A Chance-Constrained Optimization Framework for Wind Farms to Manage Fleet-Level Availability in Condition Based Maintenance and Operations

Fallahi, F.[a], Bakir, I.[b], Yildirim, M.[a,*], Zhisheng, Y.[c]

[a] *Industrial and Systems Engineering, Wayne State University, Detroit, MI, USA*

[b] *Department of Operations, Faculty of Economics and Business, University of Groningen, Groningen, Netherlands*

[c] *Department of Industrial Systems Engineering & Management, National University of Singapore, Singapore*

Abstract

Operations and maintenance (O&M) is a key contributor to wind farm expenditures. To increase competitiveness, wind farm operators are increasingly looking into leveraging real-time sensor data from condition monitoring (CM) systems. CM provides significant insights on evolving asset failure risks for wind turbines. To date, these insights have not been fully leveraged in wind farm O&M due to ad-hoc connections to decision-making. Specifically, CM applications in wind farms have been limited to detection of turbines with imminent failure risks that require immediate replacement. In reality, wind farm maintenance requires a careful proactive orchestration of O&M dependencies across turbines along with multiple sources of uncertainty associated with asset availability, operational and market conditions. This paper proposes a unified condition-based maintenance and operations scheduling approach for wind farms that models uncertainties related to turbine availability, wind power output and market price. The proposed formulation explicitly considers the turbine-to-turbine dependencies in operations and maintenance, such as opportunistic maintenance, to identify the O&M decisions that are optimal for multiple wind farms. The problem is formulated as a chance-constrained stochastic programming model to maximize operational revenue while ensuring high levels of turbine availability and generation. To make the chance constraints tractable, two approximations are proposed with a focus on sample average approximation (SAA) and prominent tail inequalities such as Markov's inequality and Chernoff bound. Our results on a comprehensive set of experiments demonstrate that the proposed approach provides significant improvements in asset availability, market revenue and maintenance costs in large scale wind farms.

Highlights

- A stochastic condition-based optimization model is proposed for wind farm operations & maintenance.
- Condition-based chance constraints are formulated to model farm-level availability risks.

[*] Corresponding author

Email addresses: farnaz.fallahi@wayne.edu (Fallahi, F.), i.bakir@rug.nl (Bakir, I.), murat@wayne.edu (Yildirim, M.), yez@nus.edu.sg (Zhisheng, Y.)

- Tractable approximations are developed for the proposed chance constraints.
- Degradation and operational data are used to build a comprehensive experimental framework.
- Proposed model significantly improves asset availability, market revenue and maintenance costs.

Keywords: Chance-constrained programming, condition-based maintenance, stochastic programming, wind farm operations and maintenance, sample average approximation

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Nomenclature

Sets:

$\mathcal{T}$	Maintenance epochs (periods)
$\mathcal{H}$	Operational sub-periods
$\mathcal{L}$	Wind farm locations
$\mathcal{G}^{\ell}$	Wind turbines at location ℓ
$\mathcal{G}^{o,\ell}$	Operational wind turbines at location ℓ
$\mathcal{G}^{f,\ell}$	Failed wind turbines at location ℓ
Ω	Uncertainty scenarios

Parameters:

$C_t^{f,i}$	Corrective maintenance cost of wind turbine i in period t
$C_t^{p,i}$	Preventive maintenance cost of turbine i in period t
$C_t^{v,\ell}$	Crew visit cost in period t at location ℓ
$C_t^{d,i}$	Dynamic maintenance cost associated with scheduling turbine i's maintenance in period t
$\Gamma_{t,h,\omega}$	Electricity price in period t and sub-period h under scenario ω
$\Phi_{t,h,\omega}^{i}$	Available wind power of turbine i in period t and sub-period h under scenario ω
M_t^p	Planned maintenance capacity in period t
M_t^o	On-the-spot maintenance capacity in period t
$\theta_{\ell,\ell'}$	Travel time from location ℓ to ℓ'
τ_ω^i	Failure time of turbine i under scenario ω
p_ω	Probability of scenario ω

Variables:

z_t^i	1, if operational turbine i is scheduled to be preventively maintained at period t
ν_t^i	1, if failed turbine i is scheduled to be correctively maintained at period t
x_t^ℓ	1, if the crew visits location ℓ in period t
$u_{s,t}^\ell$	1, if the crew visits location ℓ between (and including) periods s and t, where $s \leq t$
$\zeta_{t,\omega}^i$	1, if turbine i is available to operate at time period t under scenario ω
$y_{t,h,\omega}^i$	Power generation of turbine i at time period t and sub-period h under scenario ω

1. Introduction

Effective Operations and Maintenance (O&M) strategies play a pivotal role in improving competitiveness of wind energy. Expenditures due to O&M account for 31% and 34% of the running expenses for onshore and offshore wind farms, respectively; and cast significant implications for a wide range of operational metrics. Among these, the number of operational turbines (*i.e.*, farm-level availability) is widely regarded as a critical metric, directly impacting dispatch capabilities and operational revenues [1, 2, 3]em. Predicting and controlling farm-level availability, however, is a complex task that revolves around fine-tuning the relationship between evolving turbine failure risks, and scheduled O&M decisions to mitigate their impact. In the absence of sensor data, wind farm operators typically rely on overly conservative measures to identify probable turbine failure instances. Sensor-driven condition monitoring (CM) systems provide increased visibility on turbine failure risks, and enables sensor-driven maintenance policies called *condition-based maintenance (CBM)*. A central focus of CBM is to leverage the insights gained from CM systems to optimally coordinate O&M decisions to maximize revenue and mitigate financial and reliability risks. The integration of insights from

CM (*e.g.*, inference of failure risks) into CBM (*e.g.*, O&M decision making) constitutes a difficult modeling problem for wind farm operations. Evidently, existing CBM approaches typically propose ad-hoc connections to decision-making that revolves around conducting immediate maintenance on turbines with imminent failure risks. To enable proactive maintenance planning, CBM approaches require a new generation of risk-based optimization models that explicitly model sensor-driven predictions on turbine failure risks, and impact of unavailability on complex wind farm operations. The objective of this paper is to address this need.

