# Debiased Uncertainty Quantification Approach for Probabilistic Transient Stability Assessment

Bendong Tan, *Student Member, IEEE*, Junbo Zhao, *Senior Member, IEEE*

***Abstract*—System instability does not occur often in practice and thus the historical data for training a machine learning method has to address the imbalanced and multi-modal probabilistic distribution in the probabilistic transient stability assessment (PTSA). This letter proposes a transient stability index (TSI) density-based weighting scheme and feature-TSI similarity regularization to address that, yielding debiased uncertainty quantification for PTSA in the presence of uncertain wind generations and loads. Numerical results on the IEEE 39-bus and Illinois 200-bus power systems demonstrate the significantly improved performance of the proposed method over other state-of-the-art machine learning approaches in PTSA.**

***Index Terms*—Probabilistic transient stability assessment, power system dynamics, machine learning, uncertainty quantification, renewable energy.**

## I. Introduction

With the high penetration of inverter-based resources (IBRs), strong and unpredictable uncertainties are presented. These uncertainties together with those from loads result in time-varying operation conditions [1]. Quantifying the impacts of uncertainties from IBRs and random loads for transient stability is vital to the secure power system operation. To this end, probabilistic transient stability assessment (PTSA) methods have been developed. [2] proposes a decision tree-based framework to perform the statistical binary transient stability assessment (TSA) analysis under different faults and topologies. When integrating with wind farms, [3] utilizes a stacked denoising autoencoder to predict the transient stability index (TSI). By using reduced-order surrogate models, [4] and [5] perform the probabilistic rotor angle trajectory prediction, which requires fewer samples. However, these studies have not considered the imbalanced and multi-modal distribution of TSA. This is because the modern power system usually has a smaller number of unstable scenarios as compared to stable ones. This may lead to highly biased uncertainty quantification of PTSA, especially for unstable scenarios.

This letter proposes a debiased uncertainty quantification method for PTSA. A TSI density-based weighting scheme is developed for neural network (NN), leading to more attention on unstable scenarios. Furthermore, the feature-TSI similarity regularization is added to the NN to enable a distinction between stable and unstable scenarios. This allows us to handle the imbalanced and multi-modal probabilistic distributions for PTSA, a challenging issue for existing works. Comparison results demonstrate the significant improvements of the proposed method over existing ones.

This work is supported by the U.S. Department of Energy Advanced Grid Modernization program (*Corresponding Author*: Junbo Zhao).

B. Tan and J. Zhao are with the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT (email: junbo@uconn.edu).

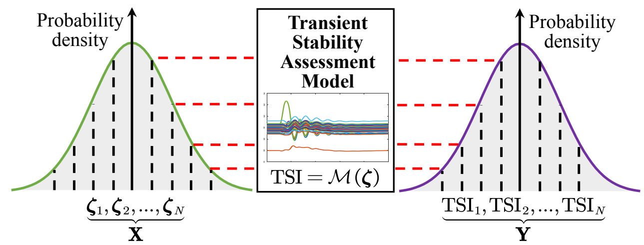


Fig. 1. Uncertainty propagation from uncertain sources to TSI.

## II. Problem Statement

The power system dynamics with IBRs under a disturbance can be described by the following stochastic differential and algebraic equation (DAE):

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{\zeta}) \\ \boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\zeta}) \end{cases} \tag{1}$$

where $\boldsymbol{f}(\cdot)$ and $\boldsymbol{g}(\cdot)$ are the differential and algebraic equations; $\boldsymbol{x}$ and $\boldsymbol{y}$ are the dynamic and algebraic variables; $\boldsymbol{u}$ is the system input; $\boldsymbol{\zeta}$ is a vector denoting all uncertain resources, i.e., wind generations and loads in this work.

