Black Hole Physics and Computer Graphics

Gabriele Bozzola, Chi-kwan Chan, and Vasileios Paschalidis, *University of Arizona, Tucson, AZ, 85721, USA*

Black holes are among the most extreme objects known to exist in nature. As such, they are excellent laboratories for testing fundamental theories and studying matter in conditions that cannot be found anywhere else in the Universe. In this article, we highlight the relevance of black holes in modern physical and astronomical research and present one of the possible paths to explain observations and probe physics with the aid of numerical simulations. We briefly review dynamical-spacetime general-relativistic magneto-hydrodynamic (GRMHD) calculations as fundamental tools to study the local properties of black holes and matter around them. Then, we discuss the need for general-relativistic radiation transport to propagate the local information about light obtained with GRMHD simulations to our telescopes. Finally, we present accretion onto binary black holes as a key area of study for testing general relativity and plasma physics. The goal of our article is to introduce the reader to some of the methods in current black hole research and to point out how improvements in hardware and software for computer graphics support advancements in the field.

Black holes, despite their name suggesting that they are dark objects, are responsible for the most luminous and energetic phenomena observed in the Universe. Since their defining feature is the immense gravity (so strong that even light cannot escape the black hole horizon), they would be nearly undetectable to our telescopes if they were completely isolated. However, most black holes in the Universe are not in vacuum but are surrounded by gas that can be accreted. In the latest stage of the gas infall toward the horizon, matter releases its immense gravitational potential energy and travels near the speed of light, becoming so hot and bright that it can outshine everything else that is nearby. In addition to this, black holes can twist magnetic fields to launch collimated, relativistic outflows that can exceed millions of light-years in length. Black holes are one of the consequences of general relativity, the gravity theory that Einstein proposed in 1915. The first signals indicating that these objects are real were observed by astronomers 50 years later, kindling interest in their study. However, it is probably the 21st century that has seen the most advancements in both theoretical and observational aspects of black hole physics.

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The flourishing of the field of gravitational-wave astronomy is one of the most significant scientific developments of the past years. Several astrophysical processes can form pairs of black holes with the two objects orbiting each other. In these configurations, the binary loses energy due to the emission of gravitational waves, which are gravitational perturbations propagating at the speed of light (or, as sometimes stated: ripples in the fabric of spacetime). These gravitational wave signals are particularly interesting because they give scientists a unique view of how extreme gravity works. The loss of energy causes the distance between the two black holes to shrink, leading to the eventual merger, which is accompanied by a burst of potentially observable gravitational waves. These signals are extremely weak, and accurate theoretical models are needed to detect them in the noise of the gravitational wave detectors. Such theoretical models require the solution of Einstein's equations of general relativity with the aid of supercomputers. This is the task of the field of *numerical relativity*. Performing simulations of binary mergers is challenging due to a series of technical and theoretical problems (e.g., handling singularities, nonlinearity and/or ill-posedness in the equations, etc.). Numerical relativity grew out of the attempts to overcome these obstacles. Finally, in 2005, the first successful simulations of binary black hole inspirals and mergers were performed, and researchers are now able to make accurate predictions. This theoretical advancement laid the foundations for the development of improved models to observe gravitational waves, which were first detected in 2015 by the Laser-Interferometer Gravitational-Wave Observatory (LIGO). We now have several tens of confirmed detections, opening a new window to observing the Universe that has never been accessible before, and enabling leaps forward in several areas of physics and astronomy.

Significant breakthroughs also happened in traditional astronomy. Thanks to progress in adaptive optics, the technology that deforms telescope mirrors to correct for atmospheric turbulence, astronomers managed to obtain high-resolution pictures of the galactic center and provide convincing evidence that our Galaxy harbors a supermassive black hole. It is now believed that most galaxies have black holes in their center that play an important role in the evolution of their host. Another example is M87, for which the Event Horizon Telescope (EHT) Collaboration released the first pseudocolor image of its black hole in 2019. This was obtained by combining data from radio telescopes distributed all over the world and led to the first measurements of some properties (e.g., magnetic fields) close to the horizon. Figure 1 reports the result of a general-relativistic radiation-transport calculation used by the collaboration to explain the observed data.

It is interesting to note the shift in black hole research. For most of the 20th century, black holes were mostly the domain of mathematicians and theoretical physicists, and the tools of investigation were primarily analytical. Nowadays, research is much more driven by technology and numerical computation. Most of the aforementioned breakthroughs were made possible by technological improvements: numerical relativity requires large supercomputers with thousands of cores; faster processors allow adaptive optics systems to correct for atmospheric turbulence better and with higher cadence, leading to sharper images; and both LIGO and EHT have to analyze petabytes of data and require large banks of expensive numerical simulations, which would have not been possible in the previous century.

Over the next decades, technological advancement is expected to precipitate additional breakthroughs. On one side, the already planned next-generation gravitational-wave observatories will be significantly more capable than the ones that are currently active. An important example is the Laser Interferometer Space Antenna (LISA), which will be the first instrument able to observe gravitational waves from binary supermassive black holes. Similarly, larger and more powerful electromagnetic telescopes are currently being built, which will detect signals that were previously unobservable and/or more frequently. As a result, the scientific community will have plenty of new experimental data on black holes, allowing studies that are currently not feasible.

