Outage Management of Power Distribution Systems with Electricity-Dependent Medically-Vulnerable Critical Loads

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Abstract— **By deploying mobile power sources and using their network topology effectively, electric utilities can enhance resilience during power outages. This paper develops an outage management approach for power distribution systems in which loads equipped with electricity-dependent medical devices are considered as critical loads. Load criticality is determined by the distance of the load from shelters, the type of medical device, the socio-economic status of the customers, and the average age of the customers at medically vulnerable load points. The optimization model based on mixed-integer linear programming optimizes network reconfiguration, the installation of mobile power sources, and the curtailing of minimum critical loads. The model includes the constraints for a three-phase network configuration for practical grid representation. Optimal restoration schemes are obtained by maximizing restored weighted loads during restoration. In order to deal with uncertainty and imprecise judgments, a fuzzy modification of the analytical hierarchical process (FAHP) is used to prioritize the loads during restoration. The proposed method is tested on the modified IEEE 34-bus distribution test system under several scenarios. The outcome of this study can provide a strategy for power distribution grid operators to manage outages in the presence of loads with electricity-dependent medical devices.**

Keywords—Distribution system restoration, electricity-dependent medical device, mixed integer linear programming, resilience, mobile power sources

I. Introduction

Extreme events, such as wildfires, floods, earthquakes, and hurricanes have a negative impact on power distribution systems, resulting in extended outages and significant losses to many communities [1, 2]. In the U.S., power contingencies due to natural disasters have cost $18 to $33 billion per year since 2002 [3]. Utility companies make sure that critical loads are resilient during times of emergencies so that they can continue to provide essential services to the community [4]. Health care facilities, water stations, data centers, and other infrastructures for basic human needs are generally considered critical loads [5]. In power system resilience analysis, service restoration to critical loads is one of the most important topics. In [3], authors utilize microgrids to minimize critical load curtailment by considering constant weighting factors. By considering interruption cost rate of customers, authors in [4] prioritize critical load restoration by incorporating expected energy not served costs in the objective function. Authors in [5] use the value of lost load (VOLL) concept to provide a priority weighting factor for all loads. The VOLL calculates the economic loss of customers impacted by an unforeseen interruption in their electricity supply. Researchers in [3-5] presented resilience-oriented methods to prioritize the recovery of critical customers, but the literature has yet to explore deeply the nature of critical customers who need to be prioritized during a post-contingency restoration. The advancements in digital health and home medical technologies have made it necessary for many homes to be powered by a reliable and steady electricity source [6, 7]. It can be life-threatening if electricity is not restored after an outage for the more than 2.5 million Americans who rely on electricity-dependent medical devices [6]. The restoration of electricity must go beyond simply prioritizing critical load points when dealing with interruptions to patients' treatments, well-being, or even survival [7]. People with home medical devices accounted for 22% of all hospital admissions in a 24-hour period following the 2003 New York blackouts [7]. Hospitals in Louisiana following Hurricane Isaac on 2012 is another example where local hospital were treating "electricity emergencies" rather medical emergencies for people who could no longer operate electricity dependent medical or enabling devices [7].

The ability of critical loads to withstand a power outage is affected by many factors, including age, socioeconomic status, and evacuation capability. As the number of critical loads with electricity-dependent medical device such as oxygen concentrators, nebulizers, ventilators, dialysis machines, and sleep apnea machines increases, electric utilities must effectively utilize available capacities and resources to guarantee the delivery of power to highly sensitive loads while meeting system operational constraints. This could include modest system upgrades to allow for low-amperage services during outages [7] and deployment of dynamic switching devices (DSDs) to enable phase reassignment in order to transfer sensitive loads between phases [8]. Changing the grid topology by altering the status of switches and deploying backup generators are also essential for prompt restoration of critical loads [9]. Despite their usefulness in reducing power disruptions, backup generators are usually installed in hospitals and shelters and are therefore inaccessible outside of healthcare settings and cannot reach those who rely on home electricity-dependent medical devices especially those with lower socioeconomic statuses. Electric utilities, however, can strategically dispatch mobile emergency resources to enhance their distribution systems' self-healing capability against extreme events [9, 10]. The aim of this paper is to develop an outage management approach for distribution systems which can minimize the impact of outages on critical medical loads through the optimal deployment of mobile power sources and reconfiguration of distribution networks. In this paper, load points with medically vulnerable customers are considered critical loads,

