

Update on Flavor Diagonal Nucleon Charges

**Sungwoo Park,^{a,*} Tanmoy Bhattacharya,^b Rajan Gupta,^b Huey-Wen Lin,^c
Santanu Mondal^{b,c} and Boram Yoon^d**

^aThomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA

^bTheoretical Division T-2, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

^cDepartment of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA

^dComputer, Computational and Statistical Science Division CCS-7, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

E-mail: sungwoo@jlab.org, tanmoy@lanl.gov, rajan@lanl.gov,
hueywen@msu.edu, santanu.sinp@gmail.com, byoon@nvidia.com

This talk provides an update on the calculation of matrix elements of flavor diagonal axial, scalar and tensor quark bilinear operators between the nucleon ground state. The simulations are done using Wilson-clover fermions on a sea of eight 2+1+1-flavor HISQ ensembles generated by the MILC collaboration. We discuss the signal in the sum of the connected and disconnected contributions for the up, down and strange quarks, control over fits to remove excited state contamination, and the simultaneous chiral-continuum fit used to extract the charges.

*The 39th International Symposium on Lattice Field Theory (Lattice2022),
8-13 August, 2022
Bonn, Germany*

*Speaker

1. Introduction

This talk updates the calculation of the matrix elements of flavor diagonal axial, scalar and tensor quark bilinear operators between the nucleon ground state from which we extract the charges $g_{A,S,T}^{u,d,s}$. The motivation for these calculations and much of the methodology used have already been published: for $g_{A,T}^q$ in Refs. [1–3], and for g_S^q in Ref. [4]. Here we will focus on describing the progress since Lattice 2021 [5]. Since the final analysis is still ongoing, all the results presented here should be considered preliminary unless otherwise stated.

The calculations have been done on eight 2+1+1-flavor HISQ ensembles generated by the MILC collaboration [6] with correlation functions calculated using Wilson-clover valence fermions with the light ($m_u = m_d$ in the isospin symmetric theory) and strange quark masses tuned to reproduce the sea (HISQ) M_π and $M_{s\bar{s}}$ values. The parameters of these ensembles are given in Table 1. This set includes one physical $M_\pi \approx 138$ MeV ensemble (labeled as $a09m130$) at $a \approx 0.09$ fm and $M_\pi L \approx 3.9$. In addition to the nucleon 2-point functions [1], we calculate the quark-line diagrams for the connected [1] and disconnected [2–4] contributions to 3-point functions illustrated in the two left panels in Fig. 1(a), and the analogous quark level diagrams in Landau gauge for calculating the renormalization constants in the RI-sMOM scheme in Fig. 1(b).

Ensemble ID	a (fm)	M_π (MeV)	$M_\pi L$	$L^3 \times T$	N_{conf}^l	N_{src}^l	N_{conf}^s	N_{src}^s	$N_{\text{LP}}/N_{\text{HP}}$
$a15m310$	0.1510(20)	320(5)	3.93	$16^3 \times 48$	1917	2000	1917	2000	50
$a12m310$	0.1207(11)	310(3)	4.55	$24^3 \times 64$	1013	10000	1013	8000	50
$a12m220$	0.1184(10)	228(2)	4.38	$32^3 \times 64$	958	11000	870	5000	30–50
$a09m310$	0.0888(8)	313(3)	4.51	$32^3 \times 96$	1017	10000	1024(*)	6000	50
$a09m220$	0.0872(7)	226(2)	4.79	$48^3 \times 96$	712	8000	847	10000	30–50
$a09m130$	0.0871(6)	138(1)	3.90	$64^3 \times 96$	1270	10000	541+453(*)	10000+4000	50
$a06m310$	0.0582(4)	320(2)	3.90	$48^3 \times 144$	808	12000	948+28(*)	10000+4000	50
$a06m220$	0.0578(4)	235(2)	4.41	$64^3 \times 144$	1001	10000	1002	10000	50

Table 1: The ensembles and the statistics used for the calculation of disconnected contributions, including updates to Refs. [2, 3, 5]. Statistics for the connected contributions are the same as in Ref. [1]. $N_{\text{conf}}^{l,s}$ is the number of gauge configurations analyzed for light (l) and strange (s) flavors. $N_{\text{src}}^{l,s}$ is the number of random sources used per configurations, and $N_{\text{LP}}/N_{\text{HP}}$ is the ratio of low/high precision measurements. The N_{conf}^s marked with (*) have been updated since 2021 [5].

