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Tidal capture of stars by supermassive black holes: implications for periodic nuclear transients and quasi-periodic eruptions

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ABSTRACT

Stars that plunge into the centre of a galaxy are tidally perturbed by a supermassive black hole (SMBH), with closer encounters resulting in larger perturbations. Exciting these tides comes at the expense of the star's orbital energy, which leads to the naive conclusion that a smaller pericentre (i.e. a closer encounter between the star and SMBH) always yields a more tightly bound star to the SMBH. However, once the pericentre distance is small enough that the star is partially disrupted, morphological asymmetries in the mass lost by the star can yield an *increase* in the orbital energy of the surviving core, resulting in its ejection – not capture – by the SMBH. Using smoothed particle hydrodynamics simulations, we show that the combination of these two effects – tidal excitation and asymmetric mass-loss – results in a maximum amount of energy lost through tides of ~ 2.5 per cent of the binding energy of the star, which is significantly smaller than the theoretical maximum of the total stellar binding energy. This result implies that stars that are repeatedly partially disrupted by SMBHs many ($\gtrsim 10$) times on short-period orbits ($\lesssim$few years), as has been invoked to explain the periodic nuclear transient ASASSN-14ko and quasi-periodic eruptions, must be bound to the SMBH through a mechanism other than tidal capture, such as a dynamical exchange (i.e. Hills capture).

Key words: hydrodynamics – galaxies: nuclei.

1 INTRODUCTION

Binary stars may form when two field (i.e. individual) stars pass close to one another on nearly parabolic trajectories. As the stars pass, tidal oscillatory modes are excited in the stars at the expense of the stars' orbital energy. If the encounter is sufficiently close, the tides dissipate enough orbital energy to bind the stars to one another (Fabian, Pringle & Rees 1975; Press & Teukolsky 1977).

A star may also be scattered into the region of parameter space, known as the 'loss cone', that brings the star's distance of closest approach very near a supermassive black hole (SMBH) in the nucleus of a galaxy (Frank & Rees 1976; Lightman & Shapiro 1977; Cohn & Kulsrud 1978). Reasoning analogously to the binary formation scenario, one is tempted to conclude that stellar orbits with distances of closest approach nearer the black hole are more strongly tidally perturbed by, and thus more tightly bound to, the SMBH. However, there is a fundamental upper limit as to how efficient this process can be, because tides cannot transfer more energy into oscillations than the binding energy of the star itself. Additionally, as the pericentre distance of the star continues to decrease, the perturbative tidal limit breaks down, and the star starts to lose a fraction of its outer envelope (known as a partial tidal disruption event, TDE). Recent simulations have demonstrated that the asymmetric ejection of tidal debris in a partial TDE results in the *unbinding* of the surviving stellar core from the SMBH (Manukian et al. 2013; Gafton et al. 2015). Therefore, the maximum binding energy a star can achieve through tidal interactions with an SMBH could be substantially smaller than the theoretical limit of the star's own binding energy. This maximum binding energy generated through tidal dissipation sets a fundamental time-scale over which one would expect repeating partial TDEs – a star that is bound to an SMBH that is partially destroyed on each pericentre passage (e.g. Zalamea, Menou & Beloborodov 2010; Campana et al. 2015; Miniutti et al. 2019; Payne et al. 2021) – to recur if the bound star is supplied through tidal dissipation.

In this paper, we present the results of smoothed particle hydrodynamics (SPH) simulations of partial TDEs from which we obtain the maximum binding energy. In Section 2.1, we describe the set-up of the simulations, and in Section 2.2 we present the results. In Section 3, we discuss the implications of our findings in the context of periodic nuclear transients (PNTs) and quasi-periodic eruptions (QPEs). We summarize and conclude in Section 4.

2 SIMULATIONS

2.1 Methodology

We performed simulations of partial TDEs with the SPH code PHANTOM (Price et al. 2018). The star was modelled as a polytrope with a $\gamma = 5/3$, adiabatic equation of state, with mass $M_\star = 1\,\mathrm{M}_\odot$ and radius $R_\star = 1\,\mathrm{R}_\odot$. In each simulation, the star was injected from a distance of $12r_\mathrm{t}$, where $r_\mathrm{t} = R_\star(M_\bullet/M_\star)^{1/3}$, from the SMBH of mass $M_\bullet = 10^6\,\mathrm{M}_\odot$ with the centre of mass on a parabolic trajectory. The impact parameter $\beta \equiv r_\mathrm{t}/r_\mathrm{p}$, where r_p is the pericentre distance of

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the star, was varied between $\beta = 0.4$ and $\beta = 0.8$ in steps of $\Delta\beta = 0.02$. Additional details of the simulation set-up can be found in Coughlin & Nixon (2015), and the algorithms for e.g. self-gravity can be found in Price et al. (2018).