O&M modeling and optimization in wind farms has a rich and growing literature. Conventional O&M approaches focus primarily on complex wind farm operations, such as transportation and routing [4, 5], logistics planning [6, 7], environmental factors [8], and expenditures [9, 10]. While providing a detailed representation for operations, these approaches typically have comparatively simpler models for conducting maintenance. Maintenance is typically modeled via a set of constraints to ensure every turbine undergoes a time-based (or periodic) repair/replacement following a pre-specified frequency (yearly or semi-annually), which are obtained through manufacturing recommendations, engineering expertise, and field observations [11, 12]. Operators collect the failure time data for certain types of turbines to develop *population-based* distributions, which are then used to predict the time of failure for specific turbines in their wind farms. Population-based failure distributions assume that every turbine degrades and fails in a similar fashion and pace. In reality, identical turbines exhibit significant differences in terms of how they fail due to variations in manufacturing & material imperfections, and operational environments. Evidently, population-based estimates result in high levels of uncertainty and inaccuracies. Operators typically address this issue by developing conservative maintenance schedules that impose frequent maintenance actions to minimize potential risks of turbine failure [13, 14]. These overly conservative maintenance policies require additional financial and maintenance resources while still resulting in a significant number of turbine failures. As operational requirements become more stringent, relying on the inefficient conventional O&M approaches is becoming an increasingly precarious position.

Recent developments in sensor technology, data processing and storage capabilities enabled CM-based approaches that leverage real-time sensor information to provide accurate predictions on turbine failure risks. These sensor-driven failure predictions are significantly different from the conventional population-based estimates, as they use sensor data from specific turbines in the field for prediction purposes [15]. Specifically, CM approaches use the streaming sensor data to capture unique degradation and failure characteristics of turbines, thus generating an *asset-specific* distribution of failure that comes with significant improvements in prediction accuracy [16]. CM predictions typically have two forms: diagnostics and prognostics. Diagnostic approaches use sensor data to estimate the current state of health [17, 18, 19], and are typically used to identify turbines with imminent failure risks. Prognostic approaches derive remaining life distributions for turbines, which require an estimate of the current state of health (as in diagnostics), as well as an accurate prediction of how the health state of the turbine is likely to evolve in the future [20, 21]. From an O&M planning perspective, prognostics have significant advantages over diagnostics, since prognostic predictions on remaining life distribution enable the operators to proactively understand and mitigate the risks associated with when to schedule maintenance actions.

In line with the existing CM approaches, literature on CBM in wind energy also focuses on two types of models: diagnostics-based maintenance, and prognostics-based maintenance. Diagnostics-based maintenance models mainly constrain their focus to identifying imminent turbine failure risks and fixing them via immediate maintenance actions, without proactive O&M planning [22]. Existing commercial solutions for CBM often use these type of policies [23, 24]. Prognostics-based maintenance incorporates remaining life distribution predictions into maintenance planning. Existing models in prognostics-based maintenance of wind farms typically focus on single-turbine systems with limited applicability to multi-turbine settings [25, 26], or rule-based opportunistic maintenance models that initiate maintenance actions for turbines when their degradation reaches a certain level of severity [27]. In recent years, there has been a growing literature in energy systems [28, 29, 30] and wind farm applications [31, 32] that uses prognostics-based costs to coordinate operations and maintenance decisions. They typically use a maintenance cost function to connect remaining life distribution to its corresponding maintenance cost values in the optimization model. These approaches are either deterministic [31, 32], or stochastic models with only operational uncertainty (*i.e.*, demand, generation) [30]. To date, wind farm O&M optimization models have not captured the uncertainty and risks associated with turbine remaining life distributions on fleet-level maintenance, operations and availability in large scale wind farms.

A significant challenge in wind farm CBM revolves around capturing turbine-to-turbine dependencies in operations and maintenance. In operations, farm-level production and revenue is a joint function of asset availability and wind power characteristics experienced by all the turbines within a farm. In maintenance, costs associated with maintenance crew deployment provides significant initiatives of grouping turbine maintenances together (*i.e.* opportunistic maintenance) to reduce the number of maintenance crew visits [27, 31, 33, 34]. Opportunistic maintenance is particularly crucial in offshore wind farm settings, where crew visits require the use of specialized boats and helicopters. The considerations associated with opportunistic maintenance in wind farm CBM is conventionally captured via fixed rule-based degradation threshold policies [27]. In reality, the opportunistic maintenance decisions are highly dynamic and evolve as a function of operational conditions (*e.g.* market price, wind speed), and asset availability & failure risks. Modeling these dynamic interactions require stochastic models that explicitly characterize operational outcomes as a function of three categories of maintenance actions. The first category, called *preventive maintenance*, denotes planned maintenance actions conducted prior to turbine failure. For this maintenance type, turbine remains unavailable during the duration of the preventive maintenance action. The second category, *planned corrective maintenance*, models the maintenance actions conducted on turbines that are already in failed stage at the time of planning. These turbines remain unavailable until their maintenance actions are completed. Finally, the third category, called *on-the-spot corrective maintenance*, is conducted on turbines that are operational at the time of planning, but fail prior to their scheduled preventive maintenance. These category of turbines are opportunistically repaired when the maintenance crew visits the location for fixing another turbine in the vicinity. In this scenario, turbines remain unavailable from their time of failure, to the completion of their opportunistic maintenance action. It is a significant challenge to model these different types of maintenance actions, the associated turbine availability conditions, and their implications on operational revenues.

In this paper, we propose a risk-based, stochastic optimization model for condition-based maintenance and operations in wind farms. The proposed stochastic optimization model leverages predictions on turbine remaining life distributions to maximize operational revenue and mitigate the risks associated with turbine availability. Unique to our approach is the explicit modeling of operational and maintenance uncertainties, and the development of chance constraints to represent turbine availability risks. To date, wind energy and renewable integration literature used chance constraints to model a wide range of operational uncertainties, such as renewable energy utilization [35, 36], power balance satisfaction [37, 38], reserve requirements [39], and line flow limits [40, 41, 42]. Our approach shifts the focus to maintenance-related uncertainties as well. The proposed chance constraints enable the optimization models to fully harness sensor-driven remaining life predictions to represent the significant failure risks and their implications on wind farm O&M. The contributions of the proposed model can be listed as follows:

- We propose a new generation of risk-based O&M optimization models that embed sensor-driven predictions on turbine remaining life distributions within a risk-based stochastic optimization model. The proposed model simultaneously characterizes operational and maintenance-related uncertainties for a large-scale wind farm, and enables the explicit modeling and control of the impact of these uncertainties on complex wind farm operations. Specifically, turbine-to-turbine dependencies in terms of operations (*e.g.*, farm-level generation) and maintenance (*e.g.*, opportunistic maintenance) are explicitly captured. The proposed model makes provision for on-the spot corrective maintenance actions that conduct opportunistic corrective maintenance when the maintenance crew visits neighboring turbines.
- We formulate sensor-driven chance constraints that adapt to remaining life distribution predictions to derive evolving turbine availability risks in a large-scale wind farm. The proposed chance constraints differ from the existing formulations that focus on operational risks, and do not incorporate prognostic predictions. The chance constraints enable the models to fully harness the value of sensor-driven predictions by representing costs, uncertainties, and risks as a function of these predictions; and restricting the unavailability of units through explicit consideration of probabilistic failures and crew visits.
- We develop two tractable approximations for the proposed chance constraints and compare their performances for large-scale models. The proposed approximations rely on tail inequalities including Markov's inequality and Chernoff bound, and sample average approximation (SAA). Employing tail inequalities enables the estimation of the constraint violation probabilities without the need for sampling. In SAA, the original chance constraints are approximated through the use of Monte Carlo simulation and integer programming reformulation methods.
- We provide a simulation framework to evaluate and compare the performance of risk-based O&M optimization models with the prominent approaches in literature. The proposed framework incorporates real-time condition monitoring data to emulate the degradation process of turbines and uses actual weather and market price data to create realistic operational environments of wind farms.