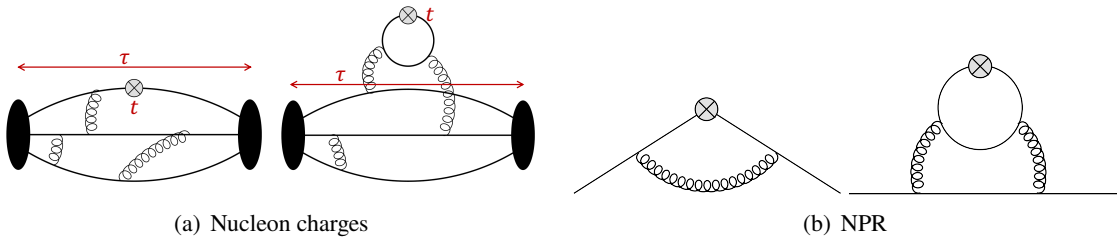


Figure 1: The connected and disconnected diagrams calculated (i) for flavor diagonal nucleon charges, and (ii) non-perturbative renormalization in the RI-sMOM scheme using quark states in Landau gauge.

2. Details of 2-point and 3-point function analysis

The parameters of quark propagators calculated on HYP smeared lattices using Wuppertal smearing are given in Ref. [1]. To construct all the 2- and 3-point correlation functions, the nucleon

interpolating operator $\mathcal{N}(x) = \epsilon^{abc} \left[q_1^{aT}(x) C \gamma_5 \frac{(1 \pm \gamma_4)}{2} q_2^b(x) \right] q_1^c(x)$ is used both at the source and the sink. To extract the charges $g_{A,S,T}^{u,d,s}$ from forward matrix elements, all 3-point functions are calculated with zero momentum projection of both the nucleon state at the sink and the operator insertion: $C_\Gamma^{3\text{pt}}(t; \tau) = \text{Tr}[\mathcal{P} \langle 0 | \mathcal{N}(\tau) O_\Gamma(t, \mathbf{q} = 0) \tilde{N}(0, \mathbf{p} = 0) | 0 \rangle]$ with $O_\Gamma^q = \bar{q} \Gamma q$, $q \in \{u, d, s\}$, and $\mathcal{P}$ is the spin projection defined in Ref. [7] for the various cases.

Flavor diagonal 3pt functions are the sum of connected (conn) and disconnected (disc) contributions illustrated in Fig. 1(a): $C_\Gamma^{3\text{pt}}(t; \tau) \equiv C_\Gamma^{\text{conn}}(t; \tau) + C_\Gamma^{\text{disc}}(t; \tau)$. For the scalar case, the disconnected contribution is calculated using the vacuum subtracted operator $O_S^q - \langle O_S^q \rangle$. The calculation of the quark loop with zero-momentum operator insertion is estimated stochastically using Z_4 random noise sources as explained in Ref. [7]. Note that in our previous works [2, 3], the fits to remove ESC in $C_\Gamma^{\text{conn}}(t; \tau)$ and $C_\Gamma^{\text{disc}}(t; \tau)$ were done separately, as was the chiral-continuum (CC) extrapolations of $g_\Gamma^{q,\text{disc}}$ and $g_\Gamma^{q,\text{conn}}$. This introduced an unquantified systematic [3] that has now been removed by making a simultaneous fit to $C_\Gamma^{3\text{pt}}(t; \tau)$ and $C_\Gamma^{2\text{pt}}(\tau)$ and extrapolating g_Γ^q .

The bare charges, $g_\Gamma^{q;\text{bare}}$, are obtained from the ground state matrix elements $\langle 0 | O_\Gamma^q | 0 \rangle$ extracted from fits to the spectral decomposition of the spin projected $C_\Gamma^{3\text{pt}}(t; \tau)$:

$$C_\Gamma^{3\text{pt}}(t; \tau) = \sum_{i,j=0} \mathcal{A}_i \mathcal{A}_j^* \langle i | O_\Gamma^q | j \rangle e^{-M_i t - M_j(t-\tau)} \quad \text{with} \quad \langle 0 | O_\Gamma^q | 0 \rangle = g_\Gamma^q. \quad (1)$$

The challenge to extracting $\langle 0 | O_\Gamma | 0 \rangle$ from fits to $C_\Gamma^{3\text{pt}}$ is removing excited state contributions (ESC) which are observed to be large at source-sink separation $\tau \approx 1.5$ fm beyond which the signal degrades due to the $e^{(M_N - 3/2 M_\pi)\tau}$ increase in noise. With the current statistics, we are only able to keep one excited state in Eq. (1), and fits leaving M_1 a free parameter are not stable in many cases.