We performed simulations with 1.25×10^5, 10^6, and 8×10^6 particles, and found very little deviation in the outcome. All results presented here are from the 8×10^6 particle runs. For the purposes of determining the properties of the surviving star, we calculate averages using only those particles that have a density greater than 1 per cent of the maximum global density within the simulation, $\rho_{\max}$; we define this subset of particles as the core. For example, the location of the core is calculated as the average position of the subset of particles that satisfy this criterion, and the distance from the SMBH, r_c, is determined from the Euclidean norm of the position. The analogous statement applies to the core velocity and speed, v_c. Over the range of β we simulated, this criterion excludes the tidally stripped tails, and reducing the fraction to 0.1 per cent does not impact the results because the tidal tails have a substantially lower density than $\sim$1 per cent of $\rho_{\max}$. Similarly, increasing the fraction to 10 per cent has a negligible effect on the results. However, values $>$ 10 per cent can lead to significant noise in the results because in this case there are too few particles that contribute to the average.

We run each simulation until the core energy has settled to a constant value that we measure for our results. For disruptions with $\beta < 0.6$, this time is $\lesssim$1 d after pericentre passage. Simulations with $\beta > 0.6$ are run up to $\sim$5 d post-pericentre to ensure that the energy has converged to a constant value. Running our simulations to later times has no effect on our results.

2.2 Results

The left-hand panel of Fig. 1 shows the specific orbital energy of the core, calculated according to

$$\epsilon_c = \frac{1}{2}v_c^2 - \frac{GM_\bullet}{r_c}. \tag{1}$$

For $0.4 \lesssim \beta \lesssim 0.62$, the orbital energy is negative, implying that the surviving core is bound to the SMBH. The global minimum energy is $\sim$ 2–3 per cent of the binding energy of the star, i.e.

$$\epsilon_{c,\min} \simeq -0.025\frac{GM_\star}{R_\star}. \tag{2}$$

The fact that the star becomes bound to the SMBH following its tidal interaction is in agreement with the classical calculations of tidal dissipation by Fabian et al. (1975) and Press & Teukolsky (1977).

Conversely, all disruptions with $\beta \gtrsim 0.62$ result in a core with a net positive energy following the interaction with the SMBH, indicating that the star is on an unbound trajectory. This result agrees with those of Manukian et al. (2013) and Gafton et al. (2015), who found that the 'kick' to the star could result in a velocity that is approximately the escape speed of the star for β very close to the value at which complete disruption occurs ($\beta \simeq 0.9$ for a 5/3 polytrope; Guillochon & Ramirez-Ruiz 2013; Mainetti et al. 2017). Therefore, if a star reaches a pericentre distance with $0.62 \lesssim \beta \lesssim 0.9$, the star is not tidally captured but is instead tidally ejected, never to return to pericentre.

With the energy-semimajor axis and energy–period relationships for a Keplerian orbit, we find that the semimajor axis of the captured star (for $\beta \lesssim 0.62$, i.e. for stars that are not ejected) is

$$a_c = -\frac{GM_\bullet}{2\epsilon_c} = \frac{1}{\eta_c}\frac{R_\star}{2}\left(\frac{M_\bullet}{M_\star}\right), \tag{3}$$

while the orbital period is

$$T_c = \frac{2\pi GM_\bullet}{(-2\epsilon_c)^{3/2}} = \frac{1}{\eta_c^{3/2}}\left(\frac{R_\star}{2}\right)^{3/2}\frac{2\pi}{\sqrt{GM_\star}}\left(\frac{M_\bullet}{M_\star}\right). \tag{4}$$

Here, we defined the specific orbital energy of the captured star as $\epsilon_c = -\eta_c GM_\star/R_\star$, where $\eta_c \lesssim 0.025$ (from Fig. 1). Using $M_\bullet/M_\star = 10^6$ in equation (3), the minimum apocentre distance of the star (with $\eta_c = 0.025$) is $\sim$1 pc for $R_\star = 1\,R_\odot$.

The right-hand panel of Fig. 1 shows the orbital period of the captured star as a function of β from the simulations and using equation (4). We see that the minimum orbital period of the captured star is $\sim 3 \times 10^4$ yr. However, because of the strong dependence on η_c in equation (4), the period can be much larger for only small changes in β.