Extensive numerical studies are performed to illustrate and validate the the performance of the risk-based O&M model in large-scale cases. We schedule maintenance and operations of 100 wind turbines in a wide range of settings and conditions. The results show significant improvements in terms of costs, reliability, availability, and renewable penetration. As a case in point, the computational experiments highlight 62%-70% reduction in average unavailability of turbines using the proposed risk-based O&M approach relative to the risk-neutral time-based (periodic) maintenance policy.

The remainder of the paper proceeds as follows. Section 2 introduces predictive analytics that explain how the sensor-data is harnessed to develop predictions on turbine remaining life distributions. Predictions of operational uncertainties are also discussed within the same section. Section 3 develops the proposed O&M optimization model that embeds the predictions on remaining life distributions within a stochastic optimization framework. Section 4 discusses the chance constraint approximations that enable tractability of the risk-based model for large-scale applications. In Section 5, a comprehensive set of computational experiments are conducted to demonstrate the performance of the proposed model compared to the existing maintenance approaches. Finally, Section 6 concludes the paper with a discussion on results and future work.

2. Predictive Analytics

The optimization model that schedules the operations and maintenance activities of wind farms (*presented in detail in Section 3*) builds on *(i)* sensor readings from wind turbines and the resulting remaining lifetime distribution (RLD) predictions, and *(ii)* turbine failure uncertainty scenarios generated as a function of the predicted turbine RLDs. The modeling of the underlying degradation processes and the Bayesian framework that is used for updating the RLD parameters with new sensor observations are described in detail in Section 2.1. Section 2.2 shifts the focus to the scenario generation method used for producing uncertainty scenarios that are good representations of the physical system.

2.1. Degradation Modeling and Bayesian Framework

The degradation of a turbine i is modeled using the degradation function, $D_i(t)$, as given in equation (1). In this function, $\phi_i(t;\kappa,\theta_i)$ and $\epsilon_i(t;\sigma)$ denote the underlying base degradation function (given the deterministic and stochastic degradation parameters κ and θ_i) and the error term (given the volatility σ) associated with turbine i, respectively.

$$D_i(t|\theta_i) = \phi_i(t;\kappa,\theta_i) + \epsilon_i(t;\sigma) \tag{1}$$

The failure time of turbine i, denoted as f_i, is the first time at which the degradation function $D_i(t)$ surpasses a pre-defined degradation threshold, Λ_i; *i.e.*, $f_i = \min\{t \geq 0 : D_i(t) \geq \Lambda_i\}$. Thus, the conditional cumulative distribution function (CDF) of the failure time can be characterized with equation (2), given that the age of turbine i at the time of observation is t_i^o.

$$F^{t_i^o}_{f_i|\theta_i}(t) = P\left(f_i \leq t|\theta_i\right) = 1 - P\left(\sup_{0\leq s\leq t} D_i(s|\theta_i) < \Lambda_i\right) = 1 - P\left(\sup_{0\leq s\leq t}\{\phi_i(s;\kappa,\theta_i) + \epsilon_i(s;\sigma)\} < \Lambda_i|\theta_i\right) \tag{2}$$

RLD of each turbine i, characterized by (2), is contingent on the value of the stochastic degradation parameter θ_i. In reality, the true value of this parameter is not known, and reliable estimates are key for accurate degradation modeling and RLD estimation. The Bayesian framework for estimating and updating this parameter starts with an initial estimate, denoted as $\pi(\theta_i)$. As new sensor readings are observed, the posterior distribution of θ_i, denoted as $\upsilon(\theta_i)$, is computed via a Bayesian update mechanism, the details of which can be found in [28]. The resulting RLD of turbine i, $F^{t_i^o}_{f_i}(t)$, can be characterized as in (3).

$$F^{t_i^o}_{f_i}(t) = P\left(f_i \leq t\right) = 1 - \int P\left(f_i > t|\theta_i\right)\upsilon\left(\theta_i\right)d\theta_i = 1 - \int P\left(\sup_{0\leq s\leq t} D_i\left(s|\theta_i\right) < \Lambda_i\right)\upsilon\left(\theta_i\right)d\theta_i \tag{3}$$

Note that the proposed degradation modeling and Bayesian framework use every new sensor observation, to generate an update on the posterior distribution $\upsilon(\theta_i)$, and the associated prediction on RLD using (3).

2.2. Scenario Generation

Incorporating prevailing operational uncertainties of wind turbines plays a crucial role in obtaining optimal operations and maintenance scheduling of wind farms. Thus, the optimization framework presented in this paper explicitly accounts for uncertainties through *scenarios* in a stochastic programming model. We consider uncertainties in *(i)* turbine failure times, *(ii)* wind power, and *(iii)* electricity price. A representative number of scenarios, which encompass all three uncertainties, are generated by using remaining life distributions and historical data.

2.2.1. Failure Time Uncertainty

In order to account for the uncertainty associated with the failure time of each turbine, we generate a set of turbine failure scenarios. A *failure scenario* represents a joint uncertainty realization of failure time, f_i, of each wind turbine i. Failure scenarios are generated according to the RLDs, which use the most recent degradation signal observations (as described in Section 2.1).

An important consideration in generating failure scenarios is that the optimization model (presented in detail in Section 3) requires discrete time periods. More specifically, the planning horizon is divided into T time periods, where each time period $t \in \{1, 2, \ldots, T\}$ represents a time interval $[t-1, t)$, where 0 marks the beginning of the planning horizon. Thus, in order to generate failure scenarios that are compatible with the discrete-time nature of the optimization model, we introduce the notion of *failure time periods*. If the uncertainty realization associated with scenario ω reveals that turbine i fails at time f_i, then the corresponding failure time period, denoted as τ_ω^i, can be computed as the time period t such that $t-1 \leq f_i < t$.

