Journal of Mathematics Teacher Education
https://doi.org/10.1007/s10857-023-09588-1

On designing better structures for feedback in practice-based professional development: Using "failure" to innovate

Amanda M. Brown[1] · **Patricio G. Herbst**[1]

Accepted: 5 June 2023

Abstract
This research article contributes to the growing literature highlighting the potential for innovation in mathematics education through design cycles that involve creative risk-taking and failure-based learning. Specifically, we explore how "failed" cycles of Story*Circles*—a practice-based professional development approach that centers on teacher collaboration—have been productive in fostering innovations within the program. Our focus is on the challenges that arose in our efforts to enable feedback mechanisms within the Story*Circles* system that support teachers' interrogation of their own instructional practice, as they collaboratively develop lessons and expand their collective knowledge base for teaching mathematics. Through examples of three challenges, we illustrate how various lesson artifacts, including those constructed by teachers in anticipation of implementation and those extracted from actual implementations, failed to serve as the sole source of feedback for supporting teachers' growth.

Keywords Instructional design · Feedback · Practice-based professional development · In-service teacher · Secondary mathematics · Teacher collaboration · Educational innovation · Teacher autonomy, creative risk, failure-based learning

Introduction

This article illustrates how "failed" cycles of Story*Circles* have been productive toward the continuous improvement of Story*Circles* as a form of teacher professional development (PD) through collaboration. Story*Circles* is a collaborative process that allows teachers to share about their practice and learn from others through scripting a lesson, visualizing it in a storyboard, and arguing about potential decisions. A cornerstone of Story*Circles* is the expectation that facilitators orient participants to each other as sources of practical

✉ Amanda M. Brown
amilewsk@umich.edu

[1] Educational Studies, University of Michigan, Ann Arbor, MI, USA

knowledge rather than dispense or validate such knowledge. Instead, Story*Circles*[1] embodies a wager that activities involving lesson anticipation can elicit individuals' practical knowledge and that lesson visualization can operate as feedback on that knowledge. To illustrate how failed cycles of Story*Circles* helped drive design innovation, we share details about three challenges that surfaced in our efforts to build feedback mechanisms for shaping the evolving interactions teachers had with each other around a lesson. This article contributes to literature examining the importance for instructional designers to embrace creative risk and learn from failure to accelerate innovation (Cropley, 2020; Henriksen et al., 2017). We illustrate how we have engaged in failure-based learning (Kapur, 2012)—leveraging failure to spur phases of ideation and innovation across three different implementations of Story*Circles.* Common to these implementations of Story*Circles* is our intention to make progress toward realizing the vision of Hiebert and Morris (2012) for a PD system that not only facilitates groups of teachers in analyzing and evaluating their own teaching practices through collaborative development of lessons, but also broadens the knowledge base for mathematics teaching. While previous studies on Story*Circles* have highlighted its potential to support teachers' professional growth, mathematical knowledge for teaching, and collaborative lesson development (Herbst et al. 2020; Milewski et al., 2018, 2020) we have also documented challenges which prompted us to innovate (Brown et al., 2021). The intentional centering on teachers and their knowledge has some important implications for the role of the facilitator. In contrast to other practice-based professional development approaches, which rely on the facilitator to provide feedback on practitioners' actions during approximations of practice (e.g., Lampert et al., 2013), Story*Circles* avoids orienting participants toward the facilitator as the source of knowledge. Instead, our approach centers on the group of teachers as a collective resource that can enhance the knowledge of each member.

Building on prior research exploring the effectiveness of lesson visualizations in aiding individual pre-service teachers' lesson design and learning (Chen, 2012), our team hypothesized that visualization—the engagement of teachers with storyboards to represent their expectations for how a lesson may unfold—could serve as a robust form of feedback to facilitate teacher collaboration. To conceptualize this process, Herbst et al. (2014) drew on Papert's constructionism, which views learning as the construction of artifacts (Papert & Harel, 1991), and Brousseau's (1997) notion of the milieu. Brousseau's theory posits that the milieu presents itself to learners as a space free of didactic intentions and thus, excludes the teacher's feedback—which, in PD alludes to the facilitator's feedback. Instead, "the student learns by adapting herself to a milieu that generates contradictions, difficulties and disequilibria" (Brousseau, 1997, p. 30). A milieu is constructed to provide feedback, utilizing semiotic resources and material artifacts to facilitate learning.

The original design of Story*Circles* was based on the idea that participants could learn exclusively from their engagement with a milieu—defined narrowly as the group's interactions about the storyboarded lesson, and they had developed by scripting or narrating their expectations for its implementation in their classrooms. We hypothesized that storyboarded representations could help teachers notice aspects of the lesson they might have overlooked during scripting and this noticing might prompt them to share their experiences, including alternative ideas and justifications for practices that provide feedback on their practical knowledge. The three challenges we present below emerged in

[1] When we use the term "Story*Circles*" we are referring to the Story*Circles* process and therefore treat the word as a singular rather than a plural.

sequence as we contended with the inadequacy of lesson artifacts (both those constructed in anticipation of a lesson implementation and those collected during an implementation of a lesson) as the sole source of feedback to provoke arguments that occasion revisions to lesson scripts. In challenge one, we demonstrate how a collective visualization of a lesson may not provide enough feedback for improvement. In challenge two, we illustrate how artifacts collected during the teaching of a lesson may also fail to support improvement. Finally, we show how designed lesson contingencies can also fail to provide adequate feedback for improvement. In each challenge, we begin by presenting evidence for how the innovation (i.e., storyboard visualization, lesson artifacts, and lesson contingencies) can facilitate teacher collaboration. Then, we describe ways the innovation sometimes failed—sharing details that exemplify how each innovation failed. Each challenge served to highlight the limitations of that operationalization of feedback and the design of modifications that aimed to maintain the original design principle of orienting teachers to each other, rather than to the facilitator as a source of knowledge. Lastly, we close by sharing some general principles we found helpful when deciding how to handle failures that emerged in our work to design a learning-through-collaboration environment.

Theoretical perspectives

Designing professional development to support teacher learning through collaboration

There is widespread agreement that teachers' professional growth is hindered by realities of schooling that conspire against meaningful collaboration (Berg, 2011; Campbell, 2009; Climent et al., 2020; Darling-Hammond, 1990; Jaworski, 2008; Kennedy, 2009). Yet, while teachers may benefit from more time to collaborate, there are reasons to suspect that not all such interactions are equally beneficial (Ben-Peretz & Schonmann, 2000; Glazier et al., 2017; Horn & Little, 2010). Little (1990) expressed some skepticism regarding the lack of conceptual rigor undergirding the field's enthusiastic uptake for teacher collaboration, saying:

> Advocates have imbued [teacher collaboration] with a sense of virtue—the expectation that any interaction that breaks the isolation of teachers will contribute in some fashion to the knowledge, skill, judgment or commitment that individuals bring to their work and will enhance the collective capacity of groups or institutions (p. 509).

If some, but not all, forms of teacher interactions are productive for supporting teachers to collaborate about problems of practice, an important question to ask is how can PD be structured to support collaboration that is productive.

This question has been the source of scholarship on teaching and teacher education over the last two decades—as the field seeks to gain conceptual clarity about the conditions underlying productive teacher collaboration (Brodie & Shalem, 2011; Chazan et al., 2009; Jaworski et al., 2017; Little & Horn, 2007; Potari, 2013; Robutti et al., 2016). An emerging theme of that work is the importance of centering teachers' interactions on what Ball and Cohen (1999) referred to as *records of practice* (e.g., videos, transcripts, student work, narrative accounts, and cases). However, merely using such records is not enough to ensure productive teacher collaboration. In their two-year observations of two teacher work groups, Horn and Little (2010) discovered "systematic, patterned differences" within the

conversational routines in the two groups, which accounted for why one group was more productive than the other. Specifically, the ways the groups handled teachers' accounts of classroom events differed—having differing propensities to turn the conversation toward or away from the teaching. Thus, even when representations of practice are used, important questions remain about how to maintain teachers' collaborative focus on instructional practice.

The introduction of a facilitator is one possible way to address these questions. Concerns exist, however, about the unequal positioning of teachers' knowledge that often prevail within approaches overly reliant on the facilitator (e.g., Goodchild, 2008; Hospesovà et al., 2006). Even when PD is centered on artifacts of practice, it is essential to consider how such artifacts are used with practicing teachers who have practical knowledge and can relate to those records. In facilitating these interactions, it is crucial to avoid "the use of artifacts of teaching as scripts that undermine teachers' abilities to exercise their judgment and adapt instruction to meet the constantly changing needs of their students and the different contexts of their work" (Zeichner, 2012, p. 379). Prescriptivism is problematic when working with practicing teachers as it ignores what teachers know and can potentially undermine, rather than promote, teachers' learning from collaboration. The prescriptive stance of a facilitator who might actively promote particular ways of relating to artifacts (e.g., defaulting to prescribing courses of action rather than trusting in teachers' capacities to exercise professional judgment, experiment with new instructional practices, and then learn from those experiences; Atweh, 2004; Smyth, 2007) is one aspect of facilitation practice that Story*Circles* strived to avoid when working with practicing teachers.

Foundations for the design of Story*Circles*

Story*Circles* is built on a foundation that combines prior scholarship with our prior practical experiences. As former secondary mathematics teachers, we realized that many innovations developed by those outside the classroom (ourselves included) fail to account for the situated nature of teaching (Chazan & Ball, 1999; Romagnano, 1994; Sherin, 2002; Silver & Stein, 1996). As teacher educators, we recognize the challenges that emerge when supporting teachers' learning, given the historical treatment of teachers as "pawns in the system" of school reform (Richardson, 1990, p. 12)—which often leads to teachers being guarded or mistrustful of our efforts. Those experiences also taught us the futility of an overly-simplistic assumption that conceives of PD as a kind of research-to-practice pipeline—deprofessionalizing teachers and exacerbating the research-practice divide (Silver & Lunsford, 2016). Finally, as researchers on teaching, we are aware that teachers, left to their own devices, do not always develop into the kind of practitioners that societies need for their children (Richardson, 1990). Our experiences left us longing for something different.

