Guide to anomaly-mediated supersymmetry-breaking QCD

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We present a careful study of the chiral symmetry breaking minima and the baryonic directions in supersymmetric QCD ($SU(N_c)$ with N_f flavors) perturbed by anomaly mediated supersymmetry breaking (AMSB). For the s-confining case of $N_f = N_c + 1$ and most of the free-magnetic phase ($N_f \leq 1.43N_c$) we find that naive tree level baryonic runaways are stabilized by loop effects. Runaways are present, however, for the upper end of the free magnetic phase ($N_f \gtrsim 1.43N_c$) and into conformal window, signaling the existence of incalculable minima at large field values of $\mathcal{O}(\Lambda)$. Nevertheless, the chiral symmetry breaking points are locally stable, and are expected to continuously connect to the vacua of QCD for large SUSY breaking. The case of $N_f = N_c$ requires particular care due to the inherently strongly coupled nature of the quantum modified moduli space. Due to the incalculability of critical Kähler potential terms, the stability of the chiral symmetry breaking point along baryonic directions cannot be determined for $N_f = N_c$. With the exception of this case, all theories to which AMSB can be applied ($N_f < 3N_c$) possess stable chiral symmetry breaking minima, and all theories with $N_f \lesssim 1.43N_c$ (aside from $N_f = N_c$) are protected from runaways to incalculable minima.

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I. INTRODUCTION

One of the greatest challenges facing particle physics and quantum field theory (QFT) is to establish the phase structure of strongly coupled gauge theories. In particular, that of ordinary quantum chromodynamics (QCD), corresponding to the observed color confinement with chiral symmetry breaking. While eventually we expect lattice simulations to settle this issue, at least for nonchiral theories, progress has been quite slow and there are very few analytic tools at our disposal. One possible approach is to use the exact results and phase structures of the supersymmetric (SUSY) versions of these theories (SQCD) worked out by Seiberg and others in the 1990s [1–3], and to add small SUSY breaking perturbations [4–20]. The exact mapping of SUSY breaking perturbations from the UV theory to its IR manifestation has been done by linking the SUSY breaking either to holomorphic quantities [15], or to conserved and anomalous currents [17–20]. While being successful in mapping UV supersymmetry breaking to the IR, in many previous attempts at studying the vacuum structure of softly broken SQCD the eventual IR phase was incalculable due to runaways and/or dependence on unknown Kähler terms. For this reason, they were of limited predictivity.

A systematic study of the phases of SUSY $SU(N_c)$ gauge theories perturbed via anomaly mediated supersymmetry breaking (AMSB) was initiated in [21], and many new results using this method have already been obtained. These include novel symmetry breaking patterns for chiral gauge theories [22–24], the description of confinement in $SO(N_c)$ theories via monopole condensation [25], and the phase structure of the $SO(N_c)$ theories while varying the number N_f of matter fields in the vector representation [26]. The result of the $SO(N_c)$ analysis was that the various exotic SUSY phases collapse as a result of SUSY breaking, and one is left only with the expected confining and chiral

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symmetry breaking phase. Interestingly, the analysis of the basic $SU(N_c)$ theories with N_f flavors of quarks turns out to be the most subtle one. A QCD-like vacuum with a chiral symmetry breaking pattern of the form $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_D$ has been identified in [21,27] which appears to be the global minimum for at least $N_f < N_c$. However, the $N_f \geq N_c$ cases are complicated by the appearance of baryonic directions, which in many cases appear to cause a runaway behavior.

The aim of this paper is to carefully examine the $SU(N_c)$ theories for $N_f \geq N_c$ and, in particular, the fate of the baryonic directions [28], thus establishing the phase structure of the $SU(N_c)$ theories, when it is calculable. We will show that for the special case of $N_f = N_c + 1$ the potential baryonic runaway is stabilized by a 2-loop AMSB effect, while for $N_f = N_c$ the theory is incalculable along these directions, and one cannot conclusively decide if the baryonic runaways are lifted or not. The lower end $N_c + 1 < N_f \lesssim 1.43 N_c$ of the free magnetic phase will again have the baryonic runaways lifted via 2-loop AMSB, and one ends up with stable, calculable vacuum with chiral symmetry breaking. Such a "QCD-like" vacuum with chiral symmetry breaking exists for any number of flavors as long as $N_f < 3N_c$ (with the possible exception of $N_f = N_c$ where its stability cannot be determined).

In contrast, for $N_f \gtrsim 1.43 N_c$ the baryonic directions will indeed contain runaways. We stress that these runaways do not invalidate the theory since they are cured once the field vacuum expectation values (VEVs) are of $\mathcal{O}(\Lambda_{\rm QCD})$. Here the IR description breaks down and one must return to the UV description, where the theory is stabilized by AMSB. Instead, they merely signal that the global minimum lies in the incalculable region where field VEVs are of $\mathcal{O}(\Lambda_{\rm QCD})$. In addition, the QCD-like minimum will persist as a local minimum, and one expects that as the magnitude of SUSY breaking is increased it will indeed take over as the true vacuum. Note also that baryonic runaway does not occur in Sp or SO gauge theories. We will discuss these cases elsewhere.

The paper is organized as follows. We first review the AMSB mechanism and then its application to the case $N_f < N_c$, where chiral symmetry breaking is observed. We then successively increase the number of flavors, exhibiting chiral symmetry breaking behavior and discussing the baryonic directions, before concluding.

II. ANOMALY MEDIATION

Anomaly mediation of supersymmetry breaking (AMSB) [29,30] (see also [17,31] for earlier work containing some important aspects of AMSB) is parameterized by a single spurion m that explicitly breaks supersymmetry in two different ways. One is the tree-level contribution based on the Kähler potential and superpotential—which is easily derived using the conformal compensator formalism [32]. It is given by

$$\begin{aligned} V_{\rm tree} &= \partial_i W g^{ij^*} \partial_j^* W^* + m^* m (\partial_i K g^{ij^*} \partial_j^* K - K) \\ &\quad + m(\partial_i W g^{ij^*} \partial_j^* K - 3W) + \text{c.c.} \end{aligned} \tag{1}$$

where g^{ij} is the inverse of the Kähler metric $g_{ij} = \partial_i \partial_j^* K$. For simplicity we will always take m to be real. Note that (1) breaks the $U(1)_R$ symmetry explicitly. When the Kähler potential is canonical, this reduces to the more familiar

$$V_{\rm tree} = m\left(\varphi_i \frac{\partial W}{\partial \varphi_i} - 3W\right) + \text{c.c.} \tag{2}$$

When the superpotential does not include dimensionful parameters, this expression identically vanishes.

