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Iterative optimal sensor placement for adaptive structural identification using mobile sensors: Numerical application to a footbridge

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ABSTRACT

This paper proposes an iterative optimal sensor placement (OSP) framework for structural identification and model updating of structural systems using a small number of mobile sensors. The model updating is performed through a Bayesian inference approach which is solved through asymptotic approximation for computational efficiency. In an iterative manner, the OSP is performed to minimize the information entropy in estimating the updating parameters of the model. Each OSP iteration is performed to find the next location of mobile sensors, where the prior probability distribution of updating parameters is assumed as the posterior probability distribution obtained from the previous iteration. This process is repeated until the uncertainties of updating parameters fall below a predetermined threshold. A forward sequential sensor placement algorithm is used to solve the OSP problem at each iteration. This algorithm provides a nearly-optimal solution and is much more efficient compared to an exhaustive search. This proposed framework is applied to a numerical case study, namely the Dowling Hall Footbridge located at Tufts University campus. Updating parameters used in this study are the added mass at different segments of the bridge deck. The purpose of using added mass is to create a realistic pseudo damage on a specific portion of the bridge. The proposed iterative system identification approach is applied for estimation of updating parameters considering different number of available sensors. This study shows that the iterative OSP approach using a small number of mobile sensors placed iteratively provides better model updating results compared to the case of using an optimal static sensor configuration involving larger number of sensors.

1. Introduction

Physics-based models such as finite element models are often used in the design as well as performance assessment of structural systems. However, such models have often large modeling errors when they are created solely based on design drawings and expected material behavior. Modeling errors can be mitigated when measurements are available on the actual built structures. The process of integrating the initial model with measured data is referred to as model updating or digital twinning [1,2]. In the updating process,

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uncertain model parameter values are tuned to minimize a cost function which is defined as the discrepancy between model prediction and measurements. Model updating has been successfully applied for large scale civil structural systems using vibration measurements or dynamic features extracted from vibration data such as modal parameters [3–6].

The accuracy of calibrated models for response prediction or performance assessment depends on (1) modeling errors, and (2) information content of measured data. While some modeling errors such as imprecision of model parameters can be mitigated in the model updating process, other types of modeling errors are inherent to modeling assumptions and model class, and cannot be addressed by model updating, e.g., simplifying assumptions regarding nonlinearity, or boundary conditions. The effectiveness of model updating process mainly depends on the information gained from the measurements about model parameters. This depends on the design of instrumentation layout which includes the type, number, and locations of sensors placed on the structure. Several researchers have proposed optimal sensor placement (OSP) approaches for structural identification and model updating. In information theoretic OSP, the aim is to determine the best sensor configuration that maximizes the information gained about the instrumented structural systems [7]. In many studies in the literature [8–17], OSP was found by maximizing the trace or determinant of the Fisher Information Matrix (FIM). Some researchers [18,19], proposed performing OSP using the Bayesian loss function in which the best sensor configuration is selected as the one that minimizes the expected Bayesian loss function which is related to the trace of the inverse of the FIM. Bayesian inference was also implemented in OSP applications [20,21] where the optimal sensor configuration was obtained using the Bayesian formulation. Informative prior probability density function (PDF) was implemented in the Bayesian approach of OSP in further studies [22].

Information entropy was introduced in [23] as a scalar measure of uncertainty. It was used in [24–26] for determining the optimal locations of sensors for the best estimation of structural parameters using a probabilistic approach. Asymptotic approximation of the information entropy for a large dataset was derived in [27] which shows the information entropy to depend on the determinant of the FIM. It was also shown that the determinant depends on the optimal sensor configuration and optimal model parameters. The information entropy was applied in optimal sensor placement studies for parameter estimation using uncertain excitations [28], parameter estimation of model classes for fault detection [29] and for determination of the excitation characteristics of linear and nonlinear models [30]. Considering the high number of possible sensor locations in a discrete optimization problem, an exhaustive search algorithm is computationally costly and inefficient. Addressing this issue, two computationally efficient and accurate heuristic algorithms, the backward sequential sensor placement (BSSP) and the forward sequential sensor placement (FSSP), for OSP were proposed in [27,29,31]. Additionally, an enhanced sequential sensor placement (ESSP) is proposed in [32] which enables to retain more than one candidate sensor configuration at each placement level. In [48], an OSP framework with cost constraint for parameter estimation and virtual sensing was proposed. The effect of spatial correlation length on the OSP was investigated in [33,48]. Also, genetic algorithms [34–36] are presented as suitable alternatives for determining optimal solutions and can be used to complement the proposed heuristic algorithms for better estimates.

In recent years, the use of mobile sensors has been investigated for structural identification [37–39]. One of the main challenges in achieving high accuracy in identification and model updating of large-scale structural systems such as bridges or buildings is the need for a large number of sensors. To address this issue, a new iterative output-only OSP methodology for model updating is proposed which is capable of providing accurate parameter estimation with a smaller number of mobile sensors. The proposed algorithm is applied on a footbridge considering 5 updating parameters which are the added masses on different portions of the footbridge. Added mass on the bridge deck provides similar effects on the dynamic response of footbridge as the loss of stiffness (i.e., damage) [40]. In each OSP iteration, the prior PDF of the model parameters is estimated based on the information gained in the previous iteration, and sensor locations are found using the heuristic FSSP algorithm. Bayesian model updating is performed using a Gaussian asymptotic approximation for the posterior PDF where the mean of the Gaussian PDF is determined with a local optimization algorithm commonly used in deterministic model updating and the covariance of the Gaussian PDF is computed numerically using finite differences. The proposed methodology is carried out using different numbers of mobile sensors and also using static sensors. Results are compared in the sense of updating-parameter uncertainties.

2. Methodology

This section provides an overview of the proposed iterative OSP for adaptive structural identification. Section 2.1 covers an overview of Bayesian inference for model parameter estimation which is the considered structural identification approach in this study. Section 2.2 outlines the proposed iterative OSP approach.

2.1. Bayesian inference for parameter estimation

The updating parameters of the structural system are estimated using Bayesian inference. Assume $\boldsymbol{\theta} \in R^{N_\theta}$ is the vector of updating parameters to be estimated using the measured data where N_θ is the number of updating parameters. Let $\mathbf{D} = \left\{\lambda_{r,\mathbf{D}} \in R^1, \boldsymbol{\phi}_{r,\mathbf{D}} \in R^{N_0};\ r = 1, \cdots, N_m\right\}$ be the vector of measured natural frequency and mode shape components of the structure, where N_0 is the number of measured degrees of freedom (DOF), and N_m is the number of considered modes. Let $\lambda_{r,\mathbf{M}}(\boldsymbol{\theta}) \in R^1$ and $\boldsymbol{\phi}_{r,\mathbf{M}}(\boldsymbol{\theta}) \in R^{N_d}$ be defined as the natural frequency and mode shape predictions obtained from the finite element (FE) model for a given value of the parameter set $\boldsymbol{\theta}$, where N_d is the total number of DOFs in the FE model. The prediction errors between natural frequencies and mode shapes obtained from the FE model and the measured data for mode r follows Eq. (1) [40]:

$$\begin{aligned} e_{\lambda,r}(\boldsymbol{\theta}) &= \lambda_{r,\mathbf{D}} - \lambda_{r,\mathbf{M}}(\boldsymbol{\theta}) \\ e_{\boldsymbol{\phi},r}(\theta) &= \frac{\boldsymbol{\phi}_{r,\mathbf{D}}}{\|\boldsymbol{\phi}_{r,\mathbf{D}}\|} - \frac{\mathbf{L}\boldsymbol{\phi}_{r,\mathbf{M}}(\boldsymbol{\theta})}{\|\mathbf{L}\boldsymbol{\phi}_{r,\mathbf{M}}(\boldsymbol{\theta})\|} \end{aligned} \tag{1}$$