Beyond our own experiences, our development of Story*Circles* draws on diverse scholarship—including research on teacher work groups (e.g., Horn & Garner, 2022; Stoll et al., 2006; Vangrieken et al., 2017), PD schools (e.g., Chazan et al., 2009; Snow-Gerono, 2009); the lesson study approach (e.g., Fernandez & Yoshida, 2012; Shimizu & Kang, 2022), professional learning communities (e.g., Brodie, 2014; Vescio et al., 2008), communities of practice (e.g., Cobb & McClain, 2006; Stein et al., 2013), teachers' practical argument (e.g., Fenstermacher & Richardson, 1993; Gholami & Husu, 2010), practice-based teacher education (e.g., Ball & Forzani, 2009; Kavanagh et al., 2020; Lampert, 2010), and action research (e.g., Jaworski, 2006). While Herbst and Milewski (2018) describe in more detail how each foundation has shaped Story*Circles*; here, we

describe how Story*Circles* addresses the goals espoused by Hiebert and Morris (2012). Specifically, Story*Circles* engages teachers in collaboratively constructing digital lesson artifacts in the form of storyboards which are shareable, durable, editable, and amenable to analysis and reflection by groups of practitioners. This approach supports continued learning, as well as the representation and transmission of practical knowledge for sharing and vetting, and ultimately contributes to the improvment of instructional practices.

Failure-based learning in educational design

Although the education field has not fully embraced failure as an ally to creativity and innovation (Henriksen et al., 2021), slogans such as *"Fail often in order to succeed sooner"* (Kelley, 2001, p. 232) have grown in popularity in fields like engineering, business, and industry (Babineaux & Krumboltz, 2013; Maxwell, 2019). However, not all failures are equal. Organizational scientists distinguish between three types of failure: (1) preventable and predictable failure caused by deviance, inadequacy, or inattention to processes within well-established systems, (2) unavoidable failures that occur within complex and uncertain systems, and (3) intelligent failures that happen at the frontiers—where systems are not yet fully understood (Edmondson, 2011). When we use the term failure here, we refer to the third kind of failure—which provide rich opportunities for productive learning about complex systems. To learn from this type of failure, designers must prioritize their own learning above their tendencies to promote innovation's success. In this paper, we illustrate ways that we have engaged in failure-based learning through iterative designs of Story*Circles.* Earlier iterations included us as facilitators and deliberately small groups of participants. In the iterations discussed here, we aimed to scale our innovation closer to the conditions of real-world PD while making progress toward our goals of developing Story*Circles* into the kind of PD system envisioned by Hiebert and Morris (2012). By doing so, we demonstrate how productive learning can occur from intelligent failure, allowing for continued innovation and improvement.

Research questions

We investigate the following research questions:

What sorts of practice-based feedback may support teachers' learning from collaboration around a lesson when:

- Teachers' reactions to the visualization of a lesson do not reliably provide feedback on their scripting of the lesson?
- Teachers' narrative accounts about their own implementation of the lesson are not readily available?
- The triggers of the feedback needed are not easily represented in storyboards or, when represented, not reliably interpreted by teachers?

Findings: Using failure to innovate

Challenge 1: Visualization alone does not guarantee productive feedback

The first challenge we encountered in the Story*Circles* approach was the limitations of a milieu for teacher learning based solely on visualizing scripted lessons. We initially believed that visualizing lessons in the form of storyboards would help teachers notice and take up problematic aspects of their practice. To test this hypothesis, we carried out the EMATHS through Lesson*Sketch* Story*Circles* project[2]—our earliest attempt to use the Story*Circles* approach at scale (with participants drawn from a statewide network of secondary mathematics teachers and with facilitators other than ourselves). In this project, 16 experienced Algebra 1 and Geometry teachers—who had previously participated in 8 days of PD focusing on the EMATHS materials[3] and were known to be using those materials in their classrooms—were invited to participate in Story*Circles*. The teachers were asked to create storyboards representing how they would teach a given task from the EMATHS curricular materials. Our goal was to promote discussions of practice at a level of specificity that included moment-to-moment decisions, using synchronous and asynchronous technologies to connect the group of participants. While we did find evidence that participants could learn through a collective focus on visually represented lessons, we also found that lesson visualization did not always help teachers notice and take up problematic aspects of their practice. This highlighted the limitations of collective visualization of scripted lessons as a sole means for providing feedback to support teacher learning, and the need to consider alternative approaches to promote more nuanced discussions and growth.

The data collected in this initial implementation of StoryCircles provided evidence of the effectiveness of storyboarded representations of teachers' scripted ideas in enhancing participants' prior PD experiences. The visualization of one teacher's idea helped the group to realize that they did not entirely agree on how to proceed with the lesson, specifically on whether to bring the student to the board to present their work (Table 1).

While, the teachers had reached some agreement about what was to come next in the lesson, namely the class would "go over" students' work, it was not until the visualization was constructed that they realized they were not quite in agreement about what that looked like, specifically whether the teacher should "bring the kid up" to accompany their work. When reflecting on the experience, Maria and Daphne acknowledged the value of the visualization in helping to surface different perspectives, as the storyboarded lesson made them aware of the need to consider alternative approaches (Table 2).

These reflections demonstrate how visualization can facilitate conversations that might otherwise not occur, and how such conversations can facilitate conversations that lead to questioning assumptions about "the right way" to handle decisions. Indeed, had teachers merely *spoken* about sharing the students' work, ambiguity as to whether the student would

[2] EMATHS (Embracing Mathematics, Assessment, Technology in High School) through Lesson*Sketch* Story*Circles* was a two-year PD intervention project funded as a Mathematics Science Partnership project awarded to Deborah Ferry at the Macomb ISD and funded through the State of Michigan. The authors participated in this project through a subcontract to the Author Institute whereby they supported EMATHS facilitators as they used Story*Circles* with teachers. All opinions in this paper are those of the authors and do not necessarily represent the views of the organizations that supported the project or other individuals involved in the project.

[3] EMATHS curricular materials were developed in the context of a previous Mathematics Science Partnership projects awarded to Deborah Ferry at the Macomb Intermediate School District and funded through the State of Michigan in the U.S. The tasks can be found at http://emathsmi.com/resources.php.

Table 1 Excerpt from PD interaction

Facilitator	So what happens after the teacher walks around and …
Daphne	So then we're gonna go over each one. So go over the table, go over the graph, and go over the equation … maybe we'll start with the table first and then the equation, talk about the rate of change, and then put it on the graph
	[Teachers discuss what questions the teacher will ask about the student's table while the storyboarder works to add the image of the student work to the storyboard.]
Storyboarder	Is the student also at the board or is it just the student's work at the board?
	[9 s pause]
Maria	I don't know
Daphne	Say it again. What was the question?
Facilitator	[The question the storyboarder asked was whether] the teacher has taken the work up to the board and is presenting it to the class or if the teacher has asked the student to come to the board and share their work with the class. Who is presenting?
Maria	I guess we might as well bring the kid up
Ziya	[giggles] Yeah, for student participation
Maria	But it just takes so long. Like how long is this lesson gonna take for such a simple idea

Table 2 Excerpt from teachers' reflection on Story*Circles* indicating potential for visualization to support teachers' learning

Daphne	Well for me, [Story*Circles*] just gives different perspectives. The way she [gesturing towards Maria] would handle herself in class, I would handle it differently but when we come together collaboratively we can reach more students
Maria	[Collectively developing a lesson storyboard] kinda reminds me that I need to open up my classroom to let them [gesturing to the other teacher participants] show me different things
Daphne	When I said 'Do you let your kids go up to the board and show their answers? You know you [sic] thinking 'You know that's the right way to do it, don't let the students go to the board' but when you get other people's perspective on it, you're like 'Well maybe!'

accompany the work onto the board or not might be hidden under different interpretations of *sharing*.

While visualization is a powerful tool for providing feedback on teaching practices, it may not always lead to productive discussions among teachers. For example, when visualizing, participants sometimes realized the actions they scripted were not desirable (e.g., evaluating a student's misconception as incorrect) but struggled to come up with better alternatives. In such situations, the facilitators sometimes resolved the issue by sharing their own classroom experience. For example, during an asynchronous forum, a group of four secondary geometry teachers coalesced around the construction of a storyboarded lesson that began with the following frame (Fig. 1-left).

The facilitator, Naomi, attempted to intervene (Table 3) when she noticed that the group had included the area formula for a rhombus on the opening slide (Fig. 1-left) of the storyboard.

The teachers' stalwart commitment to maintaining the formulas on the board suggests that they desired the lesson to unfold this way. This interaction came to a head in the next meeting (Table 4).

