Limitations of RIS-Aided Localization: Inspecting the Relationships between Channel Parameters

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***Abstract*—Reconfigurable intelligent surface (RIS) is expected to play a crucial role in exploiting the synergies across communication, localization, and sensing in future 6G networks. However, in order to effectively utilize these synergies, the fundamental limits of RIS-aided systems need to be better understood. Inspired by this, we derive the Fisher information matrix (FIM) for the channel parameters of the downlink of a single-cell RIS-assisted wireless system. We show that the derived FIM can be decomposed into i) the information provided by the receiver, ii) the information provided by the transmitter, and iii) the information provided by the RIS components. We also derive the Bayesian FIM and study the impact of the uncertainty in estimating the complex path gains (nuisance parameters) on estimating the geometric RIS-related parameters (angles of reflection and incidence). This impact is analytically characterized through the Bayesian equivalent FIM (EFIM) to have a recognizable structure. Two key insights are obtained from our analysis. First, from the Fisher information point of view, the presence of the AoIs (angle of incidence) and AoRs (angle of reflection) in the parameter vector overparameterizes the model. Hence, the AoIs and AoRs can not be separately estimated. Second, localization of a single antenna UE through the signals received from reflections from a single RIS to the UE is not feasible in the far-field.**

I. Introduction

The use of Reconfigurable intelligent surfaces (RIS) has been explored separately for enhancing communication, localization, and sensing functions of wireless networks [2]–[5]. However, since RISs control the propagation environment that is common to all these functions, they have the potential to enable a synergistic design of 6G networks in which the synergies across these functions are exploited for a more efficient operation. However, this will come at the expense of increased complications while estimating the geometric channel parameters. Localization through the parametric estimation of the geometric channel parameters has been explored in [1]–[10]. Although most of these works present Fisher information matrices (FIM) for the geometric channel parameters, the structure of these FIMs has not been investigated.

Hence, in this paper, we rigorously investigate the structure of the Fisher information derived by considering the channel parameter vector, which consists of the geometric and nuisance parameters. *We present two main theoretical contributions for the case in which the RIS reflection coefficients remain constant across all received orthogonal frequency division multiplexing (OFDM) symbols*. First, from the Fisher information point of view, the presence of the AoIs (angle of incidence) and AoRs (angle of reflection) in the parameter vector overparameterizes the resulting model and produces a rank-deficient FIM. Second, through a Bayesian theoretical analysis of the FIM, we show that localization of a single antenna UE through the signals received at the UE from reflections by a single RIS is not feasible in the far-field *when the RIS reflection coefficients remain constant across all OFDM symbols*. To provide these contributions, we derive the FIM of a multi-RIS-aided system and show that the derived FIM can be decomposed into: i) information provided by the receiver, ii) information provided by the transmitter, and iii) information provided by the RIS elements. We use this decomposition to show that the inclusion of both the AoIs and AoRs in the parameter vector results in a rank-deficient FIM. Next, we quantify the information loss associated with the geometric channel parameters due to the unknown channel path gains. This quantification is achieved by deriving the Bayesian Equivalent FIM (EFIM) for the geometric channel parameters. We show that the resulting information loss has a specific structure, and when no prior information is available about the complex path gains, the corresponding submatrix in the Bayesian EFIM related to the RIS geometric parameters is a zero matrix. Due to this absence of information, localization with a single RIS is not feasible in the far-field. In order to put the importance of this result in proper context, it is useful to note here that one cannot reach such a conclusion by simply inspecting the number of unknowns and the geometric conditions (since not every geometric parameter is useful for localization). *Although this work is limited to the case when the RIS reflection coefficients remain constant across all OFDM symbols, extensions to the general case when RIS reflection coefficients change across different OFDM symbols is presented in [1].*

II. System Model

We consider the downlink of an RIS-assisted single-cell MIMO system consisting of a single BS with N_T antennas, a UE of interest with N_R antennas, and M_1 distinct RISs. The m^{th} RIS is assumed to contain $N_L^{[m]}$ reflecting elements where $m \in \mathcal{M}_1 = \{1, 2, \ldots, M_1\}$. We further assume OFDM for this transmission. The BS has an arbitrary but known array geometry with its centroid located at $\mathbf{p}_{BS} \in \mathbb{R}^3$. The UE is defined by its position $\mathbf{p} \in \mathbb{R}^3$, orientation (θ_0, ϕ_0), and an arbitrary but known array geometry. We use notations θ and

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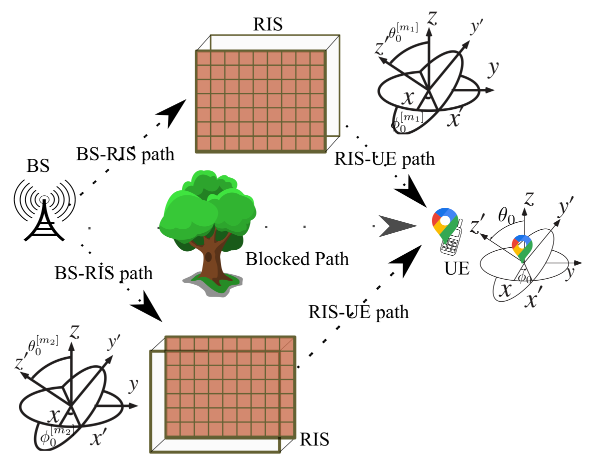

Figure 1: An illustration of the system model.

ϕ, respectively, with appropriate subscripts and superscripts for all the elevation and azimuth angles. The set of RISs is also defined by their positions $\mathbf{p}^{[m]} \in \mathbb{R}^3$ and orientation angles $(\theta_0^{[m]}, \phi_0^{[m]})$, for $m \in \mathcal{M}_1$. The geometry of each RIS is known but arbitrary. The LOS path between the BS and UE is considered blocked to focus on the information provided by the RISs, and the NLOS paths generated by the natural scatters and reflectors are ignored (see Fig. 1). Hence, the only considered paths between the BS and UE are the virtual LOS paths created by the M_1 distinct RISs.

A. Far-Field Channel Model

All paths are described in part by their angle of departure (AoD) at the BS, angle of arrival (AoA) at the UE, and time of arrival (ToA) as specified by $(\theta_{\mathrm{t,u}}^{[m]}, \phi_{\mathrm{t,u}}^{[m]})$, $(\theta_{\mathrm{r,u}}^{[m]}, \phi_{\mathrm{r,u}}^{[m]})$, and $\tau^{[m]}$, respectively. The array vector at the transmitter and receiver is specified by

$$\mathbf{a}_{\mathrm{t,u}}^{[m]}\left(\theta_{\mathrm{t,u}}^{[m]}, \phi_{\mathrm{t,u}}^{[m]}\right) \triangleq e^{-j\Delta_{\mathrm{t,u}}^{\mathrm{T}}\mathbf{k}\left(\theta_{\mathrm{t,u}}^{[m]}, \phi_{\mathrm{t,u}}^{[m]}\right)} \in \mathbb{C}^{N_T}, \tag{1}$$

$$\mathbf{a}_{\mathrm{r,u}}^{[m]}\left(\theta_{\mathrm{r,u}}^{[m]}, \phi_{\mathrm{r,u}}^{[m]}\right) \triangleq e^{-j\Delta_{\mathrm{r,u}}^{\mathrm{T}}\mathbf{k}\left(\theta_{\mathrm{r,u}}^{[m]}, \phi_{\mathrm{r,u}}^{[m]}\right)} \in \mathbb{C}^{N_R}, \tag{2}$$

respectively, where $\mathbf{k}(\theta, \phi) = \frac{2\pi}{\lambda}[\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta]^{\mathrm{T}}$ is the wavenumber vector, λ is the wavelength, $\mathbf{\Delta}_{\mathrm{r,u}} \triangleq [\mathbf{u}_{\mathrm{r,u},1}, \mathbf{u}_{\mathrm{r,u},2}, \ldots, \mathbf{u}_{\mathrm{r,u},N_R}]$. Here $\mathbf{u}_{\mathrm{r,u},n} \triangleq [x_{\mathrm{r,u},n}, y_{\mathrm{r,u},n}, z_{\mathrm{r,u},n}]^{\mathrm{T}}$ is a vector of Cartesian coordinates of the n^{th} receiver element and N_R is the number of receiving antennas. Similarly defined are the parameters $N_T, \mathbf{\Delta}_{\mathrm{r,u}}$ and $\mathbf{u}_{\mathrm{t,u},n}$ which are related to the transmit vector. The array response due to the AoR (angle of reflection) and AoI (angle of incidence) at the m^{th} RIS can be written as