The nucleon spectrum M_i and amplitude $\mathcal{A}_0$ needed to analyze $C_\Gamma^{3\text{pt}}(t; \tau)$ are obtained from the spectral decomposition of the 2pt function, $C_\Gamma^{2\text{pt}}(\tau) = \sum_{i=0} |\mathcal{A}_i|^2 e^{-M_i \tau}$, truncated at four states. We carry out two types of analyses: (i) The “standard” fit to $C_\Gamma^{2\text{pt}}(\tau)$ uses wide priors for all the excited-state amplitudes, $\mathcal{A}_i$, and masses, M_i , i.e., the priors are only used to stabilize the fits. In these fits, $M_1 \gtrsim 1.5$ GeV. (ii) The “ $N\pi$ ” fit in which a narrow prior is used for M_1 with the central value given by the non-interacting energy of the lowest allowed $N\pi$ or $N\pi\pi$ state on the lattice. The resulting values of $\mathcal{A}_0$ and the M_i are then used as inputs in the analysis of the 3-point functions. In practice, we fit $C_\Gamma^{2\text{pt}}(\tau)$ and $C_\Gamma^{3\text{pt}}(t; \tau)$ simultaneously. The important point is that the mass gap, $M_1 - M_0$, in the two analyses is significantly different, however, the augmented χ^2 minimized in the fits is essentially the same. Since the two fit strategies are not distinguished by the χ^2 , we examine the sensitivity of the results for the charges to the two M_1 and use the difference to appropriately estimate an associated systematic uncertainty.

Also, the ESC analysis is repeated to quantify model variation of results by choosing data with different set of (τ, t) values and the number of excited states (2- or 3-state) in the ansatz of Eq. (1). The final ESC analysis results are taken to be the weighted average with the Akaike information criteria weight $\exp[-(\chi^2 - 2N_{\text{dof}})/2]$ [8].

Another challenge to distinguishing between “standard” and “ $N\pi$ ” analysis strategies is that the difference in the corresponding M_1 becomes significant only for $M_\pi \lesssim 200$ MeV, which in our setup means only in the $a091m130$ ensemble. Previous works show that the difference in axial and tensor charges, $g_{A,T}$, from the two strategies is small [9]. For the isoscalar scalar charge g_S^{u+d} , χPT

suggests a large contribution from the $N\pi$ and $N\pi\pi$ states [4]. As explained in Ref. [4], this leads to a large difference in the value of the pion-nucleon sigma term. The intent of doing the full analysis with both strategies is to quantify these differences and understand which states contribute.

The renormalization of the axial, scalar and tensor operators is carried out in the 3-flavor theory (we have explicitly evaluated the 3×3 matrices accounting for flavor mixing) using the RI-sMOM lattice scheme and then converting to $\overline{\text{MS}}$ scheme at 2 GeV as described in Ref. [5].

3. Update on results for $g_{A,S,T}$

Examples of ESC fits to $a091m130$ data are shown in Fig. 2, with the largest difference between the two strategies observed in g_S . For the charges with strange flavor, $g_{A,T,S}^s$, the leading multihadron excited state is expected to be ΣK , which has a large mass gap, so we consider the “standard” analysis more appropriate for it. The chiral-continuum (CC) fits to the renormalized charges are shown in Figs. 3, 4 and 5. Possible finite-volume corrections are ignored in this analysis. The final preliminary results are summarized in Fig. 7 and 9 and Table 2. Some details are as follows:

Axial charges, $g_A^{u,d,s}$:

Compared to the “standard” analysis for $g_A^{u,d}$, the “ $N\pi$ ” analysis finds larger ESC especially in the physical smaller pion mass ensemble as shown in Fig. 2. CC fits in Fig. 3 for both g_A^u and g_A^d show similar dependence on a and M_π , with larger uncertainty in the $N\pi$ analysis. There is a significant slope versus M_π^2 that adds for g_A^{u+d} and almost cancels for g_A^{u-d} . g_A^u is almost unchanged with or without “ $N\pi$ ”, while $|g_A^d|$ is $\approx 7\%$ larger with “ $N\pi$ ” analysis. Our final extrapolated g_A^q with the “ $N\pi$ ” analysis of ESC (as motivated by the isovector axial form factor analysis described in [9]) are summarized in Fig. 7 (and Table 2) along with determinations from other collaborations taken from the FLAG review 2021 [10]. The difference between the extrapolated values with the “standard” and “ $N\pi$ ” analysis data can be viewed as an additional systematic error. Results for $g_A^{u,d,s}$ are consistent with those published in Ref. [3].

Tensor charges, $g_T^{u,d,s}$: The magnitude of ESC in g_T^u and g_T^d is similar. The “standard” and “ $N\pi$ ” analysis of ESC, shown in Fig. 2, give consistent central values. The statistical quality of the data on the physical pion mass ensemble, $a09m130$, is poor as shown in Fig. 2 and Ref. [2]. In g_T^s , when there is no clear ESC pattern in $C_T^{3\text{pt},s}(t; \tau)$, the central value is taken from a constant fit to the middle points. The CC fits are shown in Fig. 4. The extrapolated g_T^q , using the “standard” analysis of ESC, are summarized in the Fig. 7 and Table 2. They are consistent with those in Ref. [2].