3 DISCUSSION

Some current models for PNTs and QPEs suggest that they can be produced by stars on bound orbits about SMBHs with periods from hours to $\sim$*few* years, with the star being partially disrupted and feeding an accretion flare every pericentre passage (Miniutti et al. 2019; King 2020; Payne et al. 2021; King 2022; Wevers et al. 2022). PNTs have been suggested to have system parameters that are comparable to $M_\bullet = 10^7\,M_\odot$, $R_\star = 1\,R_\odot$, $M_\star = 1\,M_\odot$, which gives, from equation (4) with $\eta_c = 0.025$,

$$T_{\rm PNT} \simeq 3 \times 10^5\ \text{yr}, \tag{5}$$

which is obviously too long to explain the observed periods $\lesssim$1 yr (Payne et al. 2021; Wevers et al. 2022). Even if one could inject the theoretical maximum energy into the star, and thus set $\eta_c = 1$ in equation (4), we would obtain $T_{\rm PNT} \simeq 10^3$ yr. As argued by Cufari, Coughlin & Nixon (2022), even in this overly optimistic scenario, PNTs need an additional mechanism to bind the star sufficiently tightly to the SMBH to produce their observed periods *in situ*.

On the other hand, it is possible that a PNT could *initially* be generated with an orbital period given by equation (5), but the period then decays through gravitational wave emission (over millions of years) to produce a period as short as $\sim$1 yr by the time that we observe it.[1] However, a problem with this interpretation is that even for the maximum value of $\eta_c = 0.025$, equation (3) for the semimajor axis of the orbit yields an apocentre distance of $\sim 2a_c \sim 10$ pc for $M_\bullet/M_\star = 10^7$. Thus, we would expect the star to interact with, and be once again perturbed by, the nuclear star cluster, and it seems very unlikely that the star will repeatedly return to the same pericentre over multiple passages.

Additionally, while we have assumed that the star is on a parabolic orbit, it could have a velocity at infinity that is $v_\infty \simeq \sigma$, with σ the galactic velocity dispersion (Miller et al. 2005). From Fig. 1, the maximum amount of energy able to be dissipated through tides is

$$\epsilon_{c,\max} \simeq 0.025\frac{GM_\star}{R_\star} \simeq \frac{1}{2}\left(\frac{M_\star}{M_\odot}\right)\left(\frac{R_\star}{R_\odot}\right)^{-1}\left(100\ \text{km s}^{-1}\right)^2. \tag{6}$$

For a $10^7\,M_\odot$ SMBH, the velocity dispersion from the M–σ relation is $\sigma \sim 110$ km s^{-1} (Marsden et al. 2020). Thus, tides may not actually be capable of dissipating the true (positive) energy of the orbit, and

[1] As pointed out in Payne et al. (2021), the rate of change of the orbital period due to gravitational waves is actually too small to explain the observed value for ASASSN-14ko, but it should be possible for gravitational waves to shrink the orbit in general and for other systems.

