The branching fraction of $B_s^0 \to K^0\overline{K}^0$: Three puzzles

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Abstract

The branching fraction of the $B_s \to K^0\overline{K}^0$ decay has been recently measured by the LHCb and Belle experiments. We study the consistency of the measured value with three relations to other decay rates and CP asymmetries which follow from the Standard Model, and from the approximate flavor $SU(3)$ symmetry of the strong interactions. We find that each of these relations is violated at a level of above 3σ. We argue that various subleading effects – rescattering, electroweak penguins and $SU(3)$ breaking – if larger than theoretically expected, can account for some of these puzzles, but not for all of them simultaneously.

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I. INTRODUCTION

The $B_s \to K^0\overline{K}^0$ decay, which proceeds via the quark transition $b \to d\bar{d}s$, is a flavor changing neutral current (FCNC) process and, as such, constitutes a sensitive probe of new physics. There are several unique properties of this process, which makes experimental measurements of the rate and CP asymmetries highly motivated:

- It is a uniquely clean and rich probe of the $\bar{b} \to d\bar{d}\bar{s}$ transition. Another decay channel that proceeds via $\bar{b} \to d\bar{d}\bar{s}$ is $B^+ \to \pi^+K^0$ for which, however, CP asymmetries are not as rich. For $\bar{b} \to d\bar{d}\bar{s}$ two body decays where the $d\bar{d}$ pair bind into a meson, there is always a contribution also from the flavor changing charged current transition $\bar{b} \to u\bar{u}\bar{s}$.

- If rescattering is not surprisingly large in this mode, the CP asymmetries provide clean null tests of the Standard Model (SM). Conversely, if the CP asymmetries are experimentally established, or even just bounded, we will draw important lessons about rescattering.

- It is related by isospin to the $B_s \to K^+K^-$ decay, where the CP asymmetries have been experimentally measured.

- It is related by U-spin, with expected only small breaking effects, to the $B^0 \to K^0\overline{K}^0$ decay.

From this list of features, it is clear that measuring the rate and CP asymmetries in $B_s \to K^0\overline{K}^0$ decay will provide new information about QCD and on new physics. Indeed, the branching fraction $\mathrm{BR}(B_s \to K^0\overline{K}^0)$ was recently measured by the LHCb experiment [1] (consistent with an earlier measurement by the BELLE experiment [2]). In what follows, we use the currently available data on this and related B-meson decays and find several puzzles which further motivate an experimental effort to obtain a more precise measurement of the rate and search for the CP asymmetries in this mode.

The time-dependent CP asymmetries in the $B_s \to K^0\overline{K}^0$ decay have been argued to provide clean tests of the Standard Model and to probe the presence of new physics in $b \to s$ transitions in Ref. [3]. In Refs. [4, 5], the potential of these measurements in probing new CP violating physics in $B_s - \overline{B_s}$ mixing was analyzed. (In our work, we use the measured value of the CP violating phase as input.) Predictions for the branching fraction and CP asymmetries in $B_s \to K^0\overline{K}^0$ were made using QCD factorization in Refs. [6, 7], and a global flavor-$SU(3)$ fit in Ref. [8]. Two recent studies of related topics in B-meson decays (which, however, do not incorporate $B_s \to K^0\overline{K}^0$ in their analysis) can be found in Refs. [9, 10].

The plan of this paper is as follows. In Section II we review the experimental data that form the basis for our analysis. In Section III we give the formalism that we use for analysing the data within the SM and list the approximations that we make. In Section IV we use three sets of data, each presenting a deviation from the SM expectation at the 3σ level. In Section V we reintroduce three effects that we neglected in the previous section, arguing that they are unlikely to provide a solution to the puzzles. In Section VI we describe the improvements in the relevant measurements that can be expected in the future from the LHCb and Belle-II experiments. We conclude in Section VII.

II. EXPERIMENTAL DATA

Before we present a theoretical analysis of the $B_s \to K^0\overline{K}^0$ decay and the isospin and U-spin related modes, we collect in Table II the relevant experimental information [11, 12].