In this case, there are the loop-level supersymmetry breaking effects from the superconformal anomaly [32]. They lead to trilinear couplings, scalar masses, and gaugino masses,

$$A_{ijk}(\mu) = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)(\mu) m \tag{3}$$

$$m_i^2(\mu) = -\frac{1}{4}\dot{\gamma}_i(\mu) m^2 \tag{4}$$

$$m_\lambda(\mu) = -\frac{\beta(g^2)}{2g^2}(\mu) m. \tag{5}$$

Here, $\gamma_i = \mu \frac{d}{d\mu} \ln Z_i(\mu)$, $\dot{\gamma} = \mu \frac{d}{d\mu}\gamma_i$, and $\beta(g^2) = \mu \frac{d}{d\mu} g^2$. When the gauge theory is asymptotically free, $m_i^2 > 0$, stabilizing the theory against runaway behavior.

Therefore, in a theory described in the UV description by an $SU(N_c)$ gauge group and N_f flavors such that $N_f < 3N_c$, AMSB prepares exactly the state we are looking for: the squarks and gauginos become massive while the massless degrees of freedom are those of non-SUSY QCD. By the UV insensitive nature of AMSB, the expressions above can be reliably used in the dual (IR) description of the theory to determine the low-energy phase.

Here we present some expressions that will be useful later on. Suppose we have a $SU(\tilde{N}_c)$ gauge theory with gauge coupling g and a superpotential $W = \lambda \mathrm{Tr} q_i M_{ij} \bar{q}_j$, where the q_i ($\bar{q}_j$) are N_f flavors of (anti)quarks and M_{ij} is a gauge-singlet flavor-bifundamental meson. The anomalous dimensions are

$$\gamma_q = \frac{C_F g^2}{4\pi^2} - \frac{N_f \lambda^2}{8\pi^2} \tag{6}$$

$$\gamma_M = -\frac{\tilde{N}_c \lambda^2}{8\pi^2} \tag{7}$$

where $C_F = (\tilde{N}_c^2 - 1)/(2\tilde{N}_c)$ is the quadratic Casimir of the dual gauge group. For the 1-loop beta functions one has

$$\beta(g^2) = -\frac{\tilde{b} g^4}{8\pi^2} \tag{8}$$

$$\beta(\lambda^2) = -(\gamma_M + 2\gamma_q)\lambda^2 \tag{9}$$

where $\tilde{b} = 3\tilde{N}_c - N_f$.

III. $N_f < N_c$: ADS SUPERPOTENTIAL

For completeness we quickly review here the results of [21] for $N_f < N_c$. The dynamics is described in terms of the meson fields M_{ij} with the Affleck-Dine-Seiberg (ADS) superpotential

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)}. \tag{10}$$

In the SUSY limit, this produces a runaway potential and hence has no ground states. When $M \gg \Lambda^2$, $M_{ij} = M\delta_{ij}$ describes the D-flat direction

$$Q = \bar{Q} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \\ \hline 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}\phi, \qquad M = \phi^2. \tag{11}$$

Here, Q and $\bar{Q}$ are the quark/antiquark superfields. The upper part is an $N_f \times N_f$ block, while the lower part is $(N_c - N_f) \times N_f$. Since the Kähler potential is canonical in the variable ϕ, one can use (2) to obtain

$$V = \left| 2N_f \frac{1}{\phi}\left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}}\right)^{1/(N_c - N_f)}\right|^2 - (3N_c - N_f)m\left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}}\right)^{1/(N_c - N_f)} + \text{c.c.} \tag{12}$$

Note that there is now a minimum at

$$M_{ij} = \Lambda^2\left(\frac{4N_f(N_c + N_f)}{3N_c - N_f}\frac{\Lambda}{m}\right)^{(N_c - N_f)/N_c}\delta_{ij}. \tag{13}$$

The minimum is indeed at $M \gg \Lambda^2$ which justifies the weakly coupled analysis. The $SU(N_f)_L \times SU(N_f)_R$ flavor symmetry is dynamically broken to $SU(N_f)_V$. The case of nonhomogeneous values for the diagonal entries of M was considered in [33]. There it was shown that the minimum is indeed found at $M_{ij} \propto \delta_{ij}$.

The massless particle spectrum consists of the Nambu-Goldstone bosons (pions) [34]. The scalar and fermion partners of the Nambu-Goldstone bosons (NGBs) have masses that grow with m. Naively increasing m beyond Λ, the only remaining degrees of freedom are massless NGBs. This matches the expectations of QCD with a small number of flavors. There is no sign of a phase transition and the two limits are likely continuously connected.

IV. $N_f = N_c$: QUANTUM MODIFIED CONSTRAINT

In this section we give a complete analysis of the case of the quantum modified constraint, finding that previous discussion requires modification.

The low-energy degrees are meson fields M_{ij} and singlet baryon/antibaryon fields B ($\bar{B}$), whose moduli space is subject to the quantum modified constraint

$$\det M - B\bar{B} = \Lambda^{2N_c}. \tag{14}$$

We first treat the general case $N_c > 2$, and treat the case $N_c = 2$ separately at the end.

There are two ways to frame the theory before the addition of AMSB. The first is to implement the constraint in the superpotential via a Lagrange multiplier field X. However due to the constraint (14), the fields have VEVs of $\mathcal{O}(\Lambda)$. Therefore, higher order terms in the Kähler potential are not suppressed relative to the canonical term and the formula (2) cannot be trusted.

Instead we should perform a nonlinear analysis using the constraint (14). For simplicity, we will use units where $\Lambda = 1$. The moduli space contains two special points of enhanced symmetry: the meson point $M = \mathbf{1}$, $B = \bar{B} = 0$ with unbroken baryon number, and the baryon point $M = 0$, $B = -\bar{B} = 1$ with unbroken flavor symmetry. We perform AMSB around each of these points.