$$\begin{aligned}
&\mathbf{a}_{\mathrm{t,l}}^{[m]}\left(\theta_{\mathrm{t,l}}^{[m]}, \phi_{\mathrm{t,l}}^{[m]}\right) \triangleq e^{-j\Delta_{\mathrm{l,m}}^{\mathrm{T}}\mathbf{k}\left(\theta_{\mathrm{t,l}}^{[m]}, \phi_{\mathrm{t,l}}^{[m]}\right)} \in \mathbb{C}^{N_L^{[m]}}, \\
&\mathbf{a}_{\mathrm{r,l}}^{[m]}\left(\theta_{\mathrm{r,l}}^{[m]}, \phi_{\mathrm{r,l}}^{[m]}\right) \triangleq e^{-j\Delta_{\mathrm{l,m}}^{\mathrm{T}}\mathbf{k}\left(\theta_{\mathrm{r,l}}^{[m]}, \phi_{\mathrm{r,l}}^{[m]}\right)} \in \mathbb{C}^{N_L^{[m]}},
\end{aligned} \tag{3}$$

where $\mathbf{\Delta}_{\mathrm{l,m}} \triangleq \left[\mathbf{u}_{\mathrm{l,m},1}, \mathbf{u}_{\mathrm{l,m},2}, \ldots, \mathbf{u}_{\mathrm{l,m},N_L^{[m]}}\right], \mathbf{u}_{\mathrm{l,m},n} \triangleq [x_{\mathrm{l,m},n}, y_{\mathrm{l,m},n}, z_{\mathrm{l,m},n}]^{\mathrm{T}}$ is a vector of Cartesian coordinates of the n^{th} RIS element. We also define $\mathbf{X}_{\mathrm{l,m}} = \mathrm{diag}([x_{\mathrm{l,m},1}, \cdots, x_{\mathrm{l,m},\mathrm{N}_\mathrm{L}^{[m]}}]^{\mathrm{T}})$. The definitions of $\mathbf{Y}_{\mathrm{l,m}}$ and $\mathbf{Z}_{\mathrm{l,m}}$ are similar. The channel at the n^{th} subcarrier during the t^{th} OFDM symbol is written as

$$\mathbf{H}_t[n] = \sum_{m=1}^{M_1} \frac{\beta^{[m]}}{\sqrt{\rho^{[m]}}} \mathbf{H}_t^{[m]}[n] e^{\frac{-j2\pi n\tau^{[m]}}{NT_S}} \in \mathbb{C}^{N_R \times N_T}, \tag{4}$$

where $\beta^{[m]}$ is the complex channel gain, $\sqrt{\rho^{[m]}}$ is the pathloss of the m^{th} path and

$$\begin{aligned}
\mathbf{H}_t^{[m]}[n] = {} & \mathbf{a}_{\mathrm{r,u}}^{[m]}\left(\theta_{\mathrm{r,u}}^{[m]}, \phi_{\mathrm{r,u}}^{[m]}\right) \mathbf{a}^{\mathrm{H}\,[m]}_{\mathrm{t,l}}\left(\theta_{\mathrm{t,l}}^{[m]}, \phi_{\mathrm{t,l}}^{[m]}\right) \mathbf{\Omega}_t^{[m]}[n] \\
& \times \mathbf{a}_{\mathrm{r,l}}^{[m]}\left(\theta_{\mathrm{r,l}}^{[m]}, \phi_{\mathrm{r,l}}^{[m]}\right) \mathbf{a}^{\mathrm{H}\,[m]}_{\mathrm{t,u}}\left(\theta_{\mathrm{t,u}}^{[m]}, \phi_{\mathrm{t,u}}^{[m]}\right).
\end{aligned} \tag{5}$$

B. Transmit Processing

We consider the transmission of T OFDM symbols each containing of N OFDM subcarriers. The BS precodes a vector of communication symbols $\mathbf{x}[n] = [x_1[n], \ldots, x_{N_{\mathrm{B}}}[n]]^{\mathrm{T}} \in \mathbb{C}^{N_{\mathrm{B}}}$ at the subcarrier level with a directional precoding matrix $\mathbf{F} \in \mathbb{C}^{N_T \times N_{\mathrm{B}}}$. After precoding, the symbols are modulated with an $N-$point inverse fast Fourier transform (IFFT). A cyclic prefix of sufficient length N_{cp} is added to the transformed symbol. In the time domain, this cyclic prefix has length $T_{cp} = N_{cp}T_s$ where $T_s = 1/B$ represents the sampling period. The directional beamforming matrix is defined as $\mathbf{F} \triangleq [\mathbf{f}_1, \mathbf{f}_2, \ldots \mathbf{f}_{N_{\mathrm{B}}}]$ where $\mathbf{f}_\ell = \frac{1}{\sqrt{N_{\mathrm{B}}}}\mathbf{a}_{\mathrm{t},b}\left(\theta_{\mathrm{t},b}^{[l]}, \phi_{\mathrm{t},b}^{[l]}\right), \quad 1 \leq \ell \leq N_{\mathrm{B}}$, is the beam pointing in the direction (θ_ℓ, ϕ_ℓ) and has the same representation as (1). In order to ensure a power constraint, we set $\mathrm{Tr}\left(\mathbf{F}^{\mathrm{H}}\mathbf{F}\right) = 1$, and $\mathbb{E}\left\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n]\right\} = \mathbf{I}_{N_{\mathrm{B}}}$, where $\mathrm{Tr}(\cdot)$ denotes the matrix trace and $\mathbf{I}_{N_{\mathrm{B}}}$ is the N_{B}-dimensional identity matrix.

C. Far-Field Receive Processing

The reflection coefficients of the m^{th} RIS during the t^{th} OFDM symbol can be decomposed into $\mathbf{\Omega}_t^{[m]} = \gamma_t^{[m]}\mathbf{\Gamma}^{[m]}$ where $\gamma_t^{[m]}$ and $\mathbf{\Gamma}^{[m]}$ are termed fast and slow varying RIS coefficients respectively because $\gamma_t^{[m]}$ varies across the T OFDM symbols, $\mathbf{\Gamma}^{[m]}$ is constant across those OFDM symbols.

After the removal of the cyclic prefix and the application of an N-point fast Fourier transform (FFT), the received signal at the n^{th} subcarrier during the t^{th} OFDM symbol is

$$\mathbf{r}_t[n] = \mathbf{H}_t[n]\mathbf{F}\mathbf{x}[n] + \mathbf{n}_t[n], \tag{6}$$

and $\mathbf{n}_t[n] \sim \mathcal{CN}(0, N_0)$ is the Fourier transform of the thermal noise local to the UE's antenna array at the n^{th} subcarrier during the t^{th} OFDM symbol and $\mathbf{x}[n]$ are pilots transmitted. To facilitate any subsequent derivations, we also write the received signal as

$$\mathbf{r}_t[n] = \boldsymbol{\mu}_t[n] + \mathbf{w}_t[n], \quad t = 1, 2, \ldots, T, \quad n = 1, 2, \ldots, N, \tag{7}$$

where $\boldsymbol{\mu}_t[n]$ is the noise-free part of $\mathbf{r}_t[n]$. Based on the received signal in (7), the vectors of the unknown channel parameters related to the RIS paths are defined as