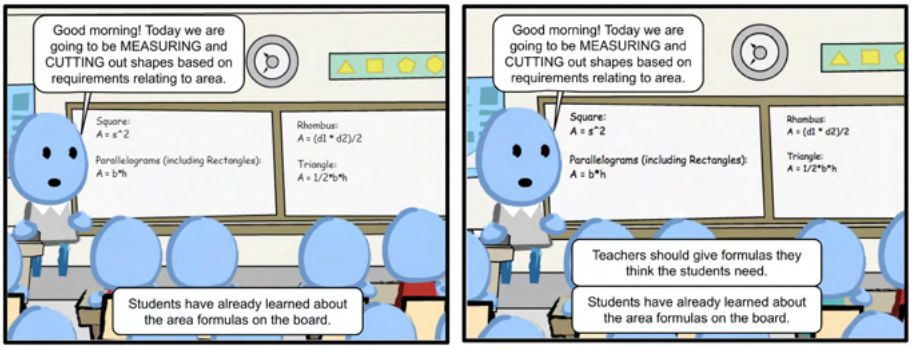

Fig. 1 Evolution to the opening storyboard frame constructed by EMATHS participants for a lesson entitled *The Wonderful World of Area. © The Regents of the University of Michigan, all rights reserved, used with permission*

Table 3 Excerpts from posts in the asynchronous forum focused on *Wonderful World of Area*

Naomi	Is it necessary to have the formula for area of a rhombus using the diagonals on the board? Or can this assignment be done without it? Do we help students with lots of formulas? Do we want students to be able to break figures into smaller pieces to find area?
Dana	[After posting the storyboard frame shown in Fig. 1-right] I guess I wasn't thinking about breaking figures down…especially in light of the student work where it looks like they are finding their area using diagonals. In some respects, I view this assignment as more of a "do you understand how and why the area formulas work" rather than "can you make these shapes." But that's my defense. I'm flexible…we could even write that it is up to the teacher which formulas they put on the board
Naomi	I believe the purpose of this activity is for students to look differently at area. Usually, students are given dimensions and have to find the area. This is a totally different process for them when they are given a needed area with different known figures and have to decide on the dimensions and construct the figure to their specifications[a]
Naomi	I would like to continue the discussion about the need for the diagonals of a rhombus area formula on the board
Dana	I guess I'm just thinking that rhombi are not understood as well as parallelograms, rectangles, squares, or triangles. So, if students are going to succeed, they need to have spent time with rhombi and that means that we would have already discussed this formula. We can scrap them if we want. I just think I would review them (because I definitely would have taught formulas before this lesson)
Naomi	We would have discussed the properties of a rhombus—diagonals perpendicular which then the students could think through this area differently without the formula
Terrie	In the last couple of years, I've actually vacillated between letting them use the formula sheet that's on the ACT[b] and on their tests and quizzes, but they would almost certainly have access to the formula before I would have done this lesson

[a]The approach to teaching area envisioned by the facilitator is similar to the one described by Walton and Randolph (2017)

[b]The ACT is a standardized test of college aptitude often used by school districts in the US to measure effectiveness of their high school mathematics programs and also by Colleges and Universities to inform the admissions process

While the teachers eventually agreed to modify the storyboard in the direction the facilitator was "pushing", the irony of a facilitator *telling* participants what to do in a

Table 4 Excerpt from third synchronous meeting

Naomi	Okay, so let's just go back to the beginning and kind of get ourselves ready for it—again because I still want to go to slide one [Fig. 1, right]
Dana	Of course you do. [smiling]
Naomi	Of course I do [smiling] because I have asked this question about five times on the forum and I have got a response from Dana and Terry, but I never heard from Sarah or Keith. I would like the four of you to decide. Is it necessary to have that rhombus formula on the board?
Dana	And that's where my answer comes. It's not—I don't know—the formulas are technically not necessary because they have them in the notebooks. That would be my assumption. However, it's just one of those—it's one more formula and we're finding this by area so that's why I threw it up there so that was my main argument—it's useful—but I love to hear [from others]
Keith	I like the options of having them there because it doesn't say you have to use this one. It's there for them to let them have choices available
Naomi	Sarah?
Sarah	I agree I like it up there. So I would say leave it up there
Dana	Naomi, I guess my question is why are you so against it. I feel like you're like [lowers his voice and growls out] 'No we shouldn't have it!' So—what's your issue? [smiling]
Naomi	This is what I am pushing: I believe that students are overwhelmed with formulas. So, there's a gazillion formulas that they're trying to figure out which one is the one that works. And from my own teaching career, I tried the least amount of formulas that I could come up with for students. That seems to work better than having a kazillion formulas. I mean, they work. You can have a formula for anything. You know that diagonal for the rhombus—it doesn't help them really understand the properties of the rhombus. And doesn't help when you're trying to get kids to take figures and dissect them, so you have two different figures. So, they're always looking for this one little formula that will always work. But that's me. But I can live with you guys wanting that formula

PD context that focused on supporting teachers to *not tell* students what to do was not lost on participants. Across their engagement, participants expressed confusion and even frustration about such choices (e.g., asking the facilitator whether the purpose of Story*Circles* was to document the facilitator's or the teachers' experiences). We observed several instances where participants abandoned their own ideas to align with the facilitator's, which raised concerns. Subsequently, some participants expressed privately to the project evaluator their wish to refocus on their own ideas rather than the facilitator's.

In our interactions with facilitators, we were also at a loss. Our failure to design the learning-through-collaboration environment to which we aspired became painfully poignant in moments where we found ourselves *telling* the facilitators *not to tell* the teachers *not to tell*. To enable participants' learning from practice, we still needed to find legitimate ways for the facilitator to organize teachers' engagement. In particular, we sympathized with Naomi's position to support participants' learning when they appeared to lack the requisite knowledge or experience needed to notice what was problematic when visualizing a lesson (Santagata, 2011).

While the canonical mechanisms in Story*Circles* for visualization to provide feedback on teachers' storyboards appeared to be insufficient for supporting development, the design of Story*Circles* did not offer a means for facilitators to modify the environment to trigger feedback without drawing excessive attention to their intentions.

In this section, we discussed the challenges we encountered while scaling our original design of Story*Circles*. Our initial design aimed to explore how visualization could enhance teacher collaboration. However, that design had drawbacks for supporting the

facilitators we partnered with to intervene. These facilitators were committed to supporting particular kinds of teaching and sometimes felt compelled to intervene directly on teachers' interactions, including challenging teachers to revise their storyboarded ideas to align with the original intent of the materials. This led to the facilitators resorting to tactics that undermine teachers' professionalism and sense making, as identified by previous researchers (Atweh, 2004; Richardson, 1990). By telling participants what to do, facilitators failed to acknowledge teachers' specialized knowledge about their practice, develop an atmosphere of collegiality, and grant teachers' sufficient autonomy in deciding what changes were significant and worthwhile (Atweh, 2004; Noddings, 1992; Richardson, 1990). Although this iteration of Story*Circles* provided evidence suggesting that visualizations enhance teacher collaboration, it also highlighted the reality that visualization alone does not guarantee productive feedback, especially when the knowledge needed to notice a problem does not emerge from the participants—as evidenced by moments in which the facilitator felt the need to intervene. Therefore, we began considering additional resources could serve as feedback on collaborative lesson development.

Challenge 2: When lesson implementation artifacts are not available to provide feedback

The first year of the EMATHS project was marked by several challenges and failures that led to a renewed iteration of the Story*Circles* design. We recognized the need to identify new resources that could provide feedback capable of challenging groups of teachers to reconsider aspects of their practice they collectively overlooked. Building on previous research, we recognized that teachers face specific tensions when implementing mathematical tasks in the classroom, such as balancing the task's intended goal with the direction students' work takes (Ball, 1993). Furthermore, we acknowledged the potential for these tensions to foster teacher learning, as highlighted in research by Stein et al. (1996). In the second year of the project, we expanded Story*Circles*' milieu to include artifacts collected by teachers within their lesson implementation. Our findings indicate that this expansion improved the Story*Circles* processes by providing additional feedback to participants. However, we also found that this enhanced design, like visualization alone, poses significant risks by relying heavily on participants' willingness and ability to document lessons, ultimately compromising the structure of PD.

In year two of EMATHS, we observed numerous incidents where implementation artifacts played a crucial role in informing teachers' revisions of the lesson scripts. During year one, a group of geometry teachers planned a lesson centered on the task shown in Fig. 2—beginning with students engaging in construction and concluding with students producing a proof. Those earlier discussions did not contain evidence that teachers recognized that neither the statement of the theorem nor the diagram adhered with the norms[4] for the situation of proving in high school geometry. Furthermore, the teachers' storyboard elided details about the transition from construction to proof.

In the project's second year, two teachers, Mac and Kelly, agreed to share their implementation of the lesson by providing artifacts. In their accounts, both teachers

[4] The problem stated in Fig. 2 fails to use the *diagrammatic register* "whereby the 'givens' and the 'prove' are stated in terms of specific objects in a diagram (i.e., using their labels)" (Herbst et al., 2013, p. 1).

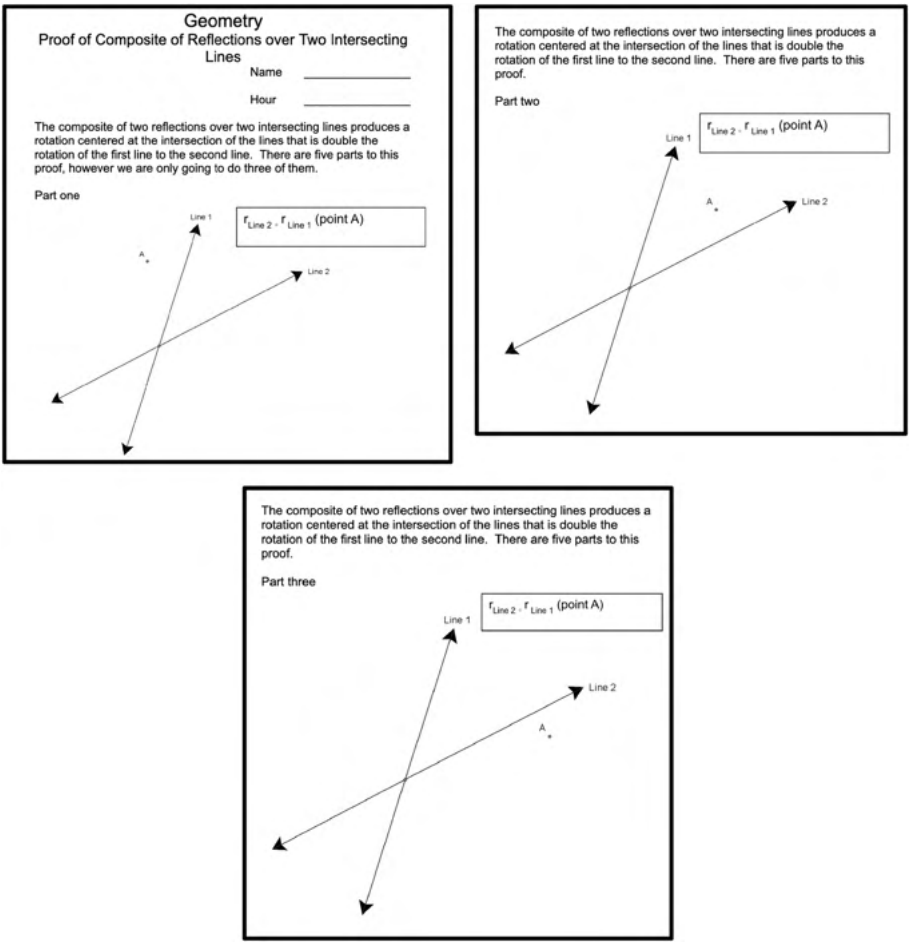

Fig. 2 Student handout containing the task around which teachers were developing a lesson