A. Meson point

To satisfy the constraint at the meson point we make the change of variables

$$M = (1 + B\bar{B})^{1/N_c} e^{\Pi} = \mathbf{1} + \frac{1}{N_c}B\bar{B} + \Pi + \frac{1}{2}\Pi^2 + \cdots \tag{15}$$

where Π is a traceless complex matrix. In what follows we will work to quadratic order. The Kähler potential is built out of flavor invariants, e.g., $\mathrm{Tr}M^\dagger M$, $(\mathrm{Tr}M^\dagger M)^2$, $\mathrm{Tr}M^\dagger M M^\dagger M$, etc. Notice that they will all contribute at quadratic order in the hadron superfields. Let us examine the Π contribution of the first term:

$$\mathrm{Tr}M^\dagger M \supset \mathrm{Tr}\Pi^\dagger\Pi + \frac{1}{2}\mathrm{Tr}\Pi^2 + \frac{1}{2}\mathrm{Tr}\Pi^{\dagger 2}. \tag{16}$$

A useful formula going forward will be the tree level AMSB potential corresponding to $K = \varphi^\dagger\varphi + \alpha/2(\varphi^2 + \varphi^{\dagger 2})$. Using the general formula (1), we get

$$V_{\rm AMSB} = \alpha^2 m^2 \varphi^\dagger \varphi + \frac{\alpha}{2} m^2 (\varphi^2 + \varphi^{\dagger 2}) = (\alpha^2 + \alpha) m^2 ({\rm Re}\varphi)^2 + (\alpha^2 - \alpha) m^2 ({\rm Im}\varphi)^2. \quad (17)$$

Setting $\alpha = 1$ corresponds to the Kähler potential for each component of Π in (16), so that the ImΠ are the massless pions, the Goldstone bosons of broken chiral flavor symmetry. Goldstone's theorem ensures that all meson flavor invariants of the Kähler potential will give contributions proportional to the right-hand side of (16). Moreover, they will (in aggregate) come with a positive sign in order for the Π to have a physical kinetic term. Thus, the ReΠ will have a positive mass, stabilizing this direction.

Turning to the baryons, things are not as clear. The most general form of the Kähler potential at quadratic order is

$$K \supset \alpha (B^\dagger B + \bar{B}^\dagger \bar{B}) + \frac{\beta}{2} (B\bar{B} + {\rm c.c.}) \quad (18)$$

where this includes contributions (15) from meson field traces. We cannot know the ratio β/α and thus are unable to determine whether the meson point is stable with respect to baryonic runaway to an incalculable minimum.

B. Baryon point

Here we parametrize the baryon and antibaryon with a single complex field b:

$$B = (1 - \det M)^{1/2} e^b \quad (19)$$

$$\bar{B} = -(1 - \det M)^{1/2} e^{-b}. \quad (20)$$

Like at the meson point, we expect to find a Goldstone boson, now from spontaneously broken baryon number. Consider for example the Kähler potential terms

$$B^\dagger B + \bar{B}^\dagger \bar{B} = 2 + (b + b^\dagger)^2 + \cdots. \quad (21)$$

Again using (17), we identify Imb as the Goldstone boson, while Reb has positive mass. Regarding the mesons however, only the quadratic term must come with a positive sign (to give positive kinetic term). The coefficients of all higher order flavor invariants in the Kähler potential are unknown. With the application of (1), these will ultimately determine if the baryon point is stable once AMSB is turned on.

In summary, we can say very little about the behavior of AMSB-deformed QCD in the singular case when $N_f = N_c$. Neither global nor local minima can be identified, though based on the behavior of theories with more or fewer flavors we can conjecture a chiral symmetry breaking minimum at the meson point. This ambiguity can be traced to the quantum modified constraint, making the theory inherently strongly coupled.

C. $N_c = 2$

When $N_c = 2$, the quarks and antiquarks belong to the same representation of the gauge group. Thus, the flavor symmetry is enhanced to $SU(4)$, with the meson M transforming in the antisymmetric representation. This meson can be decomposed into the meson, baryon, and antibaryon of the unenhanced flavor symmetry. The quantum modified constraint becomes $M^a M^a = 1$, with $a = 1, \ldots, 6$, meaning the moduli space has 5 complex dimensions. The constraint breaks the flavor symmetry to $Sp(4)$, leading to 5 Goldstone modes. Due to the kinetic term positivity arguments made above, their scalar partners have positive mass.

Thus, the enhanced symmetry causes the chiral symmetry breaking minimum to be stable in the case of $N_c = 2$. Similar results were found in [18]. Note that $N_c = 2$ is a special case of the Sp gauge theories that will be discussed elsewhere.

V. $N_f = N_c + 1$: S-CONFINEMENT

For this case we find a stable chiral symmetry breaking minimum, and demonstrate that there are no runaway directions. At the leading order we take a canonical Kähler potential for low energy fields B, $\bar{B}$, and M, which is justified when $B, \bar{B}, M \ll \Lambda$ where the theory is weakly coupled. The superpotential is

$$W = \alpha B M \bar{B} - \beta \det M \quad (22)$$

where we are again working in $\Lambda = 1$ units and α and β are unknown order one numbers used to make the Kähler canonical. The potential obtained is

$$V_{\rm SUSY} = \alpha^2 (|(M\bar{B})_a|^2 + |(BM)_a|^2) + |\alpha \bar{B}_a B_b - \beta \det M (M^{-1})_{ab}|^2 \quad (23)$$

$$V_{\rm AMSB} = -(N_c - 2)\beta m \det M + {\rm c.c.} \quad (24)$$

Seeking the minimum of this potential, we look along the direction

$$B = \begin{pmatrix} b \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \bar{B} = \begin{pmatrix} \bar{b} \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad M = \begin{pmatrix} x & & & \\ & v & & \\ & & \ddots & \\ & & & v \end{pmatrix}. \quad (25)$$

Using flavor rotations the baryon and antibaryon take this form without loss of generality. They break the flavor symmetry to $SU(N_c)_L \times SU(N_c)_R$, justifying the inhomogeneous diagonal VEVs of M. For fixed $\det M$, any off-diagonal terms would simply increase $V_{\rm SUSY}$, justifying their omission. Finally, given that we are taking m real, it is enough to look for minima with all fields real.