In what follows, we consider the following ratios of rates:

$$\begin{aligned}
R_{KK}^{ss} &\equiv \frac{\Gamma(B_s \to K^0\overline{K}^0)}{\Gamma(B_s \to K^+K^-)} = 0.66 \pm 0.13, \\
R_{KK}^{sd} &\equiv \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\Gamma(B_s \to K^0\overline{K}^0)}{\Gamma(B^0 \to K^0\overline{K}^0)} = 0.61 \pm 0.13, \\
R_{\pi K}^{ud} &\equiv \frac{\Gamma(B^+ \to \pi^+K^0)}{\Gamma(B^0 \to \pi^-K^+)} = 1.12 \pm 0.05,
\end{aligned} \tag{1}$$

where we use the measured values of the branching ratios from Table II and take into account the lifetimes of the various bottom mesons [11] when translating ratios of branching ratios to ratios of rates:

$$\begin{aligned}
\tau(B_s) &= (1.516 \pm 0.006) \times 10^{-12}\ \mathrm{s}, \\
\tau(B^0) &= (1.519 \pm 0.004) \times 10^{-12}\ \mathrm{s}, \\
\tau(B^+) &= (1.638 \pm 0.004) \times 10^{-12}\ \mathrm{s}.
\end{aligned} \tag{2}$$

We define the CKM combinations

$$\lambda_{bq}^{q'} = V_{q'b}^* V_{q'q}, \quad R_{uq'}^{bq} = |\lambda_{bq}^u / \lambda_{bq}^{q'}| \quad (q' = u, c, t,\ q = d, s). \tag{3}$$

Process	Branching ratio	Refs.	CP asymmetries	Refs.
$B_s \to K^0\overline{K}^0$	$(1.76 \pm 0.31) \times 10^{-5}$	[1, 2]	$-$	
$B_s \to K^+K^-$	$(2.66 \pm 0.22) \times 10^{-5}$	[13–15]	$C^s_{K^+K^-} = +0.172 \pm 0.031$ $S^s_{K^+K^-} = +0.139 \pm 0.032$	[16, 17]
$B^0 \to K^0\overline{K}^0$	$(1.21 \pm 0.16) \times 10^{-6}$	[18, 19]	$C^d_{K^0\overline{K}^0} = +0.0 \pm 0.4$ $S^d_{K^0\overline{K}^0} = -0.8 \pm 0.4$	[18, 20]
$B^+ \to K^0\pi^+$	$(2.37 \pm 0.08) \times 10^{-5}$	[18, 19, 21]	$A^u = -0.017 \pm 0.016$	[18, 19, 22, 23]
$B^0 \to K^+\pi^-$	$(1.96 \pm 0.05) \times 10^{-5}$	[19, 21, 24]	$A^d = -0.0834 \pm 0.0032$	[17, 19, 23, 25–27]

TABLE I. Experimental data from the PDG [11]. The sub-index on C and S represents the final state, while the super-index u, d, or s corresponds to an initial B^+, B^0, or B_s.

Parameter	Value	Parameter	Value	Parameter	Value
$\lvert V_{ub}\rvert$	0.0037 ± 0.0001	$\lvert V_{tb}\rvert$	0.99912 ± 0.00004	α	$(85.2^{+4.8}_{-4.3})^o$
$\lvert V_{us}\rvert$	0.2250 ± 0.0007	$\lvert V_{ts}\rvert$	0.0411 ± 0.0008	γ	$(65.9^{+3.3}_{-3.5})^o$
$\lvert V_{ud}\rvert$	0.9743 ± 0.0002	$\lvert V_{td}\rvert$	0.0086 ± 0.0002		

TABLE II. CKM parameters from the PDG [11]

The CKM phases are defined as follows:

$$\gamma = \arg\left(\lambda^u_{bs}/\lambda^c_{bs}\right), \qquad \alpha = \arg\left(-\lambda^t_{bd}/\lambda^u_{bd}\right). \tag{4}$$

We will need

$$\lambda^u_{bs}/\lambda^t_{bs} = -R^{bs}_{ut} e^{i\gamma}, \qquad \lambda^u_{bd}/\lambda^t_{bd} = -R^{bd}_{ut} e^{-i\alpha}. \tag{5}$$

The experimental ranges of the relevant CKM parameters are presented in Table II. They lead to the following combinations which play a role in our analysis:

$$\begin{aligned} &R^{bs}_{ut} = 0.0203 \pm 0.0007, \qquad R^{bd}_{ut} = 0.420 \pm 0.016, \qquad |V_{td}/V_{ts}| = 0.209 \pm 0.006, \\ &\sin\alpha \simeq 1, \qquad \sin\gamma = 0.91 \pm 0.02, \qquad \cot\gamma = 0.45 \pm 0.07. \end{aligned} \tag{6}$$

