

ANALYSIS OF POLAR FIRN DENSITY AND GRAIN SIZE MODELS USING AVAILABLE DATA

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ABSTRACT

This paper provides a brief analysis of existing depth-dependent models of the physical properties of the polar firn using available data for subsurface density and grain radius. Results show that across inland Antarctica, firn density increases exponentially with depth. The density has gaussian fluctuations with negative damping. Grain radius increases linearly with depth for up to a few hundred meters. A standardized dataset for in-situ measurements of density and grain radius is formed and input parameters for their depth-dependent models are summarized in this paper.

Index Terms— In-situ measurements, Antarctic firn, ice models, physical properties

1. INTRODUCTION

Analysis of ice sheets provides important information on past, present, and future characteristics of Earth's climate and water cycle. Although there have been extensive research and data collection efforts regarding the Antarctic Ice Sheet, at present, there is no standardized dataset which compiles available measurements; thus, the validation of ice property models available in literature and computation of their input parameters can be challenging. Additionally, available data is often local and fails to provide a complete picture for all of Antarctica. Studies that seek to bridge this issue require these parameters. For example, remote sensing studies use these parameters as inputs to radiation models and utilize their boundaries to justify retrievals of ice sheet properties.

In this study, available data for Antarctic firn density and grain size was collected and analyzed with respect to the existing models. This paper aims to provide a brief analysis of current depth-dependent models and data for Antarctic firn subsurface densities and grain radii.

In Section 2, we provide an overview of the existing ice density and grain radius models used in this study. In Section 3, we discuss our analysis of data collected from U.S. Antarctic Program Data Center (USAP-DC) and summarize the resulting input parameters for the depth-dependent

models found through regression. Finally, we conclude our findings in the conclusion.

This study was conducted as a part of the project “*Characterization of Antarctic firn by Multi-Frequency Passive Remote Sensing from Space*”. All data discussed in this study will be made available on the project site [1].

2. EXISTING ICE MODELS

2.1. Density model

Average firn density $\bar{\rho}(z)$ is described as increasing exponentially with depth z ($z > 0$).

$$\bar{\rho}(z) = \rho_{\infty} - (\rho_{\infty} - \rho_0)e^{-\frac{z}{\beta}} \quad (1)$$

where ρ_0 is the near surface density, ρ_{∞} is the density of compacted ice, and β is a factor that controls the saturation rate of density profiles. A vertical density profile $\rho(z)$ is formed by considering finer scale density fluctuations as correlated damped Gaussian noise $\tilde{\rho}(z)$ [2].

$$\rho(z) = \tilde{\rho}(z) + \rho_{\infty} - (\rho_{\infty} - \rho_0)e^{-\frac{z}{\beta}} \quad (2)$$

However, in this paper we consider a simpler vertical density profile with uncorrelated noise with a damping factor α .

2.2. Grain radius model

Grain radius $r^n(z)$ is modeled as increasing linearly with depth z ,

$$r^n(z) = r_{surf}^n + Q_n z \quad (3)$$

where r_{surf}^n is the grain radius near surface, n is the growth component, and Q_n is the grain size gradient. For this grain size model, Brucker et al considered a depth of up to 10 meters [3].

3. DATA COLLECTION AND REGRESSION

There are two main types of measurements available for the Antarctic firm layers – in-situ measurements from boreholes and ice cores and radar measurements. Due to the high uncertainty of radar measurements, they are largely unconsidered in this study, with a few exceptions. To validate current models, we fit data collected from USAP-DC to the existing density and grain size models discussed previously using least squares regression.

3.1. Density fits

For density we calculated the surface density using measurements at 0 to 0.1 meters of depth by taking a mean where the data was available. Using these surface density values, we derived the remaining parameters using regression. These parameters are summarized in Table 1 below.

Location	ρ_0 (kg/m ³)	ρ_∞ (kg/m ³)	β (m)	α (m)
Wilkins Ice Shelf	468.0	925.1	11.57	-0.12393
WAIS	449.2	919.8	43.00	-0.006659
Taylor Mouth	308.5	932.5	20.65	-0.0001603
South Pole	393.5	908.9	70.79	-0.004228
Siple Dome	270.1	869.2	18.78	-0.01562
Newall Glacier	301.3	899.8	17.34	-0.03002

Table 1. Density parameters. Data used: Wilkins Ice Shelf [4], WAIS Divide (radar data) [5, 6], Taylor Mouth (radar data) [7], South Pole [8], Siple Dome [9,10], Newall Glacier [11].

We found that the density of compacted ice is greater than 917 kg/m³ at some locations. This density of deep ice is usually taken to be 917 kg/m³ but this is only true at 0° Celsius and in the upper layers of ice sheets and mountain glaciers. Furthermore, this only applies to pure glacier ice [12]. Thus, lower temperatures and ice impurities can both contribute to this density being greater.