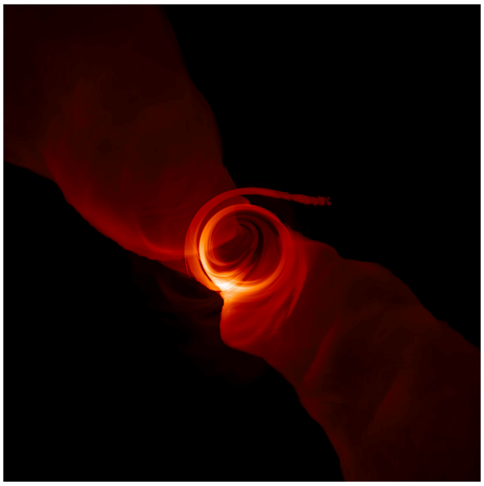

FIGURE 1. Image of accretion onto a black hole obtained combining a GRMHD simulation with general-relativistic radiation transfer, as implemented in GRay.[1] The features of this image are explained by relativistic effects and the extreme properties of the plasma (e.g., the bright ring with an intense spot). Images like this are critical in modeling observable signals that originate near the black hole.

In this article, we review some aspects of black hole physics. We start by illustrating its importance in current physical and astronomical research. Next, we discuss numerical relativity and dynamical-spacetime GRMHD simulations, one of the main tools of investigation. Then, we focus on the next step: from general-relativistic magneto-hydrodynamic (GRMHD) simulations to observable quantities and models. We present general-relativistic ray tracing and radiation transport, and we highlight its connection with common techniques in computer graphics. We discuss why graphical processing units (GPUs) are optimally suited to solve this problem and how we take advantage of them. We also touch upon some state-of-the-art ways to interact with and visualize the data. Finally, we discuss accretion onto binaries, which, thanks to the new data, will be relevant in the coming years.

RELEVANCE OF BLACK HOLES IN CURRENT PHYSICAL AND ASTRONOMICAL RESEARCH

From the time since when they were theorized, in 1916, until the first observational evidence in the 1960s, it was believed that black holes were mathematical oddities with no place in nature. Today, we know that these objects play a role of paramount importance in physics, astronomy, cosmology, and several other fields.

The extreme nature of black holes makes them particularly appealing to scientists interested in learning how the Universe works on a fundamental level. Laws of nature are best tested at their limits: studying them in their extreme conditions is a critical step in the falsification process and guides our investigation toward better models. Therefore, black holes are perfect testbeds for fundamental theories that explain how the Universe works on both the largest and the smallest scales. These theories, such as quantum field theory and general relativity, have shaped our modern world and have a direct impact on our daily life. Black holes are also ideal for the study of some applied theories. For instance, many details of plasma physics are not completely understood, and these astronomical objects give researchers a chance to shed more light on the topic.

Black holes are responsible for several different signals that are observed in the sky, so they are critically important to astronomers. For instance, a star encountering a black hole can be stripped of its matter, which, falling into the black hole, emits X-rays. A second important example is quasars, the farthest objects that can currently be observed from Earth. Quasars are powered by the energy released during accretion onto supermassive black holes and are so far away from Earth that, considering the finite speed of light, they are effectively billions of years in the past. As such, quasars are invaluable cosmological tools that allow scientists to understand more about the evolution of the Universe. Other types of signals that can be directly traced back to black holes include radio, ultraviolet, optical, and infrared. Therefore, understanding these objects is needed to properly characterize what we detect with our telescopes.

Cosmologists and astronomers are also interested in black holes because of their role in the evolution of galaxies, most of which host supermassive objects in their centers. A galaxy and its black holes co-evolve over cosmic history, leading to several correlations between the features of the galaxy and the ones of its supermassive guest. Studying this evolution helps researchers understand the formation of stars, which in turn are responsible for the formation of planetary systems, and possibly life. In addition to this, some black holes are born from the death of massive stars in supernova events. So, black holes can inform researchers about both the birth and death of stars.

Finally, black holes are part of popular culture and often play key roles in science fiction. As such, in addition to providing a source of entertainment, they have an important broad impact on science and society in general. Their mysterious aura and extreme properties intrigue and fascinate people. Thus, black holes can be a point of contact between the society at large and the scientific community, possibly inspiring students to pursue scientific careers and bringing the general public closer to science.

NUMERICAL-RELATIVITY AND GRMHD SIMULATIONS

General relativity describes black holes and what happens to matter and other fields, such as the electromagnetic, around them. One of the key ingredients of this theory is the *spacetime*, which is both the "stage" where all physical phenomena take place, but also an important "actor," as it interacts with matter and electromagnetic fields, and it deforms in response to how they behave. According to general relativity, massive bodies, like black holes, induce curvature in the spacetime, which affects the motion of the gas and light. This is also true for binary black holes, that, while inspiraling, interact with each other and deform the spacetime in a time-dependent way. Therefore, modeling accretion onto black holes requires the solution of two coupled problems: 1) how matter and electromagnetic fields behave on the curved spacetime, and 2) how this spacetime evolves. In this section, we explore numerical-relativity and dynamical-spacetime GRMHD simulations as solutions to these problems.

Let us first consider gas and electromagnetic fields. The state of matter around black holes is such that it can be described with an ideal GRMHD approach. In this framework, the material is assumed to be a fluid with negligible electrical resistivity moving in a curved spacetime. As a result, magnetic field lines are tied to the fluid and vice versa (hence, the "magneto-hydro" in GRMHD). The goal of GRMHD simulations is to find the local thermodynamic state of the matter and electromagnetic fields: rest-mass density, internal energy, pressure, velocity, etc. As we will see in the next section, this information is used to compute absorption/emission properties, which are needed to determine the observable electromagnetic signal.