We also use the following combinations:

$$\begin{aligned}
&S^s_{K^+K^-}\cot\gamma = 0.06 \pm 0.02,\\
&2R^{ut}_{bs}\cos\gamma = 0.016 \pm 0.002,\\
&2R^{ut}_{bs}\sin\gamma = 0.036 \pm 0.001,\\
&[(C^s_{K^+K^-})^2 + (S^s_{K^+K^-})^2]^{1/2} = 0.22 \pm 0.05.
\end{aligned} \tag{7}$$

III. THE SM: FORMALISM AND APPROXIMATIONS

In what follows we assume $SU(3)$ flavor symmetry and employ the diagrammatic approach of Ref. [28]. Our starting point is the analysis of Ref. [29]. The relevant amplitudes are written as follows:

$$\begin{aligned}
\mathcal{A}(B_s \to K^0\overline{K}^0) &= \lambda^t_{bs}(P + P_A),\\
\mathcal{A}(B_s \to K^+K^-) &= -\lambda^t_{bs}(P + P_A) - \lambda^u_{bs}(T + E),\\
\mathcal{A}(B^0 \to K^0\overline{K}^0) &= \lambda^t_{bd}(P + P_A),\\
\mathcal{A}(B^+ \to \pi^+K^0) &= \lambda^t_{bs}P + \lambda^u_{bs}A,\\
\mathcal{A}(B^0 \to \pi^-K^+) &= -\lambda^t_{bs}P - \lambda^u_{bs}T,
\end{aligned} \tag{8}$$

where P and P_A refer to penguin and penguin annihilation diagrams, and T, E and A refer to spectator tree, exchange and annihilation diagrams.

In writing the relations in Eqs. (8) three effects are neglected:

- $SU(3)$ breaking [30];
- Rescattering contributions, T_{RS} [31];
- Electroweak (EW) penguin contributions, P_{EW} [32].

We discuss these effects in Section V.

The smallness of $R^{ut}_{bs} = |\lambda^u_{bs}/\lambda^t_{bs}|$ implies that the decays which proceed via the quark transitions $\bar{b} \to \bar{q}q\bar{s}$ ($q = d$ or u) are dominated by the gluonic penguin contributions proportional to λ^t_{bs}. Setting $\lambda^u_{bs} \to 0$ in Eqs. (8) leads to the following predictions concerning CP asymmetries:

$$C^s_{K^+K^-} = S^s_{K^+K^-} = 0, \tag{9}$$

and ratios of decay rates:

$$R^{ss}_{KK} = R^{sd}_{KK} = R^{ud}_{\pi K} = 1. \tag{10}$$

Taking into account the λ^u_{bs} terms in Eqs. (8) leads to small deviations from these predictions. To first order in R^{ut}_{bs}, we obtain, for the CP asymmetries,

$$C^s_{K^+K^-} = 2R^{ut}_{bs}\sin\gamma \times \mathcal{I}m[(T+E)/(P+P_A)], \tag{11}$$

$$S^s_{K^+K^-} = 2R^{ut}_{bs}\sin\gamma \times \mathcal{R}e[(T+E)/(P+P_A)], \tag{12}$$

and for the ratios of rates,

$$R^{ss}_{KK} = 1 + 2R^{ut}_{bs}\cos\gamma \times \mathcal{R}e[(T+E)/(P+P_A)], \tag{13}$$

$$R^{ud}_{\pi K} = 1 + 2R^{ut}_{bs}\cos\gamma \times \mathcal{R}e[(T-A)/P], \tag{14}$$

$$R^{sd}_{KK} = 1. \tag{15}$$

IV. PUZZLES INVOLVING $B_s \to K^0\overline{K}^0$

Using the above theoretical relations that assume the SM and the $SU(3)$ flavor symmetry of QCD to analyze the experimental data, we identify three puzzles involving the $B_s \to K^0\overline{K}^0$ decay rate. We present them in the following subsections.