$$\begin{aligned}
&\boldsymbol{\theta}_{\mathrm{r,u}} \triangleq \left[\theta_{\mathrm{r,u}}^{[1]}, \ldots, \theta_{\mathrm{r,u}}^{[M_1]}\right]^{\mathrm{T}}, \quad \boldsymbol{\phi}_{\mathrm{r,u}} \triangleq \left[\phi_{\mathrm{r,u}}^{[1]}, \ldots, \phi_{\mathrm{r,u}}^{[M_1]}\right]^{\mathrm{T}}, \\
&\boldsymbol{\theta}_{\mathrm{t,l}} \triangleq \left[\theta_{\mathrm{t,l}}^{[1]}, \ldots, \theta_{\mathrm{t,l}}^{[M_1]}\right]^{\mathrm{T}}, \quad \boldsymbol{\phi}_{\mathrm{t,l}} \triangleq \left[\phi_{\mathrm{t,l}}^{[1]}, \ldots, \phi_{\mathrm{t,l}}^{[M_1]}\right]^{\mathrm{T}}, \\
&\boldsymbol{\theta}_{\mathrm{r,l}} \triangleq \left[\theta_{\mathrm{r,l}}^{[1]}, \ldots, \theta_{\mathrm{r,l}}^{[M_1]}\right]^{\mathrm{T}}, \quad \boldsymbol{\phi}_{\mathrm{r,l}} \triangleq \left[\phi_{\mathrm{r,l}}^{[1]}, \ldots, \phi_{\mathrm{r,l}}^{[M_1]}\right]^{\mathrm{T}}, \\
&\boldsymbol{\theta}_{\mathrm{t},u} \triangleq \left[\theta_{\mathrm{t,u}}^{[1]}, \ldots, \theta_{\mathrm{t,u}}^{[M_1]}\right]^{\mathrm{T}}, \quad \boldsymbol{\phi}_{\mathrm{t},u} \triangleq \left[\phi_{\mathrm{t,u}}^{[1]}, \ldots, \phi_{\mathrm{t,u}}^{[M_1]}\right]^{\mathrm{T}}, \\
&\boldsymbol{\beta} \triangleq \left[\beta^{[1]}, \ldots, \beta^{[M_1]}\right]^{\mathrm{T}}, \qquad \boldsymbol{\tau} \triangleq \left[\tau^{[1]}, \ldots, \tau^{[M_1]}\right]^{\mathrm{T}}.
\end{aligned}$$

Definition 1. *Based on a set of observations* $\mathbf{r}$, *the Bayesian Fisher information of a parameter vector* $\boldsymbol{\eta}$ *is written as*

$$\begin{aligned}
\mathbf{J}_{\boldsymbol{\eta}} &\triangleq \mathbb{E}_{\mathbf{r}|\boldsymbol{\eta}}\left[-\frac{\partial^2 \ln \chi(\mathbf{r}[n];\boldsymbol{\eta})}{\partial\boldsymbol{\eta}\partial\boldsymbol{\eta}^{\mathrm{T}}}\right] \\
&= -\mathbb{E}_{\mathbf{r}|\boldsymbol{\eta}}\left[\frac{\partial^2 \ln \chi(\mathbf{r}[n]|\boldsymbol{\eta})}{\partial\boldsymbol{\eta}\partial\boldsymbol{\eta}^{\mathrm{T}}}\right] - \mathbb{E}_{\mathbf{r}|\boldsymbol{\eta}}\left[\frac{\partial^2 \ln \chi(\boldsymbol{\eta})}{\partial\boldsymbol{\eta}\partial\boldsymbol{\eta}^{\mathrm{T}}}\right] \qquad (8) \\
&= \mathbf{J}_{\boldsymbol{\eta}}^{\mathrm{D}} + \mathbf{J}_{\boldsymbol{\eta}}^{\mathrm{P}},
\end{aligned}$$

where χ *is the probability density function (PDF),* $\mathbf{J}_{\boldsymbol{\eta}}^{\mathrm{D}}$, *and* $\mathbf{J}_{\boldsymbol{\eta}}^{\mathrm{P}}$ *are the FIMs related to the likelihood and the prior.*

Definition 2. *If the FIM of a parameter* $\boldsymbol{\eta} = [\boldsymbol{\eta}_1^{\mathrm{T}} \quad \boldsymbol{\eta}_2^{\mathrm{T}}]^{\mathrm{T}}$ *is specified by*

$$\mathbf{J}_{\boldsymbol{\eta}} = \left[\begin{array}{cc} \mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_1} & \mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_2} \\ \mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_2}^{\mathrm{T}} & \mathbf{J}_{\boldsymbol{\eta}_2\boldsymbol{\eta}_2} \end{array}\right], \qquad (9)$$

where $\boldsymbol{\eta} \in \mathbb{R}^N, \boldsymbol{\eta}_1 \in \mathbb{R}^n, \mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_1} \in \mathbb{R}^{n\times n}, \mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_2} \in \mathbb{R}^{n\times(N-n)}$, *and* $\mathbf{J}_{\boldsymbol{\eta}_2\boldsymbol{\eta}_2} \in \mathbb{R}^{(N-n)\times(N-n)}$ *with* $n < N$, *then the EFIM [11] of parameter* $\boldsymbol{\eta}_1$ *is given by*

$$\mathbf{J}_{\boldsymbol{\eta}_1}^{\mathrm{e}} = \mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_1} - \mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_1}^{nu} = \mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_1} - \mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_2}\mathbf{J}_{\boldsymbol{\eta}_2\boldsymbol{\eta}_2}^{-1}\mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_2}^{\mathrm{T}}. \qquad (10)$$

Note that the term $\mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_1}^{nu} = \mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_2}\mathbf{J}_{\boldsymbol{\eta}_2\boldsymbol{\eta}_2}^{-1}\mathbf{J}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_2}^{\mathrm{T}}$ *describes the loss of information about* $\boldsymbol{\eta}_1$ *due to uncertainty in the nuisance parameters* $\boldsymbol{\eta}_2$.

III. Fisher Information for RIS Paths

The unknown channel parameters are represented by the vector

$$\boldsymbol{\eta} \triangleq \left[\boldsymbol{\theta}_{\mathrm{r,u}}^{\mathrm{T}}, \boldsymbol{\phi}_{\mathrm{r,u}}^{\mathrm{T}}, \boldsymbol{\theta}_{\mathrm{t,l}}^{\mathrm{T}}, \boldsymbol{\phi}_{\mathrm{t,l}}^{\mathrm{T}}, \boldsymbol{\theta}_{\mathrm{r,l}}^{\mathrm{T}}, \boldsymbol{\phi}_{\mathrm{r,l}}^{\mathrm{T}}, \boldsymbol{\theta}_{\mathrm{t,u}}^{\mathrm{T}}, \boldsymbol{\phi}_{\mathrm{t,u}}^{\mathrm{T}}, \boldsymbol{\tau}^{\mathrm{T}}, \boldsymbol{\beta}_{\mathrm{R}}^{\mathrm{T}}, \boldsymbol{\beta}_{\mathrm{I}}^{\mathrm{T}}\right]^{\mathrm{T}}, \qquad (11)$$

where $\boldsymbol{\beta}_{\mathrm{R}} \triangleq \Re\{\boldsymbol{\beta}\}$, and $\boldsymbol{\beta}_{\mathrm{I}} \triangleq \Im\{\boldsymbol{\beta}\}$ are the real and imaginary parts of $\boldsymbol{\beta}$, respectively. We define the geometric channel parameters $\boldsymbol{\eta}_1 \triangleq \left[\boldsymbol{\theta}_{\mathrm{r,u}}^{\mathrm{T}}, \boldsymbol{\phi}_{\mathrm{r,u}}^{\mathrm{T}}, \boldsymbol{\theta}_{\mathrm{t,l}}^{\mathrm{T}}, \boldsymbol{\phi}_{\mathrm{t,l}}^{\mathrm{T}}, \boldsymbol{\theta}_{\mathrm{r,l}}^{\mathrm{T}}, \boldsymbol{\phi}_{\mathrm{r,l}}^{\mathrm{T}}, \boldsymbol{\theta}_{\mathrm{t,u}}^{\mathrm{T}}, \boldsymbol{\phi}_{\mathrm{t,u}}^{\mathrm{T}}, \boldsymbol{\tau}^{\mathrm{T}}\right]^{\mathrm{T}}$ and the nuisance parameter as $\boldsymbol{\eta}_2 \triangleq \left[\boldsymbol{\beta}_{\mathrm{R}}^{\mathrm{T}}, \boldsymbol{\beta}_{\mathrm{I}}^{\mathrm{T}}\right]^{\mathrm{T}}$. To derive the FIM of $\boldsymbol{\eta}$, we define the PDF as $\chi(\mathbf{r}_t[n];\boldsymbol{\eta}) = \chi(\mathbf{r}_t[n]|\boldsymbol{\eta})\chi(\boldsymbol{\eta})$ where $\chi(\boldsymbol{\eta}) = \chi(\boldsymbol{\eta}_1)\chi(\boldsymbol{\eta}_2)$. The FIM of the channel parameters due to the observation $\mathbf{r}$ is an $11M_1 \times 11M_1$ matrix which can be viewed as a collection of $M_1 \times M_1$ submatrices