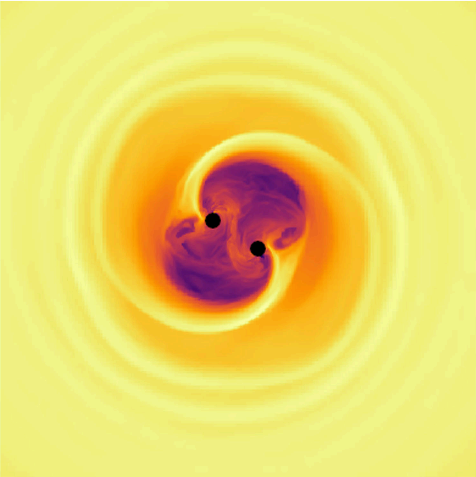

FIGURE 2. Snapshot of a numerical-relativity GRMHD simulation of accretion onto binary black holes. This simulation was performed on `Frontera` with the `Einstein Toolkit`,[2] using the `Lean`[3] and `IllinoisGRMHD`[4] solvers and visualized with `kuibit`.[5] In this simulation, we solve Einstein's equations to find the spacetime, and the GRMHD equations to find how matter and electromagnetic fields evolve. The black disks are the horizons of the black holes, which are not directly evolved, but they are found in the grid-solving specific differential equations. The ripple in the accretion disks is due to the torque of the binary. This image is not what we would observe with our telescopes. The question "what is the observable emission from this system?," requires modeling the emission and absorption of photons and their propagation, with the inclusion of all the relativistic effects.

In Figure 2, we show a snapshot of one of our GRMHD simulations of accretion onto binary black holes. The simulation was performed on `Frontera` with the `Einstein Toolkit`[2] using the `Lean`[3] and `IllinoisGRMHD`[4] solvers and visualized with `kuibit`.[5] The evolution starts with a binary surrounded by a hollow torus-like gas disk that is threaded by magnetic fields. Magnetic torques are responsible for the so-called magnetorotational instability, which drives the disk out of equilibrium and triggers the accretion of matter onto the binary. The system evolves and interesting features appear, such as the accretion streams onto the black holes, and the ripples in the disk due to the binary tidal torque.

Algorithmically, GRMHD codes tend to share several features with traditional computational-fluid dynamical simulations (e.g., a common approach is to use Eulerian finite-volume methods in a conservative formulation in conjunction with high-resolution shock-capturing schemes). Nonetheless, it should be remarked there are several differences compared to Newtonian computational-fluid dynamical calculations. For example, due to the curved spacetime, the differential equations are more in number and more non-linear, requiring dedicated methods to be solved.

The second aspect one needs to take care of is the spacetime itself. One of the key features of black holes is that, mathematically, they are characterized by the existence of a singularity, where physical quantities go to infinity, and a horizon, which constitutes a causal boundary: everything that happens inside the horizon stays there. The mathematical properties of black holes pose significant problems in numerical schemes. We will distinguish the case of a single black hole from that with binaries since the approach differs.

There exists a family of solutions to the equations of general relativity that describe the most general stationary single black hole. Hence, one can compute all the spacetime quantities analytically and directly implement a GRMHD scheme. In doing this, due to the singularities and the causal properties of the above-mentioned horizon, one needs to choose coordinates wisely. Fortunately, there are well-behaved coordinate systems that are free of singularities and can be used for stable, long-term GRMHD simulations onto single black holes.

For binary systems, the solution of the spacetime has been traditionally a difficult problem. In this case, the singularities and the other technical problems are much more severe. Efforts to evolve binary black hole spacetimes led to the development of *numerical relativity*, a subfield of physics that is concerned with the numerical integration of problems in relativistic gravity. Today, researchers can perform numerical-relativity simulations and solve accurately for the spacetime of binary black holes whose masses do not differ substantially. One of the breakthroughs that allowed this is the treatment of coordinates: to ensure stability the coordinates have to evolve, too, and researchers found families of coordinate choices that are well-suited for the numerical integration of the Einstein equations.

The terms "numerical-relativity" or "dynamical-spacetime" simulation are used when the spacetime is being solved for, and this may or may not include a GRMHD sector. In some cases, the two are strongly coupled: 1) the matter determines how the spacetime evolves and 2) the spacetime determines how matter moves. However, in simulations of accretion onto (binary) black holes as astrophysical sources, the accreted gas is not massive enough to affect gravity. Thereby, the spacetime evolution is assumed to be driven solely by the black holes. The snapshot shown in Figure 2 is an example of a dynamical-spacetime GRMHD simulation where the matter self-gravity is neglected. This simulation evolved a cubic box of size of 3000 black hole radii for 6000 black hole timescales and required approximately 18,000 node-hours on `Frontera`.

Numerical-relativity simulations solve a coupled system of highly nonlinear partial differential equations to find the time evolution of the metric tensor, the fundamental quantity in general relativity that measures the distance between two spacetime points and contains information about the curvature. We remark that there are fundamental differences between how black holes are treated in (pseudo-)Newtonian calculations and in numerical relativity, which provides the correct description. In particular, unlike (pseudo-)Newtonian approaches, numerical relativity does not treat black holes as "sinks," where one imposes some boundary conditions. Instead, horizons are found from the metric tensor, which allows them to move and grow, as described by general relativity.[a]

Once we have performed numerical-relativity, GRMHD simulations, we are still left with an important question: *what would instruments observe?* Gravitational waves are pure spacetime effects that interact weakly with matter, so little postprocessing will immediately produce an observable signal with all the relativistic effects naturally included. However, the situation is different for light; numerical-relativity simulations compute the local state of the spacetime and the matter, but do not contain explicit information on how photons are produced, absorbed, and how they propagate. Therefore, learning about what happens near black holes is not enough to make predictions or explain observations. For example, Figure 2 is not what our telescopes would observe, but it is the starting point for such calculations. In the next section, we address this issue.