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which are weighted by considering four criteria: distance from shelters, type of medical device, socioeconomic status, and average age of customers at medically vulnerable load points. The weights are fed into a mixed-integer linear programming model, to give priority to medically vulnerable load points. Using CVX environment and MOSEK solver, the optimization model for a modified IEEE 34-node three-phase distribution system is assessed in various cases. The contributions of this paper are summarized as follows:

1) The study presents an approach for prioritizing the restoration of loads with electricity-dependent medical devices during power distribution outages. This will provide an opportunity for the electric industry to enhance the resilience of medically vulnerable loads to prolonged outages.
2) FAHP frameworks are developed to prioritize medically vulnerable loads during an outage, which can integrate input from industry experts and key informants, especially where there is a lack of data.
3) This study highlights the importance of harnessing the flexibility of the distribution grid and distributed resources to deliver life-saving services to critical loads and to improve the quality of life of communities.

The rest of this paper is organized as follows. Section II introduces the three-phase restoration model. Fuzzy analytical hierarchical process (FAHP) is presented in section III. A test network using two IEEE 34 systems is introduced in section IV to provide a numerical result of the proposed model. Finally, in section V, the paper is summarized and concluded.

II. Restoration Methodology

To minimize the power outage impact on critical loads in a distribution system, the priority customer service restoration is formulated mathematically as a mixed integer linear programming problem.

A. Problem Objective

The objective function of the restoration model is formulated to maximize the weighted total loads as follows:

$$\min \sum_{i\in\Psi_B} w_i * \sum_{f\in\Psi_f} x_{i,f} * P_{Di,f} \quad (1)$$

where Ψ_B and Ψ_f represent the set of nodes and phases $\{a,b,c\}$, respectively. w_i is the weighting factor of load at node i, $P_{Di,f}$ is the served power demand at node i and phase f, $x_{i,f}$ is the continuous variable that indicates the percentage of the curtailed load at node i and phase f. The objective function (1) is subject to the following constraints.

B. Constraints

AC power flow equations are formulated by the linearized DistFlow branch model [11, 12]:

$$v_{i,f}^2 - v_{j,f}^2 \geq 2 * \sum_{f'\in\Psi_f} (r_{ij,f,f'} * p_{ij,f'} - x_{ij,f,f'} * q_{ij,f'}) + (1-\alpha_{ij}) * M \quad (2)$$

$$v_{i,f}^2 - v_{j,f}^2 \leq 2 * \sum_{f'\in\Psi_f} (r_{ij,f,f'} * p_{ij,f'} - x_{ij,f,f'} * q_{ij,f'}) - (1-\alpha_{ij}) * M \quad (3)$$

$$f \in \Psi_f, \quad ij \in \Psi_L, \quad i \text{ and } j \in \Psi_B$$

where Ψ_L represent the set of lines, α_{ij} is the binary indicator for connection status of the distribution line between node i and node j ($\alpha_{ij} = 1$ if connected). $p_{ij,f}$ and $q_{ij,f}$ are the active and reactive power flow on branch (i,j) and phase f, respectively. M is a large enough positive number. v_i and v_j are the voltage magnitude of nodes i and j, respectively. The left-hand side inequality of (2) and (3) are linearized by substituting $v_{i,f}^2$ and $v_{j,f}^2$ by another variable without squaring them.

Real and reactive power balance equations for each node are:

$$\alpha_{DGi,f} * P_{DGi,f} - P_{Di,f} * (1 - x_{i,f}) = \sum_{j\epsilon N_i} p_{ij,f} \quad (4)$$

$$\alpha_{DGi,f} * Q_{DGi,f} - Q_{Di,f} * (1 - x_{i,f}) = \sum_{j\epsilon N_i} q_{ij,f} \quad (5)$$

$$f \in \Psi_f, \quad i \in \Psi_B$$

N_i is the set of branches connected to node i. $P_{Di,f}$ and $Q_{Di,f}$ are the real and reactive demand at node i, respectively. $P_{DGi,f}$ and $Q_{DGi,f}$ are the active and reactive power output of distributed generators (e.g., mobile power sources) at node i, respectively. $\alpha_{DGi,f}$ is binary indictor for availability of generators at node i and phase f.