$$\mathbf{J}_{\eta}^{\mathrm{D}} \triangleq \left[\begin{array}{cccc} \mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\theta}_{\mathrm{r,u}}} & \mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\phi}_{\mathrm{r,u}}} & \cdots & \mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\beta}_{\mathrm{I}}} \\ \mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\phi}_{\mathrm{r,u}}}^{\mathrm{T}} & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ \mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\beta}_{\mathrm{I}}}^{\mathrm{T}} & \cdots & \cdots & \mathbf{J}_{\boldsymbol{\beta}_{\mathrm{I}}\boldsymbol{\beta}_{\mathrm{I}}} \end{array}\right] \qquad (12)$$

in which $\mathbf{J}_{\boldsymbol{\eta}_{\mathbf{v}_1}\boldsymbol{\eta}_{\mathbf{v}_2}} \triangleq \frac{2}{\sigma^2}\sum_{n=1}^{N}\sum_{t=1}^{T}\Re\left\{\frac{\partial\boldsymbol{\mu}_t[\boldsymbol{n}]^{\mathrm{H}}}{\partial\boldsymbol{\eta}_{\mathbf{v}_1}}\frac{\partial\boldsymbol{\mu}_t[\boldsymbol{n}]}{\partial\boldsymbol{\eta}_{\mathbf{v}_2}}\right\}$, where $\boldsymbol{\eta}_{\mathbf{v}_1}$, $\boldsymbol{\eta}_{\mathbf{v}_2}$ are both dummy variables that stand for the parameters of interest, and σ^2 is the signal-to-noise-ratio which incorporates the pathloss and composite noise power.

A. Entries of the FIM from the Observations

After taking the first derivative, and using the definitions in the previous sections, the autocovariance terms in the FIM in (12), which are related to elevation AoA parameter vectors are

$$\begin{aligned}
\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\theta}_{\mathrm{r,u}}} = \frac{2}{\sigma^2}\Re\Big\{&\left(\mathbf{B}^{\mathrm{H}}\mathbf{K}_{\mathrm{r,u}}^{\mathrm{H}}\mathbf{K}_{\mathrm{r,u}}\mathbf{B}\right) \odot \left(\mathbf{k}_{\mathrm{l}}\mathbf{k}_{\mathrm{l}}^{\mathrm{H}}\right) \odot \left(\mathbf{D}_{\gamma}^{\mathrm{H}}\mathbf{D}_{\gamma}\right) \odot \\
&\left(\mathbf{A}_{\mathrm{t,u}}^{\mathrm{H}}\mathbf{F}\mathbf{F}^{\mathrm{H}}\mathbf{A}_{\mathrm{t,u}}\right)^{\mathrm{T}} \odot \mathbf{R}_0\Big\},
\end{aligned} \qquad (13)$$

$$\begin{aligned}
\mathbf{J}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}} = \mathbf{J}_{\boldsymbol{\beta}_{\mathrm{I}}\boldsymbol{\beta}_{\mathrm{I}}} = \frac{2}{\sigma^2}\Re\Big\{&\left(\mathbf{A}_{\mathrm{r,u}}^{\mathrm{H}}\mathbf{A}_{\mathrm{r,u}}\right) \odot \left(\mathbf{k}_{\mathrm{l}}\mathbf{k}_{\mathrm{l}}^{\mathrm{H}}\right) \odot \left(\mathbf{D}_{\gamma}^{\mathrm{H}}\mathbf{D}_{\gamma}\right) \odot \\
&\left(\mathbf{A}_{\mathrm{t,u}}^{\mathrm{H}}\mathbf{F}\mathbf{F}^{\mathrm{H}}\mathbf{A}_{\mathrm{t,u}}\right)^{\mathrm{T}} \odot \mathbf{R}_0\Big\},
\end{aligned} \qquad (14)$$

where $\odot$ represents elementwise multiplications, $\mathbf{B} = \mathrm{diag}(\boldsymbol{\beta})$, $\mathbf{A}_{\mathrm{r,u}} \triangleq \left[\mathbf{a}_{\mathrm{r,u}}^{[1]}, \mathbf{a}_{\mathrm{r,2}}^{[2]}, \ldots, \mathbf{a}_{\mathrm{r,u}}^{[M_1]}\right]$,

$$\mathbf{K}_{\mathrm{r,u}}^{[m]} \triangleq \mathrm{diag}\left(\frac{\partial}{\partial\theta_{\mathrm{r,u}}^{[m]}}\Delta_{\mathrm{r,u}}^{\mathrm{T}}\mathbf{k}\left(\theta_{\mathrm{r,u}}^{[m]}, \phi_{\mathrm{r,u}}^{[m]}\right)\right), \qquad (15)$$

$$\mathbf{K}_{\mathrm{t,l}}^{[m]} \triangleq \mathrm{diag}\left(\frac{\partial}{\partial\theta_{\mathrm{t,l}}^{[m]}}\Delta_{\mathrm{l,m}}^{\mathrm{T}}\mathbf{k}\left(\theta_{\mathrm{t,l}}^{[m]}, \phi_{\mathrm{t,l}}^{[m]}\right)\right), \qquad (16)$$

$$\mathbf{K}_{\mathrm{r,l}}^{[m]} \triangleq \mathrm{diag}\left(\frac{\partial}{\partial\theta_{\mathrm{r,l}}^{[m]}}\Delta_{\mathrm{l,m}}^{\mathrm{T}}\mathbf{k}\left(\theta_{\mathrm{r,l}}^{[m]}, \phi_{\mathrm{r,l}}^{[m]}\right)\right), \qquad (17)$$

$$\mathbf{K}_{\mathrm{r,u}} \triangleq \left[\mathbf{K}_{\mathrm{r,u}}^{[1]}\mathbf{a}_{\mathrm{r,u}}^{[1]}, \mathbf{K}_{\mathrm{r,u}}^{[2]}\mathbf{a}_{\mathrm{r,u}}^{[2]}, \ldots, \mathbf{K}_{\mathrm{r,u}}^{[M_1]}\mathbf{a}_{\mathrm{r,u}}^{[M_1]}\right]. \qquad (18)$$

The corresponding terms for the AoD at the BS can be obtained by replacing r *with* t *in (15) and (18). In addition to replacing* $\Delta_{\mathrm{r,u}}$ *with* $\Delta_{\mathrm{l,m}}$, *the term related to the elevation AoI defined as* $\mathbf{K}_{\mathrm{r,l}}^{[m]}$ *can be obtained by replacing* $\theta_{\mathrm{r,u}}^{[m]}$ *with* $\theta_{\mathrm{r,l}}^{[m]}$. *The corresponding terms related to the AoR can be obtained from the terms related to the AoI by replacing* r *with* t. Other terms relating to AoI and AoR at the RISs are $\tilde{\mathbf{a}}_{\mathrm{r,l}}^{[m]} \triangleq \boldsymbol{\Gamma}^{[m]}\mathbf{a}_{\mathrm{r,l}}^{[m]}$, $\tilde{\mathbf{a}_{k}}_{\mathrm{r,l}}^{[m]} \triangleq \mathbf{K}_{\mathrm{r,l}}^{[m]}\tilde{\mathbf{a}}_{\mathrm{r,l}}^{[m]}$, $\mathbf{a}_{k_{\mathrm{t,l}}}^{[m]} \triangleq \mathbf{K}_{\mathrm{t,l}}^{[m]}\mathbf{a}_{\mathrm{t,l}}^{[m]}$,