OBSERVING BLACK HOLES

State-of-the-art numerical-relativity codes do not implement radiation physics modules with enough sophistication to directly determine what telescopes would observe. While in the future, it might be possible to perform simulations with full-time-dependent and self-consistent general-relativistic radiation transport to do so, the problem is currently best tackled with a different strategy. As discussed in the previous section, numerical relativity finds the state of the spacetime, matter, and electromagnetic fields throughout the evolution. We can obtain accurate models in a postprocessing step by using these data to study light emission, absorption, and propagation. Importantly, this approach allows researchers to explore different prescriptions for those quantities that are still not well understood (e.g., plasma properties) without having to re-run the expensive dynamical-spacetime GRMHD simulations. This is also the strategy adopted by the EHT Collaboration in their successful modeling and interpretation efforts of the M87 data.[6] In the following, we focus on one of the techniques developed within the collaboration and implemented in the open-source `GRay` and `GRay2`[1,7] codes; other approaches, such as the ones based on Monte Carlo methods (e.g., `grmonty` [8]), exist.

Barring relativistic effects, the problem of modeling emission of light from the vicinity of black holes shares many features with the rendering of a scene in computer graphics: We have a 3-D object and we want to know how it would look like in the image space. Therefore, it should not come as a surprise that GPUs and graphical frameworks can be useful. In this section, we highlight this connection by discussing how to obtain observables from simulations (e.g., images, spectra, etc.). First, we review the relativistic effects that need to be considered and that constitute the fundamental difference compared to traditional volume rendering; then, we discuss the details of how general-relativistic ray tracing and radiation transport is done; finally, in the third section, we show how interactive visualizations can help researchers understand the problem better and gain additional insight.

Relativistic Effects in the Propagation of Light

Near black holes, relativistic effects play a critical role in the propagation of photons. This is the topic of this section, where we discuss light bending, Doppler shifting, and relativistic beaming.

One of Einstein's first predictions after the formulation of the theory of general relativity was that light does not follow straight trajectories in the presence of strong gravity. The measurement of this effect during the total solar eclipse of 1919 led to the first experimental confirmation of the theory. The observation consisted of the detection of a 2″ (two arcseconds) deflection of the light coming from a distant star due

[a]Numerically, while several codes implement finite volume methods (which allows for easy integration of the GRMHD part), other approaches, like spectral methods or discontinuous Galerkin, are employed too.

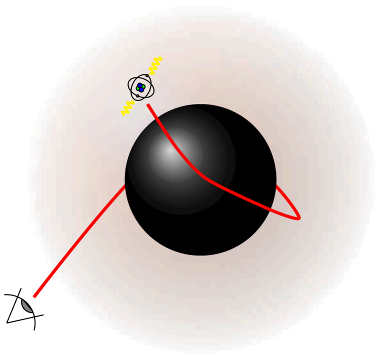

FIGURE 3. Near massive objects, light does not follow rectilinear paths and photons can orbit multiple times the black hole before reaching the telescope. Along their trajectory, the intensity of light can increase or decrease depending on the local emission and absorption properties. In our approach, we do not track the individual emission/absorption events, but use the local thermodynamic properties to determine how the intensity carried by a ray changes.

to the Sun's gravitational field. This effect is minuscule: 2″ corresponds roughly to the angular size of the One World Trade Center, as observed from the West Coast. On the other hand, black holes can lead to a much more extreme bending. For instance, an observer standing near the horizon would be able to see the back of their head. In Figure 3, we show a cartoon to illustrate this effect. The light ray that is observed does not come directly from the source following a rectilinear path, but it circles the black hole once. We will discuss the mathematical aspects of this phenomenon in the next section.

Light deflection has important consequences on how black holes affect detectable signals. One of these is *gravitational lensing*, which consists of the distortion or duplication of images of astronomical sources and is routinely observed in the sky. This is similar to the formation of mirages, optical features that are the result of a medium with varying index of refraction. In fact, the bending of light in gravitational fields is similar to the continuous deflection that a ray experiences traversing an inhomogeneous atmosphere. For our discussion, however, the most significant consequence of this phenomenon is that it affects the intensity of rays. This can be readily understood: the effect influences the origin of the photons that telescopes observe and the amount of material that they have to traverse to reach them. Photons that go around the black hole multiple times have to go through a lot of matter, so the intensity of light will be attenuated, and it may not be detectable at Earth. On the other hand, the ones that originate from regions near the black hole, which are hotter, will be more energetic compared to those that come from further away. This is shown in Figure 3: the red cloud represents gas around the black hole, and the light ray has to travel through it to reach the observer, possibly changing its intensity.

The second group of relativistic phenomena that need to be captured are the relativistic beaming and Doppler effect—a change in the observed frequency and intensity due to both general and special relativity. The frequency shift has two components. The first is classical (but with relativistic corrections): signals that are emitted by a source in motion with respect to the observer will have their frequency changed, as in the familiar case of the siren of a moving ambulance, which is more high pitched as the ambulance approaches, and becomes more low pitched as the ambulance moves away. The second component is gravitational: photons that emerge from a gravitational potential well lose energy and their frequency becomes lower. Since telescopes observe only specific or relatively narrow bands of frequencies (for example, the EHT instruments observe at 230-GHz), one needs to properly account for this effect to make accurate models. Relativistic beaming consists of the focusing of the emission of a source moving near the speed of light. This modulates the intensity depending on the observation angle and the relative direction of motion.