TSI is a commonly-used index to assess power system transient stability and is usually defined as:

$$\mathrm{TSI} = 100 \times \frac{360 - \delta_{\max}}{360 + \delta_{\max}} \tag{2}$$

where $\delta_{\max}$ is the maximum rotor angle difference between any two generators during the dynamic simulations. If TSI is larger than 0, the system is stable while those with negative values mean system instability. The transient stability assessment model $\mathcal{M}(\cdot)$, i.e., the relationship between TSI and $\boldsymbol{\zeta} = [\boldsymbol{P}_w \ \ \boldsymbol{P}_L \ \ \boldsymbol{Q}_L \ \ \boldsymbol{P}_G]^\mathsf{T}$, can be formulated as: $\mathrm{TSI} = \mathcal{M}(\boldsymbol{\zeta})$, where $\boldsymbol{P}_w$, $\boldsymbol{P}_L$, $\boldsymbol{Q}_L$ and $\boldsymbol{P}_G$ are respectively the active power of wind generations, the active power of loads and reactive power of loads, and the active power of synchronous generators. Monte Carlo sampling (MCS) is the most widely used method in the literature to investigate the impacts of uncertain sources on PTSA. Specifically, under certain operating conditions, i.e., conditioned on $\boldsymbol{P}_G$, it samples $\mathbf{X} = \{\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, ..., \boldsymbol{\zeta}_N\}$ from the probability density function (PDF) of uncertain sources and performs extensive transient stability simulations to gather all possible outcomes $\mathbf{Y} = \{\mathrm{TSI}_1, \mathrm{TSI}_2, ..., \mathrm{TSI}_N\}$, as shown in Fig. 1, where $N$ is the number of samples. Consequently, uncertainties propagated from uncertain sources can be quantified under model $\mathrm{TSI} = \mathcal{M}(\boldsymbol{P}_w, \ \boldsymbol{P}_L, \ \boldsymbol{Q}_L | \boldsymbol{P}_G)$. MCS is accurate if the number of samples is sufficient and the uncertain resource PDFs are accurate (challenging to obtain), but it is very time-consuming and thus not applicable for online decision makings. [5] proposes the surrogate model-based approach to mitigate this issue. However, system instability does not often occur in practice, leading to the imbalance between stable and unstable data. On the other hand, if the system has lost its stability, the rotor angle of the out-of-step generator will increase quickly and yield a large TSI, while the scenarios around the stability boundary $\mathrm{TSI}=0$ are rare. Consequently, TSI distribution is multi-modal. These cause significant challenges for existing machine learning approaches.

## III. Proposed Debiased PTSA Method

This letter proposes to handle imbalanced and multi-modal TSI distribution via the TSI density-based weighting scheme and ranking similarity regularization.

### A. TSI Density-based Weighting Scheme

For the surrogate model-based PTSA, it tends to behave better in stable scenarios since there is an adequate number of samples for training the approach, while it is difficult to quantify the uncertainty of limited unstable scenarios. Thus, it is intuitive to assign different weights to various scenarios to build a surrogate model. For example, large weights for unstable scenarios and small weights for stable ones can be designed. This is achieved by calculating a weight for each sample inversely proportional to the probability of the TSI value's occurrence.

To weight each sample according to the rarity of the corresponding TSI value, the weight function is defined as $\mathit{\Psi}(\mathrm{TSI})$. The TSI PDF is used to quantify the rarity, allowing distinction between rare and frequent TSI value ranges. The TSI PDF can be established by a non-parametric inference method using a kernel density estimator (KDE):

$$p(\mathrm{TSI})=\frac{1}{Nh}\sum_{i=1}^{N}\Phi\left(\frac{\mathrm{TSI}-\mathrm{TSI}_i}{h}\right) \tag{3}$$

where $h$ is the bandwidth of the estimator and it is generally set as $1.06\sigma_y N^{-0.2}$ and $\sigma_y$ is the sample standard deviation; $\Phi$ is the kernel smoothing function, i.e., standard Gaussian kernel utilized in the letter. All samples' density values in the training set are normalized into a range between 0 and 1 via:

$$p'(\mathrm{TSI})=\frac{p(\mathrm{TSI})-\min p(\mathbf{TSI})}{\max p(\mathbf{TSI})-\min p(\mathbf{TSI})} \tag{4}$$