highlighted the difficulties that they faced in transitioning the class from construction to proof (as depicted in Fig. 3a, b). The teachers observed that while students had no trouble with the construction portion of the task, they struggled to identify what was given and what to prove because they lacked precise ways of formulating conjectures from the construction (as depicted in Fig. 3c). Furthermore, the variety of diagrams produced by students made it challenging for teachers to use them to transition to proof. In short, the outcome of the construction activity was insufficient to support the work that lay ahead. Mac overcame this situation by encouraging students to label their diagrams and use those labels to formulate conjectures. However, he found it challenging to represent this resolution in the storyboard, stating that "I don't think we can resolve this for the purposes of this depiction because every kid is gonna have their own points." He went on to say:

> *It's difficult … because obviously in that group they would give me an answer. Or they would say something to the effect of 'I don't have any points' and that's*

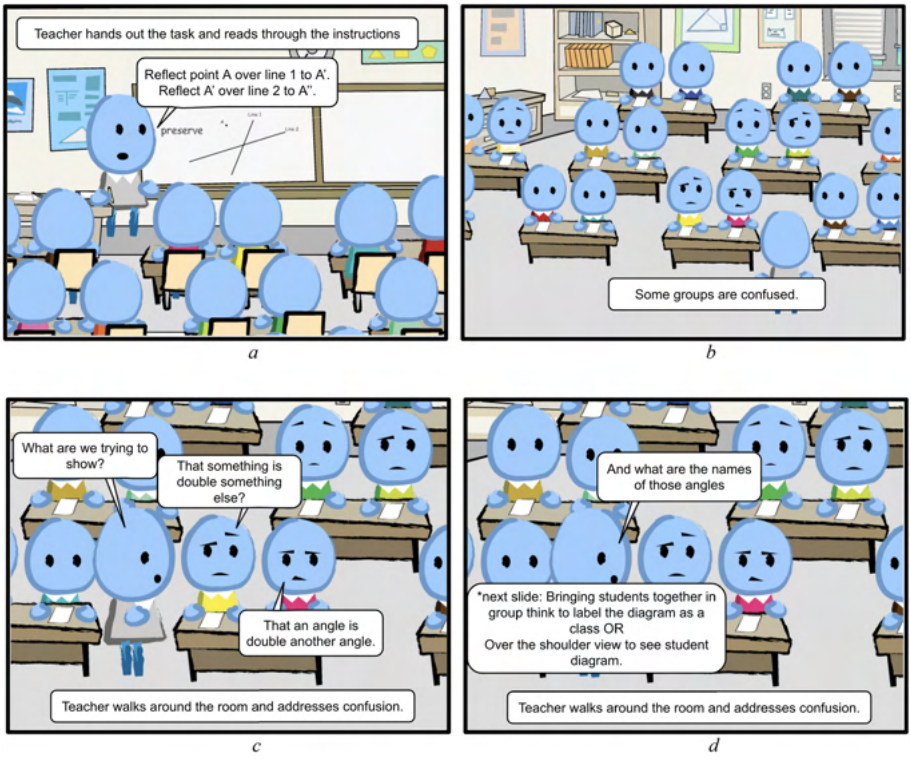

Fig. 3 Teachers' representation regarding the confusion that emerged from students' conjectures following the activity in which students constructed and labeled their own diagrams. © The Regents of the University of Michigan, all rights reserved, used with permission © 2023, The Regents of the Author Institute, all rights reserved, used with permission

> *where I would—I wouldn't even say anything, I would give them a look like 'Well maybe you better put some points on there?'*

Ultimately, the group decided to include a frame showing an "over the shoulder view" of a student's diagram with labels to demonstrate this resolution (as shown in Fig. 3d). The visualization of this resolution sparked new rounds of argumentation about alternatives. Terrie, another teacher, suggested a different way to resolve the issue, stating, "if the class begins to reach general consensus that they need to have a name for something that they are talking about, I might temporarily bring them all back together" in the hopes that "one group comes up with the idea, 'Hey maybe we should put some points on here'." Terrie's alternative was also captured in the same caption—prompting teachers to provide input on how to proceed.

The facilitator asked the group to "continue the discussion about whether we want to have the entire class label the points the same or let groups label as they wish" in the next week's asynchronous forum—where concerns emerged about Mac's approach to letting each student use their own chosen points to formulate the conjecture. Other teachers worried about the need for students to communicate in ways understandable to students with different diagrams. Eventually, the teachers resolved the issue by agreeing to allow students to label their diagrams as they wish but having the teacher color the diagram to give

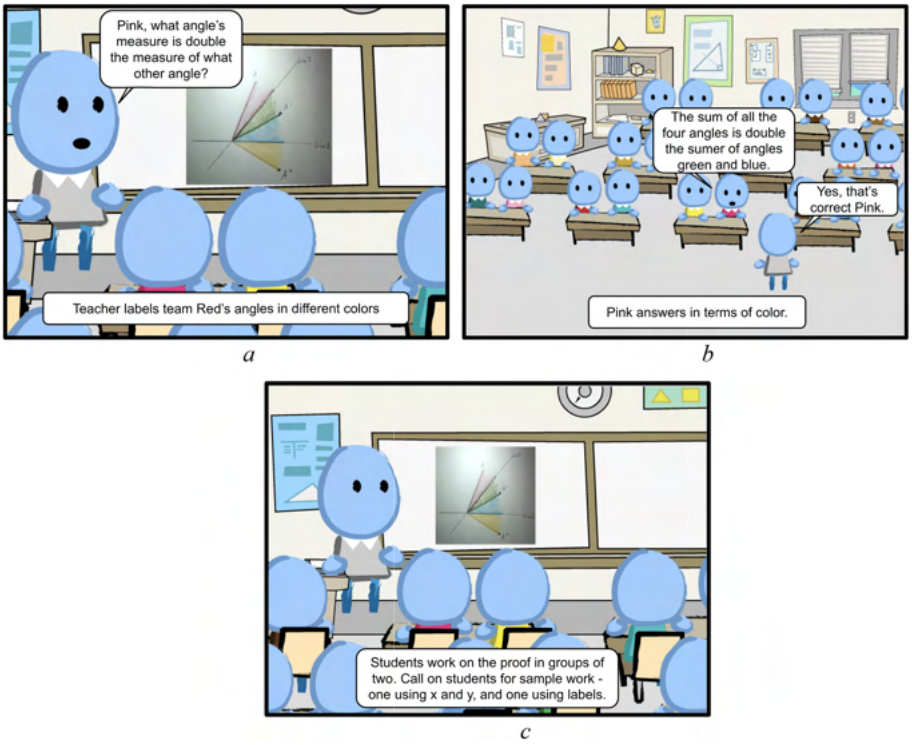

Fig. 4 Teachers' representation of how to handle the differences between the student-generated diagrams with colors. © The Regents of the University of Michigan, all rights reserved, used with permission

students a common way of referring to its constituent parts. This suggestion was used to collectively improve the lesson storyboard (as depicted in Fig. 4). Thus, we see evidence of how lesson implementation and visualization helped teachers to identify and wrestle with additional aspects of the lesson they had previously glossed over, without facilitator intervention.

Although records of classroom implementation were useful, we also experienced challenges leveraging teachers' classroom implementation as feedback. First, despite teachers' willingness to implement the lessons, they did not always manage to do so in time for the group to use those records. Second, teachers did not always bring back records from those implementations; and even when they did, the records weren't always sufficient for generating ideas to revise the scripts. To overcome these obstacles, we encouraged teachers to share accounts of the challenges that they faced during implementation, in the hopes of ushering the group's attention onto aspects of the lesson needing revision. Nonetheless, teachers' narratives were not consistently structured to highlight problems of practice the group could effectively address (e.g., one account featured the school building's fire alarm going off as a problem).