The relativistic effects described in this section play a critical role in astronomical observations of signals originating from the vicinity of black holes. Clearly, they need to be properly accounted for when imaging horizons (see Figure 1), as in the case of the EHT. For other targets for which instruments currently lack resolution, we still need these simulations to characterize observational properties and answer questions like: What is the expected flux at the various frequencies? How does it evolve with time? Are there periodicities? All these questions (and others) require accurate general-relativistic calculations. In the next section, we dive into the details of the mathematical formulation and of the algorithms employed in such simulations.

General-Relativistic Ray-Tracing and Radiation-Transport

General-relativistic ray tracing and radiation transfer are needed to extract observable signals from GRMHD simulations. This step consists of finding the path of

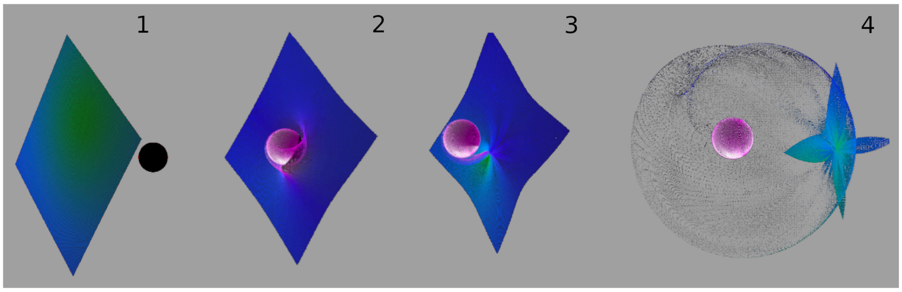

FIGURE 4. Series of screenshots of GRay in its interactive mode, which allows a user to integrate and visualize the photon positions in real time using OpenGL. Consumer-grade GPUs can integrate the geodesic equation and render the output in real time with a large number of photons. 1. Initial conditions are set up: a grid of photons at a large distance traveling toward the black hole. Some of the photons, shown in pink here, are trapped near the horizon, others, in green, are deflected at high angles (2.,3.). 4. How the grid is deformed after the impact. Photons are deflected in all directions, some move back toward the origin.

light rays and the intensity they carry at each frequency to produce images and other integrated quantities. In this section, we discuss the details of the mathematical framework and the algorithms behind these calculations. We will present a summary of the process in Figure 5, at the end of this section.

In general relativity, the trajectory of light is obtained by solving the *geodesic equation*

$$\dot{v}^\mu = -\sum_{\alpha,\beta=1}^{4} \Gamma^\mu_{\alpha\beta}(\mathbf{x}) v^\alpha v^\beta \text{ for } \mu \in \{0,\, 1,\, 2,\, 3\} \tag{1}$$

where $\mathbf{\Gamma}^\mu(\mathbf{x})$ are a set of four 4×4 position-dependent symmetric matrices known as Christoffel symbols, $\mathbf{v}$ is the 4D velocity of the photons, $\mathbf{x}$ is the spacetime position, and the dot indicates a derivative with respect to the parameter of the path. This equation describes the change in the velocity of the rays depending on the local $\mathbf{\Gamma}^\mu$. Since photons travel at the speed of light, the variation in $\mathbf{v}$ corresponds to a change in direction (hence the deflection).

The Christoffel symbols $\mathbf{\Gamma}^\mu(\mathbf{x})$ encode information about the curvature of the spacetime. Therefore, in the absence of massive bodies, they are all identically zero (in Cartesian coordinates), and the geodesic equation is trivially solved by $v^\mu = v^\mu_{\text{initial}}$. This solution is what we expect from Newtonian optics and Fermat's principle: light rays propagate in straight lines (i.e., with constant velocity). For isolated black holes, the Christoffel symbols are known analytically, and for binaries, they are determined by numerical-relativity simulations. Once the Christoffel symbols are known, one can use standard integration methods to solve (1).

Solving (1) and finding the trajectories of photons in curved spacetime is usually known as *general-relativistic ray tracing*. The problem is an example of nonlinear ray tracing and is analogous to finding the path of a photon through an atmosphere with varying index of refraction (e.g., due to a temperature gradient or stratification). In Figure 4, we show an example of such calculation (performed with GRay). The first panel shows a grid of photons, representing the array of pixels of a camera, placed far away from the black hole and moving toward the central object. Some photons are captured (pink ones), others are deflected at high angles compared to the incidence angle (green ones). The fourth panel shows how the grid is deformed after the encounter with the black hole: some rays are trapped near the horizon, and the others are deflected in all directions, even toward the place where they originated from.

The geodesic equation is time-reversible: one can look at the screenshots in Figure 4 from left to right to see where light rays that hit the camera originated from. This is the starting point for producing images like Figure 1: the equation is integrated backward in time to find what photons are relevant to form an image and what their trajectory was.