A. The $R^{sd}_{KK} = 1$ puzzle.

R^{sd}_{KK} is a ratio of two decay rates that are connected by U-spin. One that proceed via the quark transitions $\bar{b} \to \bar{d}d\bar{s}$ and the other one via $\bar{b} \to \bar{s}s\bar{d}$. They have neither tree (T), nor annihilation (A) nor exchange (E) contributions, hence the $R^{sd}_{KK} = 1$ prediction in Eq. (15). The experimental range, $R^{sd}_{KK} = 0.61 \pm 0.13$, shows a 3σ deviation from the SM prediction. The deviation of the experimental value of R^{sd}_{KK} from 1 constitutes the first puzzle.

B. The $R^{ss}_{KK} - S^s_{K^+K^-}$ puzzle

R^{ss}_{KK} is the ratio between two rates related by isospin. Eqs. (12) and (13) lead to the following prediction:

$$R^{ss}_{KK} = 1 + S^s_{K^+K^-}\cot\gamma. \tag{16}$$

Using Eq. (7) we obtain the following range for the right hand side of Eq. (16):

$$1 + S^s_{K^+K^-} \cot\gamma = 1.06 \pm 0.02. \tag{17}$$

Thus, the experimental range, $R^{ss}_{KK} = 0.66 \pm 0.13$, shows a 3σ deviation from the SM prediction. This $R^{ss}_{KK} - S^s_{K^+K^-}$ inconsistency constitutes the second puzzle.

C. The $R^{ss}_{KK} = R^{ud}_{\pi K}$ puzzle

Eqs. (13) and (14) lead to the following relation between R^{ss}_{KK} and $R^{ud}_{\pi K}$

$$R^{ss}_{KK} - R^{ud}_{\pi K} = 2R^{ut}_{bs}\cos\gamma \times \mathcal{R}e[(T+E)/(P+P_A) - (T-A)/P]. \tag{18}$$

As mentioned above, it is expected that the penguin annihilation contribution is suppressed compared to the penguin contribution, $|P_A/P| \ll 1$, and that the exchange and annihilation contributions are suppressed compared to the spectator tree contribution, $|E/T| \ll 1$ and $|A/T| \ll 1$. To first order in these small hadronic parameters, Eq. (18) leads to the following relation:

$$R^{ss}_{KK}/R^{ud}_{\pi K} = 1 + 2R^{ut}_{bs}\cos\gamma \times \mathcal{R}e\left[(T/P)(E/T + A/T - P_A/P)\right]. \tag{19}$$

To estimate the deviation of the double ratio $R^{ss}_{KK}/R^{ud}_{\pi K}$ from unity, we first use the known values of the weak parameters to calculate $2R^{ut}_{bs}\cos\gamma \approx 0.016$. The hadronic part has a large factor, T/P, multiplied by a small factor, $E/T + A/T - P_A/P$. As concerns T/P, we use Eqs. (11) and (12) to zeroth order in $|E/T|$ and $|P_A/P|$, and the values of the observables given in Eqs. (6) and (7), and find

$$\left|\frac{T}{P}\right| \approx \frac{\left[(C^s_{K^+K^-})^2 + (S^s_{K^+K^-})^2\right]^{1/2}}{2R^{ut}_{bs}\sin\gamma} \approx 6.0 \pm 1.4. \tag{20}$$

As concerns the hadronically suppressed part, while we do not assign a strict upper bound on its value, we assume that it is of order

$$E/T + A/T - P_A/P \sim f_B/m_B \sim 0.05. \tag{21}$$

See Ref. [33] for a recent discussion. We thus expect

$$\mathcal{R}e\left[(T/P)(E/T + A/T - P_A/P)\right] \lesssim 1. \tag{22}$$

We conclude that the deviation of the double ratio $R^{ss}_{KK}/R^{ud}_{\pi K}$ from unity is predicted to be highly CKM-suppressed and without hadronic enhancement, and we estimate it to be of order 0.01. The LHS of Eq. (19) is experimentally measured to be

$$R^{ss}_{KK}/R^{ud}_{\pi K} = 0.59 \pm 0.12. \tag{23}$$

This experimental range shows a 3.4σ deviation from the SM prediction of 1. This $R^{ss}_{KK} - R^{ud}_{\pi K}$ inconsistency constitutes the third puzzle.

V. RESCATTERING, EW PENGUINS, AND $SU(3)$ BREAKING

As mentioned above, the analysis of Ref. [29] is conducted in the limit of $SU(3)$-flavor symmetry and neglects the contributions from rescattering and from electroweak penguins. In this section we discuss whether these missing pieces in the analysis can account for the various puzzles presented in the previous section.