where $\mathbf{L} \in R^{N_0 x N_d}$ is the Boolean matrix which consists of zeros and ones, and maps the model DOFs to the measured DOFs, i.e., determines the sensor locations. The prediction error $e_{\lambda,r}(\boldsymbol{\theta})$ is assumed to be Gaussian with zero mean and standard deviation $\sigma_{\lambda_m} = \sigma_\lambda \lambda_{r,\mathbf{D}}$, where $\sigma_\lambda = \sigma$, measuring the intensity of the error between the measured and model predicted eigenvalue relative to the model predicted eigenvalue, is an unknown parameter to be estimated from the data. Also, the prediction error $\mathbf{e}_{\boldsymbol{\phi},r}(\boldsymbol{\theta})$ is assumed to have a Gaussian distribution with zero mean and covariance matrix $\boldsymbol{\Sigma}_t \in R^{N_0 x N_0}$. To account for the effect of spatial correlation on the prediction error in the parameter inference and the OSP formulation, i.e., data from adjacent sensors are not necessarily independent, the covariance matrix $\boldsymbol{\Sigma}_t$ is chosen as a non-diagonal matrix with elements Σ_{ij} given as [33]:

$$\Sigma_{ij} = \sigma_{\boldsymbol{\phi}}^2 R(\delta_{ij}) \tag{2}$$

where $\sigma_{\boldsymbol{\phi}}$ is a measure of the intensity of the errors assumed to be the same for all mode shape components, and $R(\delta_{ij})$ is an exponential function of the spatial distance δ_{ij} between DOFs i and j which is assumed as:

$$R(\delta_{ij}) = e^{(-\delta_{ij}/\lambda)} \tag{3}$$

where λ is a measure of spatial correlation length. Herein it is assumed that $\sigma_\phi = \alpha\sigma_\lambda = \alpha\sigma$, with α set to one in this study.

Applying the Bayes' theorem and assuming that the prediction error values for different modes are independent from each other, the posterior probability density function for updating parameter set $\boldsymbol{\theta}$ and σ given data $\mathbf{D}$ collected from the sensor configuration $\mathbf{L}$ is [40]:

$$p(\boldsymbol{\theta}, \sigma^2|\mathbf{D}, \mathbf{L}) \propto \frac{1}{(\sigma^2)^{N_m(N_0+1)} \left(\sqrt{\det(\mathbf{R})}\right)^{N_m}} exp\left[-\frac{1}{2\sigma^2} J(\boldsymbol{\theta}|\mathbf{D})\right] p(\theta) \tag{4}$$

where $p(\boldsymbol{\theta})$ is the prior probability density function of updating parameters and

$$J(\boldsymbol{\theta}|\mathbf{D}, \mathbf{L}) = \sum_{r=1}^{N_m} \left(\frac{e_{\lambda,r}(\boldsymbol{\theta})}{\lambda_{r,\mathbf{D}}}\right)^2 + \frac{1}{\alpha^2} \sum_{r=1}^{N_m} \mathbf{e}_{\boldsymbol{\phi},r}^T(\boldsymbol{\theta}) \mathbf{e}_{\boldsymbol{\phi},r}(\boldsymbol{\theta}) \tag{5}$$

which represents the fit between measured natural frequencies and mode shapes and their counterparts obtained from the FE model for all N_m identified modes.

Numerical techniques such as sampling methods [41–43] or Laplace approximation [44] can be used to estimate the posterior distribution. Among these methods, Laplace approximation is chosen for this study due to its computational efficiency. Specifically, the posterior distribution is asymptotically approximated by a Gaussian distribution centered at the most probable value $(\overline{\boldsymbol{\theta}}, \overline{\sigma}^2)$ of the posterior distribution with covariance matrix equal to the inverse of the Hessian of negative logarithm of the posterior distribution evaluated at the most probable value. Using the Gaussian asymptotic approximation, one derives the marginal posterior distribution $p(\boldsymbol{\theta}|\mathbf{D})$ of the structural model parameters $\boldsymbol{\theta}$, to be a normal distribution centered at the most probable value $\overline{\boldsymbol{\theta}}$ and with the posterior covariance matrix $\boldsymbol{\Sigma}_\mathbf{L}$ of updating parameters $\boldsymbol{\theta}$ be obtained as:

$$\boldsymbol{\Sigma}_\mathbf{L}^{-1} = \frac{1}{\overline{\sigma}^2} \mathbf{H}_s(\overline{\boldsymbol{\theta}}) + \boldsymbol{\Sigma}_\mathbf{p}^{-1} \tag{6}$$

where, $\overline{\sigma}^2 = J(\overline{\boldsymbol{\theta}}|\mathbf{D}, \mathbf{L})/[N_m(N_0+1)]$, $\mathbf{H}_s(\overline{\boldsymbol{\theta}})$ is the Hessian matrix of the function $J(\boldsymbol{\theta}|\mathbf{D})$ evaluated at the optimal values $\overline{\boldsymbol{\theta}}$, and $\boldsymbol{\Sigma}_\mathbf{p}$ is the prior covariance. The components of the Hessian matrix $\mathbf{H}_s(\overline{\boldsymbol{\theta}})$, given as [45]:

$$\mathbf{H}_s(\overline{\boldsymbol{\theta}})_{i,j} = \left.\frac{\partial^2 J(\boldsymbol{\theta}|\mathbf{D})}{\partial\theta_i \partial\theta_j}\right|_{\boldsymbol{\theta}=\overline{\boldsymbol{\theta}}} \tag{7}$$

is calculated by the finite difference method [46].

2.2. Iterative OSP

The posterior PDF $p(\boldsymbol{\theta}|\mathbf{D})$ of the model parameters $\boldsymbol{\theta}$ provides the spread of the uncertainty in the parameter space given the measured data $\mathbf{D}$. The information entropy is a scalar measure of the uncertainty for the updating parameters. For given sensor configuration $\mathbf{L}$, the information entropy $H(\mathbf{L}|\mathbf{D})$ given the data $\mathbf{D}$ takes the form:

$$H(\mathbf{L}|\mathbf{D}) = E_{\boldsymbol{\theta}}(-\ln(p(\boldsymbol{\theta}|\mathbf{D}, \mathbf{L}))) = -\int \ln(p(\boldsymbol{\theta}|\mathbf{D}, \mathbf{L}))p(\boldsymbol{\theta}|\mathbf{D}, \mathbf{L})d\boldsymbol{\theta} \tag{8}$$

where $E_{\boldsymbol{\theta}}$ is the mathematical expectation with respect to model parameters $\boldsymbol{\theta}$. It is noted that the information entropy depends on the