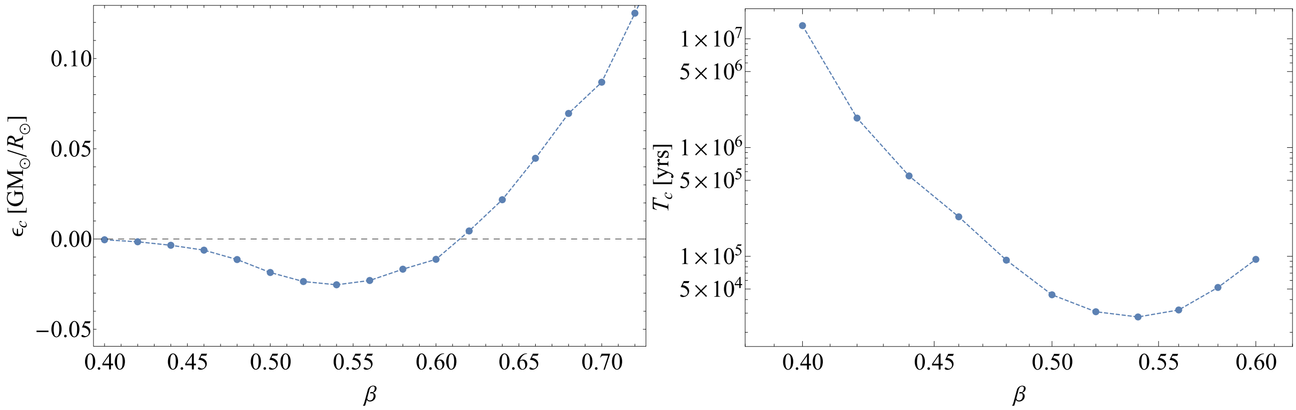

Figure 1. Left: The energy imparted to the star as a function of the impact parameter β. Stars with $\epsilon_c < 0$ are bound to the black hole. There is a small range of β for which mass-loss occurs on first pericentre passage (i.e. $\beta > 0.5$) and for which $\epsilon_c < 0$, the necessary condition for a repeating partial disruption to occur. Right: The period of the orbit of the star for encounters that yield a captured star as a function of the impact parameter β. The minimum period corresponds minimum orbital energy (i.e. the maximum binding energy), and for these parameters is $>10^4$ yr.

hence binding the star through this mechanism may be impossible in the first place.

QPEs – with periods of the order hours – are typically modelled with an $\sim 10^5\,\mathrm{M}_\odot$ SMBH partially disrupting a white dwarf star. For such a system, equation (4) with $M_\star = 0.6\,\mathrm{M}_\odot$, $R_\star = 0.011\,\mathrm{R}_\odot$ (Nauenberg 1972), $M_\bullet = 4\times10^5\,\mathrm{M}_\odot$, and $\eta_c = 0.025$ gives

$$T_{\mathrm{QPE}} \simeq 28\ \mathrm{yr}, \quad (7)$$

which shows that it is not possible to bind a star to an SMBH through tidal dissipation and immediately reproduce the observed orbital periods of QPEs. As for PNTs, it is none the less possible that gravitational wave emission shrinks the orbit[2] to $\sim$ hours before we detect them. Because the binding energy of a white dwarf is substantially larger than that of a main-sequence star, the minimum apocente distance is correspondingly smaller (from equation 3), and the maximum imparted energy through tides is substantially larger than the energy at infinity. Thus, without further investigation that is outside the scope of this work, we cannot conclusively state whether or not an additional mechanism is required for producing the observed orbital periods in QPEs (under the paradigm that they are powered by repeatedly partially disrupted white dwarfs).

On the other hand, the tidal breakup of a binary star system (i.e. Hills capture; Hills 1975) can bind the star with a substantially shorter period (compared to just tidal dissipation) if the binary is sufficiently tight. As demonstrated by Cufari et al. (2022), this is a plausible explanation for the origin of the $\sim$114 d period of ASASSN-14ko. An additional argument against the gravitational wave inspiral scenario for explaining ASASSN-14ko is that the mass lost by the star must be $\sim$1 per cent of the mass of the star to power the emission (Cufari et al. 2022), and hence it cannot have survived many ($\gtrsim$100 s) interactions prior to the $\sim$10 that have been observed since the initial detection. Similarly, if the white dwarf binary separation is not much larger than the radius of the white dwarf itself, then equation (5) from Cufari et al. (2022) with $a_\star = 0.04\,\mathrm{R}_\odot$, $M_\bullet = 4\times10^5\,\mathrm{M}_\odot$, and $M_\star = 0.6\,\mathrm{M}_\odot$ yields an orbital period of $\sim$8.3 h for the period of the bound star following a Hills capture event. As also suggested in Cufari et al. (2022), it therefore seems plausible that the periods of QPEs can be generated with a dynamical exchange process without the need for additional dissipation through other means.

[2]The orbital period may also be reduced through the interaction with a pre-existing AGN disc (Syer, Clarke & Rees 1991; Cufari et al. 2022; Lu & Quataert 2023).

4 SUMMARY AND CONCLUSIONS

We presented SPH simulations of the interaction between a star and an SMBH to determine the maximum degree by which a star can be bound to an SMBH through tidal dissipation. Two competing mechanisms prevent the star from becoming arbitrarily bound to the SMBH. As the distance of closest approach between the star and SMBH shrinks, energy from the star's orbit is expended in exciting dynamical tides in the star. However, once the star reaches a distance of closest approach comparable to its tidal radius, the star is kicked to positive energies as a result of asymmetry in the tidal tails that are liberated from the star. The competition between these two physical effects results in a minimum possible orbital energy of the star following the tidal encounter. For the parameters we simulated here, the location of this minimum is at $\beta \sim 0.55$ (pericentre distance of $r_t/0.55$ with r_t the usual tidal radius) and the binding energy of the orbit is $\sim$ 2.5 per cent of the star's binding energy; see equation (2) specifically. This minimum energy is significantly smaller than the theoretical maximum, being the entirety of the stellar binding energy.