$$\begin{aligned}
\mathbf{k}_{\mathrm{l}} &\triangleq \left[\mathbf{a}^{\mathrm{H}}{}_{\mathrm{t,l}}^{[1]}\tilde{\mathbf{a}}_{\mathrm{r,l}}^{[1]}, \mathbf{a}^{\mathrm{H}}{}_{\mathrm{t,l}}^{[2]}\tilde{\mathbf{a}}_{\mathrm{r,l}}^{[2]}, \ldots, \mathbf{a}^{\mathrm{H}}{}_{\mathrm{t,l}}^{[M_1]}\tilde{\mathbf{a}}_{\mathrm{r,l}}^{[M_1]}\right]^{\mathrm{H}}, \\
\mathbf{k}_{\mathrm{t,l}} &\triangleq \left[\mathbf{a}^{\mathrm{H}}{}_{k_{\mathrm{t,l}}}^{[1]}\tilde{\mathbf{a}}_{\mathrm{r,l}}^{[1]}, \mathbf{a}^{\mathrm{H}}{}_{k_{\mathrm{t,l}}}^{[2]}\tilde{\mathbf{a}}_{\mathrm{r,l}}^{[2]}, \ldots, \mathbf{a}^{\mathrm{H}}{}_{k_{\mathrm{t,l}}}^{[M_1]}\tilde{\mathbf{a}}_{\mathrm{r,l}}^{[M_1]}\right]^{\mathrm{H}}, \qquad (19) \\
\mathbf{k}_{\mathrm{r,l}} &\triangleq \left[\mathbf{a}^{\mathrm{H}}{}_{\mathrm{t,l}}^{[1]}\tilde{\mathbf{a}_{k}}_{\mathrm{r,l}}^{[1]}, \mathbf{a}^{\mathrm{H}}{}_{\mathrm{t,l}}^{[2]}\tilde{\mathbf{a}_{k}}_{\mathrm{r,l}}^{[2]}, \ldots, \mathbf{a}^{\mathrm{H}}{}_{\mathrm{t,l}}^{[M_1]}\tilde{\mathbf{a}_{k}}_{\mathrm{r,l}}^{[M_1]}\right]^{\mathrm{H}}.
\end{aligned}$$

The angle-related expressions in (13) - (19) are related to the elevation angles, similar expressions for azimuth angles are obtained by replacing $\mathbf{k}$ *and* $\mathbf{K}$ *with* $\mathbf{p}$ *and* $\mathbf{P}$, *respectively. These expressions are more thoroughly expressed in [1].* Also, the scalar part of the RIS reflecting coefficient which is used for multipath separation is arranged as $\boldsymbol{\gamma}^{[m]} = [\gamma_1^{[m]}, \gamma_2^{[m]}, \ldots, \gamma_T^{[m]}]^{\mathrm{T}}$, and can be arranged in a matrix as $\mathbf{D}_{\gamma} = [\boldsymbol{\gamma}^{[1]}, \boldsymbol{\gamma}^{[2]}, \ldots, \boldsymbol{\gamma}^{[M_1]}]$. This matrix, henceforth referred to as a *sequence matrix* provides control in both spatial and temporal domains through the fast-varying part of reflecting coefficients, $\gamma_1^{[m]}$, of the RISs. The signal factor representing the effect of the transmitted beams is specified by $[\mathbf{R}_k]_{uv} \triangleq \sum_{n=1}^{N} \left(2\pi n/\left(NT_s\right)\right)^k \mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n] e^{-j2\pi n \frac{\tau^{[v]}-\tau^{[u]}}{NT_s}}$, where $k \in \{0,1,2\}$.

Remark 1. *In general, we notice from the structure of (13) that all submatrices of the FIM in (12) can be written similarly, and have a general form of* $\mathbf{J}_{\mathbf{v}_1\mathbf{v}_2} = \Re\{(Rx\ factor) \odot (RIS\ gain) \odot (RIS\ correlation) \odot (Tx\ factor) \odot (signal\ factor)\}$. *From this equation, we notice that the FIM decreases with SNR and all other submatrices in (12) are presented in [1]. We notice that the submatrices can be decomposed into: i) information provided by the receiver specified by some combination of the terms* $\{\mathbf{P}_{\mathrm{r,u}}, \mathbf{K}_{\mathrm{r,u}}, \mathbf{B}, \mathbf{A}_{r,u}\}$*, ii) information provided by the transmitter specified by some combination of the terms* $\{\mathbf{P}_{\mathrm{t,u}}, \mathbf{K}_{\mathrm{t,u}}, \mathbf{F}, \mathbf{A}_{\mathrm{t,u}}\}$*, iii) information provided by the RIS gain specified by some combination of the terms* $\{\mathbf{k}_{\mathrm{l}}, \mathbf{k}_{\mathrm{t,l}}, \mathbf{k}_{\mathrm{r,l}}, \mathbf{p}_{\mathrm{t,l}}, \mathbf{p}_{\mathrm{r,l}}\}$*, iv) the correlation across the RIS specified by the product* $\mathbf{D}_\gamma^{\mathrm{H}}\mathbf{D}_\gamma$*, and v) the transmit signal factor.*

Assumption 1. *The fast-varying coefficients of distinct RISs are assumed to be orthogonal over the T OFDM symbols such that* $\mathbf{D}_\gamma^{\mathrm{H}}\mathbf{D}_\gamma = \mathbf{I}_{M_1}$.

B. Relationship between AoI and AoR Under Assumption 1

Based on the angle relationships and the coordinate translations, we provide the relationship between the AoI and AoR for an RIS deployed as a passive uniform rectangular array (URA).

Lemma 1. *The matrix specified by*

$$\mathbf{V} = \begin{bmatrix} \boldsymbol{\nu}_1 & \boldsymbol{\nu}_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta_{\mathrm{t,l}}^{[m]})\sin(\phi_{\mathrm{t,l}}^{[m]}) & \sin(\theta_{\mathrm{t,l}}^{[m]})\cos(\phi_{\mathrm{t,l}}^{[m]}) \\ \cos(\theta_{\mathrm{t,l}}^{[m]})\cos(\phi_{\mathrm{t,l}}^{[m]}) & -\sin(\theta_{\mathrm{t,l}}^{[m]})\sin(\phi_{\mathrm{t,l}}^{[m]}) \end{bmatrix} \tag{20}$$

is a full rank matrix. Hence, the 2D vectors $\boldsymbol{\nu}_3 = \begin{bmatrix} \cos(\theta_{\mathrm{r,l}}^{[m]})\sin(\phi_{\mathrm{r,l}}^{[m]}) & \cos(\theta_{\mathrm{r,l}}^{[m]})\sin(\phi_{\mathrm{r,l}}^{[m]}) \end{bmatrix}^{\mathrm{T}}$ *and* $\boldsymbol{\nu}_4 = \begin{bmatrix} \sin(\theta_{\mathrm{r,l}}^{[m]})\cos(\phi_{\mathrm{r,l}}^{[m]}) & -\sin(\theta_{\mathrm{r,l}}^{[m]})\sin(\phi_{\mathrm{r,l}}^{[m]}) \end{bmatrix}^{\mathrm{T}}$ *can be obtained as a linear combination of* $\boldsymbol{\nu}_1$ *and* $\boldsymbol{\nu}_2$.

Proof. See Appendix B. □

The following corollaries establish relationships between the information provided by the FIMs of various channel parameters. The first corollary is a vital step in showing dependence among some of the FIMs of the geometric channel parameters. More specifically , Corollary 1 follows from Lemma 1 and it is used to show Corollary 2. Corollary 2 shows that the columns in (12) which correspond to the AoIs can be obtained as a linear combination of the columns in (12) that corresponds to the AoR.

Corollary 1. *For a RIS deployed as a passive URA with a normal in the z-direction, there exist scalars* α_1, α_2, α_3, *and* α_4 *such that*[1]

$$\alpha_1\mathbf{K}_{\mathrm{t,l}}^{[m]} + \alpha_2\mathbf{P}_{\mathrm{t,l}}^{[m]} = \mathbf{K}_{\mathrm{r,l}}^{[m]}, \;\; \alpha_3\mathbf{K}_{\mathrm{t,l}}^{[m]} + \alpha_4\mathbf{P}_{\mathrm{t,l}}^{[m]} = \mathbf{P}_{\mathrm{r,l}}^{[m]}. \tag{21}$$

Proof. See Appendix C. □

Corollary 2 shows that the information provided by the AoI can be obtained as linear combination of the information provided by the AoR.