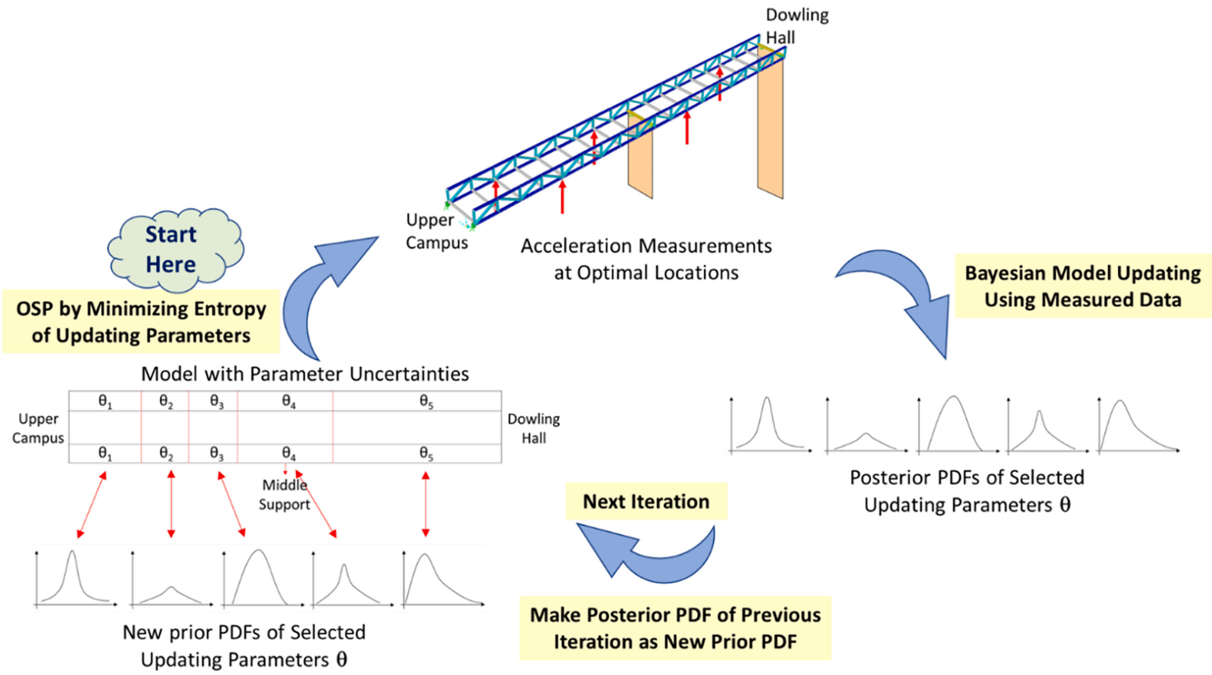

Fig. 1. Proposed methodology.

Fig. 2. Dowling Hall Footbridge [40].

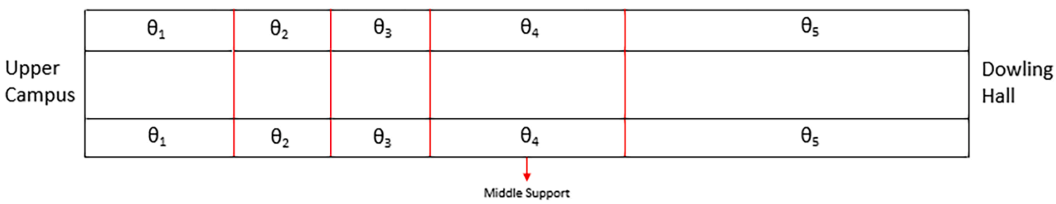

Fig. 3. Updating parameters.

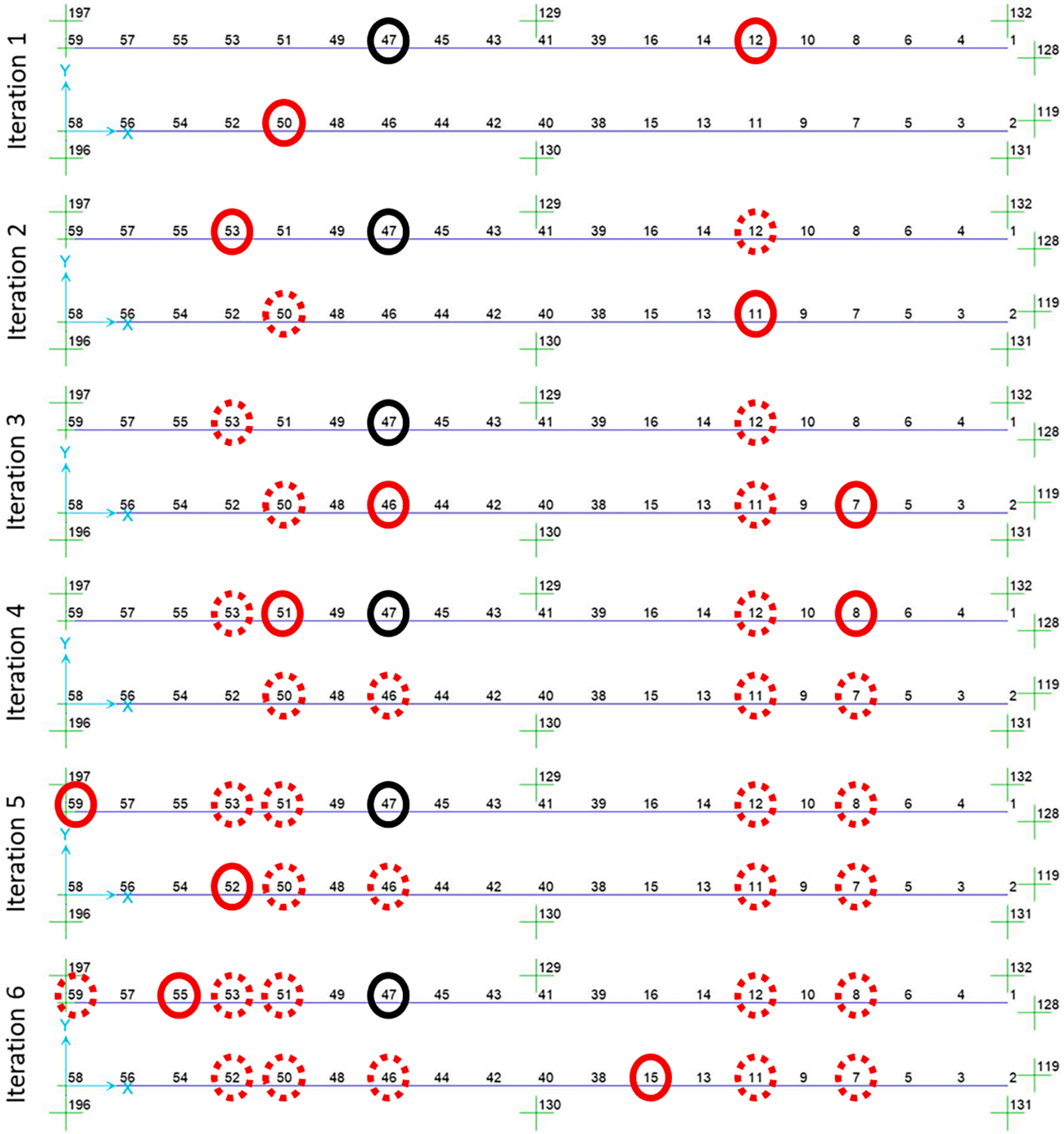

Fig. 4. Sensor locations for all 6 iterations.

measured data **D**. However, calculation of the information entropy in this integral is challenging. An asymptotic approximation of the information entropy, which is valid when the number of data points is large, i.e., $N_0 N_m \rightarrow \infty$, is used in this study. The information entropy can be asymptotically approximated as [33]:

$$H(\mathbf{L}|\mathbf{D}) = \frac{1}{2} N_\theta \ln(2\pi) - \frac{1}{2} \ln\left(\det\left[\mathbf{Q}(\mathbf{L}|\overline{\boldsymbol{\theta}}) + (\boldsymbol{\Sigma}_p)^{-1}\right]\right) \tag{9}$$

where $\overline{\boldsymbol{\theta}}$ represents the optimal values of updating parameters and **Q** denotes the Fisher Information Matrix (FIM) given by:

$$\mathbf{Q}(\mathbf{L}|\overline{\boldsymbol{\theta}}) = \frac{1}{\sigma^2} \sum_{r=1}^{N_m} \left[\frac{1}{\lambda_{r,\mathbf{M}}^2(\boldsymbol{\theta})} \left(\nabla_{\boldsymbol{\theta}} \lambda_{\mathbf{r},\mathbf{M}}(\overline{\boldsymbol{\theta}})\right)^T \left(\nabla_{\boldsymbol{\theta}} \lambda_{\mathbf{r},\mathbf{M}}(\overline{\boldsymbol{\theta}})\right) + \frac{1}{\alpha^2} \left(\mathbf{L} \nabla_{\boldsymbol{\theta}} \frac{\boldsymbol{\phi}_{\mathbf{r},\mathbf{M}}(\overline{\boldsymbol{\theta}})}{\|\boldsymbol{\phi}_{\mathbf{r},\mathbf{M}}(\overline{\boldsymbol{\theta}})\|}\right)^T \left(\mathbf{L}\mathbf{R}\mathbf{L}^{\mathbf{T}}\right)^{-1} \left(\mathbf{L} \nabla_{\boldsymbol{\theta}} \frac{\boldsymbol{\phi}_{\mathbf{r},\mathbf{M}}(\overline{\boldsymbol{\theta}})}{\|\boldsymbol{\phi}_{\mathbf{r},\mathbf{M}}(\overline{\boldsymbol{\theta}})\|}\right) \right] \tag{10}$$