Allowable range of node voltages and maximum and minimum power generations are as follows:

$$(v_{i,f}^{min})^2 \leq (v_{i,f})^2 \leq (v_{i,f}^{max})^2 \quad (6)$$

$$\alpha_{DGi,f} * P_{DGi,f}^{min} \leq P_{DGi,f} \leq \alpha_{DGi,f} * P_{DGi,f}^{max} \quad (7)$$

$$\alpha_{DGi,f} * Q_{DGi,f}^{min} \leq Q_{DGi,f} \leq \alpha_{DGi,f} * Q_{DGi,f}^{max} \quad (8)$$

$$f \in \Psi_f, \quad i \in \Psi_B$$

$P_{DGi,f}^{min}$ and $P_{DGi,f}^{max}$ is the minimum and maximum active power generation at node i and phase f, respectively. $Q_{DGi,f}^{min}$ and $Q_{DGi,f}^{max}$ is the minimum and maximum reactive power generation at node i and phase f, respectively. $v_{i,f}^{min}$ and $v_{i,f}^{max}$ are the minimum and maximum voltage magnitude at node i and phase f.

Branch current capacity is given by (9).

$$p_{ij,f}^2 + q_{ij,f}^2 \leq \alpha_{ij} \cdot S_{ij,f}^2 \quad (9)$$

$$ij \in \Psi_L$$

$S_{ij,f}$ is the apparent power capacity of branch ij and phase f. The power flow in disconnected branches is set to zero using α_{ij}. Expression (9) presents a circular constraint and can be presented as a linear constraint using the quadratic constraint linearization method. Two square constraints are used to linearize the circular constraint of (9) as follows [13].

$$-\alpha_{ij} \cdot S_{ij,f} \leq p_{ij,f} \leq \alpha_{ij} \cdot S_{ij,f} \quad (10)$$

$$-\alpha_{ij} \cdot S_{ij,f} \leq q_{ij,f} \leq \alpha_{ij} \cdot S_{ij,f} \quad (11)$$

$$-\sqrt{2}\alpha_{ij} \cdot S_{ij,f} \leq p_{ij,f} + q_{ij,f} \leq \sqrt{2}\alpha_{ij} \cdot S_{ij,f} \quad (12)$$

$$-\sqrt{2}\alpha_{ij}\cdot S_{ij,f}\leq p_{ij,f}-q_{ij,f}\leq\sqrt{2}\alpha_{ij}\cdot S_{ij,f} \quad (13)$$

$$f\in\Psi_f,\qquad ij\in\Psi_L$$

The accuracy of the approximation increases with the use of more square constraints; however, for engineering applications, two square constraints are sufficient.

For the distribution network configurations to be radial, the following constraints [14] must be met:

$$\lambda_{ij}+\lambda_{ji}=\alpha_{ij}\quad ij\in\Psi_L \quad (14)$$

$$\sum_{j\in N_i}\lambda_{ij}=1\qquad i\in\Psi_B \quad (15)$$

$$\lambda_{0j}=0\quad j\in N(0) \quad (16)$$

Equation (14) indicates that a line between node i and node j is in the spanning tree $\alpha_{ij}=1$ if either node j is the parent of node i (λ_{ij}=1), or node i is the parent of node j (λ_{ji}=1). Equation (15) indicates every node except the substation node has exactly one parent, and equation (16) indicates that the substation node has no parents [9]. N_i and $N(0)$ are set of branches connected to node i and set of branches connected to substation node, respectively.

Phase Voltage Unbalance Rate (PVUR) is formulated using phase voltage magnitudes based on the IEEE standard [15]:

$$\%\,PVUR=\frac{\mathcal{M}_i}{v_i^{AVG}}*100 \quad (17)$$

where,

$$\mathcal{M}_i=\max(|v_{i,f}-v_i^{AVG}|) \quad (18)$$

$$v_i^{AVG}=\frac{\sum_{f\in\Psi_f}v_{i,f}}{3} \quad (19)$$

$$f\in\Psi_f,\qquad i\in\Psi_B$$

Since the left-hand side inequality of (2) and (3) are linearized by substituting $v_{i,f}^2$ by $u_{i,f}$, we need to rewrite the equation (18) to (19) based on $u_{i,f}$. Since $v_{i,f}=\sqrt{u_{i,f}}$ is a nonconvex (concave) equation, v_i will be linearized by substituting $\sqrt{u_{i,f}}$ by its first order Taylor approximation. Assuming that node voltages in a distribution system vary around the substation voltage, we can approximate the $\sqrt{u_{i,f}}$ at the equilibrium point equals to the substation voltage. Assuming $S=(u_{s,a},u_{s,b},u_{s,c})$ as the equilibrium point, the approximation can be calculated by equations (20)-(22). Three additional variables ($uu_{i,a}$, $uu_{i,b}$, $uu_{i,c}$) represent the linearized form of square root of the phase voltages.

$$uu_{i,f}=\sqrt{u_{i,f}}=\sqrt{u_{s,f}}+\frac{1}{2*\sqrt{u_{s,f}}}*(u_{i,f}-u_{s,f}) \quad (20)$$

$$f\in\Psi_f,\qquad i\in\Psi_B$$

The new variable of $uu_{i,f}$ are substituted into $v_{i,f}$ in equations (18) and (19) as follows:

$$\mathcal{M}_i=\max(|uu_{i,f}-v_i^{AVG}| \quad (21)$$

$$v_i^{AVG}=\frac{\sum_{f\in\Psi_f}uu_{i,f}}{3} \quad (22)$$

$$f\in\Psi_f,\qquad i\in\Psi_B$$

III. Fuzzy Analytical Hierarchical Process

One of the most important issues in modern management is making decisions when faced with multiple options and criteria (quantitative and qualitative) [16, 17]. In these cases, decision makers face a variety of options that need to be examined based on multiple criteria, both internal and external. AHP, which was first developed by Saaty [16], integrates experts' opinions and evaluation scores into a simple elementary hierarchy system by decomposing complicated problems from higher hierarchies to lower ones [16]. There is a great deal of importance given to expert opinion and key informants' knowledge in multicriteria problems when limited amounts of data are available to make decisions. With AHP, decision makers can not only measure the consistency of their judgement, but also engage in pairwise comparisons. Problems with too much uncertainty and fuzziness are generally not handled well by AHP [17].

FAHP is used in this paper to determine the priority degree of critical loads. The first step in an AHP-based procedure is to define effective criteria. In case of a severe disaster, it is imperative that critical loads that are far away from medical facilities and cannot reach shelters receive high priority, especially if the transportation network is disrupted. Another criterion at each load point is the type of medical device that relies on electricity. Medically vulnerable populations are categorized into three main classes [7]: Those reliant on electricity for independence, (e.g., electric wheel-chairs), those reliant on electricity for survival, (e.g., those with ventilators, oxygen concentrators, reliance on exceptional temperature stability), and those susceptible to heat/cold, or with limited mobility to leave home in a blackout. Socio-economic status and average age of the customers at electricity dependent load points are considered as the third and fourth criteria, respectively. After developing some effective criteria for achieving the goal, a pairwise comparison matrix [17] will be created. The matrix outlines the relative importance of different criteria, whose elements are determined by industry experts, community key informants, and the outcome of focused group surveys. In the pairwise comparison of AHP, fuzzy sets are introduced to express uncertain comparison ratios. The triangular fuzzy members, because of their popularity, are used to fuzzify the AHP. The membership function of a triangular fuzzy number on $\Re=(-\infty,+\infty)$ can be described as follows [18]:

$$M(x)=\begin{cases}\frac{x-l}{m-l}, & l<x<m\\ \frac{u-x}{u-m}, & m<x<u\\ 0, & \text{otherwise}\end{cases} \quad (23)$$

where $M:\Re\rightarrow[0,1]$ is the membership function, l, m, and u are considered as the lower bound, the mean bound, and the upper bound of the membership function, respectively. Pairwise comparison matrix consisting triangular fuzzy numbers are given below where the linguistic terms, introduced in Table I, are used to complete the comparison matrices.