The generation of uncertainty realizations is done via a Monte Carlo sampling procedure. For each scenario ω and each turbine i, a uniform random variate U_ω^i is generated, and the failure time period corresponding to that variate is obtained using the RLD of turbine i. The failure time period corresponding to U_ω^i is $t \in \{1, 2, \ldots, T\}$ such that $F_{f_i}^{t_i^o}(t-1) \leq U_\omega^i < F_{f_i}^{t_i^o}(t)$. An artificial time period $T+1$ is added to the optimization model in order to denote cases where a turbine does not fail within the planning horizon; *i.e.*, if $U_\omega^i \geq F_{f_i}^{t_i^o}(T)$, then the failure period of turbine i under scenario ω is recorded as $T+1$. With this sampling method, independent and identically distributed samples of failure scenarios are generated.

2.2.2. Wind Energy & Market Price Prediction Uncertainties

Wind power and electricity price forecast errors can have a significant impact on the stochastic operations and profitability of the wind farm [43]. It is shown that these two factors impact the extent to which grouping maintenance actions together is beneficial [31]. Thus, these two sources of uncertainty are also considered within the uncertainty scenarios of the proposed stochastic programming model.

To incorporate the uncertainty of the wind power into our proposed decision-making framework, the available wind power of each turbine is considered in the uncertainty scenarios. In generating the *production capacity scenarios*, wind speed is assumed to follow a wn $W(\kappa, \lambda)$, where $\kappa > 0$ is the shape parameter and $\lambda > 0$ is the scale parameter. Then, historical data is used to obtain the shape and scale parameters of the distribution, and the production capacity scenarios for each turbine i is generated according to the turbine-specific power curves[44].

In generating the *electricity price scenarios*, the electricity price forecast error is assumed to follow a Gaussian distribution [45, 46]. Once the distribution parameters are estimated using historical data, a Monte Carlo simulation is employed to generate scenarios, similarly to the failure scenario generation procedure described in Section 2.2.1.

3. Sensor-Driven Adaptive Opportunistic Maintenance & Operations Scheduling Model

In this section, we develop an optimization model for the joint maintenance and operations scheduling of wind turbines in a wind farm, with explicit consideration of unexpected turbine failures, as well as electricity price and generation uncertainty. We formulate this problem as a mixed integer stochastic program, and refer to it as the *Stochastic Adaptive Opportunistic Maintenance & Operations Scheduling (SAOMOS)* model.

In the proposed model, a set of wind farm locations, $\mathcal{L}$, is considered, and the set of wind turbines for each location $\ell \in \mathcal{L}$ is denoted as $\mathcal{G}_\ell$. Based on the status of turbines at the time of planning, the set $\mathcal{G}_\ell$ is partitioned into two subsets, $\mathcal{G}_\ell^o$ and $\mathcal{G}_\ell^f$, which respectively denote the set of wind turbines that are either operational or under maintenance, and the set of failed turbines. Additionally, $\mathcal{G}^o = \bigcup_{\ell \in \mathcal{L}} \mathcal{G}_\ell^o$ and $\mathcal{G}^f = \bigcup_{\ell \in \mathcal{L}} \mathcal{G}_\ell^f$ denote the sets of all wind turbines that are operational and failed, respectively, at the time of planning. The SAOMOS model spans a time horizon $\mathcal{T}$, consisting of T time periods, and at each time period $t \in \mathcal{T}$, decisions regarding maintenance and operations must be made. In order to increase the time granularity for operational decisions, a time period t is further divided into a set of operational time periods, $\mathcal{H}$. This way, maintenance decisions are made for every time period $t \in \mathcal{T}$ (*e.g.*, every day), but operational decisions are made for every operational period $h \in \mathcal{H}$ (*e.g.*, every hour) of every period $t \in \mathcal{T}$.

Preventive maintenance, which can only be scheduled for operational turbines, is denoted with a binary decision variable z_t^i for each time period t and each operational wind turbine i. This decision variable will take the value 1 if preventive maintenance is initiated on turbine i at period t. A failed turbine can only undergo corrective maintenance. To incorporate this in SAOMOS, a binary decision variable ν_t^i for each time period t and each failed turbine i is defined. This decision variable takes the value 1 if turbine i is scheduled for a corrective maintenance at period t. A binary variable x_t^ℓ is defined for each time period t and each wind farm location ℓ to denote the maintenance crew visits. This decision variable will take the value 1 if the maintenance crew visits wind farm location ℓ at period t.

The failure time uncertainty of the turbines is incorporated into the model through the notion of scenarios [39, 47]. We assume that there is a finite number of possible joint uncertainty realizations, *i.e.*, scenarios, regarding the failure times of turbines that are operational at the time of planning, wind power, and energy price (see Section 2.2 for the details). Let Ω denote the set of these scenarios, where each scenario $\omega \in \Omega$

has a probability p_ω of occurring, and $\sum_{\omega\in\Omega} p_\omega = 1$. Turbine availability is denoted with the binary decision variable $\zeta^i_{t,\omega}$, which takes the value 1 if turbine i is operational at time period t under scenario ω. Furthermore, the amount of power (in MW) generated by turbine i in operational period h of period t under scenario ω is denoted with the non-negative continuous decision variable $y^i_{t,h,\omega}$.

In what follows, the objective and the constraints of the optimization model are described.

3.1. Objective Function

The objective of SAOMOS is to maximize the expected profit of wind farms as a function of decisions for operations and maintenance by utilizing sensor data. The objective function, given in (4), consists of expected operational revenue and maintenance costs, as well as crew visit costs.

$$\begin{aligned}\max_{\boldsymbol{z},\boldsymbol{\nu},\boldsymbol{x},\boldsymbol{y}} \quad & \sum_{\omega\in\Omega} p_\omega \sum_{i\in\mathcal{G}^o}\Bigg(\sum_{t\in\mathcal{T}}\sum_{h\in\mathcal{H}} \Gamma_{t,h,\omega}\cdot y^i_{t,h,\omega} - \sum_{t=1}^{\tau^i_\omega-1} C^{p,i}_t \cdot z^i_t - \sum_{t=\tau^i_\omega}^{T} C^{f,i}_t\cdot z^i_t\Bigg)\\ & -\sum_{\ell\in\mathcal{L}}\sum_{t\in\mathcal{T}} C^{v,\ell}_t\cdot x^\ell_t - \sum_{i\in\mathcal{G}^f}\sum_{t\in\mathcal{T}} C^{f,i}_t\cdot \nu^i_t - \sum_{i\in\mathcal{G}^o}\sum_{t\in\mathcal{T}} C^{d,i}_t\cdot z^i_t \end{aligned} \tag{4}$$