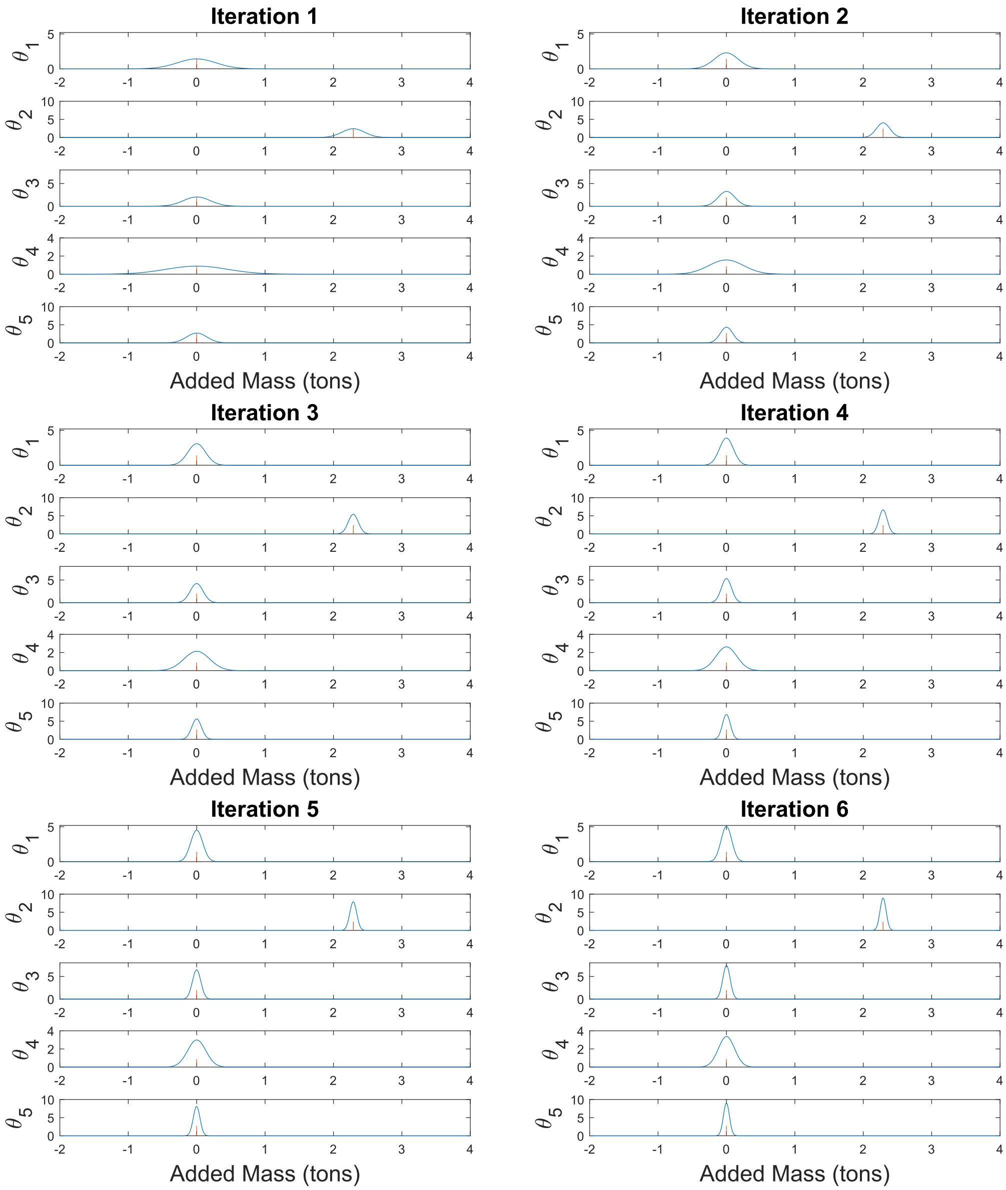

Fig. 5. Posterior distributions for all 6 iterations.

where $\nabla_{\theta} = [\partial/\partial\theta_1 \cdots \partial/\partial\theta_{N_\theta}]$ is the gradient operator with respect to the parameter vector **θ**. It should be noted that the asymptotic estimate of the information entropy in Eq. (9) is independent of the measured data **D** and depends only on the sensor locations in **L** and the sensitivity of the modal properties with respect to the model parameter set **θ**.

The aim is to minimize the information entropy with respect to the sensor locations in **L**. This is equivalent to minimizing the parameter identification uncertainty or maximizing the information gained from measurements for reliably estimating the model parameters **θ**. FE models of large-scale structures often have many DOFs which can be considered as candidates for sensor locations. The problem of finding the optimal sensor configuration involving a given number of sensors may result in massive computational cost since it requires the evaluation of the information entropy for a prohibitively large number of sensor configurations [27]. Therefore, two heuristic algorithms for sequential sensor placement (SSP) were defined in [33] which are forward SSP (FSSP) and backward SSP (BSSP). In FSSP, one sensor is placed in each iteration based on the highest reduction in the information entropy. In the second iteration, the location of the second sensor is found assuming the location of the first sensor is known. This process continues until the total number of sensors is reached. In BSSP, initially all DOFs in the FE are assumed to have sensors and at each iteration one sensor which causes the smallest increase in the information entropy is removed. In this study, the FSSP algorithm is used.

The discussed OSP approach is proposed as an iterative process. In the first step of the proposed iterative OSP methodology, the

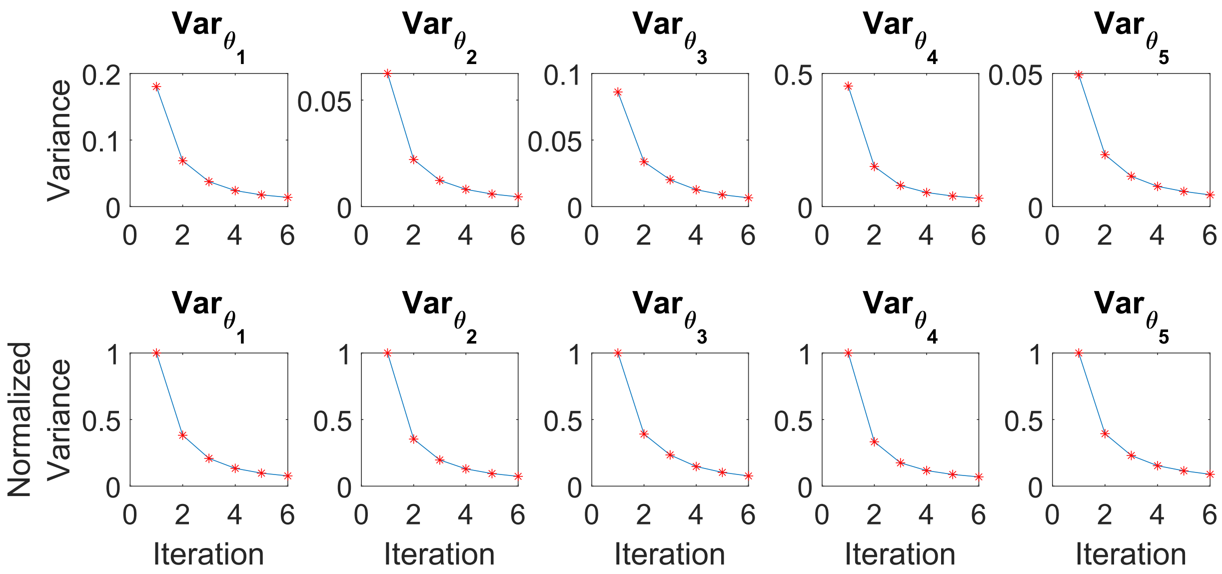

Fig. 6. The change in variances and normalized variances of updating parameters $\boldsymbol{\theta}$ using 3 sensors.

initial FE model of the bridge is created. The prior PDF of the updating parameters of the initial FE model is assumed to follow a uniform distribution. In this case the inverse of the covariance matrix, $(\boldsymbol{\Sigma}_p)^{-1}$, in Eq. (6) is set to zero. Then, the information entropy based OSP is performed to find the optimal locations of sensors given their numbers. Once the sensors are placed on their optimal locations and data collection is completed (say for 5-minutes), Bayesian model updating is performed to estimate the posterior PDF of the model parameters. The process is repeated at the next iteration by using the calculated posterior PDF of the previous step as the prior PDF of the next one. This process continues until the uncertainties of updating parameters fall below a predetermined threshold. The framework for the proposed methodology is illustrated in Fig. 1.