$$\begin{bmatrix} (1,1,1) & (l_{12}, m_{12}, u_{12}) & \cdots & (l_{1m}, m_{1m}, u_{1m}) \\ (l_{21}, m_{21}, u_{21}) & (1,1,1) & \cdots & (l_{2m}, m_{2m}, u_{2m}) \\ \vdots & \vdots & (1,1,1) & \vdots \\ (l_{m1}, m_{m1}, u_{m1}) & (l_{m2}, m_{m2}, u_{m2}) & \cdots & (1,1,1) \end{bmatrix}$$

TABLE I. THE LINGUISTIC SCALE AND CORRESPONDING TRIANGLE FUZZY NUMBER

Scale	Definition	Membership Function
1	Equally important	(1, 1, 1)
3	Moderately more important	(2, 3, 4)
5	Strongly more important	(4, 5, 6)
7	Very strongly more important	(6, 7, 8)
9	Exceedingly more important	(9, 9, 9)

Fig. 1 shows the Saaty's scales [16] expressed as fuzzy numbers.

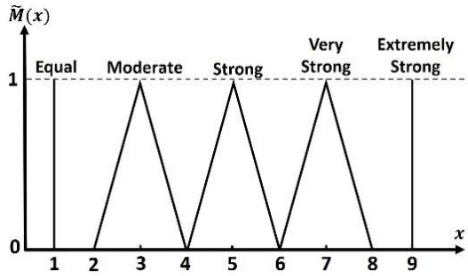

Fig. 1. Membership function of triangle fuzzy numbers corresponding to the linguistic scale [17, 18]

More details about Fuzzy arithmetic operations can be found in [17] and [19]. After creating the pairwise comparison matrix, we will find the final weight of each criterion. In order to find the weights, the geometric mean of each criterion is calculated as follows:

$$\left[\prod_{j=1}^{m} c_{ij}\right]^{1/m} \quad , i = 1,2, \dots, m \tag{24}$$

The final normalized weight of each criterion can be obtained by having the final weighting vector of each criterion divided by their column sum. All the calculations are done based on the fuzzy arithmetic rules. Criteria fuzzy weight vectors will be de-fuzzified in order to achieve normalized crisp score of the criteria. The next step is to create comparison matrix for each criterion and compare alternatives (critical load points) with respect to that criterion. Similar to the criterion calculation, the geometric means of fuzzy comparison values and relative fuzzy weights of critical load points for each criterion are calculated. Using normalized relative weights of criteria and normalized weights of alternatives with respect to each criterion, individual score of each alternative is calculated.

$$\begin{bmatrix} R_{A_1C_1} & R_{A_1C_2} & \cdots & R_{A_1C_m} \\ R_{A_2C_1} & R_{A_2C_2} & \cdots & R_{A_2C_m} \\ \vdots & \vdots & \vdots & \vdots \\ R_{A_nC_1} & R_{A_nC_2} & \cdots & R_{A_nC_m} \end{bmatrix} \begin{bmatrix} W_{C_1} \\ W_{C_2} \\ \vdots \\ W_{C_m} \end{bmatrix} = \begin{bmatrix} S_{A_1} \\ S_{A_2} \\ \vdots \\ S_{A_n} \end{bmatrix}$$

$R_{A_nC_m}$ is the normalized weight of the n^{th} alternative with respect to the m^{th} criterion, W_{C_m} is the weight of the m^{th} criterion. S_{A_n} is the final score for the n^{th} alternative. The scores will represent the priority of each alternative.

IV. CASE STUDIES

A test network using two IEEE 34-bus [20] systems is formed to validate the model, as shown in Fig. 2. Five tie lines, one healthcare facility, and five electricity-dependent medically vulnerable loads are added to the test system. Feeders 1 and 2 are connected through the tie line between node 18 and node 34 as shown in Fig. 2.