A. The $R^{sd}_{KK} = 1$ puzzle: Rescattering

In this subsection we argue that the $R^{sd}_{KK} = 1$ puzzle cannot be explained by $SU(3)$ breaking or electroweak penguins. It can, however, be explained by rescattering.

The $B_s \to K^0\overline{K}^0$ and $B^0 \to K^0\overline{K}^0$ decays are related by U-spin. Since the final state of the two decays is the same and, furthermore, does not include pions, there is no U-spin breaking proportional to a factor of f_K/f_π. The remaining effects are theoretically expected to be small, of order m_s/m_b. This expectation was recently confirmed by relations between $B_s \to K^+K^-$ and $B^0 \to \pi^+\pi^-$ [34].

Given that the electromagnetic charge of the s and d quarks are the same, U-spin implies that the electroweak penguin contributions to the $B_s \to K^0\overline{K}^0$ and $B^0 \to K^0\overline{K}^0$ decays are also equal, and thus they do not affect the $R^{sd}_{KK} = 1$ prediction.

Rescattering contributes to $B_s \to K^0\overline{K}^0$ via $\bar{b} \to \bar{u}u\bar{s}$ followed by $u\bar{u} \to d\bar{d}$. Rescattering contributes to $B^0 \to K^0\overline{K}^0$ via $\bar{b} \to \bar{u}u\bar{d}$ followed by $u\bar{u} \to s\bar{s}$. Thus, in the presence of rescattering, Eq. (8) is modified:

$$\begin{aligned}
\mathcal{A}(B_s \to K^0\overline{K}^0) &= \lambda^t_{bs}(P + P_A) + \lambda^u_{bs} T_{RS}, \\
\mathcal{A}(B^0 \to K^0\overline{K}^0) &= \lambda^t_{bd}(P + P_A) + \lambda^u_{bd} T_{RS}.
\end{aligned} \tag{24}$$

Consequently, neglecting P_A/P and R_{bs}^{ut} compared with R_{bd}^{ut} but keeping all orders in T_{RS}/P, we find

$$R_{KK}^{sd} = [1 - 2(R_{bd}^{ut}\cos\alpha - R_{bs}^{ut}\cos\gamma)\mathcal{R}e(T_{RS}/P) + (R_{bd}^{ut}|T_{RS}/P|)^2]^{-1}. \tag{25}$$

The experimental value of R_{KK}^{sd} can be accounted for with

$$1.4 \lesssim |T_{RS}/P| \lesssim 2.4. \tag{26}$$

Given $|T/P| \approx 6$ from Eq. (20), Eq. (26) implies that, in order to explain the experimental value of R_{KK}^{sd}, we need

$$|T_{RS}/T| \sim 1/3. \tag{27}$$

While this value is somewhat large, it is not unacceptably so. It implies that rescattering is a subleading effect and can very well solve the puzzle.

If, indeed, the rescattering contribution enhances $\Gamma(B^0 \to K^0\overline{K}^0)$ in a significant enough way to suppress R_{KK}^{sd} from unity to $\mathcal{O}(0.6)$, then either or both (depending on the phase of T_{RS}/P) time-dependent CP asymmetries, $C_{K^0\overline{K}^0}^d$ and $S_{K^0\overline{K}^0}^d$, are large. Measuring these asymmetries would thus provide a crucial test of this scenario.

For the time-dependent CP asymmetries in $B_s \to K^0\overline{K}^0$, we can formulate a sum rule:

$$\left[(C_{K^0\overline{K}^0}^s)^2 + (S_{K^0\overline{K}^0}^s)^2\right]^{1/2} = 2R_{bs}^{ut}\sin\gamma \times |T_{RS}/P|. \tag{28}$$

Taking into account Eq. (26), we conclude that, if rescattering explains the puzzle, at least one of the CP asymmetries should be of order a few percent:

$$0.05 \lesssim \left[(C_{K^0\overline{K}^0}^s)^2 + (S_{K^0\overline{K}^0}^s)^2\right]^{1/2} \lesssim 0.09. \tag{29}$$

B. The $R_{KK}^{ss} = 1 + S_{K^+K^-}^s \cot\gamma$ puzzle: Electroweak penguins

In this subsection we argue that the $R_{KK}^{ss} = 1 + S_{K^+K^-}^s \cot\gamma$ puzzle cannot be explained by $SU(3)$ breaking or rescattering. It can, however, be explained by electroweak (EW) penguins, but at the cost of tension with other observables.