3. Numerical Application: Case study and approach

The considered case study is a numerical model of the Dowling Hall footbridge which is located at Tufts University Campus in Medford, Massachusetts. The bridge consists of two 22 m spans with width of 3.9 m. It is composed of steel frame and reinforced concrete deck. A picture of the Dowling Hall Footbridge is shown in Fig. 2.

The bridge was loaded with 2.29 tons of concrete blocks to simulate a local damage on the bridge deck [40]. The goal is to accurately estimate the location and amount of added mass on the bridge using a small number of sensors. The added masses on different segments of the footbridge deck are chosen as the five updating parameters. For each updating parameter, the added mass is assumed to be uniformly distributed along the length of the considered bridge segment and on the two sides of the deck as shown in Fig. 3.

The OSP study is performed using 3 mobile sensors as well as 5 mobile sensors to compare the influence of the number of sensors on the identification results. The results of using mobile sensors are then compared with those from using 10 optimally located static sensors to show the efficiency of the proposed iterative method. The presented OSP results are based on the assumption that the signal-to-noise ratio (SNR) of all the accelerometers are the same and thus the value of SNR does not affect the results. In the case where different types of sensors with different accuracies are considered, SNR would influence the OSP results.

4. Numerical Application: Results

The initial FE model of the footbridge is created in MATLAB according to the design drawings and then calibrated based on the in-situ measurements using a deterministic FE model updating [47]. The calibrated model is assumed as the undamaged reference model.

Three Sensor Setup: Three mobile sensors are first considered in this section. Using the reference model, the first iteration of OSP is performed using the FSSP algorithm. The spatial correlation length for OSP is chosen as 40% of the footbridge span length. Sensor locations are obtained to maximize the information about updating parameters $\boldsymbol{\theta}$. The optimal location of these 3 sensors on the bridge are shown in the top row of Fig. 4. Using this sensor setup, measurements on the "damaged structure" are simulated using the calibrated model with the considered added mass of $\boldsymbol{\theta}_2 = 2.29 \times 10^3$ kg. An asymptotic approximation is performed for Bayesian model updating using the natural frequencies and mode shapes of the first six vibration modes. The prediction error is assumed to be normally distributed with zero mean and a covariance matrix based on Equation (2) which updated in each iteration. The prior probability distribution for updating parameters is assumed as noninformative for the first iteration. The posterior PDF of updating parameters is estimated at the completion of the first iteration.

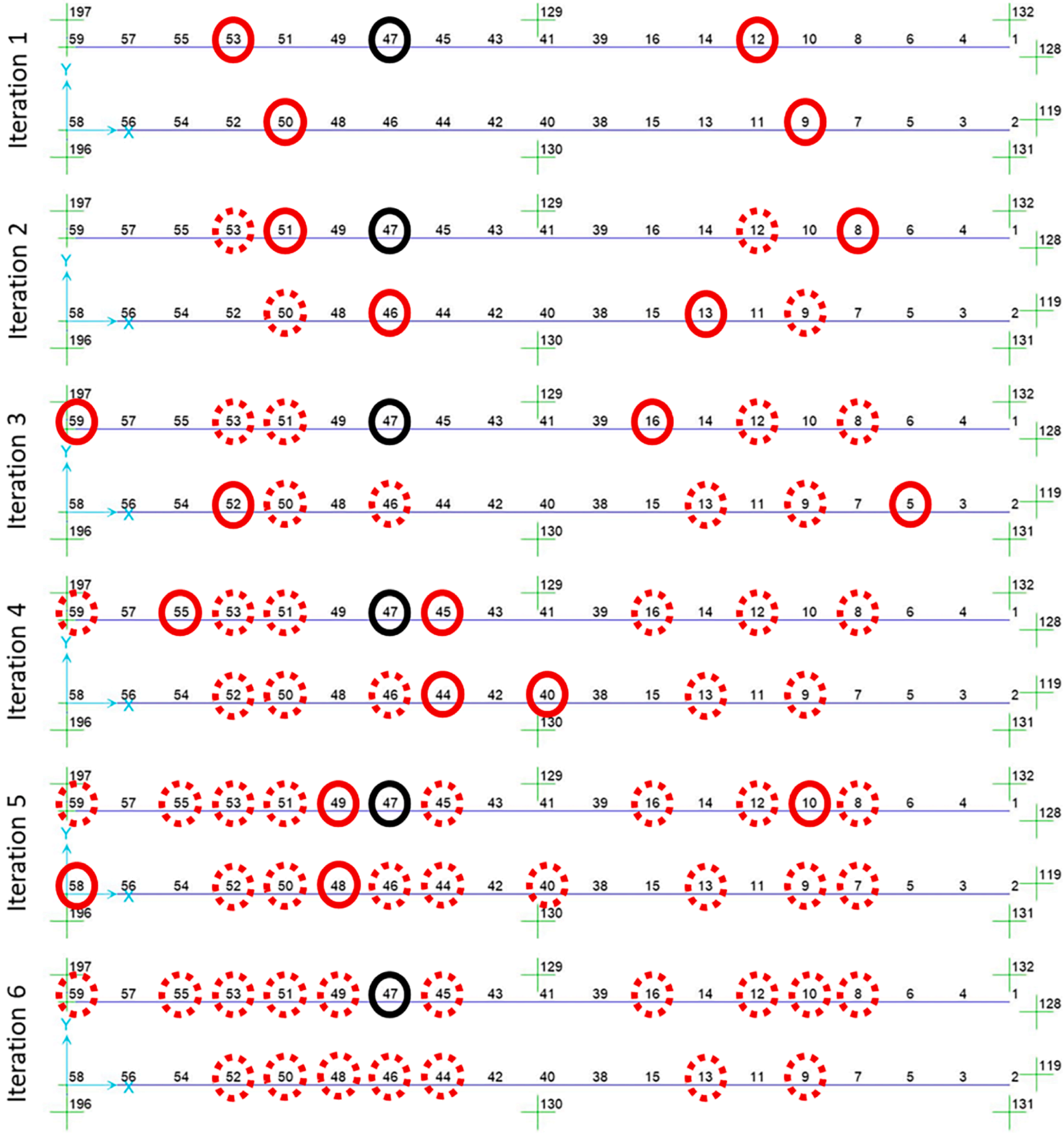

Fig. 7. Sensor locations for all 6 iterations.

In the next iterations, OSP is performed for only two of the three sensors as a common/reference sensor needs to stay fixed for normalizing the mode shapes from different iterations. The first sensor that the OSP algorithm gives is chosen as the reference sensor since it provides the most informative data. The selected reference sensor is shown with a black circle in Fig. 4 (node 47). The prior distribution of updating parameters at the second iteration is considered as their posterior distribution estimated at the first iteration. The locations of sensors for all 6 iterations are given in Fig. 4. Previous locations of sensors are represented with dashed circles. The iterations continue the until uncertainty of updating parameters falls below a threshold. For this study, the threshold is chosen as 80% decrease in the variance of each updating parameter.