In this section, we have examined the opportunities and challenges of utilizing teachers' implementation of lessons as feedback for collective lesson development. We overcame many of these challenges with redundancies (having multiple teachers cued up to implement a lesson) and support (providing human and material resources to

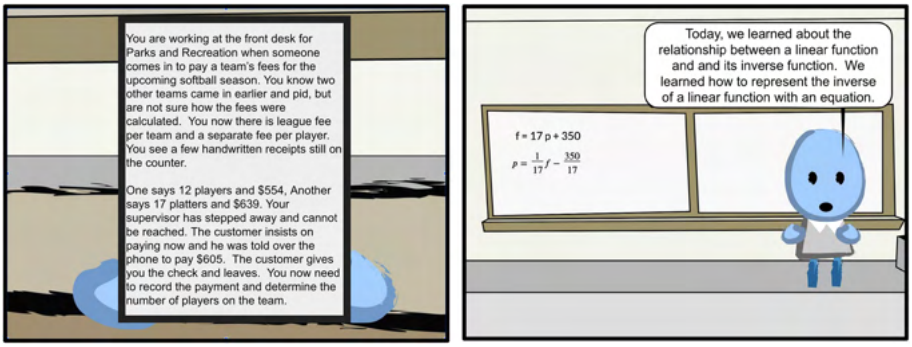

Fig. 5 One of the starting (left) and ending (right) storyboard frames provided to teachers. © The Regents of the University of Michigan, all rights reserved, used with permission

support lesson documentation). This iteration yielded promising evidence that teachers' implementation of lessons can complement storyboards providing feedback on scripted lessons, hence improving Story*Circles*' capacity to promote collaborative learning. Yet, this design has its challenges: Specifically, there are dangers in becoming overly reliant on participants' lesson documentation. This reliance would hinder our ability to implement Story*Circles* during certain seasons, such as summer months, when teachers are available to participate in more intensive PD. We realized that we had yet to create a model that could effectively surface the kinds of practical problems teachers can be expected to experience in practice without relying on a facilitator's guidance or the participants' ability and willingness to implement lessons. These realizations led us to conclude that we need to explore alternative approaches to provide feedback on participants' lesson anticipation in order to achieve success with our design.

Challenge 3: Using lesson contingencies as sources of feedback

In this section, we introduce an innovation called *contingency cards* for addressing the challenge of providing reliable sources of challenge to Story*Circles*[5] participants' lesson anticipations. Drawing inspiration from PD literature demonstrating that artifacts of practice can support teacher learning (Fennema et al., 1996), we introduced *contingency cards* (Brown et al., 2021)—storyboard frames that usher teachers into potential implementation scenarios and prompt discussion around what actions a teacher may need to take to manage such scenarios. We share evidence demonstrating the potential of contingency cards to provide facilitators with a resource to challenge teachers' collaboration and prompt them to share knowledge that they are otherwise prone to leave unpacked. We also share evidence for the challenges facilitators sometimes faced when introducing the cards—with participants sometimes misunderstanding the facilitators' intentions which led them to treat the cards as a non-negotiable aspect of the lesson or as a separate activity unto itself. Both of these treatments were problematic—the former violated the principle of supporting

[5] Developed in the context of a project entitled "Managing Students Contributions to Mathematical Work in Whole Class Discussions in High School: How Do Teachers Decide What to Do?" funded by the James S. McDonnell Foundation awarded to Patricio Herbst.

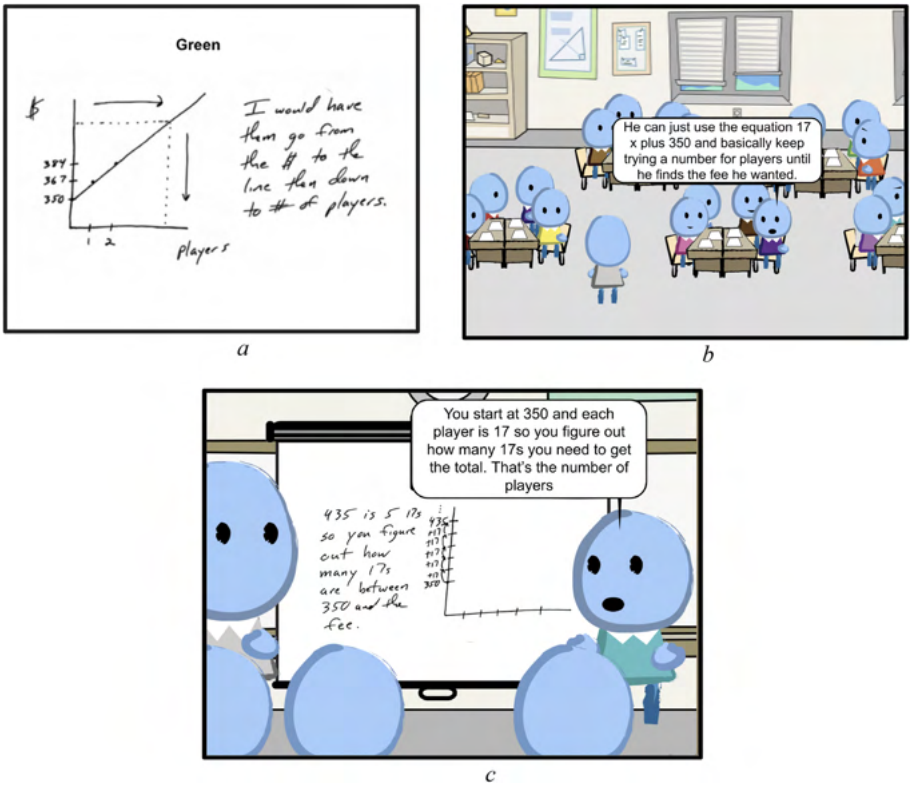

Fig. 6 Sample contingency cards representing student conceptions. © The Regents of the University of Michigan, all rights reserved, used with permission

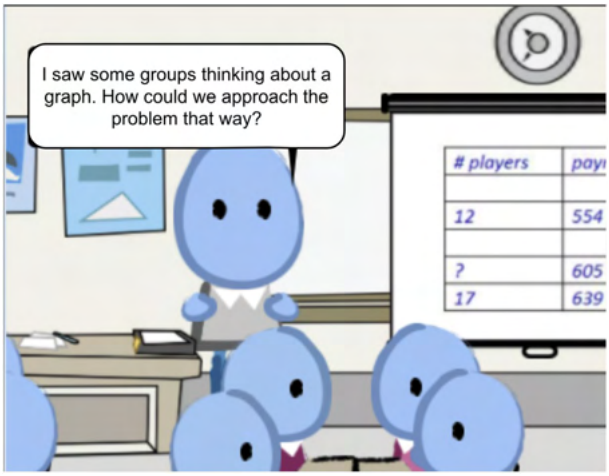

Fig. 7 One of the lesson storyboard frames the teachers had constructed. © The Regents of the University of Michigan, all rights reserved, used with permission

teachers' agency and learning from collective practical experiences, while the latter derailed our goal of centering activities around the collective design of lessons.

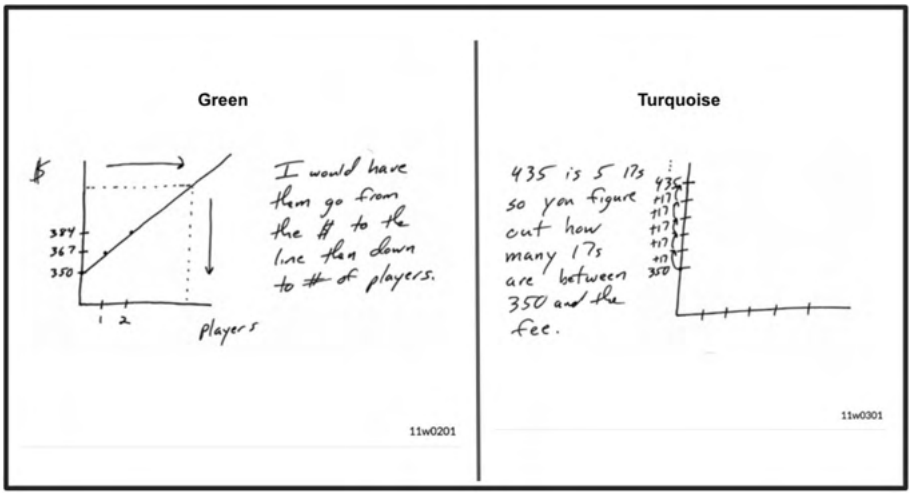

Fig. 8 Contingency cards used by the facilitator to support teachers to specify storyline details. © The Regents of the University of Michigan, all rights reserved, used with permission

To illustrate both the potential and the challenges of contingency cards, we share data from a 6-week Algebra Story*Circle* which included 8 practicing teachers. The group worked on a lesson storyboard that started with a given task (Fig. 5-left) and concluded with the teacher asserting the learning goals that avowedly had been accomplished through a discussion of students' work on the task (Fig. 5-right).

Drawing on previously collected records of practice and the literature on student conceptions (Milewski et al., 2020), we developed cards representing various conceptions that teachers might confront within a lesson implementation. To explore the potential of these cards, we intentionally represented student conceptions in different ways (as shown in Fig. 6).

Like the previously described innovations, we found ample evidence that contingency cards hold potential for supporting teachers to collaborate about aspects of practice they may otherwise overlook. For example, during the group's first meeting, participants decided the teacher would feature two pieces of student work—a table and a graph—after providing students with time to work on the problem. However, due to time constraints, the group was unable to script the details of the graph; with Fig. 7, being the final frame, they constructed during the meeting.

The facilitator, Benard, closed by asking them to continue the storyline in the asynchronous forum. When reviewing what teachers produced, Bernard noticed that while they shared ideas to further the storyline, they did so in ways that left details about the student work unspecified—drafting dialogue about "the graph" in vague terms making it difficult to envision (e.g., suggesting the teacher ask "What do we notice about the graph?" without describing the graph they were envisioning).

Knowing the details of the graph were crucial, Bernard focused the next meeting around two contingency cards (Fig. 8). During the meeting, Bernard asked the teachers, "If these [two cards] are the things that the teacher has seen … what is it going to look like to then move on from this point in the story?"