Scalar charges $g_S^{u,d,s}$: Chiral PT gives two differences in the chiral behavior of flavor diagonal scalar charges. First, the CC ansatz $g_S^{u,d} = d_0 + d_a a + d_1 M_\pi + d_2 M_\pi^2 + M_\pi^2 \log M_\pi^2 + \dots$, has the chiral behavior starting with a term proportional to M_π [11]. Second, the contribution of $N\pi$ and $N\pi\pi$ excited states is large in $g_S^{u,d}$ [4] as observed in the data shown in Fig. 2. The CC fits are shown in Fig. 5. Our current estimates of g_S^q are summarized in Table 2. For $g_S^{u,d}$, the quoted results are from the “ $N\pi$ ” analysis motivated by Ref. [4], and for g_S^s , we use the “standard” analyses as the lowest multihadron state should be the ΣK .

The nucleon sigma term $\sigma_{\pi N}$ and the strangeness content σ_s : The analysis of $\sigma_{\pi N}$ in Ref. [4] used the renormalization independent $\sigma_{\pi N} = m_l^{\text{bare}} g_S^{u+d, \text{bare}}$. The χ PT analysis suggests that 4–5 M_π -dependent terms contribute significantly [4]. With data at three values

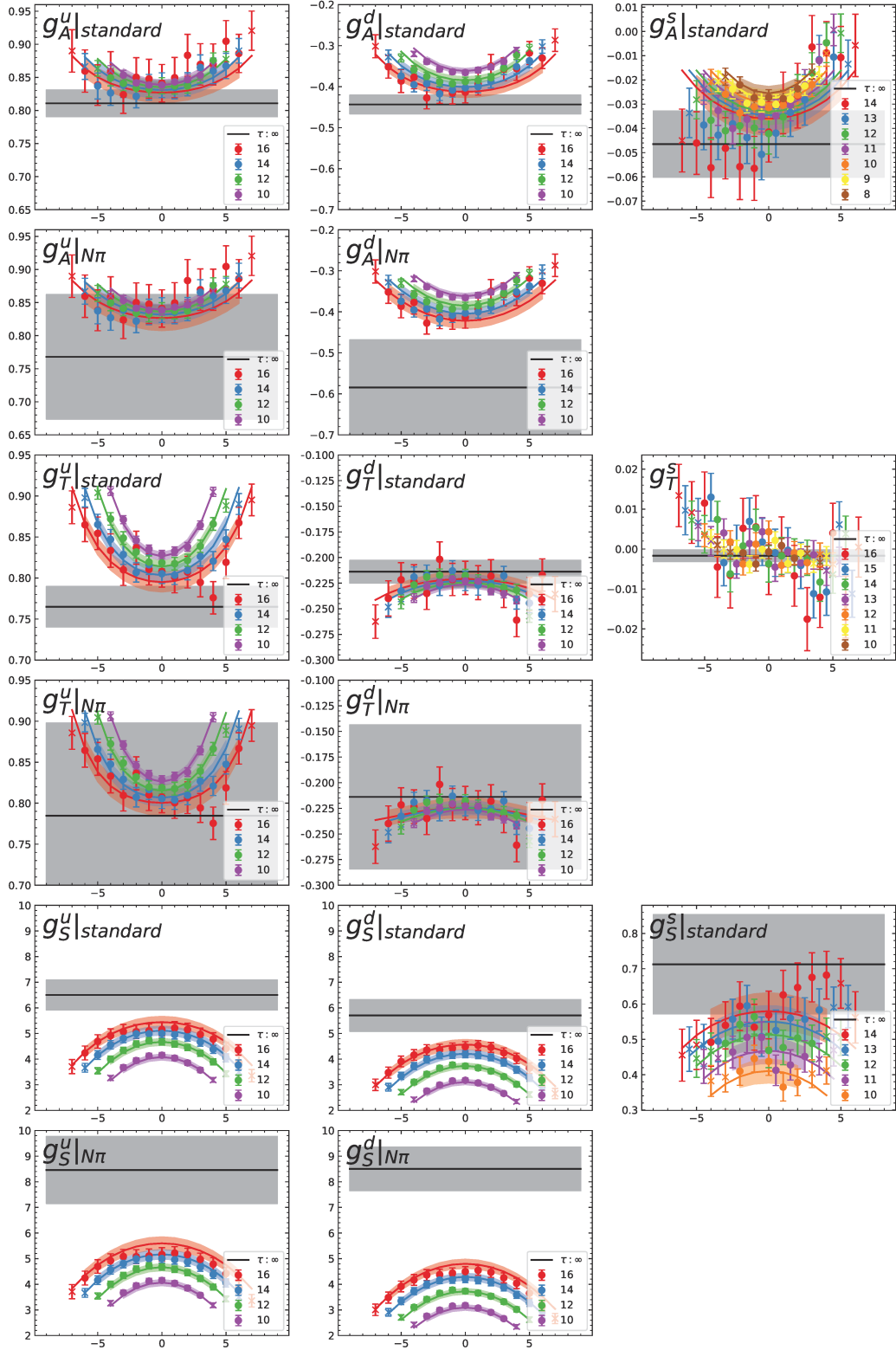