Alongside the paths followed by rays, one has to find the intensity of the light at various frequencies, obtained by solving the equation for *general-relativistic radiation transport*. This can be written in the following form:

$$\dot{\mathcal{I}}_\nu = j_\nu - \kappa_\nu \mathcal{I}_\nu \tag{2}$$

where $\mathcal{I}_\nu$ is the Lorentz-invariant specific intensity of radiation at frequency ν, j_ν, and κ_ν are the Lorentz-invariant specific emissivity and absorptivity. Ignoring the technicalities of relativity, the physical content of (2) is straightforward: at a given frequency ν, the

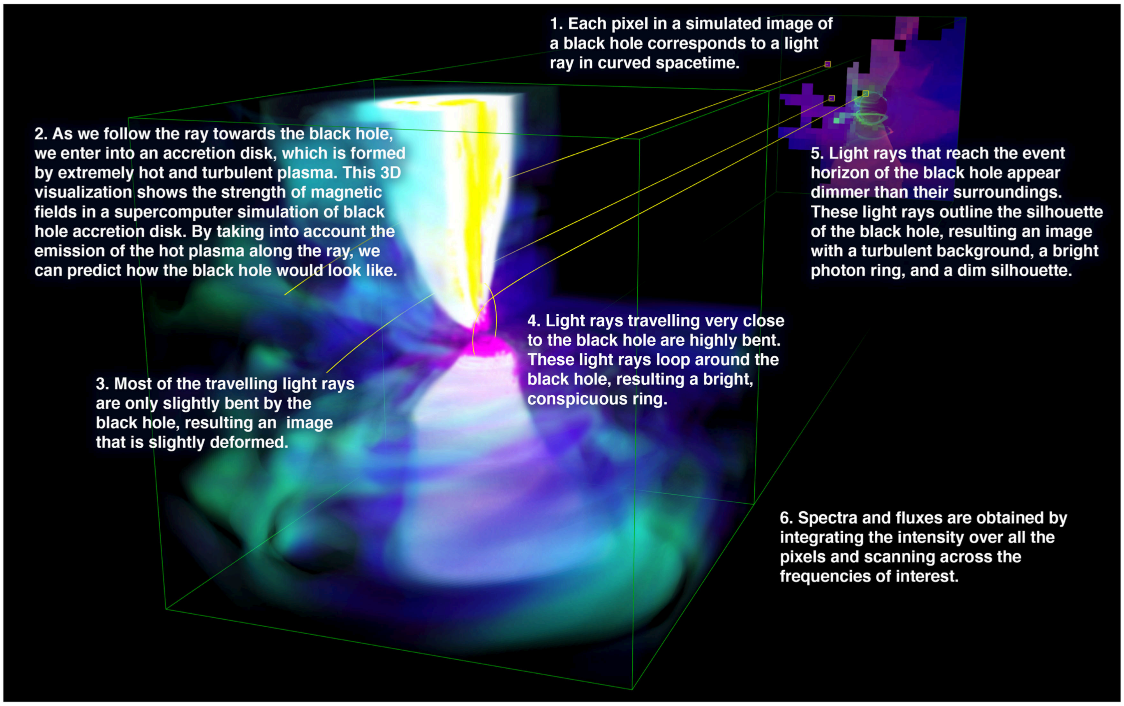

FIGURE 5. Summary of our scheme. On the left, we show a volume rendering of the gas around the black hole; on the right, its image is generated with a general-relativistic ray tracing and radiation-transfer scheme. Figure 1 is generated in the same way.

intensity changes ($\dot{\mathcal{I}}_\nu$) due to additional local emission (j_ν), or absorption from what the ray already had ($\kappa_\nu \mathcal{I}_\nu$). These two quantities, j_ν and κ_ν depend on the local state of the matter (e.g., density and temperature) and of the electromagnetic fields. It is important to point out that there are two frames of reference in this problem: one is the observer's, and the other is the fluid's. The relativistic transformation between the two takes care of the relativistic effects: for example, frequency ν in the fluid frame will correspond to a different value in the other one (similarly for the beaming effects).

The geodesic (1) and radiation transport (2) equations form a coupled system that describes the emission, absorption, and propagation of light taking care of all relativistic effects. Solving this system allows us to find observable signals and make pictures, such as that in Figure 1. In the following, we go into the details of how this is done by describing the overall algorithm implemented in `GRay2`.

First, we set up the initial data. We place our camera far away and discretize the field of view with an array of pixels. In this context, a camera represents the detector of a telescope, which is very far from the black hole system considered (of the order of 1000 black hole radii). From each pixel, we shoot light rays back toward the system. Equation (1) determines how the rays move and (2) how their intensity is enhanced or attenuated, as they propagate through matter. We use a Runge–Kutta scheme to integrate the equations by a Δt. The right-hand side of (2) contains α_ν and κ_ν, which are determined by interpolating the matter variables (velocity, temperature, magnetic fields, etc.) at the new position of the ray. The specific form of α_ν and κ_ν depends on the physical processes that one wants to include. The rays also carry information about the optical depth, which is a measure of how deep into the matter the ray is. When the optical depth reaches a threshold value, there is no point in following the rays further because anything that is beyond that threshold will have a negligible impact on the rays' intensity. We keep integrating (1) and (2) one Δt at a time, until the threshold value of the optical depth is reached or one of two following conditions are met: 1) the ray falls into the horizon; 2) the ray leaves the computational domain, outside of which we assume that nothing interesting happens.

The result of the computation is an image like Figure 1: a 2-D array of values, where each pixel contains the intensity of the ray at that frequency. If we integrate the values over the entire array, we obtain the expected flux from the source. We obtain the spectrum (intensity versus frequency of light) by scanning through different frequencies. Both quantities (flux and spectrum) can be functions of time. Up to some modulation due to the propagation from the source to us, these are the signals that telescopes observe. In Figure 5, we report a high-level summary of the entire process that takes information from GRMHD simulations to build an image of the black hole system.