Corollary 2. *Under Assumption 1 and with Corollary 1, there exist scalars* α_1, α_2, α_3, *and* α_4 *such that*

$$\begin{aligned} \alpha_1\mathbf{J}_{\mathbf{v}_1\boldsymbol{\theta}_{\mathrm{t,l}}} + \alpha_2\mathbf{J}_{\mathbf{v}_1\boldsymbol{\phi}_{\mathrm{t},l}} &= \mathbf{J}_{\mathbf{v}_1\boldsymbol{\theta}_{\mathrm{r,l}}}, \\ \alpha_3\mathbf{J}_{\mathbf{v}_1\boldsymbol{\theta}_{\mathrm{t,l}}} + \alpha_4\mathbf{J}_{\mathbf{v}_1\boldsymbol{\phi}_{\mathrm{t},l}} &= \mathbf{J}_{\mathbf{v}_1\boldsymbol{\phi}_{\mathrm{r,l}}}, \end{aligned} \tag{22}$$

where $\mathbf{v}_1 \in \boldsymbol{\eta}$.

Proof. See Appendix D. □

Theorem 1. *From the purely information-theoretic point of view, the presence of both the AoIs and AoRs in a parameter vector,* $\boldsymbol{\eta}$*, overparameterizes the model.*

Proof. The proof follows from Corollary 2, by observing that the FIM of the channel parameters $\boldsymbol{\eta}$ specified by $\mathbf{J}_{\boldsymbol{\eta}}^{\mathrm{D}}$ is rank deficient and non-invertible due to the elevation and azimuth AoIs. More specifically, if there are M_1 RISs, the resultant FIM $\mathbf{J}_{\boldsymbol{\eta}}^{\mathrm{D}}$ has a rank of atmost $11M_1 - 2M_1$. □

C. Entries of the General FIM

To incorporate any prior knowledge about the unknown channel parameters, the Bayesian FIM is also analyzed. Similar to (12), the Bayesian FIM of the channel parameters $\boldsymbol{\eta}$ is also an $11M_1 \times 11M_1$ matrix which can also be viewed as several $M_1 \times M_1$ submatrices such that its entries are written as $\tilde{\mathbf{J}}_{\mathbf{v}_1\mathbf{v}_2} = \mathbf{J}_{\mathbf{v}_1\mathbf{v}_2} + \mathbf{J}_{\mathbf{v}_1\mathbf{v}_2}^{\mathrm{P}}$ where $\mathbf{v}_1, \mathbf{v}_2 \in \boldsymbol{\eta}$. We assume uninformative *a priori* information about the geometric channel parameters $\boldsymbol{\eta}_1$. However, *a priori* information is available about the nuisance parameters (complex path gains). This *a priori* information about the real and imaginary part of the complex gains are independent. Now, due to the nature of the *a priori* information, the block entries in the Bayesian FIM can be written as $\tilde{\mathbf{J}}_{\mathbf{v}_1\mathbf{v}_2} = \mathbf{J}_{\mathbf{v}_1\mathbf{v}_2}$, $\tilde{\mathbf{J}}_{\mathbf{v}_3\mathbf{v}_4} = \mathbf{J}_{\mathbf{v}_3\mathbf{v}_4}$, $\tilde{\mathbf{J}}_{\mathbf{v}_3\mathbf{v}_3} = \mathbf{J}_{\mathbf{v}_3\mathbf{v}_3} + \mathbf{J}_{\mathbf{v}_3\mathbf{v}_3}^{\mathrm{P}}$, where $\mathbf{v}_1, \mathbf{v}_2 \in \boldsymbol{\eta}_1$, $\mathbf{v}_3, \mathbf{v}_4 \in \boldsymbol{\eta}_2$, and $\mathbf{v}_3 \neq \mathbf{v}_4$.

D. Structure of General EFIM

In this section, we analyze the structure of the Bayesian EFIM under Assumption 1.

Lemma 2. *Except the information loss terms related to the cross covariance between the receive/transmit angles of elevation and azimuth, all other off-diagonal information loss terms related to the ToA, AoAs, and AoDs in the Bayesian*

[1] $\mathbf{P}_{\mathrm{t,l}}^{[m]}$ is related to the azimuth AoR defined similarly to the definition related to the elevation AoR in (16). $\mathbf{P}_{\mathrm{r,l}}^{[m]}$ is related to the azimuth AoI defined similarly to the definition related to the elevation AoI in (17).

EFIM matrix have a structure of Bayesian FIM of $\boldsymbol{\beta}_{\mathrm{R}}\times$ FIM of $\boldsymbol{\beta}_{\mathrm{R}}\times$ FIM of parameters of interest. More specifically,

$$\mathbf{J}^{\mathrm{nu}}_{\mathbf{v}_1\mathbf{v}_2} \triangleq \tilde{\mathbf{J}}^{-1}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}\mathbf{J}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}\mathbf{J}_{\mathbf{v}_1\mathbf{v}_2}, \tag{23}$$

where $\mathbf{v}_1 \in \left[\boldsymbol{\theta}^{\mathrm{T}}_{\mathrm{r,u}}, \boldsymbol{\phi}^{\mathrm{T}}_{\mathrm{r,u}}, \boldsymbol{\theta}^{\mathrm{T}}_{\mathrm{t,u}}, \boldsymbol{\phi}^{\mathrm{T}}_{\mathrm{t,u}}, \boldsymbol{\tau}^{\mathrm{T}},\right]^{\mathrm{T}}$, $\mathbf{v}_2 \in \boldsymbol{\eta}_1$, $\mathbf{v}_1 \neq \mathbf{v}_2$. Also If $\mathbf{v}_1 = \boldsymbol{\theta}_{\mathrm{r,u}}$ then $\mathbf{v}_2 \neq \boldsymbol{\phi}_{\mathrm{r,u}}$. Again for the transmit angles, if $\mathbf{v}_1 = \boldsymbol{\theta}_{\mathrm{t,u}}$ then $\mathbf{v}_2 \neq \boldsymbol{\phi}_{\mathrm{t,u}}$.

Proof. See Appendix F. □

Lemma 3. *The information loss terms related to the AoI and AoR (both elevation and azimuth) in the Bayesian EFIM matrix have a structure of Bayesian FIM of $\boldsymbol{\beta}_{\mathrm{R}}\times$ FIM of $\boldsymbol{\beta}_{\mathrm{R}}\times$ FIM of parameters of interest. More specifically, $\mathbf{J}^{\mathrm{nu}}_{\mathbf{v}_1\mathbf{v}_2} \triangleq \tilde{\mathbf{J}}^{-1}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}\mathbf{J}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}\mathbf{J}_{\mathbf{v}_1\mathbf{v}_2}$, where $\mathbf{v}_1 \in \left[\boldsymbol{\theta}^{\mathrm{T}}_{\mathrm{t,l}}, \boldsymbol{\phi}^{\mathrm{T}}_{\mathrm{t,l}}, \boldsymbol{\theta}^{\mathrm{T}}_{\mathrm{r,l}}, \boldsymbol{\phi}^{\mathrm{T}}_{\mathrm{r,l}},\right]^{\mathrm{T}}$ and $\mathbf{v}_2 \in \boldsymbol{\eta}_1$.*

Proof. See Appendix F. □

Lemma 4. *When the* a priori *information of the nuisance parameters is uninformative, the Bayesian EFIM related to the RIS geometric angle parameters is an all zero matrix. More specifically,*

$$\mathbf{J}^{\mathrm{e}}_{\boldsymbol{\theta}_{\mathrm{t,l}}\boldsymbol{\theta}_{\mathrm{t,l}}} = \mathbf{J}^{\mathrm{e}}_{\boldsymbol{\phi}_{\mathrm{t,l}}\boldsymbol{\phi}_{\mathrm{t,l}}} = \mathbf{J}^{\mathrm{e}}_{\boldsymbol{\theta}_{\mathrm{r,l}}\boldsymbol{\theta}_{\mathrm{r,l}}} = \mathbf{J}^{\mathrm{e}}_{\boldsymbol{\phi}_{\mathrm{r,l}}\boldsymbol{\phi}_{\mathrm{r,l}}} = \mathbf{0}.$$

This indicates that with no prior information about the complex channel path gains, the AoI and AoR can not be estimated.