Figure 2: The physical M_π ensemble ($a09m130$) data for the sum of the connected and disconnected contributions at different $\{\tau, t\}$ are plotted versus $(t - \tau/2)/a$. Different panels show excited-state fits to data for bare g_A , g_T , and g_S using the “standard” and $N\pi$ strategies defined in the text. Result of the fit is shown by lines of the same color as the data for various τ/a listed in the label, and the $\tau \rightarrow \infty$ value is given by the gray band.

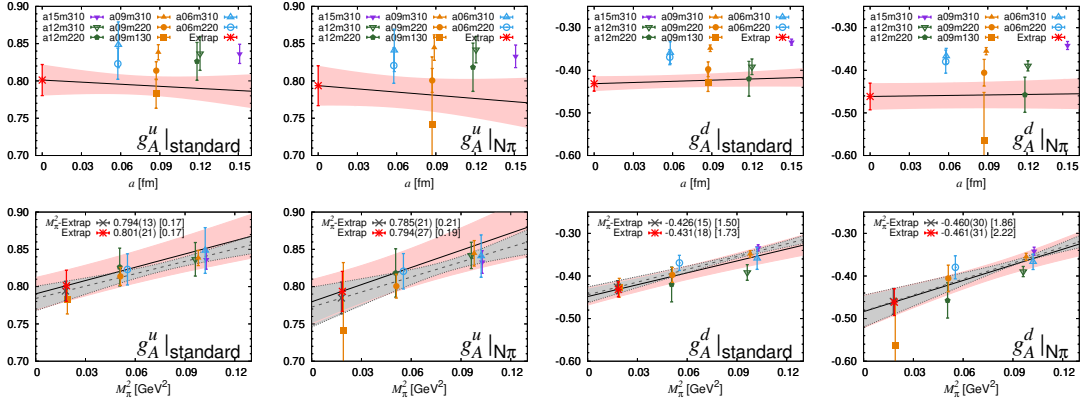


Figure 3: CC fits to g_A^u (left two panels) and g_A^d (right two panels) obtained with standard and $N\pi$ strategies using the ansatz $d_0 + d_a a + d_2 M_\pi^2$. Fit result is plotted versus a in top row and versus M_π^2 in bottom row.

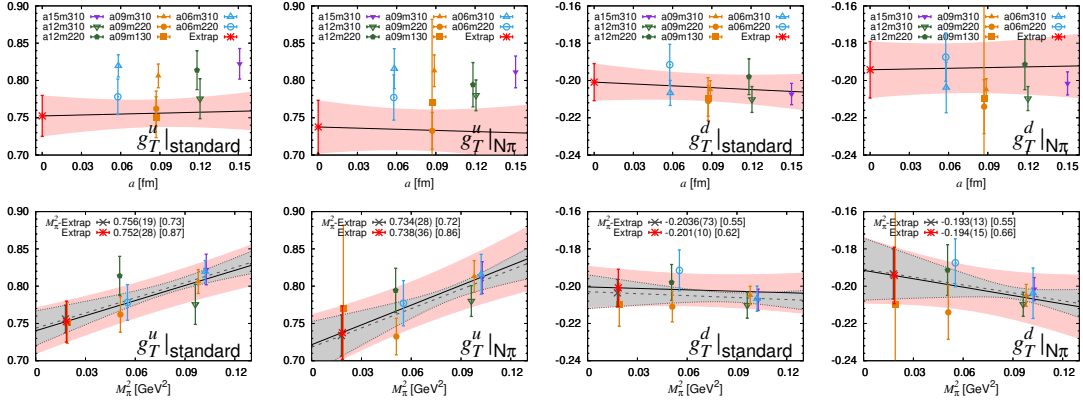


Figure 4: CC fits to g_T^u (left two panels) and g_T^d (right two panels) obtained with standard and $N\pi$ strategies using the ansatz $d_0 + d_a a + d_2 M_\pi^2$. Fit result is plotted versus a in top row and versus M_π^2 in bottom row.