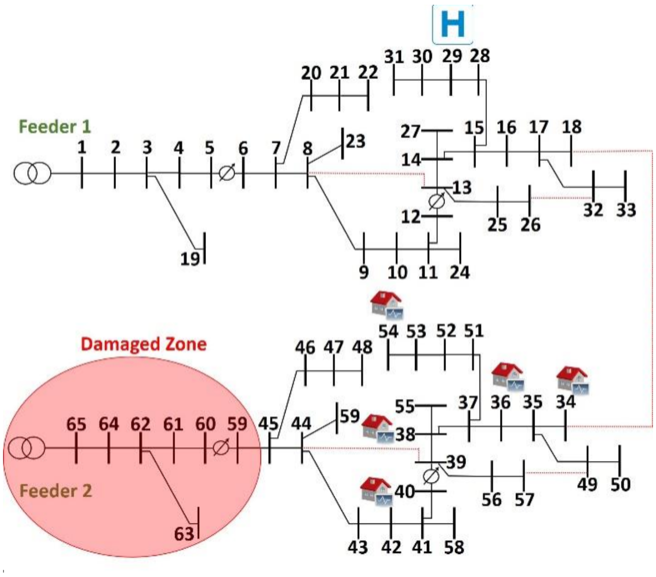

Fig. 2. Two connected 34-Bus distribution networks with medically vulnerable loads and tie lines

It is assumed that feeder 2 is disconnected from the substation, thus the load points located outside the damaged zone need to be restored through the feeder 1 substation. We assume feeder 1, in case of an emergency, will first serve its own loads, and allocate the remaining capacity to the critical loads of feeder 2. The radial test network is a three-phase distribution system. Each feeder has a total active load of 1,769 kW, and a total reactive load of 1,044 kVar. The substation line-to-line voltage is 24.9 kV. The minimum and maximum allowable voltages at each node were considered to be 0.9 and 1.1 per unit, respectively. The test network also benefits from three step voltage regulators in order to help with regulating voltages at each phase. One mobile power source is assumed to be available for serving the critical loads during emergencies. Other network parameters and load data are provided in [20]. Using MOSEK [21] in a desktop computer with Intel Core i7 2.8 GHz and 16 GB RAM, the mixed integer optimization problems are solved. In Fig. 2, node 29 is assumed to be a node connected to a healthcare facility. In the event of an emergency, if individuals are able to evacuate and reach the facility, this facility can shelter those in need of medical support. Five load points, 34, 36, 38, 42, and 54 are considered to be the critical load points and the five alternatives for the FAHP. The model in III is used to determine the weights required for prioritizing the load points during the restoration process. Considering the four criteria (load point distance from the healthcare facility, type of medical device at the load point, and socio-economic status, and average age of the connected customers to the load point), the comparison matrix for criteria is formed as follows:

TABLE II. COMPARISON MATRIX OF CRITERIA (FW: FINAL WEIGHT, NFS: NORMALIZED FUZZY SCORES, NCS: NORMALIZED CRISP SCORE), AND C: CRITERIA

C	C_1	C_2	C_3	C_4	FW	NFS	NCS
C_1	1,1,1	2,3,4	4,5,6	9,9,9	2.9,3.4,3.8	0.4,0.56,0.75	0.57
C_2	$\frac{1}{4},\frac{1}{3},\frac{1}{2}$	1,1,1	2,3,4	6,7,8	1.3,1.6,2	0.2,0.27,0.3	0.28
C_3	$\frac{1}{6},\frac{1}{5},\frac{1}{4}$	$\frac{1}{4},\frac{1}{3},\frac{1}{2}$	1,1,1	4,5,6	0.64,0.76,0.9	0.9,0.1,0.2	0.13
C_4	$\frac{1}{9},\frac{1}{9},\frac{1}{9}$	$\frac{1}{8},\frac{1}{7},\frac{1}{6}$	$\frac{1}{6},\frac{1}{5},\frac{1}{4}$	1,1,1	0.2,0.24,0.26	0.03,0.04,0.05	0.04

After the NCS calculation for criteria, the technique in III will be repeatedly applied to find the final scores for alternatives with respect to each criterion. Table III shows the NCSs of alternatives (critical load points) with respect to each criterion.

TABLE III: NCS OF EACH ALTERNATIVE FOR EACH CRITERION

Alternatives	C_1	C_2	C_3	C_4
A_1	0.04	0.4	0.4	0.1
A_2	0.06	0.09	0.03	0.39
A_3	0.14	0.4	0.05	0.03
A_4	0.52	0.09	0.4	0.39
A_5	0.28	0.04	0.14	0.1

Using NCS of criteria and NCS of alternatives with respect to each criterion, individual score of each alternative is calculated as follows:

$$\begin{pmatrix} 0.04 & 0.40 & 0.40 & 0.10 \\ 0.06 & 0.09 & 0.03 & 0.39 \\ 0.14 & 0.4 & 0.05 & 0.03 \\ 0.52 & 0.09 & 0.40 & 0.39 \\ 0.28 & 0.04 & 0.14 & 0.10 \end{pmatrix} \begin{pmatrix} 0.57 \\ 0.28 \\ 0.13 \\ 0.04 \end{pmatrix} = \begin{pmatrix} 0.19 \\ 0.08 \\ 0.20 \\ 0.39 \\ 0.19 \end{pmatrix}$$

The normalized individual score of alternatives (load points 34, 36, 38, 42, and 54) will be 0.18, 0.08, 0.19, 0.37, and 0.18, respectively. Using the determined restoration weights for the critical load points the following scenarios are studied.