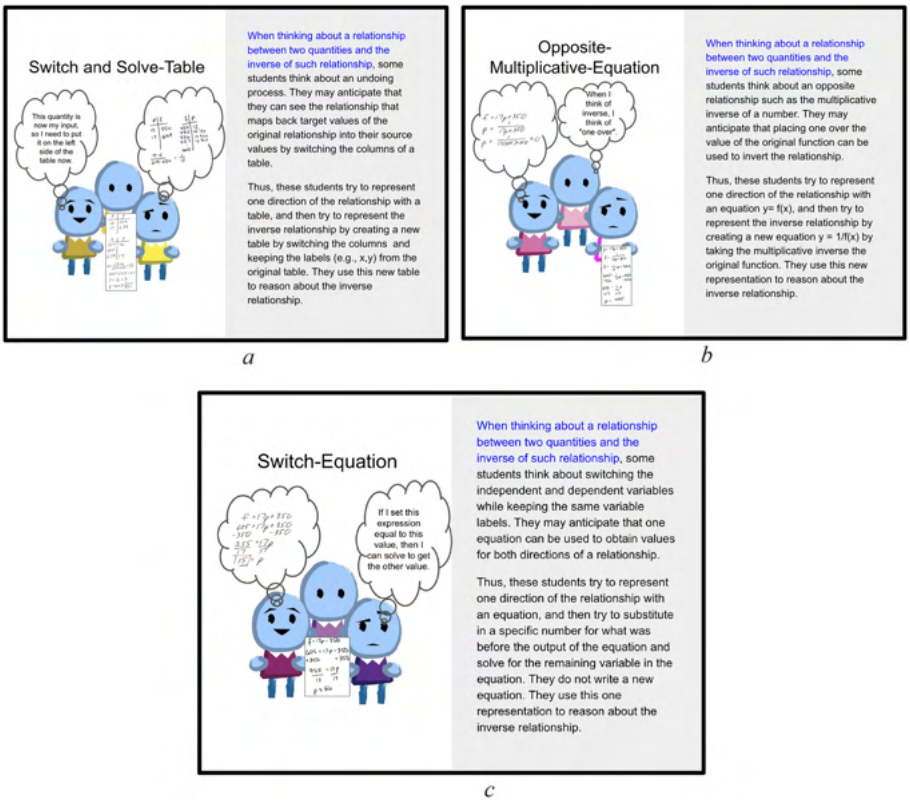

Fig. 9 Three conception biography cards. © The Regents of the University of Michigan, all rights reserved, used with permission

When presented with the two contingency cards, participants expressed uncertainty about the student on the right, indicating that they did not interpret the student's work as a graph—saying it "almost looks like what you might have on a table." The contingency cards helped participants realize that not all graphs were suitable for their purposes. Also, the cards facilitated a more productive discussion about the details of the graph on the left, focusing their attention on how to best handle it in their lesson.

The implementation of the contingency cards, however, was not without its challenges.

Prior to the fourth meeting, Bernard expressed concerns regarding the participants' use of a 'show and tell' approach for organizing the discussion (Stein et al., 2008)—failing to prioritize certain ideas over others. To address this, Bernard felt the need to focus teachers' attention on strategically selecting and sequencing students' work toward a coherent mathematical storyline that achieves the lesson goal. We wondered whether the nature of the contingency cards used thus far (Fig. 6) was contributing to the problem. It seemed possible that participants mistakenly perceived Bernard's use of the cards as mandating certain aspects of the lesson rather than recognizing the cards as resources for collaborative scripting. Because of this, we introduced a different kind of contingency card, the *conception biography cards* (Fig. 9)—which contain more decontextualized descriptions of student conceptions.

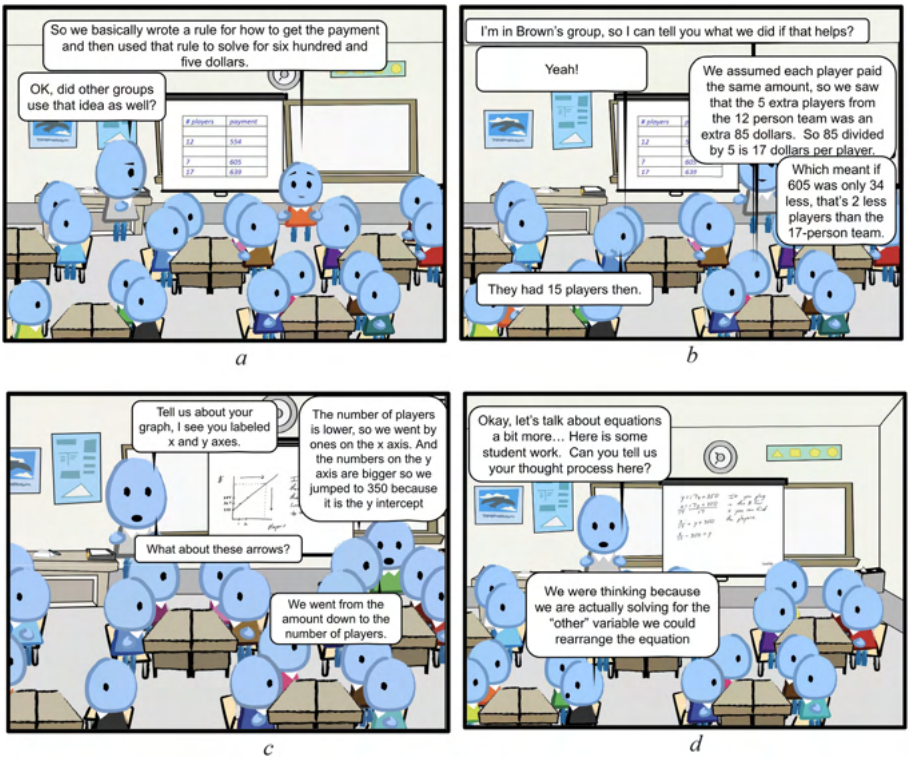

Fig. 10 Four student contributions reviewed by participants. © The Regents of the University of Michigan, all rights reserved, used with permission

Because Bernard wanted to encourage participants to think strategically about selecting and sequencing student work; he intentionally selected some cards representing conceptions, and he believed teachers would disprefer (Fig. 9b). At the beginning of the meeting, Bernard emphasized the importance of keeping their focus on the lesson goal and posed the question, "If we're going to try to do this—in, say a 60 or 70 min class period, which of these conceptions might feel really essential to talk about?" Bernard asked the teachers to react to the cards by sharing which ones they thought were essential and would realistically emerge from students. However, the teachers continued to talk about the conceptions in ways that suggested they found them all realistic and essential. For instance, when discussing the Opposite-Multiplicative-Equation card, one teacher said, "I have some students—very high functioning—they might look at it that way." The teachers were unwilling to differentiate between the conceptions and identify which ones were essential to include in the whole class discussion—which already featured four pieces of student work.

Realizing the cards, as implemented, were not serving his goals, Bernard shifted teachers' attention back to the storyline. After reminding the participants of the session goal ("paring down the story that we have") and the lesson goal (Fig. 5-right), Bernard asked participants to review the existing lesson storyline—considering whether each student contribution (Fig. 10) was essential to achieve the lesson goal.

Unlike the prior interactions with the conception biography cards, Bernard was able to use these cards (Fig. 10) to elicit the kinds of reasoning that he was hoping for. Out of the four teachers, only two considered the second piece of work (Fig. 10b) crucial for the lesson's objective, and none of them deemed the third piece (Fig. 10c) essential. When reviewing the third piece of work, teachers justified their decision to exclude it by saying things like:

> *I think that what we've already talked about in the lesson is already covered in this … They probably have this on their papers, I could have walked around and already acknowledged that this is on their papers. We don't really need to have a slide that says this happened, I think it's already happened.*

This reasoning differed from that elicited by the previous conception biography card activity, where Bernard struggled to convince the teachers to prioritize certain aspects of the lesson.

Thus, the contingency cards seem to have enabled teachers to reason about the role of the student contribution in the context of the lesson timeline and to identify which pieces of work required less emphasis. This finding highlights the usefulness of contingency cards for facilitating teacher reasoning and decision-making in the classroom.

After that meeting, Bernard reflected on why the conception biography cards failed to achieve their intended purpose.

> *We just weren't talking at a level of 'Okay when kids are talking about inverses, kids sometimes think this and sometimes they think that.' It was very much like 'Well sure if they say this, I'd love to hear what they have to say, of course. I want to hear every group.' And I was like 'Well, okay' [shrugs] and I didn't know how to push back on that.*

One possible explanation for the failure is that Bernard saw the cards as resources to script a leaner storyline, whereas the teachers understood the cards as structuring a more decontextualized activity that focused on how they would handle individual students. The disconnect could have resulted from the decontextualized nature of the cards or the decontextualized manner in which the cards were presented. Another possible explanation is that both Bernard and the teachers saw the cards as structures for an activity, but they had different understandings of that activity: Bernard understanding the cards as emphasizing selecting and sequencing practices, while the teachers thinking the cards were emphasizing responding practices. The ambiguity of the cards' purpose and introduction may have left too much open to interpretation—failing to provide interlocutors with a shared understanding of the target practice.

Finally, there is a chance that participants misunderstood Bernard's intention—perceiving him as insisting they discuss the cards, rather than offering alternative approaches for consideration. This possible misinterpretation left us wondering about alternative ways to introduce the contingency cards into the flow of participants' activities that are less reliant on the facilitator, such as having rules for when cards are drawn and used by participants at different points in the lesson development.

In this section, we shared about some of the challenges that arose during our attempt to use contingency cards as feedback sources on teachers' lesson anticipation. Although the cards sometimes served their intended purposes, we also faced challenges in designing the cards and activity structures to ensure their consistent effectiveness. This predicament led us to realize that we have not yet achieved our goals of supporting a Story*Circles* facilitator with reliable resources for focusing participants'

attention on specific aspects of practice. To address this particular failure, we need a deeper understanding of how teachers interpret and attend to different designs and applications of contingency cards.