Using the fact that for fixed $b\bar{b}$, the quantity $b^2 + \bar{b}^2$ is minimized when $b = \bar{b}$, the potential is

$$V = 2\alpha^2 x^2 b^2 + (\alpha b^2 - \beta v^{N_c})^2 + N_c \beta^2 x^2 v^{2(N_c-1)} - 2(N_c - 2)\beta m x v^{N_c}. \quad (26)$$

Again we treat the general case $N_c > 2$ first, and the case $N_c = 2$ separately afterwards.

The crucial observation implicit above is that the baryon fields do not acquire tree-level SUSY breaking whose mass originates from AMSB and they do not induce threshold corrections when they are integrated out, called "non-decoupling effects" in [32].

A. Baryon number conserving direction, $b = 0$

For the baryon number conserving direction $b = 0$, one finds a minimum

$$v = x = \left(\frac{(N_c - 2)m}{N_c \beta}\right)^{\frac{1}{N_c-1}}, \qquad V_{\min} = -\mathcal{O}(m^{2N_c/(N_c-1)}). \quad (27)$$

This is the chiral symmetry breaking minimum that we hope to be continuously connected to that of non-SUSY QCD. First we must check that it is not disturbed by loop effects coming from the marginal Yukawa term in (22). The baryons acquire a mass αv, and integrating them out and using (4) yields a 2-loop mass for the meson

$$m_M^2 = \frac{(2N_c + 3)\alpha(v)^4 m^2}{(16\pi^2)^2}. \quad (28)$$

Along the direction we are considering, this gives a potential

$$V_{\text{2-loop}} = \frac{(N_c + 1)(2N_c + 3)\alpha(v)^4}{(16\pi^2)^2} m^2 v^2. \quad (29)$$

Notice that at the point (27), this is also $\mathcal{O}(m^{2N_c/(N_c-1)})$. However, since it is 2-loop suppressed, it does not destabilize the chiral symmetry breaking minimum.

We should finally check the effects of higher order terms in the Kähler potential, the leading ones being $(\mathrm{Tr} M^\dagger M)^2$ and $\mathrm{Tr} M^\dagger M M^\dagger M$ with unknown coefficients (including signs). Using (1), we find that these give potential terms $\sim m^2 v^4$. At the point (27), these are higher order in m and can be neglected.

B. Baryon number breaking direction, $b \neq 0$

In general one can minimize (26) with respect to b and x, finding

$$b^2 = \frac{\beta}{\alpha} v^{N_c} - 2x^2 \quad (30)$$

$$x = \frac{(N_c - 2)m}{2\alpha}. \quad (31)$$

Plugging these in we find the runaway potential found in [33]

$$V|_{b,x} = -\frac{(N_c - 2)^2 \beta}{2\alpha} m^2 v^{N_c}. \quad (32)$$

However, we must account for loop corrections. The bottom N_c components of B and $\bar{B}$ acquire a mass αv, so we integrate them out. This gives, to all but the upper-left component M_{11}, the 2-loop mass (28). At this point, the remaining superpotential is simply $W = \alpha B_1 M_{11} \bar{B}_1$. M_{11} then obtains a mass at the lower scale $\sqrt{2}\alpha b$. Integrating it out results in 2-loop AMSB masses for B_1 and $\bar{B}_1$

$$m_b^2 = \frac{3\alpha(b)^4 m^2}{(16\pi^2)^2}. \quad (33)$$

Adding up these contributions along the direction we are considering, this gives a potential

$$V_{\text{2-loop}} = \frac{m^2}{(16\pi^2)^2}[N_c(2N_c + 3)\alpha(v)^4 v^2 + 6\alpha(b)^4 b^2]. \quad (34)$$

Clearly the first term is dominant. Importantly however, this is the same order in m as the tree level runaway (32) and lower order in v since $N_c > 2$. While it is loop (logarithmically) suppressed, this is a smaller effect than the power suppression of (32). Therefore, around the origin where $v \ll 1$, the loop effects stabilize the tree level runaway.

In this case there is also a trilinear AMSB term coming from (3) that goes as $\sim m b^2 x$ with 1-loop suppression. Like the second term in (34), this is subdominant. Finally, subleading terms in the Kähler lead to power suppressed potential terms that can be neglected.

What we have shown is remarkable: the chiral symmetry breaking point for small m is stable and the AMSB loops effects play a subleading role. However, when we consider a possible runaway direction, the loops come in to save the day. While we cannot be sure of what happens when the fields are $\mathcal{O}(\Lambda)$, there are no runaways from the origin and the chiral symmetry breaking point stands a good chance of being the global minimum.

C. $N_c = 2$

In this case tree-level AMSB vanishes because the superpotential is marginal. Due to the positive 2-loop masses, the meson and baryon fields are pushed to the origin of moduli space, where the theory experiences confinement without chiral symmetry breaking. This does not match expectations of non-SUSY QCD and we expect a different global minimum to emerge in the large SUSY breaking limit. A similar phenomenon was seen for a Standard-Model-like chiral $SU(5)$ gauge theory in [35].

VI. $N_c + 1 < N_f \leq 3/2N_c$: FREE MAGNETIC PHASE

For this range of flavors the SUSY theory is in the free magnetic phase and the IR is described by an $SU(\tilde{N}_c)$ ($\tilde{N}_c = N_f - N_c$) gauge theory with quarks and antiquarks in representations $q_i(\bar{\square}, \mathbf{1})$ and $\bar{q}_j(\mathbf{1}, \square)$ of the $SU(N_f) \times SU(N_f)$ flavor group, respectively. Additionally, the magnetic theory has a gauge-singlet meson M_{ij} in the $(\square, \bar{\square})$ of the flavor symmetry. The superpotential is given by

$$W = \lambda \mathrm{Tr} q_i M_{ij} \bar{q}_j \tag{35}$$

where all fields have already been normalized to have canonical Kähler potentials. Importantly, only the deep IR behavior of the theory is specified and we do not have control over the relative strengths of the gauge interaction and the Yukawa interaction λ in Eq. (35).