Consequently, the sample in the most densely populated part of **TSI** is assigned a value of 1, while that in the most sparsely populated part of **TSI** is assigned a value of 0. Based on this normalized PDF, a basic weighting function can be defined:

$$\mathit{\Psi}'(\mathrm{TSI})=\max\left(1-\alpha p'(\mathrm{TSI}),\epsilon\right) \tag{5}$$

where $\alpha\in[0,1]$. The small, positive, and real-valued constant $\epsilon$ is to make sure the weight of each sample is larger than 0 so that the information in the dataset won't be wasted. Finally, to correct the impacts of different scales of $\mathit{\Psi}'(\mathrm{TSI})$ values, $\mathit{\Psi}(\mathrm{TSI})$ can be obtained:

$$\mathit{\Psi}(\mathrm{TSI})=\mathit{\Psi}'(\mathrm{TSI})\Big/\left(\frac{1}{N}\sum_{i=1}^{N}\mathit{\Psi}'(\mathrm{TSI})\right) \tag{6}$$

Combined with (6), the loss function of NN is formulated as:

$$\mathcal{L}_1=\frac{1}{N}\sum_{i=1}^{N}\mathit{\Psi}(\mathrm{TSI}_i)\left\|\mathrm{TSI}_i-\overline{\mathrm{TSI}_i}\right\| \tag{7}$$

where $\overline{\mathrm{TSI}_i}$ is the predicted TSI by the NN. By applying (7), the minority of the datasets, namely the unstable samples, can be paid more attention by NN.

### B. Ranking Similarity Regularization

Except for the TSI weighting scheme, this letter also improves the uncertainty quantification performance from the view of the feature similarity. From (1), it can be found that if $\|\boldsymbol{\zeta}_1-\boldsymbol{\zeta}_2\|<\varepsilon$ and $\lim\varepsilon=0$, the dynamic trajectories corresponding to $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ may be similar. This indicates that similar inputs to the power system can lead to similar TSI outcomes. Inspired by that, we can mitigate the distribution imbalance issue by matching features with close TSIs. This is also beneficial for multi-modal distribution. **Note that the features $\boldsymbol{z}$ used here are the outputs of the final full connection layer in the NN**. Let's denote it as $\boldsymbol{z}=\varphi(\boldsymbol{\zeta})$, where $\mathrm{TSI}=\phi(\boldsymbol{z})$.

Firstly, we need to define a ranking function $\boldsymbol{RK}(\cdot)$ for matching operations:

$$\boldsymbol{RK}(\boldsymbol{a})_i=1+\sum_{j=1,j\neq i}^{N} sign(\boldsymbol{a}_j>\boldsymbol{a}_i) \tag{8}$$

where $\boldsymbol{RK}(\boldsymbol{a})_i$ denotes the $i$-th element in $\boldsymbol{RK}(\boldsymbol{a})$ and $\boldsymbol{a}$ is the input of $\boldsymbol{RK}$, i.e., TSI or feature similarity. Taking an example to illustrate $\boldsymbol{RK}$, if $\boldsymbol{a}=[4,6,2,8]$, $\boldsymbol{RK}(\boldsymbol{a})_2$ can be calculated as 2 since 6 is the second-largest number in $\boldsymbol{a}$.