The first term of the objective captures the expected operational revenue minus the maintenance costs of the turbines that are operational at the time of planning. The electricity price at operational period h of period t under scenario ω is denoted with $\Gamma_{t,h,\omega}$. Furthermore, τ^i_ω denotes the time period at which turbine i fails under scenario ω, and $C^{p,i}_t$ and $C^{f,i}_t$ denote the preventive and corrective maintenance costs of turbine i in period t, respectively. By using the failure time period τ^i_ω in the limits of the summations, we ensure that the corrective maintenance cost is incurred if the turbine is maintained after its failure time, otherwise the preventive maintenance cost is incurred under scenario ω. The second and third terms of the objective account for the cost of crew visits to wind farm locations and the corrective maintenance cost of the turbines that were at a failed state at the time of planning, respectively. The parameter $C^{v,\ell}_t$ denotes the cost of a maintenance crew visit to wind farm location ℓ at period t.

The last term of the objective represents the *dynamic maintenance cost* of conducting maintenance at time period t. The proposed maintenance cost $C^{d,i}_t$ casts a balance between *(i)* premature (early) maintenance that inefficiently uses the equipment lifetime, and *(ii)* late maintenance that increases failure risks. The cost, $C^{d,i}_t$, associated with conducting maintenance of a partially degraded turbine i at time period t is given as follows:

$$C^{d,i}_t = U^{f,i}\int_{s=0}^{t}(t-s)P(\tau_i = s)ds + U^{p,i}\int_{s=t}^{\infty}(s-t)P(\tau_i=s)ds, \tag{5}$$

where τ_i denotes the random variable defining the failure time period of turbine i, and $P(\tau_i = t')$ denotes the probability that turbine i fails in time period t'. In defining the dynamic maintenance cost, $U^{f,i}$ and $U^{p,i}$ denote the costs per unit time of conducting maintenance after and before the time of failure, respectively. The proposed function outputs a penalty when the maintenance time t deviates from the time of failure τ_i: the first and second terms penalize deviation due to late and premature maintenances, respectively. In this formulation, $U^{f,i} >> U^{p,i}$, meaning that late maintenance (which causes failure) is penalized significantly higher than a premature maintenance; therefore enabling the maintenance function to remain conservative.

Note that the probabilities $P(\tau_i = t')$ are computed using the RLDs, and thus maintenance cost $C_t^{d,i}$ evolves as a function the turbine RLD predictions that are introduced in Section 2.1. As a result, updates to turbine RLDs are integrated into the objective function through this maintenance cost function.

3.2. Maintenance Coordination Constraints

Constraints (6)–(8) coordinate and limit turbine maintenance decisions.

$$\sum_{t\in\mathcal{T}} z_t^i = 1, \qquad \forall i \in \mathcal{G}^o \tag{6}$$

$$\sum_{t\in\mathcal{T}} \nu_t^i \le 1, \qquad \forall i \in \mathcal{G}^f \tag{7}$$

$$\sum_{i\in\mathcal{G}^o} z_t^i + \sum_{i\in\mathcal{G}^f} \nu_t^i \le M_t^p, \qquad \forall t \in \mathcal{T} \tag{8}$$

Constraints (6) ensure that the operational turbines are maintained once during the planning horizon. Constraints (7) limit the number of corrective maintenance actions on a turbine that was in a failed state at the time of planning to at most one. Note that constraint set (7) does not enforce corrective maintenance on failed turbines; rather, it allows turbines to be idle for some time so that corrective maintenance actions can be grouped opportunistically with more maintenance actions. Constraints (8) are the labor capacity constraints, where M_t^p denotes the planned maintenance capacity at period t.

3.3. Maintenance Crew Coordination Constraints

In this section, constraints that establish relationships between the different maintenance decision variables are presented. In addition to the decision variables introduced in Section 3, let the binary decision variable $u_{s,t}^\ell$, defined for all wind farm locations $\ell \in \mathcal{L}$, all periods $t \in \mathcal{T}$ and all periods $s \le t$, denote whether or not a maintenance crew visits wind farm location ℓ between periods s and t. This variable will take the value 1 if there is a maintenance crew visiting location ℓ between periods s and t (including periods s and t).

$$z_t^i \le x_t^\ell \qquad \forall \ell \in \mathcal{L}, i \in \mathcal{G}_\ell^o, t \in \mathcal{T} \tag{9}$$

$$\nu_t^i \le x_t^\ell \qquad \forall \ell \in \mathcal{L}, i \in \mathcal{G}_\ell^f, t \in \mathcal{T} \tag{10}$$

$$x_t^\ell \le \sum_{i\in\mathcal{G}_\ell^o} z_t^i + \sum_{i\in\mathcal{G}_\ell^f} \nu_t^i \qquad \forall \ell \in \mathcal{L}, t \in \mathcal{T} \tag{11}$$

$$\sum_{\ell\in\mathcal{L}} x_t^\ell \le 1 \qquad \forall t \in \mathcal{T} \tag{12}$$

$$x_t^\ell + x_{t'}^{\ell'} \le 1 \qquad \forall \ell, \ell' \in \mathcal{L}, \ell \ne \ell', t \in \{1, \ldots, T - \theta_{\ell,\ell'}\}, \forall t' \in \{t, \ldots, t + \theta_{\ell,\ell'}\} \tag{13}$$

$$x_{t'}^\ell \le u_{s,t}^\ell \qquad \forall \ell \in \mathcal{L}, t \in \mathcal{T}, s \in \{1, \ldots, t\}, t' \in \{s, \ldots, t\} \tag{14}$$

$$u_{s,t}^\ell \le \sum_{t'=s}^{t} x_{t'}^\ell \qquad \forall \ell \in \mathcal{L}, t \in \mathcal{T}, s \in \{1, ..., t\} \tag{15}$$

Constraints (9)–(11) enforce that the maintenance crew visits wind farm location ℓ if and only if at least one wind turbine in location ℓ is scheduled for preventive or corrective maintenance. Constraints (12) enforce that at most one wind farm location can be visited in a time period. Constraints (13) enforce the travel time between wind farm locations: If the maintenance crew visits location ℓ in period t, it cannot visit another location ℓ' before the travel time between the two locations, $\theta_{\ell,\ell'}$, passes. Constraints (14) and (15) establish the relationship between variables $u^{\ell}_{s,t}$ and $x^{\ell}_{t'}$, *i.e.*, $u^{\ell}_{s,t}$ takes the value 1 if and only if there is at least one crew visit to location ℓ between periods s and t.