The nature of general-relativistic ray tracing and radiation transfer makes it optimally suited for GPUs.[b] The reason for this is not that modern GPUs come with hardware specialized for ray tracing (we will touch upon this in the outlook section), but because the task of creating an image is trivially parallelizable (each photon in independent), the calculations involved are relatively simple algebraic manipulations with little to no conditional branching, and the problem can be expressed naturally using some computer graphics concepts.

Graphical libraries have a hardware-optimized data type that are perfectly suited for spacetime calculations: several quantities in computer graphics are represented as four-vectors (e.g., homogeneous coordinates, or color with transparency), which are what we use to represent spacetime quantities like position and velocities, as well as polarization states of light (the Stokes parameters). Since libraries provide optimized methods to act on these vectors, this leads to simpler, but highly efficient, code. Another common operation that is performed multiple times at each time step is the interpolation of volumetric data. Since the rays do not move on a grid, we need to interpolate the data onto the location of the photon. This is both needed to solve the geodesic equation and to determine the local emission/absorption. Given that interpolation of textures is a common problem in computer graphics, graphical frameworks offer natural and efficient ways to perform these operations that we can use as long as we postprocess data from GRMHD simulation to logically Cartesian grids. Once again, this leads to clean and efficient code.

In addition to this, with the growing adoption of GPUs for machine learning, hardware is becoming increasingly faster in performing matrix operations. A clear example of this is the introduction of the tensor cores in the latest NVIDIA cards. Matrix operations are the base of the integration of the geodesic equation (1). Therefore, GPUs yield significant boosts in computational performance.

GRay makes use of several of these features to maximize its speed. For example, the movie *Interstellar* contains scenes involving a black hole that are based on physically accurate ray tracing (but additional effects were tweaked to increase visual appealing, e.g., adding the camera lens flare). The code used required tens of computer hours for each frame. On the other hand, GRay can accomplish the same underlying task (general-relativistic ray tracing) in real time. The additional performance allows us to add and explore more physics. Thus, GRay has been used to perform large parameter surveys[11] that have not been matched by traditional codes yet.

Furthermore, GPUs allow for extremely fast single-precision operations. While these might not be enough for some scientific applications, they result in a quick feedback loop. Some of this is explored in the next section, as graphical frameworks can also help us with improved visualizations.

New Visualization and Interaction Techniques

In addition to predicting emissions and images from black hole systems, there is another area where computer graphics has a significant impact on black hole research. Because of the light-bending effect and the dynamic nature of the accretion flows, even though we have access to all information in black hole simulations, it is still difficult to interpret and understand the resulting images. For example, is the bright filament seen in Figure 1 a gravitational or a plasma effect?

Real-time interactive visualization has proven to be a useful tool to inspect simulations and allow scientists to develop their physical insights. Using GPU-accelerated computing frameworks provides a natural way to integrate graphic libraries to visualize GRMHD simulation data in real time.

Because geodesic integration of millions of photons can now be done interactively on GPUs, we extended GRay with OpenGL to visualize all the photons in real time. We even integrate the LeapMotion[c] and PrimeSense (now discontinued) controllers, which are devices that users can interact using hand gestures, to control these visualizations. This provides an intuitive way for us to follow the photons in interactive 3-D visualizations like Figure 4 and understand the formation of black hole shadows.

[b]In addition to our own GRay[1] and GRay2,[7] there are other GPU-based codes publicly available (e.g., RAPTOR[9] and Odyssey[10]). On the other hand, GRMHD simulations are not embarrassingly parallel, so the adoption of GPUs in numerical relativity is much more limited.

[c][Online]. Available: https://leapmotion.com/

To understand the complicated features seen in black hole images, we also explored virtual reality using Oculus headsets. We put volume rendering of the accretion flows and ray-traced images together, as in Figure 5, to interactively compare them, untangling general-relativistic effects from the turbulent plasma flow. Other groups have also created 360 videos of general-relativistic ray-traced movies of supermassive black holes.[12]

ONE OF THE CHALLENGES OF THE NEXT DECADE: BINARY BLACK HOLES

Considering that researchers anticipate massive amounts of data on binary black holes from facilities that will be coming online in the near future, the problem of accretion onto binary black holes is particularly timely. Specifically, the LISA is expected to start operations in the 2030s and will open up the possibility of observing gravitational waves from mergers of supermassive black holes with masses 10^6–10^7 times the mass of our Sun. Gravitational waves are significantly different from electromagnetic ones and they often carry complementary information. Hence, detecting both at the same time, in a so-called *multimessenger observation*, enables leaps in understanding these systems. In addition to that, the EHT Collaboration is breaking new grounds in imaging black holes, and binaries may be observed in the future. Despite the scientific relevance, the field of accretion onto binaries is still in its infancy, and better and more numerical simulations are needed to build theoretical models and make predictions that future observatories will be able to test. In this section, we discuss the work that we are currently doing to obtain accurate models of observable signals.