Discussion

This article outlines the challenges we encountered when designing structures within the Story*Circles* process to provide teachers with the necessary feedback to provoke them to learn from practice and each other. These challenges raise a crucial question regarding the design of practice-based professional development: *How can we design structures that can stimulate teachers to learn from practice when the knowledge needed for practice is not present among the participants, or when access to actual classrooms is limited?* While we are optimistic about the potential of contingency cards for helping us address this question in our continued design of Story*Circles*, we still have much to learn. Problems of practice emerged that either we or the facilitator wanted to address, but none of the contingency cards seem adequate to tackle. For instance, we have not yet designed cards suitable for addressing deficit perspectives teachers sometimes hold about children.

Beyond illustrating challenges, we have demonstrated how these challenges sensitized us to the ways that our design of Story*Circles* failed to measure up to our aspirations. When we say that our design failed, we are not suggesting that it failed in an overall sense—even our earlier versions of Story*Circles* were successful in supporting teacher collaboration and growth (Milewski et al., 2018; Herbst et al., 2020). Instead, we mean that the design processes failed to address specific situations that we anticipate will arise with some regularity. If these situations are not adequately addressed, future users of Story*Circles* may resort to less satisfactory solutions that undermine the fundamental principles of Story*Circles.*

It is crucial to acknowledge that not all the challenges we encountered prompted us to pivot our design. While the challenges we featured here accumulated in ways that suggested our design failed in crucial circumstances, this was not always the case. Furthermore, the challenges we highlighted are specifically related to principled elements of our design—teacher learning is intended to occur primarily through discussion about practice supported by interactions around a lesson rather than through direct intervention of a facilitator. When we initially encountered these challenges, we did not believe that they warranted a pivot in our design. Instead, we focused on enhancing the visualization phase to enable feedback that would support richer discussions—incorporating artifacts from real-world practice or introducing lesson contingencies. We addressed these challenges incrementally, rather than assuming they indicated fundamental flaws in the design.

This approach to challenges is common among entrepreneurs and businesses (McMullen, 2015). A recent systematic literature review on pivot decisions distinguishes between different types of pivots. Pivot design, which involves making changes in the strategic design or execution of a design process, is considered risky due to irreversible commitments and unknowable outcomes (Chaparro & de Vasconcelos Gomes, 2021). Nonetheless, organizations and their stakeholders are expected to pivot design regularly to support ongoing innovation. To handle these decisions responsibly, individuals should (1) only make pivot design decisions when compelling evidence suggests such changes are necessary and (2) follow a structured process that includes problem formation, ideation, preparation, assessment, and scaling up (Chaparro & de Vasconcelos Gomes, 2021).

Conclusion

In conclusion, this article contributes to two important areas of literature. Firstly, it contributes to the literature on the importance of embracing creative risk and failure-based learning for instructional designers to accelerate innovation. Secondly, it adds to the body of scholarship on innovative approaches for supporting teacher learning through collaborative lesson planning (i.e., lesson study, teacher work groups, professional learning communities). Using data from three different Story*Circles* implementations, we have demonstrated how failure can be leveraged to spur ideation and innovation. We hope that this contribution will inspire other instructional designers to embrace challenges and failures as opportunities for innovation, while also providing the reader with valuable insights into what failure-based learning can look like. In our experience, embracing and learning from failure necessitates both a readiness to endure discomfort and a willingness to make incremental changes, rather than hastily abandoning principled decisions. These dispositions are further strengthened by adopting a broader perspective on design failures, acknowledging that advancing the field of teaching necessitates more than one-teacher-at-a-time approaches that profess at a sluggish pace (Hiebert & Morris, 2012). Instead, with patience, we elect to embrace a failure—constantly reminding ourselves of the aspirations we hold for Story*Circles.*

This work enriches the existing literature on teacher learning through collaborative lesson planning by presenting a system that elicits and documents teachers' practical knowledge in a format that is accessible, adaptable, and continually enhanced. In this regard, every failure we encountered during the development of Story*Circles* served as unvaluable feedback, enabling us to refine our skills as designers and enhance the quality of our own work. By focusing on teachers and their knowledge as the central resource, we aim to disrupt the conventional approach to professional development—offering a more collaborative and collective approach to advancing professional growth. Nevertheless, this disruption necessitates a shift in the facilitator's role, transforming them from that of expert to that of co-learner (Schwarts et al., 2022), which can be challenging and may result to additional failures. However, documenting these "failures at the frontier" has provided us with the needed insights to refine Story*Circles* and enhance its feedback mechanisms. We hope that sharing our experiences and insights will help other designers of professional development programs to gain deeper understandings of the potential and limitations of lesson visualization, artifacts, and contingencies for fostering teachers' learning from practice and from each other within a collaborative context.

Funding This work and writing of this paper is done with support of the National Science Foundation (Grant DRK12-2201087) to A. Brown and P. Herbst and the James S. McDonnell Foundation (Grant 220020524) to P. Herbst. All opinions are those of the authors and do not represent the views of the foundations.

References

Atweh, B. (2004). Understanding for changing and changing for understanding. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education* (pp. 187–205). Springer.

Babineaux, R., & Krumboltz, J. (2013). *Fail fast, fail often: How losing can help you win*. The Penguin Group.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal, 93*(4), 373–397. https://doi.org/10.1086/461730

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession* (pp. 3–31). Jossey-Bass.

Ball, D. L., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60*(5), 497–511. https://doi.org/10.1177/0022487109348479

Ben-Peretz, M., & Schonmann, S. (2000). *Behind closed doors: Teachers and the role of the teachers' lounge*. SUNY Press.

Berg, C. V. (2011). In-service teachers' professional development: Which systemic aspects are involved? *Research in Mathematics Education, 13*(2), 223–224. https://doi.org/10.1080/14794802.2011.585832

Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics, 85*(2), 221–239. https://doi.org/10.1007/s10649-013-9507-1

Brodie, K., & Shalem, Y. (2011). Accountability conversations: Mathematics teachers' learning through challenge and solidarity. *Journal of Mathematics Teacher Education, 14*(6), 419–439. https://doi.org/10.1007/s10857-011-9178-8

Brousseau, G. (1997). *Theory of didactical situations in mathematics.* N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield (Eds. & Trans.). Kluwer.

Brown, A., Stevens, I., Herbst, P., & Huhn, C. (2021). Confronting teachers with contingencies to support their learning about situation-specific pedagogical decisions in an online context. In K. Hollebrands, R. Anderson, & K. Oliver (Eds). *Online Learning in Mathematics Education.* (pp. 291-316). Springer Cham.

Campbell, M. P. (2009). Mathematics teachers and professional learning communities: understanding professional development in collaborative settings. In S. L. Swars, D. W. Stinson, & S. Lemons-Smith (Eds.), *Proceedings of the 31st annual meeting of the North American chapter of the international group for the psychology of mathematics education* (pp. 956–964). PME.

Chen, C. (2012). Learning to teach from anticipating lessons through comics-based approximations of practice. Unpublished dissertation, The Author Institute.

Climent, N., Codes, M., & Carrillo, J. (2020). Mathematics teachers' professional development: principles and challenges in a collaborative setting. In H. Borko & D. Potari (Eds.) *Teachers of mathematics working and learning in collaborative groups: Proceedings of the 25th ICMI Study. The University of Lisbon.* (pp. 270–277). http://icmistudy25.ie.ulisboa.pt/.

Chaparro, X. A. F., & de Vasconcelos Gomes, L. A. (2021). Pivot decisions in startups: A systematic literature review. *International Journal of Entrepreneurial Behavior and Research, 27*(4), 884–910. https://doi.org/10.1108/IJEBR-12-2019-0699

Chazan, D., & Ball, D. (1999). Beyond being told not to tell. *For the Learning of Mathematics, 19*(2), 2–10.

Chazan, D., Callis, S., & Lehman, M. (2009). *Embracing reason: Egalitarian ideals and the teaching of high school mathematics*. Routledge.

Cobb, P., & McClain, K. (2006). The collective mediation of a high stakes accountability program: Communities and networks of practice. *Mind, Culture, and Activity, 13*, 80–100. https://doi.org/10.1207/s15327884mca1302_2

Cropley, A. (2020). Creativity-focused technology education in the age of industry 4.0. *Creativity Research Journal, 32*(2), 184–191. https://doi.org/10.1080/10400419.2020.1751546

Darling-Hammond, L. (1990). Teacher professionalism: Why and how? In A. Lieberman (Ed.), *Schools as collaborative cultures: Creating the future now* (pp. 25–50). Falmer Press.

Edmondson, A. C. (2011). Strategies for learning from failure. *Harvard Business Review, 89*(4), 48–55.

Fennema, E., Carpenter, T., Franke, M., Levi, L., Jacobs, V., & Empson, S. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal of Research in Mathematics Education, 27*(4), 403–434. https://doi.org/10.5951/jresematheduc.27.4.0403

Fenstermacher, G. D., & Richardson, V. (1993). The elicitation and reconstruction of practical arguments in teaching. *Journal of Curriculum Studies, 25*(2), 101–114. https://doi.org/10.1080/0022027930250201

Fernandez, C., & Yoshida, M. (2012). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. Routledge.

Gholami, K., & Husu, J. (2010). How do teachers reason about their practice? Representing the epistemic nature of teachers' practical knowledge. *Teaching and Teacher Education, 26*(8), 1520–1529. https://doi.org/10.1016/j.tate.2010.06.001

Glazier, J. A., Boyd, A., Bell Hughes, K., Able, H., & Mallous, R. (2017). The elusive search for teacher collaboration. *The New Educator, 13*(1), 3–21. https://doi.org/10.1080/1547688X.2016.1144841

Goodchild, S. (2008). A quest for 'good' research. In B. Jaworski & T. Wood (Eds.), *The International Handbook of Mathematics Teacher Education (Vol 4) The mathematics teacher educator as a developing professional* (pp. 201–220). Sense Publisher.