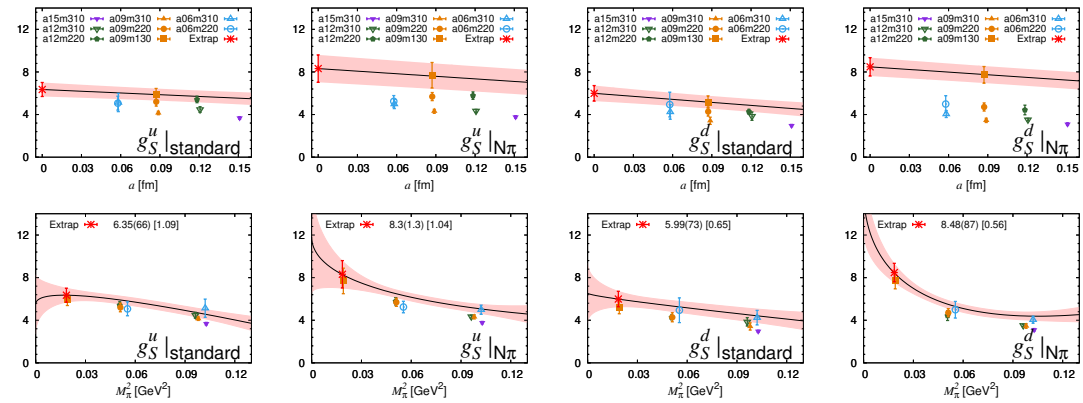


Figure 5: CC fits to g_S^u (left 2 panels) and g_S^d (right 2 panels) obtained with standard and $N\pi$ strategies using the ansatz $d_0 + d_a a + d_1 M_\pi + d_2 M_\pi^2$ to $g_S^{u,d}$. Fit result is plotted versus a in top row and versus M_π^2 in bottom row.

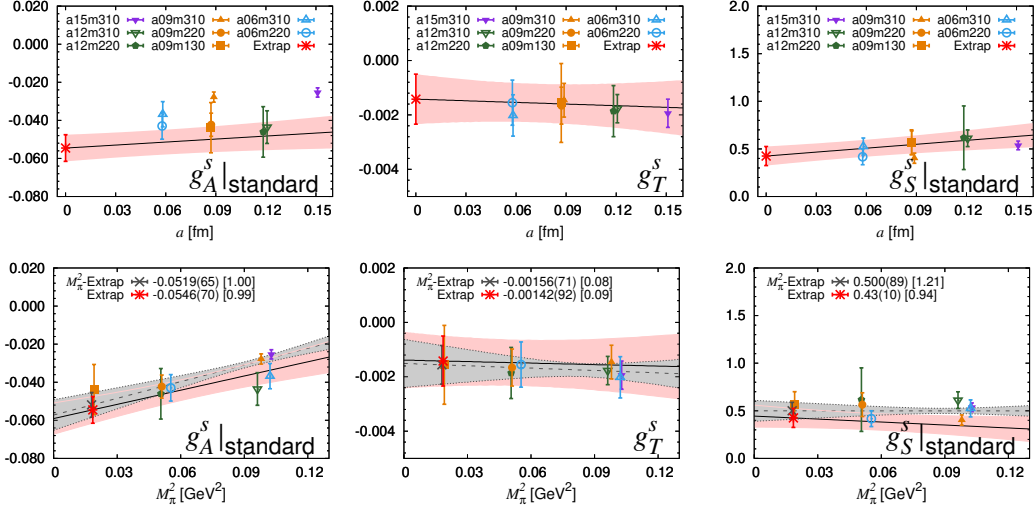


Figure 6: CC fits to $g_{A,T,S}^S$ using the ansatz $d_0 + d_a a + d_2 M_\pi^2$ to g_S^S . We neglect dependence on M_K since m_s is tuned to its physical value.

q	This work (Preliminary)			PNDME'18	
	g_A^q	g_T^q	g_S^q	g_A^q [3]	g_T^q [2]
u	0.79(3)(1)	0.75(3)(1)	8.3(1.3)	0.777(25)(30)	0.784(28)(10)
d	-0.46(3)(3)	-0.20(1)(1)	8.5(9)	-0.438(18)(30)	-0.204(11)(10)
s	-0.055(7)	-0.0014(9)	0.41(10)	-0.053(8)	-0.00319(72)

Table 2: Updated preliminary results for the flavor diagonal charges compared to results published in [2, 3].

($M_\pi \approx 135, 220, 310$ MeV), we have used different CC fit ansatz with some of the coefficients fixed to their values from χ PT [4]. Two such fits are shown in Fig. 8 to updated data with new analysis.