The case of the free magnetic phase is very subtle, and so far has not been properly analyzed. In fact, this phase is expected to be beset by baryonic runaway directions, so that no useful information can be obtained. We show that for the majority of the free magnetic phase ($N_c + 1 < N_f \lesssim 1.43N_c$) the baryonic runaway directions are lifted, and the chiral symmetry breaking minimum is stable and likely the global minimum of the theory. The analysis itself is quite involved, as one has to examine several branches, which we will present below.

We proceed by first analyzing the baryonic direction, where the entire dual gauge group is Higgsed. As mentioned, the free magnetic phase for $N_f \lesssim 1.43N_c$ is free of runaways in this direction. We next exhibit the chiral symmetry breaking minimum along the mesonic direction. Finally, we check the mixed directions, where only some meson VEVs are turned on, to ensure that they contain no runaways.

A. RG analysis and baryonic branches

In a small neighborhood of the origin of moduli space, the theory is allowed to run into the deep IR. As suggested by the name, the theory is IR free, with both the gauge coupling g and Yukawa coupling running to zero. However, their coupled beta functions make them run asymptotically to the IR attractor given by

$$0 = \frac{d}{d\log\mu}\frac{g^2}{\lambda^2}. \tag{36}$$

This allows λ to be written in terms of g, and we can use (4) to find the 2-loop masses of the dual squarks and the mesons

$$m_q^2 = \frac{(-\tilde{b})g^4}{(16\pi^2)^2}\frac{N_f^2 - 3N_f\tilde{N}_c - \tilde{N}_c^2 + 1}{2N_f + \tilde{N}_c}m^2 \tag{37}$$

$$m_M^2 = \frac{(-\tilde{b})\tilde{N}_c\lambda^2 g^2}{(16\pi^2)^2}m^2 \tag{38}$$

where $\tilde{b} = 3\tilde{N}_c - N_f$ is negative. The mesons maintain a positive mass throughout the free magnetic window, as do the dual squarks for most of the window. However, at the upper end $N_f \gtrsim 1.43N_c$ (in the large N_c limit), the dual squark mass turns negative and we expect a baryonic runaway toward an uncalculable minimum.

Concretely, for $N_f \gtrsim 1.43N_c$ we consider giving D-flat VEVs to the dual squark

$$q = \tilde{B}\begin{pmatrix} 1_{\tilde{N}_c\times\tilde{N}_c} \\ 0_{\tilde{N}_c\times N_c}\end{pmatrix}. \tag{39}$$

The effect of this is to Higgs the dual gauge group at the scale $\tilde{B}$, and to give masses to the dual antiquarks and some of the mesons. Substituting their equations of motion eliminates the superpotential. Equation (37) then translates into a tachyonic mass for $\tilde{B}$, where the gauge coupling is evaluated at the scale $\tilde{B}$.

The first detailed exploration of baryonic runaways with SUSY breaking applied consistently between the UV and IR was undertaken in [15] (see also the more recent [19]). In both of these works, which used different mechanisms to break SUSY, baryonic runaways were present throughout the free-magnetic phase. It is encouraging that AMSB, while not eliminating them, lifts these directions for most of the phase.

B. Mesonic branch

In this section we give the meson a VEV with full rank, repeating the analysis of [21]. This gives masses to the dual quarks and antiquarks. Without their effects, the beta function of the gauge theory flips sign, allowing the theory to generate a new IR dynamical scale given by

$$\Lambda_L^{3\tilde{N}_c} = \tilde{\Lambda}^{3\tilde{N}_c - N_f}\det M. \tag{40}$$

The usual superpotential of pure SYM is generated:

$$W = \tilde{N}_c\Lambda_L^3 = \tilde{N}_c(\det M)^{1/\tilde{N}_c} \tag{41}$$

where as usual we have set $\tilde{\Lambda} = 1$. Upon adding tree level AMSB, the minimum can be found along the homogeneous direction $M = v\mathbf{1}$ with the potential

$$V = N_f|v^{N_f/\tilde{N}_c - 1}|^2 + (N_f - 3\tilde{N}_c)mv^{N_f/\tilde{N}_c} + \text{c.c.} \tag{42}$$

at the point

$$v=\left(\frac{(3\tilde{N}_c-N_f)m}{N_f-\tilde{N}_c}\right)^{\frac{\tilde{N}_c}{N_f-2\tilde{N}_c}},\quad V_{\min}=-\mathcal{O}\left(m^{2\frac{N_f-\tilde{N}_c}{N_f-2\tilde{N}_c}}\right). \quad (43)$$

The 2-loop potential from (38) contributes at the same order in m, however it is loop suppressed. We find that the chiral symmetry breaking minimum is stable.

C. Mixed branches

Instead of turning on all of the meson VEVs, we can choose to turn on only some of them. These will reveal tree level AMSB contributions within the free magnetic phase with tree level runaways. However, as in the case of s-confinement, the AMSB loop effects will stabilize these directions.

We begin by writing the meson matrix as

$$M=\begin{pmatrix}\tilde{M}_{R_f\times R_f} & 0\\ 0 & \hat{M}_{(N_f-R_f)\times(N_f-R_f)}\end{pmatrix} \quad (44)$$

and without loss of generality we look for minima at diagonal M. We then give the lower component $\hat{M}$ a VEV. This gives masses to N_f-R_f flavors of quarks, leaving an $SU(\tilde{N}_c)$ gauge theory with R_f massless flavors and a new dynamical scale

$$\Lambda_L^{3\tilde{N}_c-R_f}=\tilde{\Lambda}^{3\tilde{N}_c-N_f}\det\hat{M} \quad (45)$$

with $\tilde{\Lambda}$ the Landau pole of the dual theory. In what follows we will set $\tilde{\Lambda}=1$. Finally, we assume that $\tilde{M}$ remains small compared to both $\hat{M}$ and the generated scale Λ_L.

For $1\leq R_f<\tilde{N}_c$, the remaining theory is of ADS-type and has the superpotential

$$W=(\tilde{N}_c-R_f)\left(\frac{\Lambda_L^{3\tilde{N}_c-R_f}}{\det N}\right)^{\frac{1}{\tilde{N}_c-R_f}}+\mathrm{Tr}\tilde{M}N \quad (46)$$

where N is the meson formed by the remaining massless dual-quarks. We have ignored λ as it will be irrelevant for this discussion. The second term comes from the Yukawa of the dual theory.