As described in the previous sections, one of the fundamental differences between the case of a single stationary black hole and binary black holes is that in the former case the Christoffel symbols in (1) are known analytically and the coordinate system is fixed. In the case of binaries, we must deal with numerical Christoffel symbols, which dramatically increase the computational cost. For this reason, state-of-the-art codes for ray tracing and radiation transfer in binary black hole spacetimes make assumptions to simplify the problem and make it tractable (e.g., using approximate analytical Christoffel symbols) or freezing the spacetime and the matter, as the light ray propagates—the so-called fast-light approximation. However, such approximations are not valid when the objects are close to the merger. The improvements in GPU technology of the past years made it possible to implement a full code without these simplifications. Hence, we are extending `GRay2`[7] to support the case with full numerical relativity data that go beyond the fast-light approximation.

The fundamental improvements that we have witnessed over the past years were three. First, GPUs have now larger on-chip memories, enough to handle this much information. We could still make use of more memory to increase the numerical resolution, but there are algorithmic ways to take the most advantage of what we have. For example, one of the tricks we employ is to utilize a nonuniform grid of coordinates, with more resolution closer to the black holes. Second, faster interconnections with the host made it possible to transfer larger amounts of data to the GPU. Transfer from the host to the device is still the slowest step in our approach, but this is not a serious problem because there are several unknowns in the problem, and we can loop over the different possible prescriptions without having to reload data. Finally, support to double-precision floating-point arithmetic improved dramatically over the past years. When dealing with spacetime data, double precision is often needed.

Thanks to these improvements, modern graphics cards are powerful and fast enough that we can tackle the problem of ray tracing on binaries with an improved version of `GRay2`. The updated version of `GRay2` will read the output of the `Einstein Toolkit`,[2] an open-source GRMHD and spacetime solver, to make an image like Figure 1, but for binaries (see Figure 2). This will push the state of the art and will allow for the first studies of emission for binary black holes near the merger. Over the coming years, this will be used to prepare models to explain existing and future observations.

The updated version of `GRay2` pays a significant performance penalty in dealing with numerical Christoffel symbols. Luckily, there is more untapped potential in modern GPUs to improve performance. For instance, technologies like NVIDIA's GPUDirect Storage can be leveraged to speed up data transfer to the device. Moreover, in `GRay2`, we chose to extend the battle-tested algorithms implemented in previous iterations of the software, which are naturally expressed in the language of vectors and matrices, and can be greatly accelerated by tensor cores. On the other hand, the code does not take advantage of ray-tracing accelerators like NVIDIA's RT Cores. The reason for this is that general-relativistic ray tracing is highly nonlinear. Given that light does not propagate on straight lines, and the emitting and absorbing material is not a solid object but is a diffuse medium, conventional methods are ineffective. While partial adoption might come in the future (e.g., hardware-tuned traversal of hierarchies might help implementing complex meshes), fully embracing the use of ray-tracing accelerators would need a significant change in the approach to the problem. This would require implementing and experimenting with novel algorithms for nonlinear ray tracing. Nonetheless, given the growing complexity of numerical-relativity simulations and the improvement in instruments and telescopes, the need for higher resolution and cadence radiation transport is expected to grow; fundamental algorithmic changes might be needed in the future. In this regard, it would be black hole physics that exploring new algorithms for nonlinear ray-tracing pushes advancements in the field of computer graphics.

CONCLUSION

In this article, we reviewed several aspects of black hole physics. In the introduction, we summarized the recent breakthroughs in the field, and why we expect further advances in the future. We highlighted the importance of the study of black holes in several areas of physics and astronomy. We presented one technique to make models and predictions: we reviewed GRMHD simulations, and we showed how we use them in combination with general-relativistic ray tracing and radiation transport to explain the data. We highlighted the role of GPUs and we discussed how advancements in hardware and frameworks for computer graphics translate into improvements in our ability to model and understand black holes. We ventured into one of the most active areas of research in the field: accretion onto binaries.

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GABRIELE BOZZOLA is currently a Ph.D. candidate with The University of Arizona, Tucson, AZ, USA and a NASA future investigator in Space Science. His research interests include black holes, compact objects, high-energy astrophysics, high-performance computing and the development of scientific software. He is a contributor to the Einstein Toolkit and other open-source projects. He worked on the problem of ray-tracing of binary black holes as part of the Frontera Fellowship by the Texas Advanced Computing Center, Austin, TX, USA. Bozzola received the M.Sc. degree in physics from the University of Milan, Milan, Italy. Contact him at gabrielebozzola@email.arizona.edu.

CHI-KWAN CHAN is currently a computational astrophysicist with The University of Arizona, Tucson, AZ, USA. His research interests include simulating and understanding accretion disks, capturing images of black holes, and visualizing astrophysical simulations in virtual reality. He has developed new algorithms to study magnetohydrodynamic turbulence, used GPUs to accelerate general relativistic ray-tracing, designed cloud computing infrastructures to handle big observational data, and applied machine learning algorithms to speed up and automate data processing. Chan received the Ph.D. degree in physics from the University of Arizona. Contact him at chanc@arizona.edu.

VASILEIOS PASCHALIDIS is currently an assistant professor of astronomy and physics with The University of Arizona, Tucson, AZ, USA. His interests include relativistic gravitation, black holes, neutron stars, white dwarves, gravitational wave astronomy, testing fundamental physics, and high-energy astrophysics. He has developed new algorithms and codes to solve the equations of general relativistic magnetohydrodynamics in dynamical spacetime, and the Einstein equations of general relativity. He is a proponent of open-source code, and has released the dynamical-spacetime general relativistic magnetohydrodynamics `IllinoisGRMHD` to the `Einstein Toolkit`. Paschalidis received the Ph.D. degree in astrophysics from the University of Chicago, Chicago, IL, USA. Contact him at vpaschal@email.arizona.edu.