Let $\boldsymbol{S}^{\mathrm{TSI}}\in\mathbb{R}^{N\times N}$ denotes the pairwise similarity matrix obtained by applying the cosine similarity function $\eta(\cdot,\cdot)$ across all sample TSIs. Thus, $(i,j)$-th element in $\boldsymbol{S}^{\mathrm{TSI}}$ is $\boldsymbol{S}^{\mathrm{TSI}}_{i,j}=\eta(\mathrm{TSI}_i,\mathrm{TSI}_j)$. Similarly, $\boldsymbol{S}^{\boldsymbol{z}}\in\mathbb{R}^{N\times N}$ is the pairwise similarity matrix for features, where $\boldsymbol{S}^{\boldsymbol{z}}_{i,j}=\eta(\boldsymbol{z}_i,\boldsymbol{z}_j)$.

Consequently, ranking similarity regularization loss can be formulated as:

$$\mathcal{L}_2=\sum_{i=1}^{N}\left\|\boldsymbol{RK}(\boldsymbol{S}^{\mathrm{TSI}})_{[i,:]}-\boldsymbol{RK}(\boldsymbol{S}^{\boldsymbol{z}})_{[i,:]}\right\| \tag{9}$$

In (9), $[i,:]$ denotes the $i$-th row in a matrix. For a sample, (9) encourages the sorted list of its neighbors in TSI space to match its neighbors in feature space, leading to fewer misspecifications both in unstable and stable scenarios.

### C. PTSA under Imbalanced and Multi-modal Distribution

The proposed method can be divided into three steps:

1) ***Dataset construction***: To obtain a dataset $\mathbf{D}=\{\boldsymbol{\zeta}_i,\mathrm{TSI}_i\}_{i=1}^{N}$ for training the NN, the active power of wind generations and the real and reactive power of loads in $\boldsymbol{\zeta}$ are sampled from Weibull and Gaussian distributions, respectively. To reflect the impacts of the active power of synchronous generators on TSI, it can be varied as:

$$\boldsymbol{P}_G=\left[\left(\sum\boldsymbol{P}_L-\sum\boldsymbol{P}_w-\sum\boldsymbol{P}_{G,b}\right)\Big/\sum\boldsymbol{P}_{G,b}+1\right]\boldsymbol{P}_{G,b} \tag{10}$$

where $\boldsymbol{P}_{G,b}$ is the base active power of synchronous generators. The corresponding TSI can be calculated by time domain simulation under predefined contingency sets.

2) ***Imbalanced and multi-modal distribution learning***: Taking $\boldsymbol{\zeta}$ and TSI as input and output respectively, NN is trained according to the following loss function:

$$\mathcal{L}=\mathcal{L}_1+\lambda\mathcal{L}_2 \tag{11}$$

TABLE I
PERFORMANCE COMPARISON FOR FAULT AT BUS 12.

| Methods | MCS | NN | SGPR |
|---|---|---|---|
| MSE | 0 | 307.21 | 3071.23 |
| Number of samples | 10000 | 5000 | 5000 |
| Training time | ≈ 0.66 h | ≈ 0.33 h | ≈ 0.40 h |
| Assessment time | ≈ 0.66 h | < 1 s | < 1 s |
| Methods | PM-D | PM-R | PM-DR |
| MSE | 232.99 | 240.00 | 192.68 |
| Number of samples | 5000 | 5000 | 5000 |
| Training time | ≈ 0.36 h | ≈ 0.65 h | ≈ 0.71 h |
| Assessment time | < 1 s | < 1 s | < 1 s |

where $\lambda$ is the regularization coefficient.

3) ***Online application***: Once the NN is trained, the uncertainties from random sources are quantified for PTSA. This is advantageous, especially when the distribution of random sources has changed.

## IV. NUMERICAL RESULTS

### A. Verification on IEEE 39-Bus System

The IEEE 39-bus system is employed to verify the proposed method (PM). Four DFIG wind farms are connected to Buses 2, 8, 11, and 21 respectively. Their generation capacities of them are 225 MW, respectively. The random parameter settings for the uncertainty sources follow [6]. Two three-phase short-circuit faults applied on Buses 12 and 26 are respectively tested with a fault duration of 180 ms. The total simulation time is 3 s. By sampling from the PDF of wind speeds and loads, 10000 samples are generated for MCS. Note that, in this letter, 20% samples of MCS are used for testing.