Henriksen, D., Mishra, P., Creely, E., & Henderson, M. (2021). The role of creative risk taking and productive failure in education and technology futures. *TechTrends*. https://doi.org/10.1007/s11528-021-00622-8

Henriksen, D., Richardson, C., & Mehta, R. (2017). Design thinking: A creative approach to educational problems of practice. *Thinking Skills and Creativity, 26*, 140–153. https://doi.org/10.1016/j.tsc.2017.10.001

Herbst, P., Kosko, K., & Dimmel, J. (2013, November). How are geometric proof problems presented? Conceptualizing and measuring teachers' recognition of the diagrammatic register. *Proceedings of the 2013 Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Available from Deep Blue at the University of Michigan http://hdl.handle.net/2027.42/97761

Herbst, P., Chieu, V., & Rougée, A. (2014). Approximating the practice of mathematics teaching: What learning can web-based, multimedia storyboarding software enable? *Contemporary Issues in Technology and Teacher Education, 14*(4). Retrieved from http://www.citejournal.org/vol14/iss4/mathematics/article1.cfm

Herbst, P., & Milewski, A. M. (2018). What StoryCircles can do for mathematics teaching and teacher education? In R. Zazkis & P. Herbst (Eds), *Mathematical Dialogue: Scripting approaches in Mathematics Education Research and Practice.* (pp. 321–364). Advances in Mathematics Education. Springer, Cham.

Herbst, P., Ko, I., & Milewski, A. (2020). A heuristic approach to assess change in mathematical knowledge for teaching geometry after a practice-based professional learning intervention. *Research in Mathematics Education. 22*(2), 188-208. https://doi.org/10.1080/14794802.2019.1704851

Hiebert, J., & Morris, A. K. (2012). Teaching, rather than teachers, as a path toward improving classroom instruction. *Journal of Teacher Education, 63*(2), 92–102. https://doi.org/10.1177/0022487111428328

Horn, I., & Garner, B. (2022). *Teacher learning of ambitious and equitable mathematics instruction: A sociocultural approach*. Routledge.

Horn, I. S., & Little, J. W. (2010). Attending to problems of practice: Routines and resources for professional learning in teachers' workplace interactions. *American Educational Research Journal, 47*(1), 181–217. https://doi.org/10.3102/0002831209345158

Hospesovà, A., Machàckovà, J., & Tichà, M. (2006). Joint reflection as a way to cooperation between researchers and teachers. In J. Novotna, E. Jarmila, H. Moraova, M.Kratka, & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International group for the Psychology of Mathematics Education* (Vol. 1, pp. 99–103). PME.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9*(2), 187–211. https://doi.org/10.1007/s10857-005-1223-z

Jaworski, B. (2008). Building and sustaining inquiry communities in mathematics teaching development: Teachers and didacticians in collaboration. In K. Krainer & T. Wood (Eds.), *The international handbook of mathematics teacher education* (Vol. 3, pp. 309–330). Rotterdam: Sense Publishers.

Jaworski, B., Chapman, O., Clark-Wilson, A., Cusi, A., Esteley, C., Goos, M., Isoda, M., Joubert, M., & Robutti, O. (2017). Mathematics teachers working and learning through collaboration. In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education* (pp. 261–276). Springer.

Kapur, M. (2012). Productive failure in learning the concept of variance. *Instructional Science, 40*(4), 651–672. https://doi.org/10.1007/s11251-012-9209-6

Kavanagh, S. S., Conrad, J., & Dagogo-Jack, S. (2020). From rote to reasoned: Examining the role of pedagogical reasoning in practice-based teacher education. *Teaching and Teacher Education, 89*(2020), 1–11. https://doi.org/10.1016/j.tate.2019.102991

Kelley, T. A. (2001). *The art of innovation: Lessons in creativity from IDEO, America's leading design firm* (Vol. 10). Broadway Business.

Kennedy, M. M. (2009). *Inside teaching*. Harvard University Press.

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education, 61*(1–2), 21–34. https://doi.org/10.1177/0022487109347321

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., Cunard, A., & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education, 64*(3), 226–243. https://doi.org/10.1177/0022487112473837

Little, J. (1990). The persistence of privacy: Autonomy and initiative in teachers. *Teachers College Record, 91*(4), 509–536. https://doi.org/10.1177/016146819009100403

Little, J. W., & Horn, I. S. (2007). "Normalizing" problems of practice: Converting routine conversation into a resource for learning in professional communities. In L. Stoll & K. S. Louis (Eds.), *Professional learning communities: Divergence, detail and difficulties* (pp. 79–92). Open University Press.

Maxwell, J. C. (2019). *Leadershift: The 11 essential changes every leader must embrace*. HarperCollins Leadership.

McMullen, J. S. (2015). Entrepreneurial judgment as empathic accuracy: A sequential decision-making approach to entrepreneurial action. *Journal of Institutional Economics, 11*(3), 651–681. https://doi.org/10.1017/S1744137413000386

Milewski, A. M., Herbst, P., Bardelli, E., & Hetrick, C. (2018). The role of virtual spaces for professional growth: Teachers' engagement in virtual professional experimentation. *Journal of Technology and Teacher Education., 26*(1), 103–126.

Milewski, A. M., Herbst, P. G., & Stevens, I. (2020). Managing to collaborate with secondary mathematics teachers at a distance: Using storyboards as a virtual place for practice and consideration of realistic classroom contingencies. In Ferdig, R. E., Baumgartner, E., Hartshorne, R., Kaplan-Rakowski, R. & Mouza, C. (Eds.) *Teaching, technology, and teacher education during the COVID-19 pandemic: Stories from the field.* (pp. 623–630). Association for the Advancement of Computing in Education. https://www.learntechlib.org/p/216903/

Noddings, N. (1992). Professionalization and mathematics teaching. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 197–208). Macmillan.

Papert, S., & Harel, I. (1991). Situating constructionism. In I. Harel & S. Papert (Eds.), *Constructionism* (pp. 1–11). Ablex.

Potari, D. (2013). The relationship of theory and practice in mathematics teacher professional development: An activity theory perspective. *ZDM Mathematics Education, 45*(4), 507–519. https://doi.org/10.1007/s11858-013-0498-2

Richardson, V. (1990). Significant and worthwhile change in teaching practice. *Educational Researcher, 19*(7), 10–18.

Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., Goos, M., Isoda, M., & Joubert, M. (2016). ICME international survey on teachers working and learning through collaboration: June 2016. *ZDM Mathematics Education, 48*(5), 651–690. https://doi.org/10.1007/s11858-016-0797-5

Romagnano, L. (1994). *Wrestling with change: The dilemmas of teaching real mathematics*. Greenwood.

Santagata, R. (2011). From teacher noticing to a framework for analyzing and improving classroom lessons. In M. G. Sherin, V. R. Jacobs, & R. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 152–168). Routledge.

Schwarts, G., Stevens, I., Herbst, P., & Brown, A. (2022). It's a different mindset here: Facilitation challenges in a practice-based professional development. In Lischka, A. E., Dryer, E. b., Hones, R. S., Lovett, J., Strayer, J. & Drown, S. (Eds). *Proceedings of the 44th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1461–1469). Middle Tennessee State University.

Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Instruction, 20*(2), 119–150. https://doi.org/10.1207/S1532690XCI2002_1

Shimizu, Y., & Kang, H. (2022). Discussing students' thinking and perspectives for improving teaching: An analysis of teachers' reflection in post-lesson discussions in lesson study cycles. *ZDM Mathematics Education, 54*(2), 419–431. https://doi.org/10.1007/s11858-022-01371-5

Silver, E. A., & Lunsford, C. (2016). Linking research and practice in mathematics education: Perspectives, problems, and possible pathways. In J. Cai (Ed.), *Compendium for Research in Mathematics Education.* National Council of Teachers of Mathematics.

Silver, E. A., & Stein, M. K. (1996). The QUASAR project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education, 30*(4), 476–521.

Smyth, J. (2007). Teacher development against the policy reform grain: An argument for recapturing relationships in teaching and learning. *Teacher Development, 11*(2), 221–236. https://doi.org/10.1080/13664530701414837

Snow-Gerono, J. L. (2009). Voices less silenced: What do veteran teachers value in school-university partnerships and initial teacher preparation? *The Teacher Educator, 44*(4), 248–267. https://doi.org/10.1080/08878730903186348

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488. https://doi.org/10.3102/00028312033002455

Stein, M. K., Silver, E. A., & Smith, M. S. (2013). Mathematics reform and teacher development: A community of practice perspective. In J. G. Greeno & S. V. Goldman (Eds.), *Thinking practices in mathematics and science learning* (pp. 27–62). Routledge.

Stoll, L., Bolam, R., McMahon, A., Wallace, M., & Thomas, S. (2006). Professional learning communities: A review of the literature. *Journal of Educational Change, 7*(4), 221–258. https://doi.org/10.1007/s10833-006-0001-8

Vangrieken, K., Meredith, C., Packer, T., & Kyndt, E. (2017). Teacher communities as a context for professional development: A systematic review. *Teaching and Teacher Education, 61*, 47–59. https://doi.org/10.1016/j.tate.2016.10.001

Vescio, V., Ross, D., & Adams, A. (2008). A review of research on the impact of professional learning communities on teaching practice and student learning. *Teaching and Teacher Education, 24*(1), 80–91. https://doi.org/10.1016/j.tate.2007.01.004

Walton, C., & Randolph, T. (2017). Alternative methods for understanding area formulas. *Illinois Mathematics Teacher, 64*(1), 40–45.

Zeichner, K. (2012). The turn once again toward practice-based teacher education. *Journal of Teacher Education, 63*(5), 376–382. https://doi.org/10.1177/0022487112445789