The SUSY equation of motion (EOM) for $\tilde{M}$ sets $N=0$. Evidently, the EOM for N is singular at this point and to compensate we must have $\tilde{M}\to\infty$. However, this violates the assumption of small $\tilde{M}$. Therefore, even before a small AMSB deformation can be applied, this branch collapses back to the mesonic branch already considered.

Next consider the case of $R_f=\tilde{N}_c$, which will have emergent meson and baryon degrees of freedom with a quantum modified constraint. Furthermore, the superpotential $W=\mathrm{Tr}\tilde{M}N$ fixes $\tilde{M}=N=0$. We thus find ourselves at the baryon point where as before the baryons are stable, but this time with the emergent meson directions stabilized by a superpotential. The only question that remains is the $\hat{M}$ dependence. For simplicity consider $\hat{M}=v\mathbf{1}$. The new dynamics will generate at leading order the Kähler potential term

$$K\supset a\Lambda_L^2=av^{2C} \quad (47)$$

where a is an $\mathcal{O}(1)$ number of unknown sign and $C=(N_f-R_f)/(3\tilde{N}_c-R_f)>1$. This will give rise to a tree level AMSB potential of $\mathcal{O}(m^2v^{2C})$. However, as before the 2-loop AMSB mass for the meson will give a positive contribution at $\mathcal{O}(m^2v^2)$, stabilizing this direction.

For $\tilde{N}_c+1\leq R_f<3\tilde{N}_c$, the IR dynamics of the remaining theory are described by a magnetic dual with gauge group $SU(R_f-\tilde{N}_c)$ (except for $R_f=\tilde{N}_c+1$ where the theory is s-confining). The superpotential is

$$W_L=\mathrm{Tr}b_iN_{ij}\bar{b}_j+\mathrm{Tr}\tilde{M}N. \quad (48)$$

The N, b, and $\bar{b}$ are dual mesons, quarks (baryons), and antiquarks (antibaryons) formed by the massless dual quarks. Again the superpotential term (35) has transformed to enforce $N=0$ in the supersymmetric limit. This means when we introduce tree-level AMSB, $N=\mathcal{O}(m)$, and we were justified in ignoring the s-confining $\det N$ term as a high power of m (assuming N is even full rank). We rescale the fields by appropriate factors of Λ_L to make them canonical. Ignoring order one factors we have

$$W_L=\mathrm{Tr}b_iN_{ij}\bar{b}_j+\Lambda_L\mathrm{Tr}\tilde{M}N. \quad (49)$$

Finally we substitute the value of Λ_L (and set $\tilde{\Lambda}=1$) to arrive at

$$W_L=\mathrm{Tr}b_iN_{ij}\bar{b}_j+(\det\hat{M})^{1/(3\tilde{N}_c-R_f)}\mathrm{Tr}\tilde{M}N. \quad (50)$$

Let all fields be real and consider the direction given by $N_{ii}=n_i$, $\tilde{M}_{ii}=x_i$, $b_{ii}=-\bar{b}_{ii}=y_i$, for $i=1,\ldots,(R_f-\tilde{N}_c)$ and with all other entries 0. Finally let $\hat{M}=v\mathbf{1}$.

The potential is

$$V=\sum_i(2y_i^2n_i^2+(v^Cx_i-y_i^2)^2+v^{2C}n_i^2+2(C-1)mv^Cn_ix_i)+\frac{C}{3\tilde{N}_c-R_f}v^{2C-2}\left(\sum\nolimits_i n_ix_i\right)^2 \quad (51)$$

where C is defined as before and remains greater than 1. Notice that the final term is smaller than the third term in the first sum by a factor of $x^2/v^2\ll 1$. Therefore, we can neglect this term and the potential splits into $R_f-\tilde{N}_c$

identical parts. In what follows, we suppress the index i. Substituting the y and n equations of motion, and using $n, x \ll \Lambda_L = v^C$ along the way, we get

$$V|_{y,n} = -(C-1)^2 m^2 x^2. \tag{52}$$

As long we keep $x \ll v^C$, we can let $x, v \to 1$, signaling a tree level minimum of $-\mathcal{O}(m^2)$ in the incalculable region where field VEVs are $\mathcal{O}(\Lambda)$. Note that in this direction all fields, baryonic and mesonic, are turned on.

However, as we saw for the s-confining runaway, the loop effects must be considered. While this tree-level runaway is power suppressed as $\mathcal{O}(x^2) \ll \mathcal{O}(v^{2C})$, the 2-loop potential gives a positive contribution with $\mathcal{O}(v^2)$. Therefore, there is again no runaway.

When $R_f \geq 3\tilde{N}_c$, the theory remains IR free and there are no tree level runaways. As long as $N_f \lesssim 1.43 N_c$ the dual quarks will have positive 2-loop AMSB mass.

In summary, we have demonstrated that there is a stable chiral symmetry breaking minimum and that for $N_f \lesssim 1.43 N_c$ there are no runaways.

VII. $3/2N_c < N_f < 3N_c$: CONFORMAL WINDOW

In the conformal window, the magnetic description is no longer IR free. Rather, it has a nontrivial fixed point, which is weakly coupled at the lower end of the window. We will first analyze the behavior of AMSB in this region and find baryonic runaways to incalculable minima. Then, we will turn to the upper end of the window where the electric theory has a weakly coupled fixed point. As demonstrated in [27], AMSB makes a relevant deformation and destroys the superconformal phase. We can only conjecture about the intermediate region where both descriptions are strongly coupled. Finally, we demonstrate local chiral symmetry breaking minima throughout the window.