The $B_s \to K^0\overline{K}^0$ and $B_s \to K^+K^-$ decays are related by isospin. Isospin breaking is very small, of $O(0.01)$, and cannot explain the puzzle.

The value of $|T_{RS}/T| \sim 1/3$, see Eq. (27), implies that, at best, rescattering can bring the central value of the right hand side of Eq. (17) to 1.03, not enough to explain the puzzle.

Due to the different electromagnetic charges of the u and d quarks, EW penguins give different contributions to the decays in question:

$$\mathcal{A}(B_s \to K^0\overline{K}^0) = \lambda^t_{bs}[P + P_A - (1/3)P_{EW}],$$
$$\mathcal{A}(B_s \to K^+K^-) = -\lambda^t_{bs}[P + P_A + (2/3)P_{EW}] - \lambda^u_{bs}(T + E). \quad (30)$$

Consequently, neglecting $|P_A/P|$ and $|E/T|$, we find

$$S^s_{K^+K^-} = 2R^{ut}_{bs} \sin\gamma \times \mathcal{R}e(T/P),$$
$$R^{ss}_{KK} = 1 + 2R^{ut}_{bs} \cos\gamma \times \mathcal{R}e(T/P) - 2\mathcal{R}e(P_{EW}/P). \quad (31)$$

Thus, in the presence of EW penguins, Eq. (16) is modified:

$$R^{ss}_{KK} = 1 + S^s_{K^+K^-} \cot\gamma - 2\mathcal{R}e(P_{EW}/P). \quad (32)$$

To explain the puzzle we thus need

$$\mathcal{R}e(P_{EW}/P) = +0.20 \pm 0.07. \quad (33)$$

While as a stand alone effect EW penguins can explain the puzzle, the required value is in contradiction with other observations. The central value is larger by about an order of magnitude than the theoretical expectations for the color-suppressed EW penguin [32]. This expectation was confirmed by an analysis of a large set of observables in B-meson decays to pairs of $SU(3)$-octet mesons (π, K, η_8) [35]. Furthermore, the EW penguin contributions would generate a similar shift in $R^{ud}_{\pi K}$, which is unacceptable. In fact, the experimental range, $R^{ud}_{\pi K} = 1.12 \pm 0.05$, implies that $\mathcal{R}e(P_{EW}/P) = -0.03 \pm 0.03$.

C. The $R^{ss}_{KK} = R^{ud}_{\pi K}$ puzzle: $SU(3)$ breaking

In this subsection we argue that the $R^{ss}_{KK} = R^{ud}_{\pi K}$ puzzle cannot be explained by rescattering or EW penguins. It is affected, in principle, by $SU(3)$ breaking, but the required size of the breaking is unacceptably large.

Neglecting P_A/P and $R^{ut}_{bs}(A/P)$, we have, in the $SU(3)$ limit,

$$\mathcal{A}(B_s \to K^0\overline{K}^0) = \mathcal{A}(B^+ \to \pi^+K^0). \quad (34)$$

Neither rescattering nor EW penguins affect this equality. Neglecting P_A/P and E/T, we have, in the $SU(3)$ limit,

$$\mathcal{A}(B_s \to K^+K^-) = \mathcal{A}(B^0 \to \pi^-K^+). \tag{35}$$

Again, neither rescattering nor EW penguins affect this equality. Hence, the $SU(3)$ prediction that $R^{ss}_{KK} \simeq R^{ud}_{\pi K}$ (up to effects that are strongly CKM suppressed), is violated by neither rescattering nor electroweak penguin contributions.

The question is then whether $SU(3)$ breaking effects can account for the experimental result (23). An analysis of $SU(3)$ breaking was presented in Ref. [30]. We consider the $SU(3)$ breaking effects for only the P and T diagrams. We neglect P_A, E and A.