3.4. Coupling Constraints for Wind Turbine Availability and Maintenance

Constraints (16) and (17) model the availability of the wind turbines, which is defined with the decision variable $\zeta^i_{t,\omega}$. This decision variable will assume the value 1 if turbine i is available to operate at time period t under scenario ω. Note that turbine availability depends on maintenance and crew visit decisions, as well as the failure scenarios.

$$\zeta^i_{t,\omega} = \begin{cases} 1 - z^i_t, & \text{if } t < \tau^i_\omega \\ \sum_{s=1}^{t-1} z^i_s, & \text{if } t = \tau^i_\omega \\ \sum_{s=1}^{\tau^i_\omega - 1} z^i_s + u^{\ell}_{\tau^i_\omega, t-1} \cdot \left(\sum_{s=\tau^i_\omega}^{T} z^i_s \right), & \text{if } t > \tau^i_\omega \end{cases}, \quad \forall \ell \in \mathcal{L}, i \in \mathcal{G}^o_\ell, t \in \mathcal{T}, \omega \in \Omega \tag{16}$$

Constraints (16) establish the availability of turbines that are operational at the time of planning. The availability of operational wind turbines under any scenario $\omega \in \Omega$ can be examined in three cases. Let cases I, II, and III correspond to the time periods before, during, and after the failure period of turbine i under scenario ω, respectively. In case I, for time periods before the failure time of turbine i under scenario ω, τ^i_ω, the turbine is available if it is not under the preventive maintenance. In case II, *i.e.*, at the time period in which turbine i fails under scenario ω, the turbine would be available only if it has gone under preventive maintenance before that time period. Lastly, case III corresponds to the time periods after the failure time of a wind turbine i under scenario ω. In this case, the turbine is available in a period t if *(i)* it has been preventively maintained before its failure time, or *(ii)* it has not been preventively maintained before its failure time but has undergone on-the-spot corrective maintenance when the maintenance crew visited the wind farm location to maintain other turbines. To model case III, the binary indicator $\sum_{s=\tau^i_\omega}^{T} z^i_s$, which takes the value 1 if preventive maintenance is scheduled after the failure time of turbine i under scenario ω, is used. If the aforementioned binary indicator is 1, and if the maintenance crew visits the wind farm location after the failure time τ^i_ω and before t (so, $u^{\ell}_{\tau^i_\omega, t-1}$ is equal to 1), then the availability variable $\zeta^i_{t,\omega}$ will be set to 1.

Constraints (17) set the value of $\zeta^i_{t,\omega}$ for the turbines that are at a failed state at the time of planning. It establishes that failed turbines are not available until they undergo planned corrective maintenance.

$$\zeta^i_{t,\omega} = \sum_{s=1}^{t-1} \nu^i_s, \qquad \forall i \in \mathcal{G}^f, t \in \mathcal{T}, \omega \in \Omega \tag{17}$$

Constraints (18) are introduced to limit the number of turbines that undergo on-the-spot corrective maintenance when the maintenance crew visits the wind farm location to maintain other turbines; a concept that is introduced with the $u^{\ell}_{s,t}$ variables, as in case III of constraint set (16). The number of turbines that fail before their scheduled preventive maintenance but are maintained through on-the-spot corrective maintenance is limited to at most M^o_t at every time period $t \in \mathcal{T}$.

$$\sum_{\ell \in \mathcal{L}} \sum_{\substack{i \in \mathcal{G}^o_\ell: \\ \tau^i_\omega < t}} u^{\ell}_{\tau^i_\omega, t-1} \cdot \left(\sum_{s=\tau^i_\omega}^{T} z^i_s \right) \leq M^o_t, \quad \forall \omega \in \Omega, t \in \mathcal{T} \tag{18}$$

Note that constraints (18) and case III of constraints (16) are non-linear. But since they involve binary variables, the linearization procedure for these constraints is relatively straightforward. For the sake of completeness, the linear counterparts of these constraints are derived in Appendix B.

3.5. Coupling Constraints for Wind Turbine Availability and Operations

Constraints (19) limit the power produced by each turbine. They ensure that an unavailable turbine cannot produce any power, and the production of an available turbine is limited by the maximum production capacity of that turbine and the weather conditions. The parameter $\Phi^i_{t,h,\omega}$ denotes the predicted production output of turbine i at operational period h of period t under scenario ω, which is obtained by using *(i)* wind speed scenarios generated with historical wind speed data, and *(ii)* wind turbine specifications.

$$y^i_{t,h,\omega} \leq \Phi^i_{t,h,\omega} \cdot \zeta^i_{t,\omega}, \quad \forall \ell \in \mathcal{L}, i \in \mathcal{G}_\ell, t \in \mathcal{T}, h \in \mathcal{H}, \omega \in \Omega \tag{19}$$

3.6. Chance constraints

In this section, we introduce the *chance constraints* of the SAOMOS formulation, which guarantee a high level of availability at each time period. Let χ^i_t be a random variable denoting the non-availability of turbine i at period t, which will take the value 1 if turbine i is *not* available in period t. Then, the chance constraints given in (20) guarantee that at each time period t, the number of turbines that may become unavailable is below a threshold $\mathcal{N}$ with a probability of at least $1-\epsilon$.

$$P\left(\sum_{i \in \mathcal{G}^o} \chi^i_t \geq \mathcal{N} \right) \leq \epsilon, \quad \forall t \in \mathcal{T} \tag{20}$$

The random variable χ^i_t, which denotes the non-availability of wind turbine i at period t, is defined in equation (21) for each turbine that is operational at the time of planning, *i.e.* $i \in \mathcal{G}^o$. The non-availability of turbine i at period t will be equal to 1 if one of the following two cases hold true: *(i)* turbine i has not failed yet but is undergoing scheduled preventive maintenance at period t, or *(ii)* turbine i has failed before period t and has not experienced on-the-spot corrective maintenance yet. In order to distinguish between these two cases, we define the random variable η^i_t, which takes the value 1 if time period t is on or after the failure period of turbine i, τ_i, and the value 0 otherwise.

$$\chi^i_t = \big(1-\eta^i_t\big) z^i_t + \eta^i_t \big(1 - u^{\ell}_{\tau_i, t-1}\big), \quad \forall \ell \in \mathcal{L}, i \in \mathcal{G}^o_\ell, t \in \mathcal{T} \tag{21}$$

The η_t^i random variable follows a Bernoulli distribution. The probability distribution of this random variable is generated from the predicted RLD of turbine i through sensors, namely $F_{f_i}^{t_i^o}(t)$. This way, constraints (20) fully adapt to sensor information.