A. Lower conformal window

We begin by considering $N_f = 3\tilde{N}_c/(1+\epsilon)$ where $\epsilon \ll 1$, and will work in the large $\tilde{N}_c$ limit and leading nontrivial order in ϵ for simplicity. For notational convenience, we define

$$x \equiv \frac{\tilde{N}_c}{8\pi^2}\lambda^2, \qquad y \equiv \frac{\tilde{N}_c}{8\pi^2} g^2. \tag{53}$$

The beta functions of the magnetic theory, including the 2-loop contribution for y, are

$$\beta(x) = x(-2y + 7x), \tag{54}$$

$$\beta(y) = -3y^2(\epsilon - y + 3x). \tag{55}$$

They admit a Banks-Zaks (BZ) fixed point at $(x_0, y_0) = (2\epsilon, 7\epsilon)$. As the theory flows to the IR, x and y will approach this point from above, along the trajectory specified by (36). Define $\delta x = x - x_0$ and $\delta y = y - y_0$. Close to the fixed point this yields

$$\delta x = \frac{2}{7}\left(1 + \frac{3}{2}\epsilon\right)\delta y. \tag{56}$$

The RG flow is

$$\beta(y) = 21\epsilon^2 \delta y \tag{57}$$

yielding

$$\delta y \sim \mu^{21\epsilon^2}. \tag{58}$$

Using (4), the meson and dual squark masses are

$$m_M^2 = \frac{3}{2}\epsilon^2 \delta y m^2 \tag{59}$$

$$m_q^2 = -\frac{3}{4}\epsilon^2 \delta y m^2. \tag{60}$$

Thus in the lower conformal window the dual squarks are tachyonic and there is a runaway to an incalculable minimum.

B. Upper conformal window

We now examine the upper conformal window via the electric description, reviewing the results of [27]. Now $N_f = 3N_c/(1+\epsilon)$, and we use all conventions of the previous section. The beta function at 2-loop is

$$\beta(y) = -3y^2(\epsilon - y) \tag{61}$$

where the BZ fixed point $y_0 = \epsilon$ is now approached from below as

$$(-\delta y) \sim \mu^{3\epsilon^2}. \tag{62}$$

From (4) and (5) we obtain the squark and gluino masses

$$m_Q^2 = \frac{3}{4}\epsilon^2(-\delta y)m^2 \tag{63}$$

$$m_\lambda = \frac{3}{2}(-\delta y)m. \tag{64}$$

As expected the squark mass is positive. As long as $3\epsilon^2 < 1$ (this bound is outside of our small ϵ limit and should be taken with a grain of salt), at some point in the RG flow the squark and gluino masses will exceed the renormalization scale. At this point the superpartners can be integrated out and the superconformal phase is destroyed. What remains is non-SUSY QCD and must be analyzed from the (albeit strongly coupled) magnetic description.

C. Chiral symmetry breaking minimum

We have shown that AMSB, at both the top and bottom of the conformal window, destroys the superconformal phase. It is reasonable to assume this is the case throughout the window. Furthermore, we demonstrated that at the bottom of the window the theory has a runaway to an incalculable minimum.

Looking instead for local minima, we examine the mesonic branch. Just as in the free magnetic phase, this gives masses to the dual quarks and generates a new dynamical scale. The superpotential is given by (41). However, unlike the free magnetic phase where the Kähler receives logarithmic wave-function renormalization (which we ignored), in the conformal window we have

$$Z_M(\mu) \sim \mu^{1-3\tilde{N}_c/N_f} \tag{65}$$

which is evaluated at $\mu = v$, where $M = v\mathbf{1}$. The result is that the scaling of the local chiral symmetry minimum is modified to [27]

$$V = -\mathcal{O}(m^\sigma), \qquad \sigma = 1 + \frac{N_f^2}{N_f^2 - 3N_f\tilde{N}_c + 3\tilde{N}_c^2}. \tag{66}$$

Note that σ goes from 4 ($N_f = \frac{3}{2}N_c$) to 5 ($N_f = 2N_c$) back to 4 ($N_f = 3N_c$).

VIII. $N_f \geq 3N_c$: FREE ELECTRIC PHASE

For large number of flavors, the 2-loop squark mass from AMSB is negative, leading to true runaway behavior. AMSB cannot be used to understand the non-SUSY theory in this case.

IX. CONCLUSIONS

We carefully analyzed the behavior of $SU(N_c)$ gauge theories with N_f flavors upon the application of AMSB, focusing on the chiral symmetry breaking minima and potential baryonic runaway directions. For $N_c + 1 \leq N_f \leq 3/2N_c$ we found that naive tree level runaways are power suppressed in comparison to loop effects, which stabilize these directions. However, a true loop level runaway was found for the upper end of the free magnetic phase, $N_f \gtrsim 1.43N_c$. This baryonic runaway continued into the lower end of the conformal window, and we cannot discount such runaways throughout the window. Such runaways point to the existence of some noncalculable minimum at large field values of $\mathcal{O}(\Lambda)$, which may or may not correspond to the global minimum of the theory.

The case of $N_f = N_c$ required particular care due to the inherently strongly coupled nature of the quantum modified moduli space. We found that the theory is best analyzed after implementing the quantum constraint. Upon application of AMSB the stability of the chiral symmetry breaking point cannot be determined. This is not due to a problem with the AMSB method, but rather because the Kähler potential terms that are critical to this determination are incalculable.

In summary we found (with the exception of the cases $N_f = N_c$ for $N_c > 2$ and $N_f = N_c + 1$ for $N_c = 2$) that stable chiral symmetry breaking minima are present for $N_f < 3N_c$ upon application of AMSB in the small SUSY-breaking limit. Furthermore, the theories with $N_f \lesssim 1.43N_c$ are protected from runaways to incalculable minima. This does not prove that there are no deeper minima with fields of $\mathcal{O}(\Lambda)$, however we take it to be strong evidence for the conjecture that in these cases the chiral symmetry breaking minima are in fact global.

Our analysis was performed in the $m \ll \Lambda$ limit, and the question remains about the behavior in the nonsupersymmetric limit of $m \gg \Lambda$. The existence of the chiral symmetry breaking minima for all flavors is indicative that these are continuously connected to the true vacua of non-SUSY QCD. Irrespective of the potential appearance of a phase transition between these two limits (see arguments based on holomorphy in [22,23], and also see Refs. [36,37]), these are the vacua that are of phenomenological interest for the study of real-world QCD.