Case I (Base Case): In this case, no mobile power sources is available to serve the critical loads. All three phases are subject to unlimited load curtailment. The weights derived by the FAHP are used to prioritize the load restoration. Voltage unbalance is considered to be less than 2% at each load point.

TABLE IV. LOAD SHEDDING, OPTIMAL TOPOLOGY, VOLTAGE UNBALANCE, AND OBJECTIVE FUNCTION OF CASE I

Node Number i	x_i^A	x_i^B	x_i^C
34	1	1	1
36	1	1	0.8038
38	1	1	0
42	1	0	0
54	1	0.8975	1
Open Switches	8-9 16-17 49-57 39-44		
Voltage Unbalance	1.77%		
Objective Function	6.06456 kW		

As it can be seen in Table IV, phase A of all critical loads are completely curtailed. At load point 34, all phases have been shed meaning that the load point has been completely disconnected. This means there won't be even one phase available for the electric utility to provide service to a medical vulnerable load point. As shown in Table IV, although the utility gives high priority to critical loads, not all loads are restored due to the technical constraints. The maximum voltage unbalance observed is 1.77% which is below the threshold of 2% for this case.

Case II: This scenario represents a case where the utility can guarantee the delivery of power to a specific load point. As shown in Table IV, all phases of the load point 34 are remained de-energized after the restoration. We now assume a scenario where the utility guarantees that phase C of load point 34 cannot be shed greater than 80 percent. Table V shows how other critical loads will be affected.

TABLE V: LOAD SHEDDING, OPTIMAL TOPOLOGY, VOLTAGE UNBALANCE, AND OBJECTIVE FUNCTION OF CASE II

Node Number i	x_i^A	x_i^B	x_i^C
34	1	1	0.8
36	1	1	0.8199
38	1	1	0
42	1	0	0
54	1	0.9421	1
Open Switches	8-9 14-15 38-39 40-41		
Voltage Unbalance	1.92%		
Objective Function	6.08156 kW		

Even though the restored loads in load points 36 and 54 are decreased, the electric utility has been able to at least maintain 20 percent of the load in phase C of load point 34. It also needs to be noted that at least one phase at each critical load point will remain partially energized after the restoration. This will provide the utility with an opportunity to switch medically vulnerable loads to the energized line as envisioned in [7]. Comparing with case I, the maximum voltage unbalance of the system has increased and the supplied loads in both phases B and C have decreased.

Case III: We consider another scenario where at least 10 percent of the load connected to phase B and phase C of each load point to be restored. Table VI shows how the other load points are affected.

TABLE VI. LOAD SHEDDING, OPTIMAL TOPOLOGY, VOLTAGE UNBALANCE, AND OBJECTIVE FUNCTION OF CASE III

Node Number i	x_i^A	x_i^B	x_i^C
34	1	0.9	0.9
36	1	0.9	0.8299
38	1	0.9	0
42	1	0.5961	0
54	1	0.9	0.9
Open Switches	11-12 13-14 37-38 40-41		
Voltage Unbalance	1.96%		
Objective Function	6.17175 kW		

Compared to case II, the minimum requirement of the service is delivered to phase B and phase C of each load point at the cost of shedding more loads. The amount of curtailed load and the voltage unbalanced has increased.

Case IV: In this case, we assume an emergency scenario where the electric utility cannot change the status of switches and one of the step voltage regulators (the regulator between nodes 12 and 13, in our case) is out of service. We assume the electric utility guarantees a continuous service for the medically vulnerable customers connected to load points of phase C. This is enabled by maintaining 5% of loads and Table VII illustrates the changes compared to the base case.