Using the results of regression (ρ_0 , ρ_∞ , and β), we considered variations between the fitted model and the in-situ measurements as noise. We define density noise as a function of depth,

$$N(0, \theta)e^{\alpha z} = N(z, \theta) \quad (4)$$

where z ($z > 0$) represents depth, θ is the standard deviation from the smooth density (calculated using regression parameters), and α is the damping factor. Below, Figures 1 – 6 show the in-situ measurements of density, fit to a smooth density curve (left) calculated using the parameters in Table 1 and the calculated noise (right). Except for data at Taylor Mouth, South Pole, and Siple Dome, (where noise is either not Gaussian or does not decrease with depth), the density measurements fit well to the exponential model defined in [2].

The damping factor α is found by considering the near surface and deep ice fluctuations.

$$\alpha = \frac{\ln \frac{N(z_{end}, \theta)}{N(0, \theta)}}{z_{end}} \quad (5)$$

Density fluctuations decrease over depth as expected and thus we note negative damping factors for all data sets.

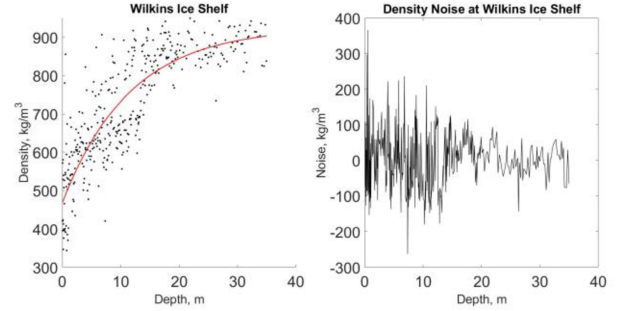


Figure 1. Density regression and calculated noise for Wilkins Ice Shelf.

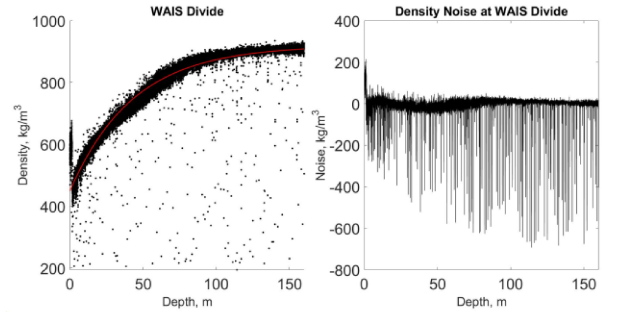


Figure 2. Density regression and calculated noise for WAIS Divide.

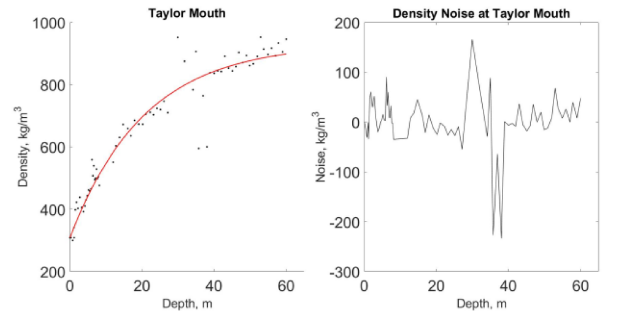


Figure 3. Density regression and calculated noise for Taylor Mouth.

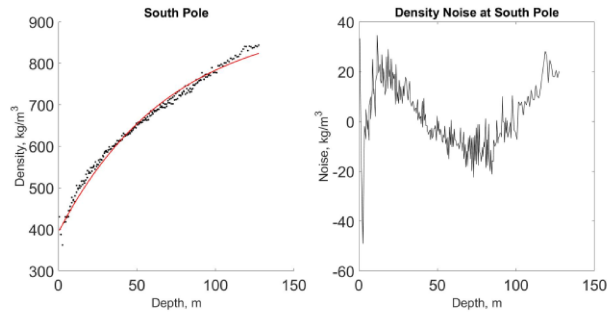


Figure 4. Density regression and calculated density noise for the South Pole.

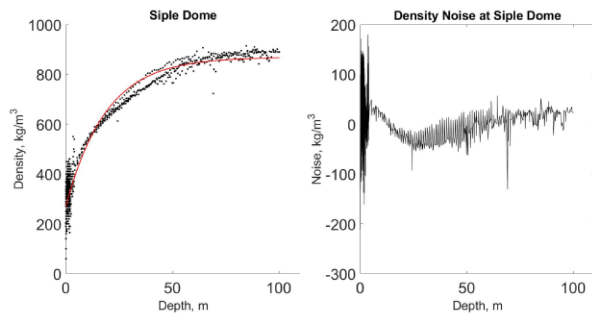


Figure 5. Density regression and calculated density noise for Siple Dome.