In this work, we use CC extrapolated renormalized scalar charges $g_S^{u,d,s}$ and renormalized quark masses $m_{l,s}$ ($N_f = 2 + 1 + 1$ results from FLAG 2021 [10]) to determine $\sigma_{\pi N}$ and σ_s . These results are summarized in Fig. 9 along with other lattice determinations. The $N\pi$ analysis gives $\sigma_{\pi N}|_{N\pi} \approx 60$ MeV, consistent with phenomenology while the standard analysis value $\sigma_{\pi N}|_{\text{standard}} \approx 40$ MeV is consistent with previous lattice estimates [4]. For σ_s , we use g_S^s from the standard analysis, which gives $\sigma_s = 38(9)$ MeV using $m_s = 93.44(68)$ MeV from FLAG [10].

4. Conclusion

Significant progress has been made in calculating χ PT predictions and using them as guides in fits to remove ESC and to do the chiral extrapolation. Unfortunately, excited-state fits to present data do not, in most cases, distinguish between standard and $N\pi$ analyses. To get percent level results for these charges with data driven methods to control ESC requires higher statistics at many more values of a and on physical pion mass ensembles.

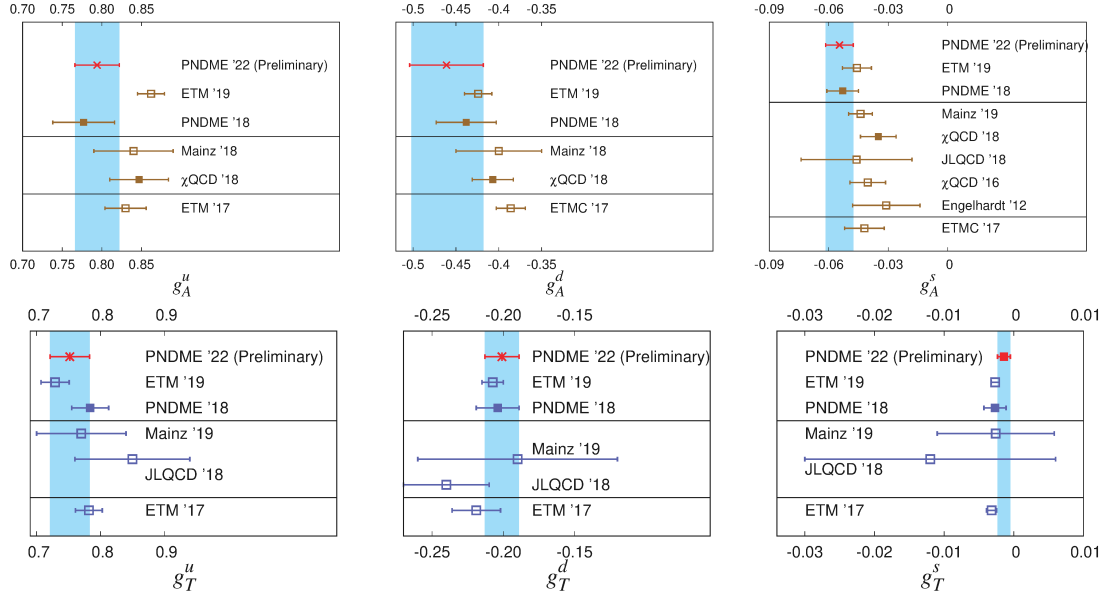


Figure 7: Preliminary (PNDME 22) and published PNDME 18 [2, 3] results for g_A^q (top) and g_T^q (bottom) added to the FLAG 2021 summary figure. See Ref. [10] for details and references to other works.

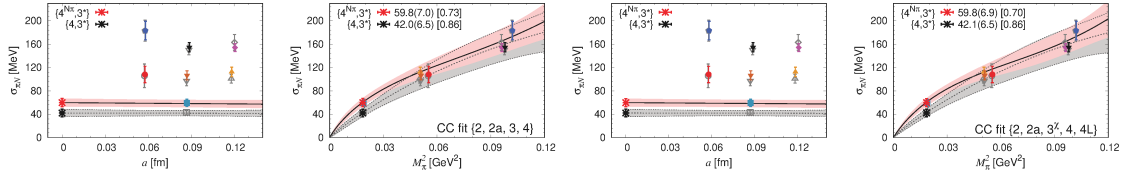


Figure 8: CC fits to $\sigma_{N\pi}$ data with 2 different chiral ansatz as explained in Fig.3 of Ref. [4].