TABLE VII. LOAD SHEDDING, OPTIMAL TOPOLOGY, VOLTAGE UNBALANCE, AND OBJECTIVE FUNCTION OF CASE IV

Node Number i	x_i^A	x_i^B	x_i^C
34	1	1	0.95
36	1	1	0.95
38	1	1	0.2211
42	1	1	0
54	1	1	0.95
Open Switches	8-13, 26-32, 49-57, 39-44		
Voltage Unbalance	2.69%		
Objective Function	6.6072 kW		

In order for the algorithm to converge we relax the voltage unbalance constraint from 2% to 3%. This increase in the maximum voltage unbalance is tolerable in emergency conditions and for a short period of time [22]. The amount of load curtailed and the objective function have also increased.

Case V: This case will consider a scenario where the electric utility has the option to restore loads using a mobile power source. Three scenarios are compared in this case. Scenario 1, 2, and 3 each represent three single-phase mobile power sources of 15kW, 20kW, and 25kW, respectively. This will provide a total capacity of 45kW, 60kW, and 75kW for each scenario. The contribution of mobile sources with and without reconfiguration is compared with the base case.

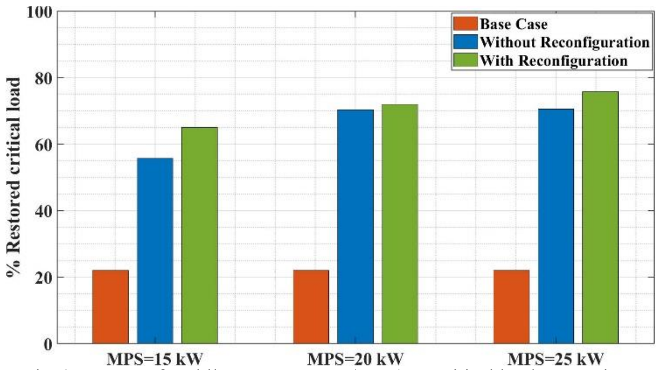

Fig. 3: Impact of mobile power source (MPS) on critical load restoration.

Results in Fig. 3 show significant improvement in the restored critical load compared to the base case. The increase in the capacity of available mobile sources along with the optimal system reconfiguration will play a significant role in enhancing the hosting capacity of the grid thus maximizing the restored critical loads. Fig. 4 shows the impact of mobile source on the unbalance voltage at each load point. We used the case IV which is the case with voltage unbalance greater than 2%. As it can be seen mobile sources will bring the unbalance below the 2% threshold. This clearly shows that using the mobile sources the electric utility can guarantee 5% of restored loads on phase C while maintaining the voltage unbalance within the standard threshold [22].

V. CONCLUSION AND FUTURE WORK

This paper has presented an outage management approach for determining the optimal topology and mobile power source utilization of the power distribution grids. The framework is modeled as a mixed-integer linear programming problem, where FAHP is used to determine the weighting factor of medically vulnerable load points as an input for the optimization engine. Through network reconfiguration and the installation of mobile power sources, critical load restoration is maximized. The proposed framework has been validated considering a modified three-phase IEEE 34-bus distribution system. Numerical results demonstrate that the proposed optimization framework can effectively ensure critical load restoration and enhance the distribution network resiliency during power outages. Future work will include the performance of various electricity-dependent medical devices under a variety of outage restoration scenarios.

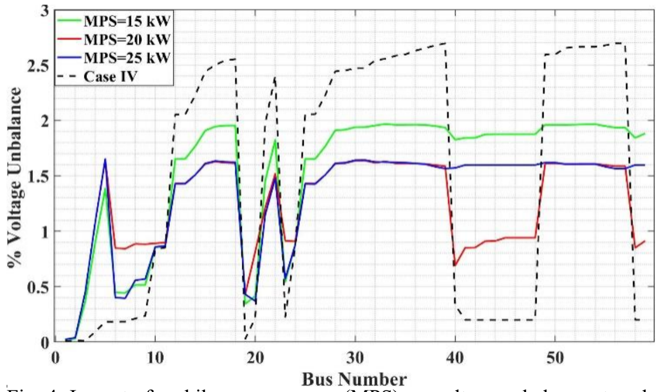

Fig. 4: Impact of mobile power source (MPS) on voltage unbalance at each load point

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