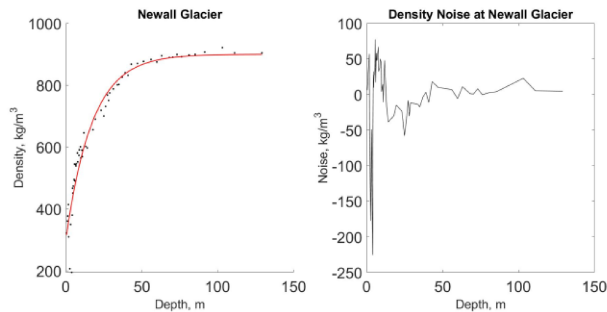


Figure 6. Density regression and calculated density noise for the Newall Glacier.

3.2. Grain radius fits

We analyzed two grain size datasets over Dome Concordia and the WAIS Divide and found that over several hundreds of meters of depth, grain radius does not follow a linear model. This is possibly due to changes in the rates of accumulation and/or significant ice densification. This may also be untrue for other locations in inland Antarctica.

That given, we found that the grain radius measurements could be reasonably fit to the linear model defined by [3] up to a few hundred meters of depth. The following are parameters found using measurements up to ~450 m depth, with growth component $n = 2$.

Location	r_{surf} (m)	Q (m)
WAIS Divide	0.678×10^{-3}	2.55×10^{-9}
Dome Concordia	0.569×10^{-3}	5.07×10^{-9}

Table 2. Grain radius parameters. Data used: WAIS Divide [13], Dome Concordia [14].

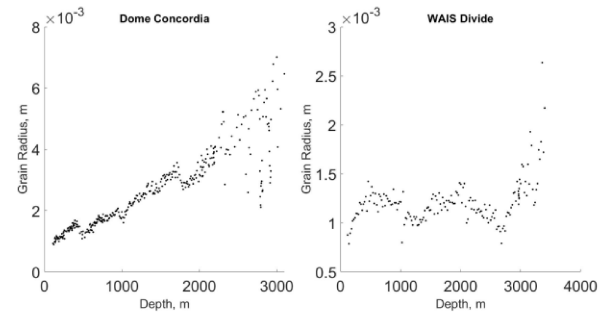


Figure 7. Grain radius measurements at Dome Concordia [14] and WAIS Divide [13].

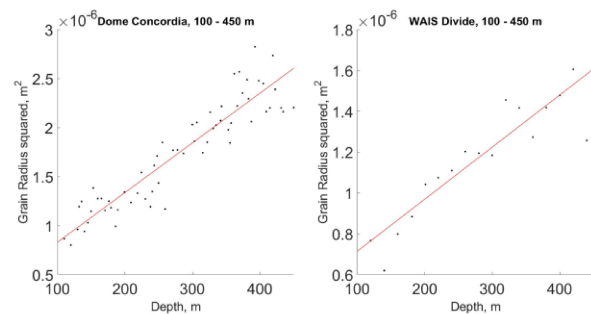


Figure 8. Fitted grain radius data for Dome Concordia (left) and WAIS Divide (right).

3.3. Goodness-of-fits

3.3.1. Density

Location	SSE (kg/m^3) ²	R^2	RMSE (kg/m^3)
Wilkins Ice Shelf	2.36×10^6	0.7062	79.1
WAIS	5.28×10^7	0.9301	32.7
Taylor Mouth	2.04×10^5	0.9288	54.3
South Pole	3.63×10^4	0.9867	12.9
Siple Dome	3.32×10^6	0.9420	53.3
Newall Glacier	1.38×10^5	0.9427	49.2

Table 3. Details for density regression.

3.3.2. Grain radius

Location (dataset)	SSE (m^4)	R^2	RMSE (m^3)
WAIS Divide	4.25×10^{-13}	0.8028	1.54×10^{-7}
Dome Concordia	2.29×10^{-12}	0.8629	1.99×10^{-7}

Table 4. Details for grain radius regression.

4. CONCLUSION

We have summarized and examined an exponential model for ice density and a linear model for ice grain radius.

For most ice density datasets analyzed in this study, subsurface density increases exponentially with depth and has noise with negative damping. There are exceptions to this, notably, South Pole, Taylor Mouth, and Siple Dome. For South Pole, the calculated noise shows an oscillating pattern indicating that the model is not a good match for the data. There are a few outlier points in the dataset for Taylor Mouth resulting in the large fluctuations noted between 30 and 40 meters of depth, most likely due to fact that the data was collected using radar detection. The errors in the fit at Siple Dome may be due to combining various datasets at similar locations. These datasets could be analyzed separately in the future to understand the location with more accuracy.

We analyzed two grain radius datasets and so supplemental data is needed for a better understanding of ice grain radius. For the available data, we found that grain radius does not increase linearly for all of depth, but a linear model can be reasonably considered for up to a few hundred meters. Thus, the grain radius parameters found in this study are relevant for studies focusing on the upper layers of ice sheets.

The ice density and grain radius models analyzed in this study largely agree with available data (with certain limitations that are mentioned above). The boundaries found in this study are by no means comprehensive of all of Antarctica, but they provide key insights into ice density and grain radius for various ice sheets for future remote sensing studies.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

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