In order to produce a repeating partial tidal disruption via a tidal dissipation mechanism, the energy kick imparted to the star must be negative, otherwise the star will be ejected on a hyperbolic trajectory. Hence, it would appear that there is a relatively small region of parameter space within which the star is only partially destroyed, not ejected, and survives for many ($\gtrsim$10) pericentre passages, specifically $0.4 \lesssim \beta \lesssim 0.5$ for the type of star considered here. Initially, non-rotating stars in this range of β will be rotating at a non-trivial fraction of breakup following the initial interaction, which will move the effective tidal radius out (Golightly, Coughlin & Nixon 2019), but provided that the pericentre (effectively unaltered because of the small ratio of the maximal angular momentum of the star to the angular momentum of the orbit itself) is still within this tidal radius, the star may transfer a small amount of mass during each additional pericentre passage. In this manner, a star may undergo many cycles of partial disruptions before being destroyed or ejected.

In our simulations, we modelled the star as a 5/3 polytrope, which is most applicable to low-mass main-sequence stars and low-mass white dwarfs. By number, most stars are thought to fall into this regime. However, more massive (radiative) stars are considerably more centrally concentrated than predicted by a 5/3 polytropic model. Here, we have shown that the minimum energy for the captured star – modelled as a 5/3 polytrope – occurs at $\beta \approx 0.55$. This result will depend somewhat on the type of star being considered. For example, fig. 3 of Manukian et al. (2013) shows that it occurs for $\beta < 1$ when the star is modelled as a 4/3 polytrope and that there is very little dependence of the result on the black hole mass. Faber, Rasio & Willems (2005) considered the tidal capture of a planet by a star in which the mass ratio was $q = 0.001$, and found a minimum binding energy of $\sim$ 14 per cent of the binding energy of the planet at $\beta = 10/19 \simeq 0.523$ (see their table 1). Kremer et al. (2022) also recently considered black hole–star systems with mass ratios closer to unity, and found a similar effect to the one described here if the mass ratio was 0.02 or 0.05 if the star was modelled as a $\gamma = 5/3$ polytrope, but that the star was able to go from bound to completely disrupted – without being ejected – as the mass ratio increased beyond 0.05 and the star was modelled with the Eddington standard model (see their fig. 1). We defer an analysis of the minimum orbital energy – and the β at which the minimum energy occurs – as a function of the mass ratio and the type of star to future work.

In the context of extreme mass-ratio inspirals, Zalamea et al. (2010) demonstrated that a white dwarf (or other compact object) completes thousands of large-eccentricity orbits before reaching the direct capture radius of the SMBH. However, their model accounts only for the orbital decay due to gravitational wave emission and omits tidal dissipation and mass-loss asymmetry effects. Likewise, our simulations omit the effects of orbital decay due to gravitational wave emission. The pericentre distance of our simulated disruptions is $>50r_{\rm g}$, so orbital decay due to general relativistic effects (at least on the first pericentre passage) is negligible. For more compact stars with smaller tidal radii nearer the event horizon, orbital decay due to general relativistic effects will be more significant over fewer orbital periods (though the change in the pericentre will still be extremely small, as all of the dissipation occurs near pericentre for these high-eccentricity systems; thus, the tidal interaction itself may be relatively unaltered, aside from the stronger tidal field of the SMBH due to relativistic gravity). Future work on partial TDEs nearer the horizon of the SMBH should incorporate the change in orbital energy due to tidal interaction and mass-loss asymmetry alongside gravitational wave emission.

Finally, here we focused on orbits that produce partial TDEs in the traditional sense; i.e. the tidal force is not sufficiently strong to destroy the star completely. However, Nixon & Coughlin (2022) found that at very high β (in their case $\beta = 16$), the compression experienced by the star near pericentre could revive self-gravity to the point that a core *reformed* with a binding energy (to the SMBH) that was much larger than the value predicted by equation (2) (though we caution that while the mass contained in the core was converged, the orbital period – and thus the binding energy – was resolution-dependent in their simulations). While encounters with high-β are rare,[3] it may be possible for tidal capture in this considerably more exotic scenario to produce shorter period orbits than through the traditional means.

[3]For example, equation (16) of Coughlin & Nixon (2022) shows that the fraction of TDEs with $\beta > 10$ for a 10^6 $M_\odot$ Schwarzschild SMBH – including general relativistic effects – is 0.0046; note that this is a factor of $\sim$4 smaller than the value derived by ignoring general relativistic effects, given by their equation (17).

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DATA AVAILABILITY

Code to reproduce the results in this paper is available upon reasonable request to the corresponding author.

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