To verify the effectiveness of the proposed TSI density-based weighting and the feature-TSI similarity regularization, the following scenarios are tested:

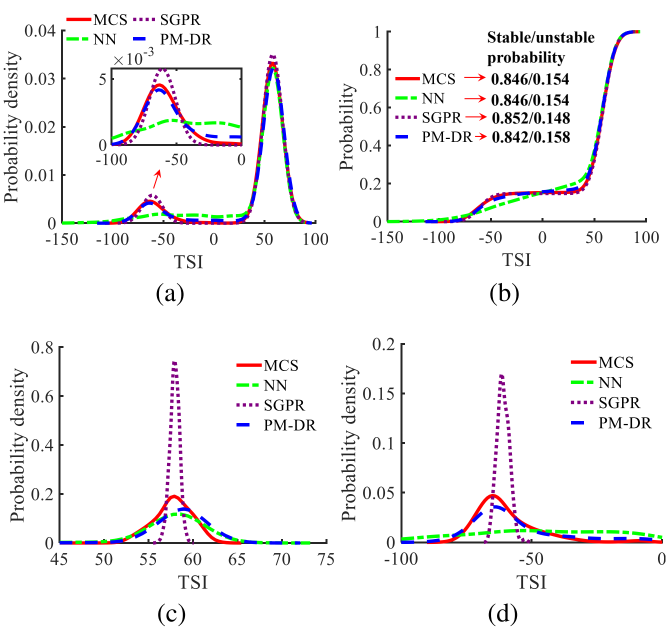


Fig. 2. Results of PTSA under fault at Bus 12. (a) Full PDF; (b) Cumulative probability; (c) PDF of stable scenarios; (d) PDF of unstable scenarios.

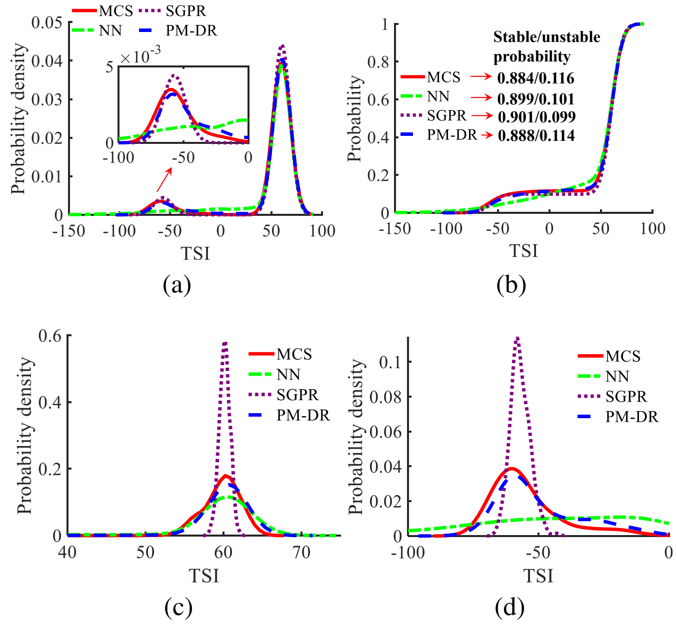


Fig. 3. Results of PTSA under fault at Bus 26. (a) Full PDF; (b) Cumulative probability; (c) PDF of stable scenarios; (d) PDF of unstable scenarios.