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[1] I. Affleck, M. Dine, and N. Seiberg, Dynamical supersymmetry breaking in supersymmetric QCD, Nucl. Phys. **B241**, 493 (1984).
[2] N. Seiberg, Exact results on the space of vacua of four-dimensional SUSY gauge theories, Phys. Rev. D **49**, 6857 (1994).
[3] N. Seiberg, Electric—magnetic duality in supersymmetric non-Abelian gauge theories, Nucl. Phys. **B435**, 129 (1995).
[4] N. J. Evans, S. D. H. Hsu, and M. Schwetz, Exact results in softly broken supersymmetric models, Phys. Lett. B **355**, 475 (1995).
[5] O. Aharony, J. Sonnenschein, M.E. Peskin, and S. Yankielowicz, Exotic nonsupersymmetric gauge dynamics from supersymmetric QCD, Phys. Rev. D **52**, 6157 (1995).
[6] N. J. Evans, S. D. H. Hsu, M. Schwetz, and S. B. Selipsky, Exact results and soft breaking masses in supersymmetric gauge theory, Nucl. Phys. **B456**, 205 (1995).
[7] E. D'Hoker, Y. Mimura, and N. Sakai, Gauge symmetry breaking through soft masses in supersymmetric gauge theories, Phys. Rev. D **54**, 7724 (1996).
[8] L. Alvarez-Gaume, J. Distler, C. Kounnas, and M. Marino, Softly broken $N = 2$ QCD, Int. J. Mod. Phys. A **11**, 4745 (1996).
[9] L. Alvarez-Gaume and M. Marino, More on softly broken $N = 2$ QCD, Int. J. Mod. Phys. A **12**, 975 (1997).
[10] N. J. Evans, S. D. H. Hsu, and M. Schwetz, Phase transitions in softly broken $N = 2$ SQCD at nonzero theta angle, Nucl. Phys. **B484**, 124 (1997).
[11] K. Konishi, Confinement, supersymmetry breaking and theta parameter dependence in the Seiberg-Witten model, Phys. Lett. B **392**, 101 (1997).
[12] L. Alvarez-Gaume, M. Marino, and F. Zamora, Softly broken $N = 2$ QCD with massive quark hypermultiplets. 1., Int. J. Mod. Phys. A **13**, 403 (1998).
[13] N. J. Evans, S. D. H. Hsu, and M. Schwetz, Controlled soft breaking of $N = 1$ SQCD, Phys. Lett. B **404**, 77 (1997).
[14] L. Alvarez-Gaume, M. Marino, and F. Zamora, Softly broken $N = 2$ QCD with massive quark hypermultiplets. 2., Int. J. Mod. Phys. A **13**, 1847 (1998).
[15] H.-C. Cheng and Y. Shadmi, Duality in the presence of supersymmetry breaking, Nucl. Phys. **B531**, 125 (1998).
[16] S. P. Martin and J. D. Wells, Chiral symmetry breaking and effective Lagrangians for softly broken supersymmetric QCD, Phys. Rev. D **58**, 115013 (1998).
[17] N. Arkani-Hamed and R. Rattazzi, Exact results for non-holomorphic masses in softly broken supersymmetric gauge theories, Phys. Lett. B **454**, 290 (1999).
[18] M. A. Luty and R. Rattazzi, Soft supersymmetry breaking in deformed moduli spaces, conformal theories, and $N = 2$ Yang-Mills theory, J. High Energy Phys. 11 (1999) 001.
[19] S. Abel, M. Buican, and Z. Komargodski, Mapping anomalous currents in supersymmetric dualities, Phys. Rev. D **84**, 045005 (2011).
[20] C. Córdova and T. T. Dumitrescu, Candidate phases for SU(2) adjoint QCD_4 with two flavors from $\mathcal{N} = 2$ supersymmetric Yang-Mills theory, arXiv:1806.09592.
[21] H. Murayama, Some Exact Results in QCD-like Theories, Phys. Rev. Lett. **126**, 251601 (2021).
[22] C. Csáki, H. Murayama, and O. Telem, Some exact results in chiral gauge theories, Phys. Rev. D **104**, 065018 (2021).
[23] C. Csáki, H. Murayama, and O. Telem, More exact results on chiral gauge theories: The case of the symmetric tensor, Phys. Rev. D **105**, 045007 (2022).
[24] D. Kondo, H. Murayama, and C. Sylber, Dynamics of simplest chiral gauge theories, arXiv:2209.09287.
[25] C. Csáki, A. Gomes, H. Murayama, and O. Telem, Demonstration of Confinement and Chiral Symmetry Breaking in $SO(N_c)$ Gauge Theories, Phys. Rev. Lett. **127**, 251602 (2021).
[26] C. Csáki, A. Gomes, H. Murayama, and O. Telem, Phases of nonsupersymmetric gauge theories: The $SO(N_c)$ case study, Phys. Rev. D **104**, 114018 (2021).
[27] H. Murayama, B. Noether, and D. R. Varier, Broken conformal window, arXiv:2111.09690.
[28] We thank Nathaniel Craig for emphasizing to us the importance of properly analyzing the baryonic directions and their potential runaways for establishing the correct vacuum of the theory.
[29] L. Randall and R. Sundrum, Out of this world supersymmetry breaking, Nucl. Phys. **B557**, 79 (1999).
[30] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, Gaugino mass without singlets, J. High Energy Phys. 12 (1998) 027.
[31] N. Arkani-Hamed, G. F. Giudice, M. A. Luty, and R. Rattazzi, Supersymmetry breaking loops from analytic continuation into superspace, Phys. Rev. D **58**, 115005 (1998).
[32] A. Pomarol and R. Rattazzi, Sparticle masses from the superconformal anomaly, J. High Energy Phys. 05 (1999) 013.
[33] A. Luzio and L.-X. Xu, On the derivation of chiral symmetry breaking in QCD-like theories and S-confining theories, J. High Energy Phys. 08 (2022) 016.
[34] The case $N_f = 1$ is special as there is no non-anomalous flavor symmetry and hence the spectrum is gapped.
[35] Y. Bai and D. Stolarski, Phases of confining SU(5) chiral gauge theory with three generations, J. High Energy Phys. 03 (2022) 113.
[36] M. Dine and Y. Yu, Challenges to obtaining results for real QCD from SUSY QCD, arXiv:2205.00115.
[37] M. Dine, On the possibility of demonstrating confinement in non-supersymmetric theories by deforming confining supersymmetric theories, arXiv:2211.17134.