It is important to note that the chance constraints (20) are intractable. Thus, to incorporate these constraints into the SAOMOS model in a tractable way, safe approximations are derived in Section 4.

4. Chance Constraint Approximations

As mentioned earlier, the chance constraints given in (20) are intractable, as is typical for chance-constrained stochastic programs [48, 49]. In this section, two tractable approximation methods for the chance constraints are presented.

4.1. Analytical Safe Tractable Approximation of Chance Constraints

A *safe approximation* is a constraint whose satisfaction guarantees the satisfaction of the original constraint. Proposition 1 provides a tractable safe approximation, (22), for the chance constraints given in (20). This means that any solution that satisfies constraints (22) is guaranteed to also satisfy constraints (20).

Proposition 1. *The deterministic linear constraint set* (22) *is a safe approximation of* (20).

$$\mathbb{E}\left[\sum_{i\in\mathcal{G}^o}\chi_t^i\right] \leq \max\left\{\mathcal{N}\epsilon,\ \max_{\alpha>0}\frac{2|\mathcal{G}^o|\left(\left(\epsilon e^{\alpha\mathcal{N}}\right)^{\frac{1}{2|\mathcal{G}^o|}}-1\right)}{e^{\alpha}-1}\right\}, \quad \forall t\in\mathcal{T} \tag{22}$$

The proof of Proposition 1 is provided in Appendix A.

4.2. Scenario Approximation for the Chance Constraints

The safe approximation presented in Section 4.1 is beneficial because it guarantees a certain level of turbine availability. However, safe approximations give no indication as to how close they are to the original constraint. In this section, we derive a simple yet effective sampling-based approximation that remains close to the original chance constraint (20), on average [50].

Let $\boldsymbol{\xi_t} = \left(\xi_t^1, ..., \xi_t^{|\mathcal{G}^o|}\right)$ denote the vector of ξ_t^i random variables, which define whether or not turbine i has failed by time period t. Then, we define a function $G_t(\mathbf{x}, \boldsymbol{\xi_t})$ as follows:

$$G_t(\mathbf{x}, \boldsymbol{\xi_t}) = \sum_{i\in\mathcal{G}^o}\chi_t^i - \mathcal{N}, \quad \forall t\in\mathcal{T} \tag{23}$$

Using (23), the chance constraints (20) can be formulated in the following way:

$$P\Big(G_t(\mathbf{x}, \boldsymbol{\xi_t}) \geq 0\Big) \leq \epsilon, \quad \forall t\in\mathcal{T} \tag{24}$$

Equivalently,

$$\mathbb{E}_{\boldsymbol{\chi}}\left[\mathbb{1}_{\{G_t(\mathbf{x},\boldsymbol{\xi}_t)\geq 0\}}\right] \leq \epsilon, \quad \forall t \in \mathcal{T}. \tag{25}$$

In inequality set (25), $\mathbb{1}$ is the indicator function, *i.e.*, $\mathbb{1}_{\{G_t(\mathbf{x},\boldsymbol{\xi}_t)\geq 0\}}$ takes the value 1 if $G_t(\mathbf{x}, \boldsymbol{\xi}_t)$ is non-negative, and 0 otherwise.

Let $\boldsymbol{\xi}_{t,1}, \ldots, \boldsymbol{\xi}_{t,|\Omega|}$ be an independently identically distributed (iid) sample of $|\Omega|$ realizations of the random vector $\boldsymbol{\xi}_t$. This sample can be used to approximate the expectation in constraints (25). Using this idea, a natural approximation of the chance constraints is given in (26).

$$\frac{1}{|\Omega|}\sum_{\omega=1}^{|\Omega|} \mathbb{1}_{\{G_t(\mathbf{x},\boldsymbol{\xi}_{t,\omega})\geq 0\}} \leq \gamma, \quad \forall t \in \mathcal{T} \tag{26}$$

Note that the proportion of realizations ω with $G_t(\mathbf{x}, \boldsymbol{\xi}_{t,\omega}) \geq 0$ approximates the probability that the number of unavailable turbines exceed $\mathcal{N}$. Although constraint set (26) constitutes a tractable approximation to the chance constraints, it is not a safe approximation, *i.e.*, a feasible solution to constraints (26) is not guaranteed to be feasible to the original chance constraints (20). Thus, the choice of γ is important for ensuring feasibility. Choosing a small enough γ ($\gamma < \epsilon$) would increase the likelihood of obtaining solutions that satisfy the original chance constraints. On the other hand, choosing a value for γ such that $\gamma > \epsilon$ can be useful for obtaining a lower bound on the objective value of the original problem.

In order to incorporate constraints (26) into the SAOMOS formulation, a new binary decision variable, $v_{t,\omega}$, is defined for each period t and uncertainty realization (scenario) $\omega \in \Omega$. The decision variable $v_{t,\omega}$ takes the value 1 if the availability requirement is violated at time period t under scenario ω. Constraints (27), where M denotes a sufficiently large positive number, ensure that $v_{t,\omega}$ takes the value 1 when $G_t(\mathbf{x}, \boldsymbol{\xi}_{t,\omega}) \geq 0$ holds, *i.e.*, when the unavailability tolerance is exceeded. Constraints (28) limit the proportion of scenarios under which the unavailability tolerance is exceeded.

$$G_t(\mathbf{x}, \boldsymbol{\xi}_{t,\omega}) - M \cdot v_{t,\omega} \leq 0, \qquad \forall t \in \mathcal{T}, \omega \in \Omega \tag{27}$$

$$\frac{1}{|\Omega|}\sum_{\omega\in\Omega} v_{t,\omega} \leq \gamma, \qquad \forall t \in \mathcal{T} \tag{28}$$

5. Computational Experiments

In this section, a comprehensive set of experiments are designed and conducted in order to *(i)* evaluate the performance of the SAOMOS model across different realistic settings, and *(ii)* demonstrate the generalizability of the findings. Section 5.1 introduces and justifies the experimental setting, data, and parameter values used in the computational experiments. Consequently, Section 5.2 presents and discusses the results of the computational experiments.