There are two relevant $SU(3)$-breaking diagrams related to the P contributions: P_1 where there is a $b \to s$ transition, and P_2 where the s quark is a spectator. Similarly, there are two relevant $SU(3)$-breaking diagrams related to the T contributions: T_1 where there is a $W \to u\bar{s}$ transition, and T_2 where the s quark is a spectator. Thus, $SU(3)$ breaking effects modify Eqs. (8) as follows [30]:

$$\begin{aligned}
\mathcal{A}(B_s \to K^0\overline{K}^0) &= \lambda^t_{bs}(P + P_1 + P_2), \\
\mathcal{A}(B_s \to K^+K^-) &= -\lambda^t_{bs}(P + P_1 + P_2) - \lambda^u_{bs}(T + T_1 + T_2), \\
\mathcal{A}(B^+ \to \pi^+K^0) &= \lambda^t_{bs}(P + P_1), \\
\mathcal{A}(B^0 \to \pi^-K^+) &= -\lambda^t_{bs}(P + P_1) - \lambda^u_{bs}(T + T_1).
\end{aligned} \tag{36}$$

We learn that, while each of the equalities (34) and (35) is violated at order P_2/P, the deviation of the double ratio $R^{ss}_{KK}/R^{ud}_{\pi K}$ from unity,

$$R^{ss}_{KK}/R^{ud}_{\pi K} = 1 + 2R^{ut}_{bs}\cos\gamma \times \mathcal{R}e\left\{(T/P)[(T_2/T) - (P_2/P)]\right\}, \tag{37}$$

is also CKM suppressed and thus very small.

Given that $2R^{ut}_{bs}\cos\gamma \times |T/P| \approx 0.10 \pm 0.03$, to explain the puzzle we would need

$$|T_2/T - P_2/P| \gtrsim 3. \tag{38}$$

We learn that, to explain the deviation of $R^{ss}_{KK}/R^{ud}_{\pi K}$ from unity, the $SU(3)$ breaking effect has to be unacceptably large, an order of magnitude larger than the naive expectation of 30%.

VI. FUTURE PROSPECTS

Understanding the origin of the puzzles reported in this paper requires the analysis of additional data. Fortunately, answers may rise from both the LHCb experiment and the Belle 2 experiment. According to Table II, the input measurements to this work which have the largest statistical uncertainties are $\mathrm{BR}(B_s \to K^0\overline{K}^0)$, $C^s_{K^+K^-}$ and $S^s_{K^+K^-}$. In addition, $C^s_{K^0\bar{K}^0}$ and $S^s_{K^0\bar{K}^0}$ have not been measured yet.

The Belle 2 experiment foresees to collect 50 ab^{-1} at the $\Upsilon(4S)$ together with a sample at the $\Upsilon(5S)$ [36]. The rapid B_s oscillations make the tagging of its initial flavor impossible at Belle 2. Information regarding CP violation can, however, be derived from the study of the lifetime distribution of this decay [36].

The LHCb experiment has gone through a first major upgrade [37] and a second one is foreseen for 2030 [38]. The integrated luminosity that is expected to be reached is 23 fb^{-1} (Run 1-3) for the first and 300 fb^{-1} for the second upgrade (Run 1-5).

At LHCb, the B_s oscillation can be resolved, as demonstrated in Ref [39]. While the branching fraction sensitivity can be directly estimated from the expected yields, the sensitivity on CP asymmetries must be extrapolated from the analogous decays $B_s \to K^+K^-$ and $B^0 \to \pi^+\pi^-$ [40].

A. The rates: $\mathrm{BR}(B^0 \to K^0\overline{K}^0)$ and $\mathrm{BR}(B_s \to K^0\overline{K}^0)$

Starting from the yields quoted in Ref. [1] and assuming the same scaling adopted in Ref. [40], a simple back of the envelope estimate gives yields of about 1500 B^0, and 4800 B_s decaying to a K_SK_S final state with 300 fb^{-1}. Assuming that background dilution is negligible, one can expect to reach a branching fraction precision of 0.09 (0.026) for the B^0 decay with 23 (300) fb^{-1}, and of 0.05 (0.014) for the B^0_s decay with 23 (300) fb^{-1}. The improvement of the precision on $\mathrm{BR}(B_s \to K^0\overline{K}^0)$ from the current 0.18 by a factor of 3.5 (12) will be of impact for all three puzzles.