TABLE II
PERFORMANCE COMPARISON FOR FAULT AT BUS 26.

| Methods | MCS | NN | SGPR |
|---|---|---|---|
| MSE | 0 | 251.31 | 2669.46 |
| Number of samples | 10000 | 5000 | 5000 |
| Training time | ≈ 0.66 h | ≈ 0.33 h | ≈ 0.40 h |
| Assessment time | ≈ 0.66 h | < 1 s | < 1 s |
| Methods | PM-D | PM-R | PM-DR |
| MSE | 164.28 | 134.03 | 122.73 |
| Number of samples | 5000 | 5000 | 5000 |
| Training time | 0.36 | ≈ 0.65 h | ≈ 0.71 h |
| Assessment time | < 1 s | < 1 s | < 1 s |

- **PM-D**: NN with the TSI density-based weighting regularization;
- **PM-R**: NN with the feature-TSI similarity regularization;
- **PM-DR**: NN with the TSI density-based weighting and the feature-TSI similarity regularization.

The NN employed by the proposed method contains three-layer neurons of 50-50-20-1. Parameters are determined by the grid-searching approach. Specifically, $\lambda$ and maximum NN training epochs are set as 0.01 and 500 respectively. Besides, $\alpha$ can be set as the proportion of the stable samples in all data since a larger $\alpha$ can lead to smaller weights on the samples with low density according to (5). As a result, $\alpha$ are respectively 0.85 and 0.88 for fault scenarios of Buses 12 and 26. The batch training size is set as 16, which is able to improve the training accuracy for the proposed method. MCS is utilized as the benchmark, while the NN without imbalanced learning and the state-of-art Sparse Gaussian Process Regression (SGPR) in [5] are used for comparisons. For fair comparisons, the same amount of samples is utilized for training NN, SGPR, and the proposed method.

Figs. 2 and 3 show the estimated PDFs by (3) for all methods under different fault scenarios. The proposed method can

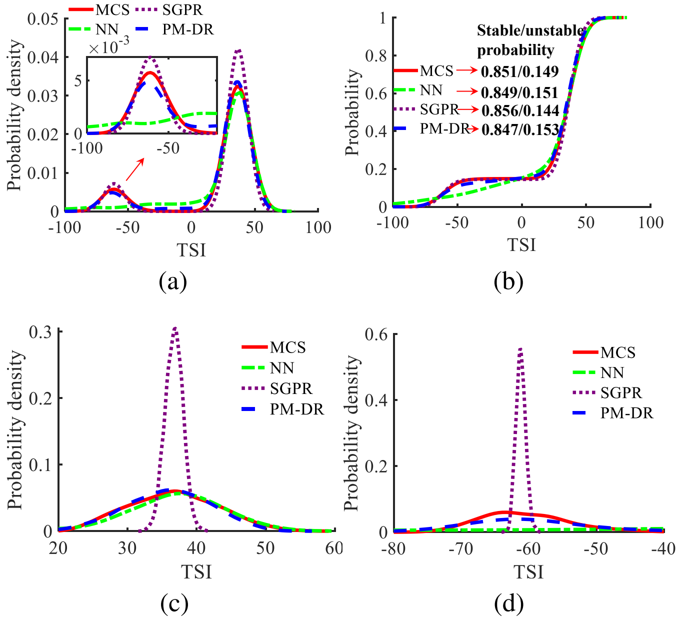


Fig. 4. Results for Illinois 200-bus system. (a) Full PDF; (b) Cumulative probability; (c) PDF of stable scenarios; (d) PDF of unstable scenarios.

quantify the uncertainties from uncertain sources accurately due to the imbalanced and multi-modal learning capability. If we separately construct the PDFs of TSI according to the stable and unstable scenarios, Figs. 2 and 3 also illustrate that SGPR has poor uncertainty quantification performance in each scenario since it tends to learn TSI that has a large probability density. Due to the smoothing effect in (3) for building a full PDF, the large estimation error of SGPR is masked. The mean square error (MSE) comparisons in Tables I and II also support these results. Note that the proposed method outperforms them in this case as well. **These indicate that the full PDF (including stable and unstable scenarios simultaneously) is not enough for uncertainty quantification under imbalanced and multi-modal TSI distributions. It is also necessary to build PDFs separately for stable and unstable scenarios to verify the effectiveness of the model.** Although the training time (including data generation and model training) of the proposed method is a little bit larger than MCS, see Tables I and II, it doesn't have to be retrained when the distributions of random sources change, leading to fast assessment. However, time-consuming MCS has to be re-performed .