The marginal posterior distributions of updating parameters θ_1- θ_5 are presented in Fig. 5 for each of the six iterations. As it can be seen from Fig. 5, the added mass is accurately estimated ($\theta_1, \theta_3, \theta_4, \theta_5 = 0, \theta_2 = 2.3\text{tons}$). However, the estimation uncertainty of the results (width of the marginal PDFs) is rather large in the early iterations. This uncertainty is reduced as new measurements are collected in the later iterations. Furthermore, the uncertainty of θ_1 and θ_4 are generally larger than that of other parameters which is

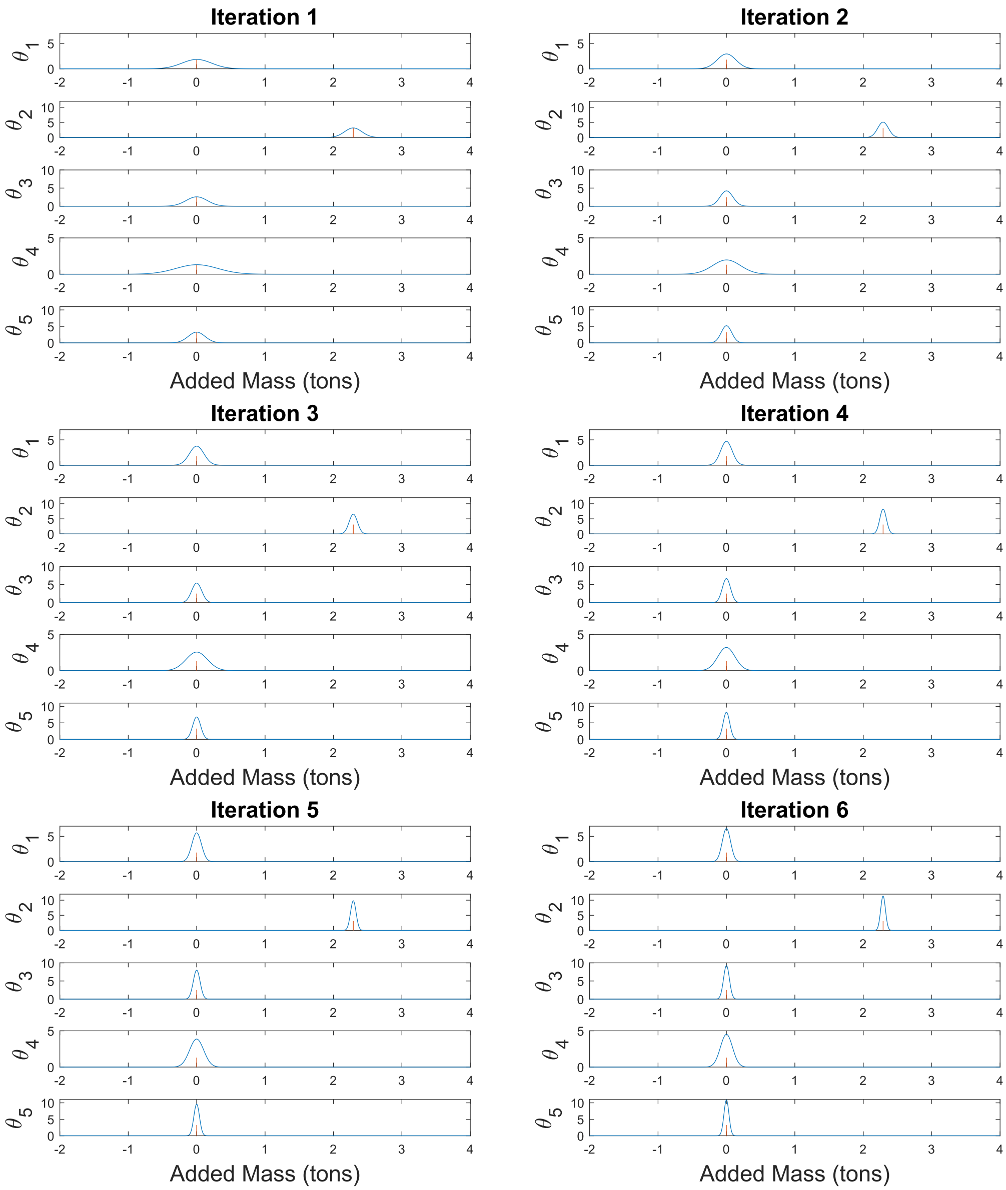

Fig. 8. Posterior distributions for all 6 iterations.

due to the fact that these two parameters are located next to the left abutment and the middle pier, where the vibration motion of bridge deck is small. However, θ_5 also includes nodes on the right support but its uncertainty is much lower than others since this parameter includes 16 nodes and a big portion of the deck and any change in θ_5 affect the structural behavior of the bridge significantly. The reduction in the uncertainty (variances and normalized variances) of updating parameters $\boldsymbol{\theta}$ as a function of OSP iteration is given in Fig. 6. The variance normalization is with respect to the first iteration variance of the same parameter and allows to compare the normalized uncertainty reduction of parameters. It can be seen that the variances of all parameters decrease as the number of iterations increase. The decrease in the first iteration is the most significant, while this reduction becomes less significant after the 4^{th} iteration.

Five Sensor Setup: The iterative OSP procedure is repeated in this section using 5 mobile sensors. Similar to the previous case, one of the five sensors is considered as a reference sensor and is kept fixed during the iterations. Fig. 7 shows the location of sensors for six iterations, allowing measurement of mode shapes at 25 (=6 × 4 + 1) locations along the bridge. Fig. 8 plots the posterior PDF of updating parameters after each iteration of OSP, data collection, and model updating. Fig. 9 represents the change in variances and normalized variances of updating parameters $\boldsymbol{\theta}$ for the 5-sensor setup. Similar to the three-sensor setup, the added mass is correctly estimated in θ_2 and its estimation uncertainty is decreasing as more iterations are performed. However, the estimation uncertainties are lower than the 3-sensor setup which is expected.

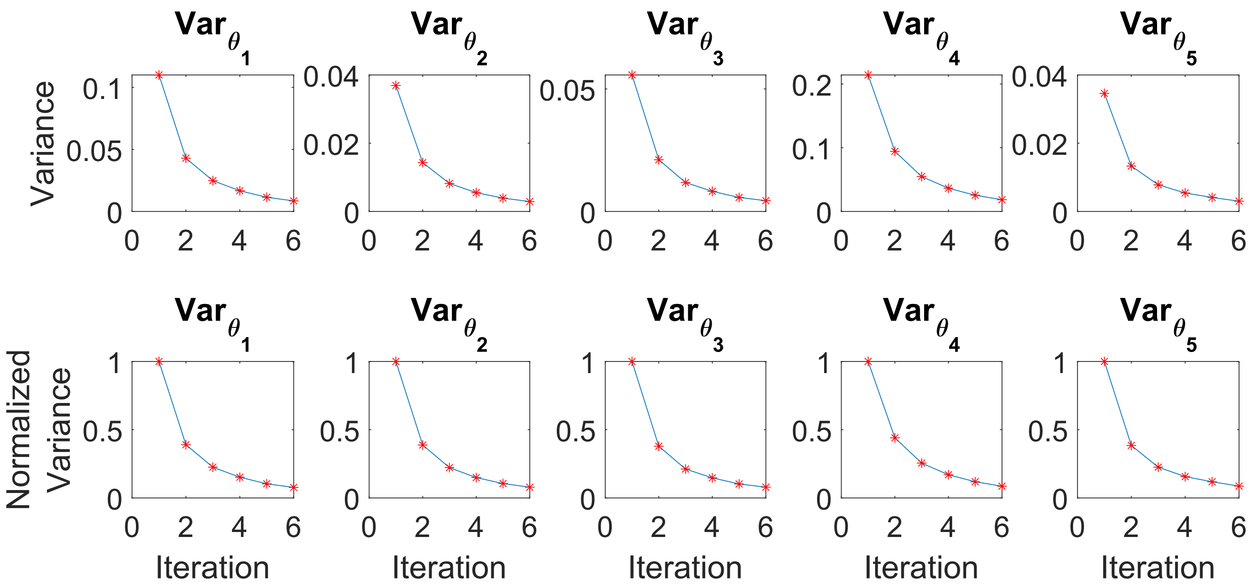

Fig. 9. The change in variances and normalized variances of updating parameters **θ** using 5 sensors.

