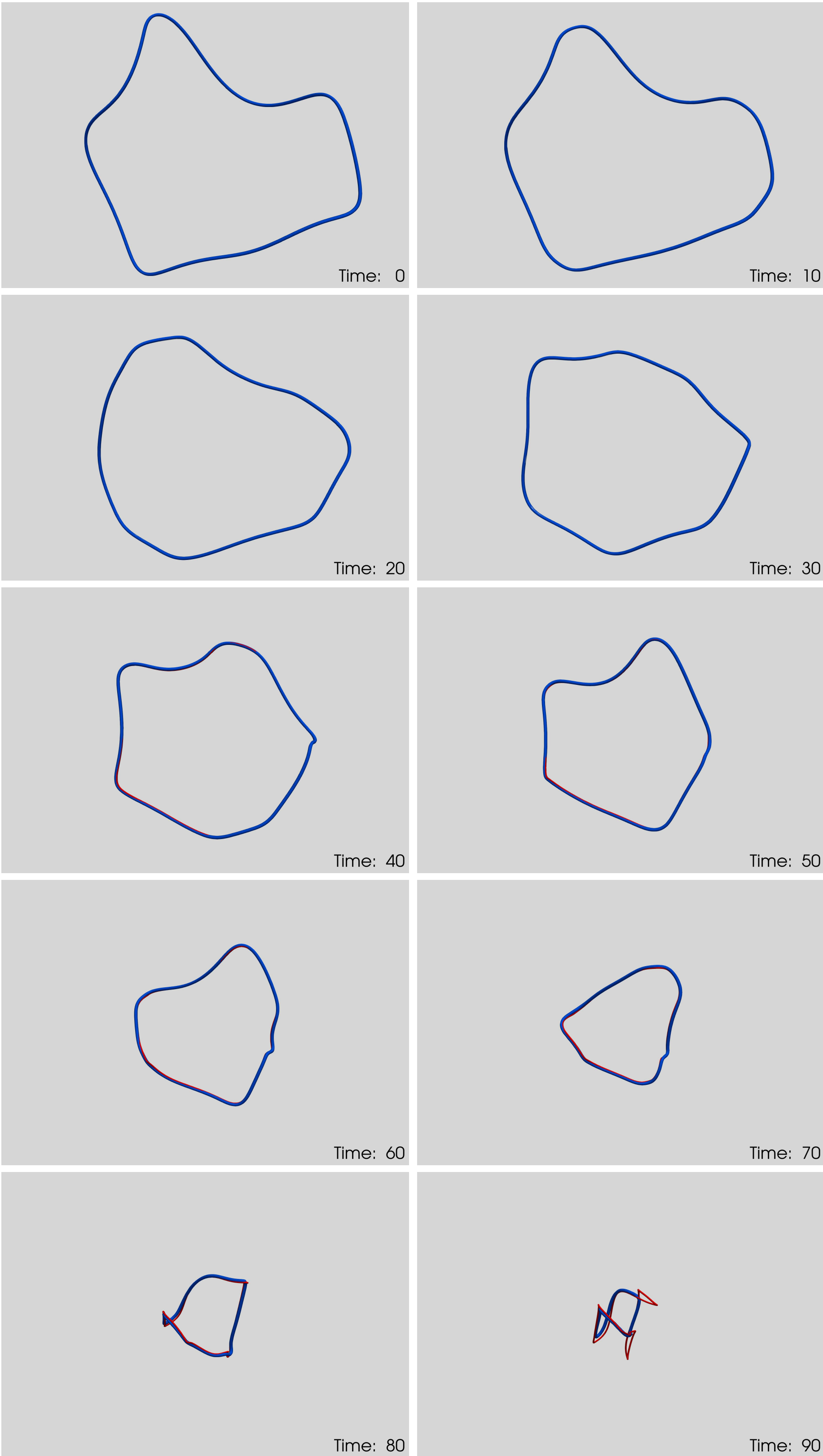

Figure 7. Snapshots of the evolution of a primordial loop. The loop starts from rest. We notice how the evolution of the field theory (in blue) is very close to the NG (red) except towards the end of evolution where the loop has shrunk by a large fraction and the NG predicts a complete overlap of the extent of the string on itself.

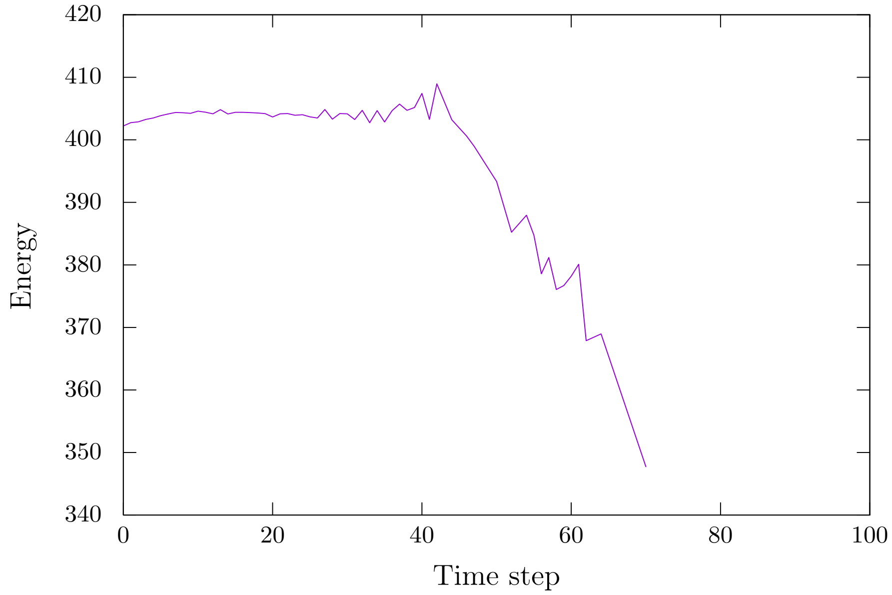

Figure 8. Total amount of invariant energy (total amount of σ) for the NG reconstruction of the field theory data of the primordial loop.

identical in both cases all the way until moments before the predicted overlap of the loop. There are no departures from the NG dynamics due to high curvature regions for much of its evolution. This is easy to understand since the loop is indeed much smoother due to the diffusion period. However, as the loop shrinks, we start seeing some deviations from the NG behaviour, although not so dramatic as in the non-primordial loops. As the loop comes close to its overlap, the difference between both field theory and NG becomes more apparent. This is also to be expected since the interaction of different regions of the string in this pathological self-intersection is of course not handled by the NG dynamics. Nevertheless, the fact that up to this point both descriptions agree with one another can be seen as a validation of both the field theory and NG reconstruction codes.

We also show in figure 8 the energy of this loop using the NG reconstruction at each moment in time. We notice that the energy is pretty much constant until $t \sim 40$, which is also the moment where there is the first signal of deviation from NG dynamics in the loop's evolution (see figs. 7). The deviation from the NG prediction happens in several places in the loop and even though they are associated with high curvature regions, they are not so obvious, at least not visually, as the ones presented earlier in the non-primordial loop. This is somewhat similar to what happens in figure 6. The energy of the loop is decreasing, and yet the dynamics of the loop seems to follow quite closely that of NG.

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