Proof. When the *a priori* information of the nuisance parameters is uninformative then $\tilde{\mathbf{J}}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}} = \mathbf{J}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}$. Substituting this and Lemma 3 into Definition 2 completes the proof. □

Theorem 2. *In the far-field, it is not feasible to localize a single antenna UE using only the signals received at the UE through reflections from a single RIS when the RIS reflection coefficients remain constant across all OFDM symbols.*

Proof. The proof follows as Lemma 4 indicates that the AoR can not be estimated in the far-field without *a priori* information about the complex path gains. Hence, because *a priori* information about the complex path gain is nontrivial to obtain and a single antenna receiver can not estimate the AoAs, only a single ToA measurement without any angle information is available. Localization is not feasible under these conditions. □

IV. Numerical Results

The CRLB of the AoR and AoA are shown in Fig. 2 as a function of the number of receive antennas. The CRLB is obtained by first inverting the FIM in (12), and subsequently taking the square root of appropriate diagonals. This figure indicates that the AoA can be more accurately estimated than the AoR. This difference in accuracy is not surprising as the lack of processing at the RIS hinders the AoR estimation, while the antenna elements at the UE enable AoA estimation.

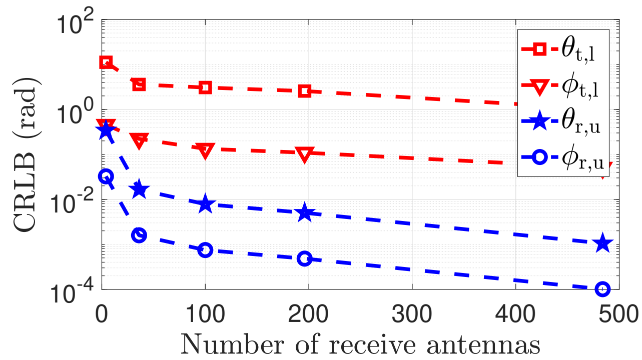

Figure 2: CRLB (AoR and AoA) vs N_R with perfect knowledge of $\boldsymbol{\beta}$ and $1/\sigma^2 = 20$ dB.

V. Conclusion

This paper has investigated the FIM for the downlink of a RIS-assisted system. We analytically showed that the FIM is decomposable into i) information provided by the receiver, ii) information provided by the transmitter, and iii) information provided by the RIS components. We also derived the Bayesian FIM. Through the Bayesian EFIM, we analytically showed that the information loss in estimating the geometric angle channel parameters due to the complex path gains takes a specific structure. This structure showed that the absence of prior information about the complex path gains makes it impossible to estimate the geometric angle channel parameters. *Hence, due to the lack of information about the AoRs, localization of a single antenna UE through the signals received at the UE from reflections by a single RIS is not feasible in the far-field when the RIS reflection coefficients remain constant across all OFDM symbols.*

Appendix

A. RIS Related Angle Definitions and Relationships

To analyze both the angle relationships and derive the FIM for positioning, we define the rotation matrix $\mathbf{Q}(\theta_0, \phi_0)$ given by $\mathbf{Q}(\theta_0, \phi_0) = \mathbf{Q}_z(\phi_0)\mathbf{Q}_{-x}(\theta_0)$, where $\mathbf{Q}_z(\phi_0)$ and $\mathbf{Q}_{-x}(\theta_0)$ define the counter-clockwise rotation around the z-axis and the clockwise rotation around the x-axis respectively. We define $\mathbf{g}^{[m]} = (\mathbf{p}^{[m]} - \mathbf{p}_{\mathrm{BS}})$, and specify the AoD at the BS as $\theta^{[m]}_{\mathrm{t,u}} = \cos^{-1}\left(g^{[m]}_z/\|\mathbf{g}^{[m]}\|\right)$, $\phi^{[m]}_{\mathrm{t,u}} = \tan^{-1}\left(g^{[m]}_y/g^{[m]}_x\right)$. Next, we translate the m^{th} RIS to the origin, and the new coordinates of the BS can be written as $\mathbf{c}^{[m]} = (\mathbf{p}_{\mathrm{BS}} - \mathbf{p}^{[m]})$, $\tilde{\mathbf{c}}^{[m]} = \mathbf{Q}^{-1}\left(\theta^{[m]}_0, \phi^{[m]}_0\right)\mathbf{c}^{[m]}$. With respect to these new coordinates, we can write $\theta^{[m]}_{\mathrm{r,l}} = \cos^{-1}\left(c^{[m]}_{\tilde{z}}/\|\tilde{\mathbf{c}}^{[m]}\|\right)$, $\phi^{[m]}_{\mathrm{r,l}} = \tan^{-1}\left(c^{[m]}_{\tilde{y}}/c^{[m]}_{\tilde{x}}\right)$. Subsequently, the translated coordinates allow the following definition $\mathbf{v}^{[m]} = (\mathbf{p} - \mathbf{p}^{[m]})$, $\tilde{\mathbf{v}}^{[m]} = \mathbf{Q}^{-1}\left(\theta^{[m]}_0, \phi^{[m]}_0\right)\mathbf{v}^{[m]}$ and we can write $\theta^{[m]}_{\mathrm{t,l}} = \cos^{-1}\left(v^{[m]}_{\tilde{z}}/\|\tilde{\mathbf{v}}^{[m]}\|\right)$, $\phi^{[m]}_{\mathrm{t,l}} = \tan^{-1}\left(v^{[m]}_{\tilde{y}}/v^{[m]}_{\tilde{x}}\right)$. Similarly, we obtain a new set of coordinates by translating the UE to the origin, and we write the following definitions $\mathbf{e}^{[m]} = -(\mathbf{p} - \mathbf{p}^{[m]})$, $\tilde{\mathbf{e}}^{[m]} = \mathbf{Q}^{-1}(\theta_0, \phi_0)\,\mathbf{e}^{[m]}$. Hence, we can write $\theta_{\mathrm{r,u}}^{[m]} = \cos^{-1}\left(e_{\tilde{z}}^{[m]}/\|\tilde{\mathbf{e}}^{[m]}\|\right)$, $\phi_{\mathrm{r,u}}^{[m]} = \tan^{-1}\left(e_{\tilde{y}}^{[m]}/e_{\tilde{x}}^{[m]}\right)$.

B. Proof of Lemma 1

By simple geometry and based on the angle definitions, we can write (20) as

$$\mathbf{V} = \begin{bmatrix} \frac{v_{\tilde{y}}^{[m]} v_{\tilde{z}}^{[m]}}{\left(\|\tilde{\mathbf{v}}^{[m]}\|\sqrt{(v_{\tilde{x}}^{[m]})^2+(v_{\tilde{y}}^{[m]})^2}\right)} & \frac{v_{\tilde{x}}^{[m]}}{\|\tilde{\mathbf{v}}^{[m]}\|} \\ \frac{v_{\tilde{x}}^{[m]} v_{\tilde{z}}^{[m]}}{\left(\|\tilde{\mathbf{v}}^{[m]}\|\sqrt{(v_{\tilde{x}}^{[m]})^2+(v_{\tilde{y}}^{[m]})^2}\right)} & \frac{-v_{\tilde{y}}^{[m]}}{\|\tilde{\mathbf{v}}^{[m]}\|} \end{bmatrix}. \tag{24}$$

Based on the property that a rank deficient matrix has a zero determinant, we obtain $-(v_{\tilde{y}}^{[m]})^2 = (v_{\tilde{x}}^{[m]})^2$ as the only condition for rank deficiency. Because this rank deficiency condition is not possible, the lemma is proved. The second part of the Lemma is obvious as $\boldsymbol{\nu}_3$ and $\boldsymbol{\nu}_4$ are 2D vectors which can be obtained from a linear combination of $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_2$.