B. The CP asymmetries for the neutral modes: $C^{d,s}_{K_SK_S}$ and $S^{d,s}_{K_SK_S}$

The fairly sizable samples also open the possibility to access the CP observables $C_{K_SK_S}$ and $S_{K_SK_S}$ in both the B^0 and B^0_s systems. In order to estimate this, we assume that the achievable precision on the C_X and S_X parameters is equal for all $B^0_{(s)}$ decays given equal signal yields. Furthermore, we assume that the flavor tagging performances are roughly the same for all hadronic $B^0_{(s)}$ decays at LHCb. Then one can scale the sensitivities reported for $B_s \to K^+K^-$ and $B^0 \to \pi^+\pi^-$ in [40] by the expected yields at a given luminosity. This computation leads to an expected precision of 0.89 and 0.48 for the B^0 and B^0_s decays, respectively, with 23 fb^{-1}. The precision on these quantities is expected to improve to 0.25 and 0.13 with 300 fb^{-1}. A better estimate of these extrapolations and assumptions will be possible once LHCb will explore the Run 3 data. Though challenging to measure, this information is key to address the $R^{sd}_{KK} = 1$ puzzle.

C. The CP asymmetries for the charged modes: $C^s_{K^+K^-}$ and $S^s_{K^+K^-}$

Ref. [40] provides an extrapolation of the statistical sensitivity for the CP violating parameters of the $B_s \to K^+K^-$ decay at LHCb. It is worth mentioning that the scaling used for these extrapolations is conservative, given that the performances of the flavor tagging, the decay-time resolution and the particle identification performance were assumed to be the same as in Run 1. One can expect to reach already precision of 0.015 for each of $C^s_{K^+K^-}$ and $S^s_{K^+K^-}$ with Run 1-3 data.The expected precision improves to 0.004 for the same observables with 300 fb^{-1}. This will contribute to shedding the light on the second and third puzzles.

VII. DISCUSSION AND CONCLUSIONS

Our starting point is the measurement of BR$(B_s \to K^0\overline{K}^0)$ by the LHCb [1] and Belle [2] experiments. Our analysis involves branching fractions and CP asymmetries in four additional B-meson decays related to $B_s \to K^0\overline{K}^0$ by $SU(3)$-flavor symmetry: $B_s \to K^+K^-$, $B^0 \to K^0\overline{K}^0$, $B^+ \to K^0\pi^+$ and $B^0 \to K^+\pi^-$. Our analysis demonstrates that the values of the CP asymmetries $S^s_{K^+K^-}$ and $C^s_{K^+K^-}$, and of the ratios of rates

$$R^{sd}_{KK} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\Gamma(B_s \to K^0\overline{K}^0)}{\Gamma(B^0 \to K^0\overline{K}^0)}, \qquad R^{ud}_{\pi K} = \frac{\Gamma(B^+ \to \pi^+K^0)}{\Gamma(B^0 \to \pi^-K^+)}, \tag{39}$$

can be accounted for with the following hierarchy of contributions to $b \to s$ transitions:

- A dominant contribution from gluonic penguin, proportional to $V^*_{tb}V_{ts}$.
- Tree level contribution of $\mathcal{O}(0.06)$ of the leading gluonic penguin to $B_s \to K^+K^-$.

- Rescattering contribution of $\mathcal{O}(0.03)$ of the leading gluonic penguin contribution to $B_s \to K^0\overline{K}^0$.

- Color suppressed electroweak penguin contributions of $\mathcal{O}(0.03)$ of the leading gluonic penguin contributions to $B_s \to K^0\overline{K}^0$ and to $B_s \to K^+K^-$.

These contributions cannot, however, explain the low value of

$$R^{ss}_{KK} = \frac{\Gamma(B_s \to K^0\overline{K}^0)}{\Gamma(B_s \to K^+K^-)} \tag{40}$$

compared to unity and, even more so, compared to $R^{ud}_{\pi K}$. In fact, they imply that $R^{ss}_{KK} \gtrsim 1$. The discrepancy is at the 3σ level.

The large deviation of R^{ss}_{KK} from unity and/or from $R^{ud}_{\pi k}$ is the combined puzzle. It cannot be accounted for even after considering various effects – rescattering, color-suppressed EW penguins and $SU(3)$ breaking – that are expected to be small.

While we did not look for possible explanations of the puzzles, we note that if $\mathrm{BR}(B_s \to K^0\overline{K}^0)$ would turn out to be 3σ higher than its experimental central value, all three puzzles that we presented will be solved, and the situation would be consistent with the expectation that rescattering and color-suppressed electroweak penguins give very small contributions to the decays in question.

The puzzles described in this work call for an experimental effort to improve the accuracy of the relevant measurements. In particular, searching for CP asymmetries in $B_s \to K^0\overline{K}^0$ and in $B^0 \to K^0\overline{K}^0$ might shed light on the solution(s) to these puzzles.

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