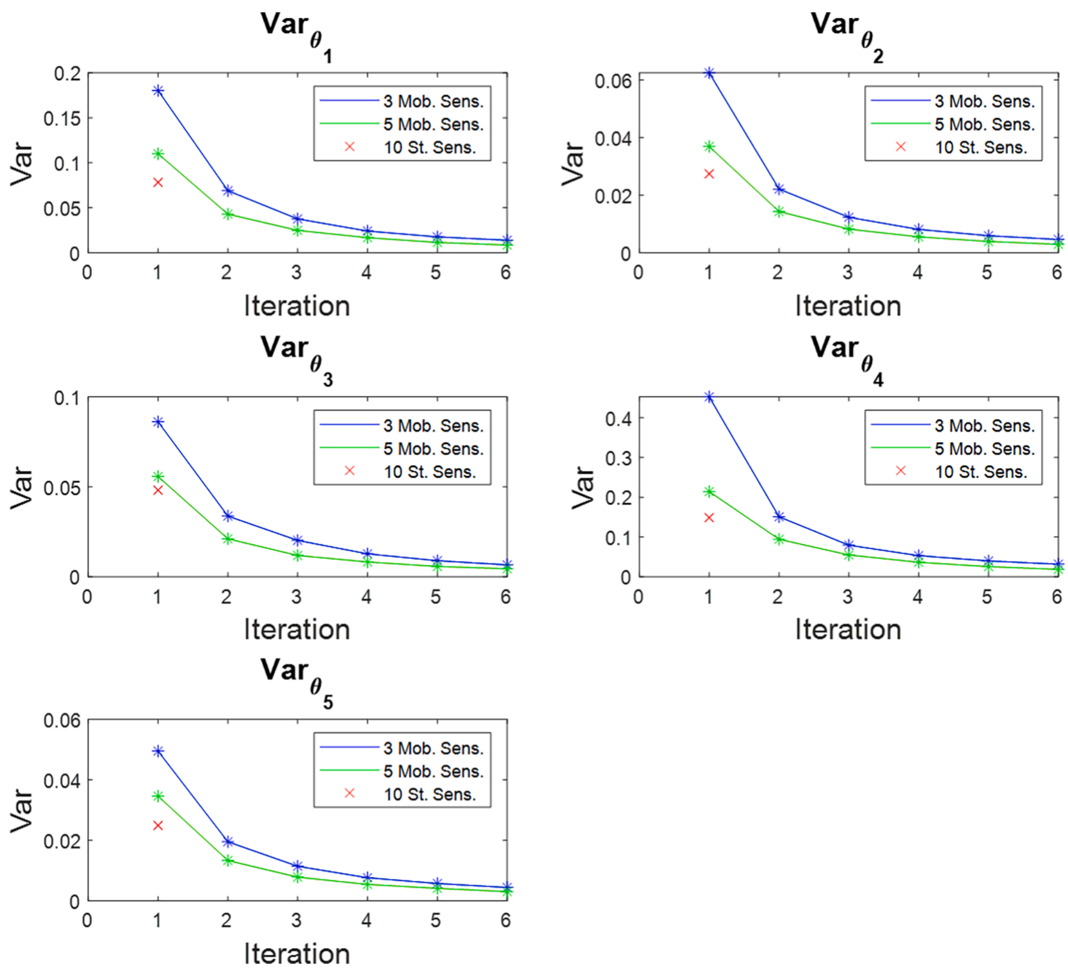

Fig. 10. The change in variances of updating parameters θ vs iteration for 3 and 5 mobile sensors as well as 10 static sensors.

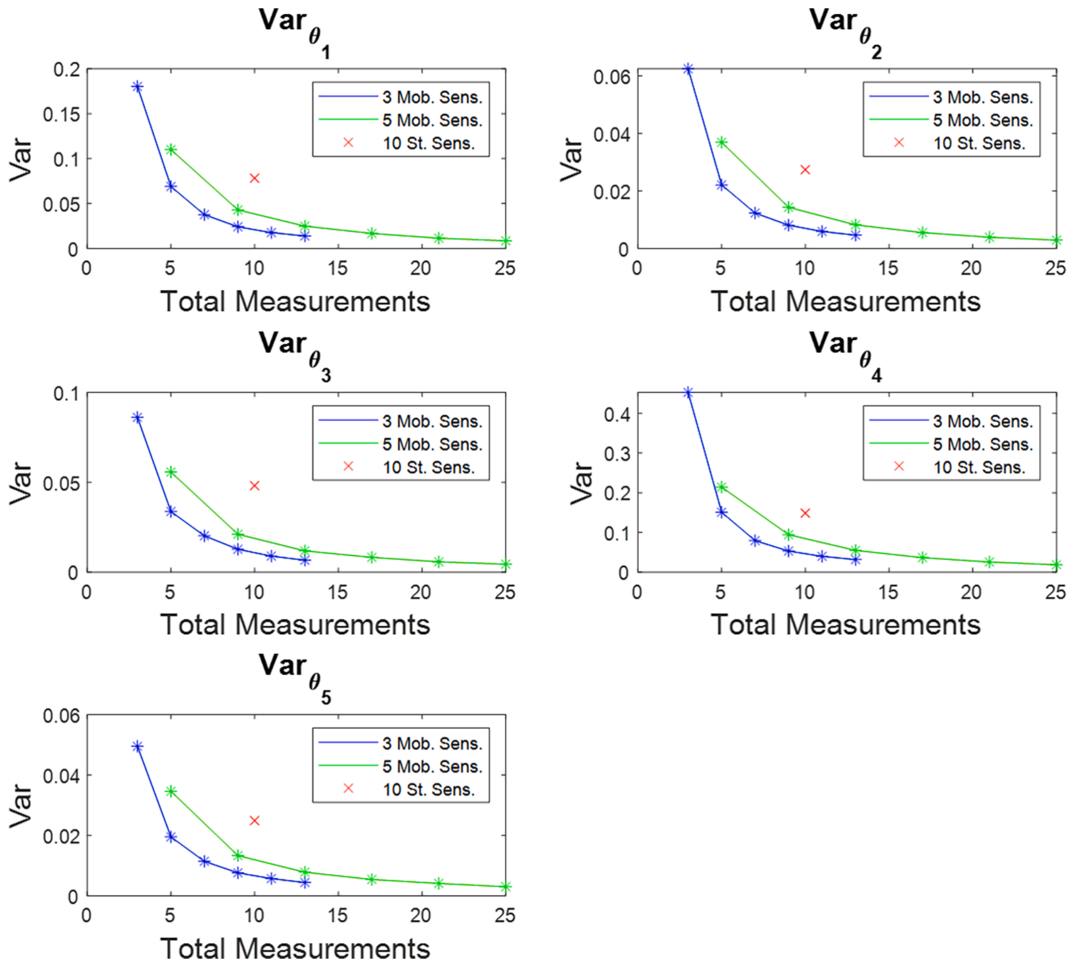

Fig. 11. The change in variances of updating parameters θ vs total number of measurement points for 3 and 5 mobile sensors and 10 static sensors.