C. Proof of Corollary 1

The diagonal matrices in (21) have a size of $(N_L^{[m]})$ and each element in these matrices can be decomposed into components in the x and y directions. The angle component of $\mathbf{K}_{\mathrm{t,l}}^{[m]}$ in the x and y direction can be shown to correspond with the elements of $\boldsymbol{\nu}_1$. More specifically $\mathbf{K}_{\mathrm{t,l}}^{[m]} = \pi\mathbf{Y}_{\mathrm{l,m}}\cos(\theta_{\mathrm{t,l}}^{[m]})\sin(\phi_{\mathrm{t,l}}^{[m]}) + \pi\mathbf{X}_{\mathrm{l,m}}\cos(\theta_{\mathrm{t,l}}^{[m]})\cos(\phi_{\mathrm{t,l}}^{[m]})$. Similarly, the angle components of $\mathbf{P}_{\mathrm{t,l}}^{[m]}$ and $\mathbf{K}_{\mathrm{r,l}}^{[m]}$ can be shown to equal $\boldsymbol{\nu}_2$ and $\boldsymbol{\nu}_3$, respectively. Hence, $\mathbf{K}_{\mathrm{r,l}}^{[m]}$ is a linear combination of $\mathbf{K}_{\mathrm{t,l}}^{[m]}$ and $\mathbf{P}_{\mathrm{t,l}}^{[m]}$. The second equation in this corollary can be proved similarly.

D. Proof of Corollary 2

First, note that the FIMs in the above corollary are diagonal matrices. Due to the properties relating Hadamard products with diagonal matrices [12], the left-hand side of the above Corollary can be decomposed as $\frac{2}{\sigma^2}\Re\Big\{\mathbf{V}_{\mathbf{v}_1} \odot \Big[\alpha_1\left(\mathbf{k}_{\mathbf{v}_1}\mathbf{k}_{\mathrm{t,l}}^{\mathrm{H}}\right) + \alpha_2\left(\mathbf{k}_{\mathbf{v}_1}\mathbf{p}_{\mathrm{t,l}}^{\mathrm{H}}\right)\Big] \odot \left(\mathbf{D}_\gamma^{\mathrm{H}}\mathbf{D}_\gamma\right)\Big\}$, where $\mathbf{V}_{\mathbf{v}_1}$ is a dummy diagonal matrix representing the common terms between $\mathbf{J}_{\mathbf{v}_1\boldsymbol{\theta}_{\mathrm{t,l}}}$ and $\mathbf{J}_{\mathbf{v}_1\boldsymbol{\phi}_{\mathrm{t},l}}$. Analyzing the diagonal elements of the matrices in the square brackets gives

$$\frac{2}{\sigma^2}\Re\Big\{v_{v_1}^{[m]} \odot \mathbf{a}^{\mathrm{H}}{}_{\mathrm{t,l}}^{[m]}\left[\alpha_1\mathbf{K}_{\mathrm{t,l}}^{[m]} + \alpha_2\mathbf{P}_{\mathrm{t,l}}^{[m]}\right]\tilde{\mathbf{a}}_{\mathrm{r,l}}^{[m]} \odot \left(\mathbf{D}_\gamma^{\mathrm{H}}\mathbf{D}_\gamma\right)\Big\}, \tag{25}$$

where $v_{v_1}^{[m]}$ represents the common terms on the m^{th} diagonal. From Corollary 1, the terms in the square brackets equals $\mathbf{K}_{\mathrm{r,l}}^{[m]}$, hence $\frac{2}{\sigma^2}\Re\Big\{v_{v_1}^{[m]} \odot \mathbf{a}^{\mathrm{H}}{}_{\mathrm{t,l}}^{[m]}\mathbf{K}_{\mathrm{r,l}}^{[m]}\tilde{\mathbf{a}}_{\mathrm{r,l}}^{[m]} \odot \left(\mathbf{D}_\gamma^{\mathrm{H}}\mathbf{D}_\gamma\right)\Big\}$, and we can write $\frac{2}{\sigma^2}\Re\Big\{\mathbf{V}_{\mathbf{v}_1} \odot \left(\mathbf{k}_{\mathbf{v}_1}\mathbf{k}_{\mathrm{r,l}}^{\mathrm{H}}\right) \odot \left(\mathbf{D}_\gamma^{\mathrm{H}}\mathbf{D}_\gamma\right)\Big\} = \mathbf{J}_{\mathbf{v}_1\boldsymbol{\theta}_{\mathrm{r,l}}}$. The second part of the corollary can be proved similarly.

E. Effect of Assumption 1 on Complex Path Gains

The submatrix $\mathbf{J}_{\boldsymbol{\eta}_2\boldsymbol{\eta}_2}$ and its corresponding entries $\{\mathbf{J}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}, \mathbf{J}_{\boldsymbol{\beta}_{\mathrm{I}}\boldsymbol{\beta}_{\mathrm{I}}}\}$ are diagonal matrices. With Assumption 1, the various RIS paths can be orthogonalized and we can write $\mathbf{J}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{I}}} = \mathbf{J}_{\boldsymbol{\beta}_{\mathrm{I}}\boldsymbol{\beta}_{\mathrm{I}}} = \frac{2}{\sigma^2}\left\{\left(\mathbf{A}_{\mathrm{r,u}}^{\mathrm{H}}\mathbf{A}_{\mathrm{r,u}}\right) \odot \left(\mathbf{k}_{\mathrm{l}}\mathbf{k}_{\mathrm{l}}^{\mathrm{H}}\right) \odot \left(\mathbf{D}_\gamma^{\mathrm{H}}\mathbf{D}_\gamma\right) \odot \left(\mathbf{A}_{\mathrm{t,u}}^{\mathrm{H}}\mathbf{F}\mathbf{F}^{\mathrm{H}}\mathbf{A}_{\mathrm{t,u}}\right)^{\mathrm{T}} \odot \mathbf{R}_0\right\}$.

F. Proof of Lemma 2 and Lemma 3

To prove Lemmas 2 and 3, we focus on the cross covariance between the elevation AoAs and the azimuth AoAs. The information loss due to the nuisance parameters which concerns the cross covariance between the receive elevation angle and the elevation angle of reflection can be written as

$$\begin{aligned}\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\theta}_{\mathrm{t,l}}}^{\mathrm{nu}} &= \mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\beta}_{\mathrm{R}}}\tilde{\mathbf{J}}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}^{-1}\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{t,l}}\boldsymbol{\beta}_{\mathrm{R}}}^{\mathrm{T}} + \mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\beta}_{\mathrm{I}}}\tilde{\mathbf{J}}_{\boldsymbol{\beta}_{\mathrm{I}}\boldsymbol{\beta}_{\mathrm{I}}}^{-1}\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{t,l}}\boldsymbol{\beta}_{\mathrm{I}}}^{\mathrm{T}} \\ &= \tilde{\mathbf{J}}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}^{-1}\left[\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\beta}_{\mathrm{R}}}\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{t,l}}\boldsymbol{\beta}_{\mathrm{R}}}^{\mathrm{T}} + \mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\beta}_{\mathrm{I}}}\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{t,l}}\boldsymbol{\beta}_{\mathrm{I}}}^{\mathrm{T}}\right],\end{aligned} \tag{26}$$

which is a consequence of Appendix E. Now, applying basic complex analysis, $\Im(\nu_1)\Im(\nu_2) + \Re(\nu_1)\Re(\nu_2) = \Re(\nu_1\nu_2^{\mathrm{H}}) = \Re(\nu_1^{\mathrm{H}}\nu_2)$, we have

$$\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\theta}_{\mathrm{t,l}}}^{\mathrm{nu}} = \tilde{\mathbf{J}}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}^{-1}\left[(\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\beta}_{\mathrm{I}}} + j\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\beta}_{\mathrm{R}}})(\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{t,l}}\boldsymbol{\beta}_{\mathrm{I}}} + j\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{t,l}}\boldsymbol{\beta}_{\mathrm{R}}})^{\mathrm{H}}\right]. \tag{27}$$

With appropriate manipulations, we have $\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\theta}_{\mathrm{t,l}}}^{\mathrm{nu}} = \tilde{\mathbf{J}}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}^{-1}\mathbf{J}_{\boldsymbol{\beta}_{\mathrm{R}}\boldsymbol{\beta}_{\mathrm{R}}}\mathbf{J}_{\boldsymbol{\theta}_{\mathrm{r,u}}\boldsymbol{\theta}_{\mathrm{t,l}}}$. Derivations of the expressions for other terms describing the information losses due to the nuisance parameters which are presented in Lemmas 2 and 3, can be obtained similarly.

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