### *B. Verification on Illinois 200-Bus Power System*

To demonstrate the scalability of the proposed method, new tests are performed on the modified Illinois 200-bus power system. Six 150-MW DFIG wind farms are connected to Buses 8, 10, 16, 30, 41, and 88 respectively. A three-phase short-circuit fault is applied on Bus 65. The hyperparameter setting is similar to Section IV-A, while $\alpha$ is 0.85 and the fault duration is 300 ms. Besides, NN inside the proposed method contains three-layer neurons of 203-50-20-1. To verify the generalization ability of the proposed method considering generation dispatch, the active power of synchronous generators in the testing data is an instance from Gaussian distribution, where its mean value is the base active power while its standard deviation is 10% of that. It can be seen from Fig. 4 and Table III that the proposed method is scalable to large power systems and it can achieve accurate PTSA in high-dimension uncertain resources. Compared with Figs. 2(c) and (d) (or Figs. 3(c) and (d)), Figs. 4(c) and (d) show that SGPR has much higher errors in stable and unstable scenarios; on the other hand, since thousands of SGPRs are built with whole rotor angle trajectories, it also has a much longer training time than the proposed method, see Table III. Therefore, compared with SGPR, the proposed method has the superior performance both in assessment accuracy and training time for PTSA.

TABLE III
PERFORMANCE COMPARISON FOR ILLINOIS 200-BUS POWER SYSTEM.

| Methods | MCS | NN | SGPR | PM-DR |
|---|---|---|---|---|
| MSE | 0 | 275.80 | 2431.27 | 191.39 |
| Number of samples | 10000 | 8000 | 8000 | 8000 |
| Training time | $\approx$ 0.94 h | $\approx$ 0.75 h | $\approx$ 1.47 h | $\approx$ 1.19 h |
| Assessment time | $\approx$ 0.94 h | $<$ 1 s | $<$ 1 s | $<$ 1 s |

## V. Conclusions

This letter proposes a debiased uncertainty quantification approach for PTSA. By using TSI density-based weighting and feature-TSI similarity regularization for NN, the proposed method can deal with imbalanced and multi-modal distribution issues. Numerical results demonstrate its excellent PTSA prediction accuracy compared with other approaches.

## References

[1] P. N. Papadopoulos, *et al.*, "Probabilistic framework for transient stability assessment of power systems with high penetration of renewable generation," *IEEE Trans. Power Syst.*, vol. 32, no. 4, pp. 3078-3088, Jul. 2017.

[2] P. N. Papadopoulos, *et al.*, "Probabilistic framework for online identification of dynamic behavior of power systems with renewable generation," *IEEE Trans. Power Syst.*, vol. 33, no. 1, pp. 45-54, Jan. 2018.

[3] T. Su, *et al.*, "Probabilistic stacked denoising autoencoder for power system transient stability prediction with wind farms," *IEEE Trans. Power Syst.*, vol. 36, no. 4, pp. 3786-3789, Jul. 2021.

[4] K. Ye, *et al.*, "Stochastic power system dynamic simulation and stability assessment considering dynamics from correlated loads and PVs," *IEEE Trans. Industry Applications*, vol. 58, no. 6, pp. 7764-7775, 2022

[5] K. Ye, *et al.*, "Physics-informed sparse gaussian process for probabilistic stability analysis of large-scale power system with dynamic PVs and loads," *IEEE Trans. Power Syst.*, 2022.

[6] S. Xia, *et al.*, "Probabilistic transient stability constrained optimal power flow for power systems with multiple correlated uncertain wind generations," *IEEE Trans. Sustainable Energy*, vol. 7, no. 3, pp. 1133-1144, Jul. 2016.