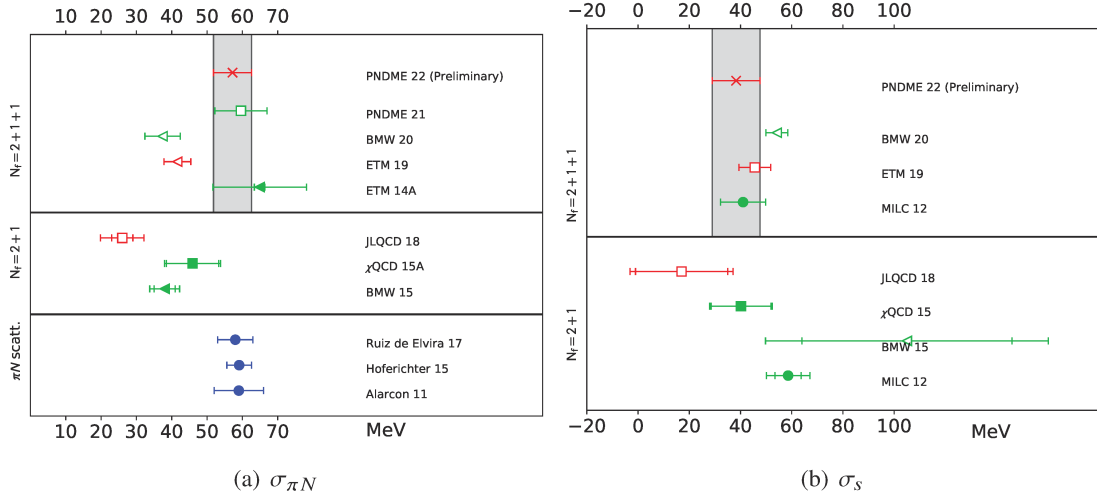


Figure 9: Our results (PNDME 22) and published PNDME 21 [4]) for the pion-nucleon sigma term and the strangeness content of the nucleon. For details and references to other lattice calculations, see Ref. [4, 10].

5. Acknowledgements

We thank the MILC collaboration for providing the 2+1+1-flavor HISQ lattices. The calculations used the Chroma software suite [12]. This research used resources at (i) NERSC, a DOE Office of Science facility supported under Contract No. DE-AC02-05CH11231; (ii) the OLCF, a DOE Office of Science User Facility supported under Contract DE-AC05-00OR22725, through ALCC awards LGT107 and INCITE awards PHY138 and HEP133; (iii) the USQCD collaboration, which is funded by the Office of Science of the U.S. DOE; and (iv) Institutional Computing at Los Alamos National Laboratory. S. Park acknowledges support from the U.S. Department of Energy Contract No. DE-AC05-06OR23177, under which Jefferson Science Associates, LLC, manages and operates Jefferson Lab. Also acknowledged is support from the Exascale Computing Project (17-SC-20-SC), a collaborative effort of the U.S. DOE Office of Science and the National Nuclear Security Administration. T. Bhattacharya and R. Gupta were partly supported by the U.S. DOE, Office of Science, HEP under Contract No. DE-AC52-06NA25396. T. Bhattacharya, R. Gupta, S. Mondal, S. Park, and B. Yoon were partly supported by the LANL LDRD program, and S. Park by the Center for Nonlinear Studies.

References

- [1] R. Gupta, Y.-C. Jang, B. Yoon, H.-W. Lin, V. Cirigliano and T. Bhattacharya, *Phys. Rev.* **D98** (2018) 034503 [1806.09006].
- [2] R. Gupta, B. Yoon, T. Bhattacharya, V. Cirigliano, Y.-C. Jang and H.-W. Lin, *Phys. Rev.* **D98** (2018) 091501 [1808.07597].
- [3] H.-W. Lin, R. Gupta, B. Yoon, Y.-C. Jang and T. Bhattacharya, *Phys. Rev.* **D98** (2018) 094512 [1806.10604].
- [4] R. Gupta, S. Park, M. Hoferichter, E. Mereghetti, B. Yoon and T. Bhattacharya, *Phys. Rev. Lett.* **127** (2021) 242002 [2105.12095].
- [5] S. Park, T. Bhattacharya, R. Gupta, H.-W. Lin, S. Mondal, B. Yoon et al., *PoS LATTICE2021* (2021) 558 [2203.09584].
- [6] MILC collaboration, *Phys. Rev.* **D87** (2013) 054505 [1212.4768].
- [7] PNDME collaboration, *Phys. Rev.* **D92** (2015) 094511 [1506.06411].
- [8] W.I. Jay and E.T. Neil, *Phys. Rev. D* **103** (2021) 114502 [2008.01069].
- [9] NUCLEON MATRIX ELEMENTS (NME) collaboration, *Phys. Rev. D* **105** (2022) 054505 [2103.05599].
- [10] FLAVOUR LATTICE AVERAGING GROUP (FLAG) collaboration, *Eur. Phys. J. C* **82** (2022) 869 [2111.09849].
- [11] M. Hoferichter, J. Ruiz de Elvira, B. Kubis and U.-G. Meißner, *Phys. Rept.* **625** (2016) 1 [1510.06039].
- [12] SciDAC, LHPC, UKQCD collaboration, *Nucl. Phys. Proc. Suppl.* **140** (2005) 832 [hep-lat/0409003].