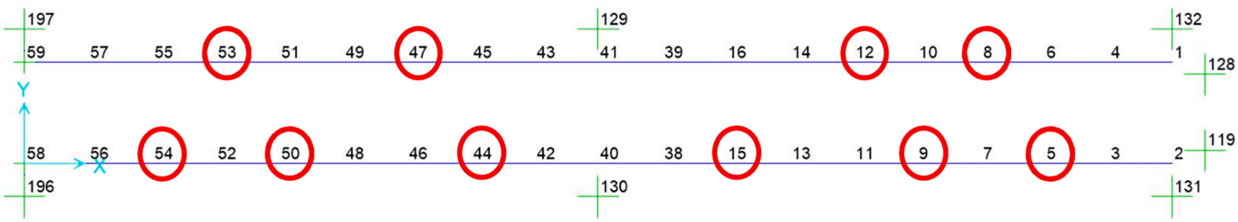

Fig. 12. Locations of 10 static sensors.

The changes in the estimation uncertainty of updating parameters versus number of iterations are plotted in Figs. 10 and 11 for the cases of 3 and 5 mobile sensors. The results for a 10-sensor static OSP is also shown for comparison. The locations of static sensors are given in Fig. 12. Fig. 10 plots the comparison of variances vs iteration number while Fig. 11 plots the comparison of variances vs the total number of measurement points. It can be seen that the iterative methods are much more effective in improving the accuracy of updating parameters even when a smaller number of total measurements are considered. Specifically, as expected, in the first iteration the optimal configurations for 3 and 5 sensors provide less information than the optimal sensor configuration of 10 sensors involved in the static case. However, in the second iteration the optimal sensor configurations for 3 mobile sensors (involving a total of 5 measurement points) and for 5 mobile sensors (involving a total of 9 measurement points) provide more information than the 10-sensor configuration in the static case. This is due to the fact that the information collected from the measurements in the first iteration was advantageously used to update the model parameters and provide a more informed prior distribution for optimizing the location of mobile sensors in the second iteration.

From the results, it is clear that for the 3 and the 5 mobile sensor cases the information entropy decreases as the number of iterations

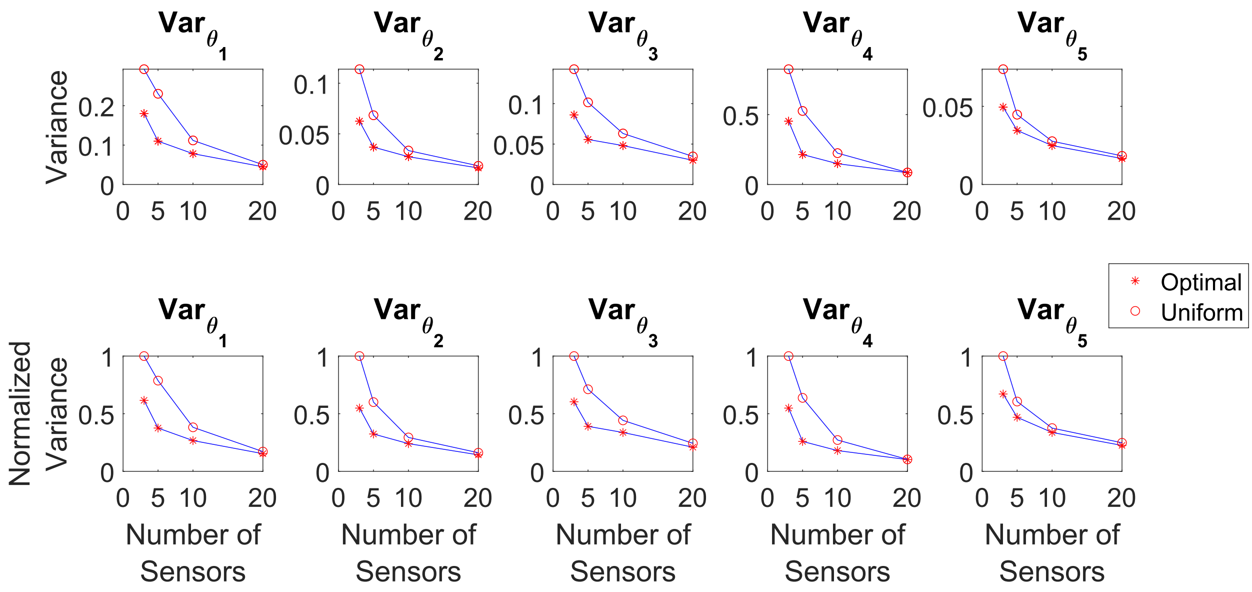

Fig. 13. Change in variances and normalized variances of updating parameters **θ** using different number of static sensors (3, 5, 10, 20).

in Fig. 10 or the number of sensors in Fig. 11 increases. Although this decrease in information entropy is significant in the first few iterations, it becomes insignificant as more iterations are considered, signifying that the iterative process should be terminated after a number of iterations. Termination of the iterative process should consider the trade-off between the small improvements in identification accuracy and the cost of field implementation of the iterative process.

In addition, from Figs. 10 and 11 it is observed that the 3-sensor case (1 static and 2 mobile) gives better results than the 5-sensor case for similar number of total measurement points, e.g., 4 iterations for the 3 sensors case (total of 9 measurements) and 2 iterations for the 5 sensors case (total of 9 measurements). It is also observed that when a larger number of iterations is possible/required, the use of a smaller number of mobile sensors (e.g., 3 sensors) would be more efficient as it can provide similar accuracy as the larger number of sensors (e.g., 5 sensors) after 5–6 iterations. On the other hand, when the number of iterations is limited, then a larger number of sensors can be considered to improve system identification accuracy. Moreover, more reduction in uncertainty can be observed for the 1^{st} and 4^{th} parameters, which are the ones with the highest level of uncertainty in the first iteration and the static case.

Fig. 13 compares the information gain (change in variance) of updating parameters from OSP using different number of static sensors (3, 5, 10, 20). The results are compared with a common sensor installation approach of uniform sensor distribution. The figure shows that the information gain slows down with increasing number of sensors. The gain is also less than what we observed through the iterative process. However, the static OSP still does a better job compared to uniform distribution of sensors, especially for the cases of 3 and 5 sensors.

5. Conclusion

In this paper a novel adaptive OSP methodology is proposed for system identification/model updating using a small number of mobile sensors. The method is evaluated when applied on a numerical model of pedestrian bridge located at Tufts University. In the iterative OSP process, model updating is performed at each iteration using collected data from mobile sensors at their current iteration locations. The computationally efficient asymptotic approximation is used for Bayesian model updating at each iteration. The iterative OSP approach relies on the prior PDFs of the updating parameters which are considered as the posterior PDF estimated from the previous iteration. It is observed that the uncertainties of the updating parameters decrease considerably at each iteration as a result of informative prior knowledge. This methodology decreases the number of sensors needed for accurate results which can reduce the cost of instrumentation. The investigation shows that the proposed iterative method provides more accurate results with a smaller number of sensors compared to the conventional OSP methodologies when static sensors are used. Furthermore, it is found that when large number of iterations is possible, use of smaller number of mobile sensors provides better identification accuracy than an optimal static sensor configuration involving the same number of sensors as the ones used with the optimal iterative sensor configuration. On the other hand, when number of iterations are limited, then a larger number of sensors can be considered to improve system identification accuracy.

Generally, OSP is prone to modeling errors since it is often performed at the instrumentation design phase and before having access to any data to mitigate modeling errors. However, the proposed iterative OSP overcomes this shortcoming by updating the FE model at each iteration using measurements and thus reducing the modeling errors. Therefore, another benefit of the proposed method over classic OSP is mitigating the modeling error gradually through sequential model updating at iterations of data collections. The prior covariance of updating parameters is shown to drastically reduce through the iterations implying the improvement of model. The proposed method can also be applied assuming the sensor measurements at all or certain locations are displacements, strains or mixed-type displacement and strains. This would require slightly modifying the formulation for model error related to the mode shape components to account for the use of displacements and strains. The proposed framework can also be used for optimizing the sensor locations for the purpose of identifying different kind of model related parameters such as stiffness and mass properties.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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