5.1. Experimental Setting and Data

The experimental framework consists of three modules: *(i)* predictive analytics module, *(ii)* planning module, and *(iii)* execution module. The *predictive analytics module* uses the degradation data to derive the RLDs of wind turbines based on new observations, and accordingly updates the associated dynamic maintenance costs and failure predictions. The *planning module* takes the output of the predictive analytics module as input, and solves the SAOMOS model to schedule maintenance and operations for a 30-day planning horizon. Then the sequence of events is simulated in the *execution module*. In these simulations, the optimal maintenance schedule obtained from SAOMOS is fixed to be executed for a number of periods (commonly referred to as the "freeze period"), and the chronology of events that occur following this maintenance schedule is simulated. By tracking the degradation signals of the wind turbines, the execution module checks to see which turbines fail before their scheduled maintenance and which successfully undergo maintenance within the freeze period. The turbines that fail during the freeze period can be correctively maintained if the maintenance crew visits the wind farm location after the failure time. Otherwise, they remain offline and should be scheduled for a corrective maintenance the next time the planning module is executed. At the end of each freeze period, the execution module calculates the resulting operational revenue of the wind farms based on updated turbine availability, observed wind profile, and energy prices. The degradation signals of turbines are then updated based on outages or new sensor observations, and the planning horizon is moved forward (commonly referred to as the "rolling horizon") to plan the next monthly schedule. The predictive analytics, planning, and execution modules are executed 15 times in a rolling horizon fashion to simulate a time horizon of 315 days. Furthermore, this procedure is repeated 10 times with different initial turbine ages and degradation signals. The metrics presented in the remainder of Section 5 are obtained by calculating the average of these ten replications.

In all cases, maintenance and operations are scheduled and simulated for 100 wind turbines, each with a rated capacity of 2 MW. The identical cut-in, cut-out, and rated speed of the turbines are 3, 30, and 12 meters per second (m/s), respectively. The planning horizon is 30 days with daily maintenance and hourly operational decisions. In all experiments, the corrective and preventive maintenance costs of turbines are $C^f = \$8\,000$ and $C^p = \$2\,000$, respectively. Unless otherwise indicated, a crew deployment cost of $C^v = \$32\,000$ per visit is assumed. The chance constraints are implemented by setting $\mathcal{N} = 10$, $\epsilon = 0.05$, and $\gamma = 0.04$, unless stated otherwise.

The real-world vibration-based degradation data from a bearing application is used to mimic the degradation process in wind turbines. The RLDs of the turbines are dynamically estimated from the data by employing the Bayesian updating technique described in Section 2.1. The sensor-driven RLDs are *(i)* discretized into daily periods to generate independently and identically distributed (iid) turbine failure scenarios, and *(ii)* transformed to derive the expected cost of deviation from optimal maintenance time of individual wind turbines (denoted as $C_t^{d,i}$). The wind speed distribution is calculated using data obtained from *National Centers for Environmental Information* [51, 52], and this distribution is used to generate production scenarios for wind turbines. For incorporating the energy price scenarios into the optimization model, real-time prices reported by the *PJM* are used. Following the works of [53, 46, 54], Gaussian distribution is used to represent the price forecast error, with the base value as the mean and 10% of the base value as the standard deviation.

The performance of the SAOMOS model is benchmarked against *(i)* a time-based opportunistic maintenance policy, and *(ii)* a sensor-driven availability-neutral opportunistic maintenance policy. The *time-based opportunistic (TBO)* maintenance policy schedules maintenance actions at fixed industrially approved time intervals, regardless of the degradation states of turbines, without using sensor information. The *sensor-driven availability-neutral opportunistic (SANO)* maintenance policy uses sensor observations and accordingly adapts maintenance schedules. However, it does not limit the number of unavailable turbines. The maintenance schedules for the SANO policy are obtained by implementing and solving SAOMOS without the chance constraints.

The details of individual case studies are outlined in Table 1. The first case conducts a benchmark analysis with 5 wind farm locations. The second case is aimed at evaluating the effect of corrective maintenance costs on the availability of wind turbines. The third case study assesses the effect of crew costs on resulting maintenance schedules. Finally, the fourth case study focuses on testing the effect of the number of wind farm locations on maintenance schedules. The experimental results associated with these four case studies are presented in Section 5.2.

Table 1: Comparative case studies

	Case Description	**Sensitivity Analysis**
Case 1	O&M scheduling of 5 wind farms	Benchmark analysis
Case 2	O&M scheduling of 5 wind farms	Corrective maintenance cost
Case 3	O&M scheduling of 5 wind farms	Crew cost
Case 4	O&M scheduling of 100 turbines	Number of wind farms

5.2. Experimental Results

Sections 5.2.1 to 5.2.4 present and discuss the results of the case studies summarizes in Table 1. In order to conduct a thorough comparison of various maintenance policies and parameter settings, the performance metrics listed and detailed below are used.

- The number of preventive and corrective maintenance actions and crew visits are recorded. The corrective maintenance actions are considered in two categories: planned and on-the-spot. Planned corrective maintenance is conducted on turbines that were already at a failed state before the planning module solves SAOMOS, and on-the-spot corrective maintenance is conducted on turbines that fail unexpectedly after planning, when a maintenance crew visits the wind farm location to maintain another turbine.

- Average unavailability (in turbine-days), average curtailed power (in MW), and the maximum number of turbines that are simultaneously unavailable in a day are reported. Since the chance constraints are enforced on the number of unavailable turbines each day, we expect that they have a profound effect on reducing the maximum number of unavailable turbines.

- Average unused life is recorded to assess the efficiency of maintenance policies. This metric reports, at the time of preventive maintenance, the number of days a turbine would have functioned if it had not gone under maintenance.

- Finally, total maintenance cost, operational revenue, and the resulting profit are presented.

5.2.1. Case 1: Benchmark Analysis

This first case focuses on comparing the performance of the SAOMOS model with the two benchmark maintenance policies, TBO and SANO. Both chance constraint approximation methods presented in Section 4 are implemented and tested. The safe (Section 4.1) and scenario (Section 4.2) approximation methods are referred to as *SAOMOS-Safe* and *SAOMOS-Scenario*, respectively, in the presentation of results.

Table 2 provides the reliability and operational metrics for the three policies (TBO, SANO, SAOMOS), with two approximation method variants of SAOMOS (SAOMOS-Safe, SAOMOS-Scenario). In this case study, five wind farms containing a total of 100 turbines are simulated for a time horizon of 315 days. The simulations are repeated 10 times and the averages of these 10 replications are presented in Table 2.

Table 2: Benchmark Analysis of Maintenance Policies

Performance Metric	TBO	SANO	SAOMOS-Safe	SAOMOS-Scenario
Preventive Actions	147.4	140	138.9	138.2
Corrective Actions - Planned	32.5	3.7	3	4.5
Corrective Actions - On-the-Spot	3.3	12.3	15.9	12.2
Crew Visits	29	25.1	32.7	25.7
Average Curtailed Power (MW)	308.74	106.71	106.22	93.96
Average Unavailability (days)	8.61	2.98	2.95	2.6
Maximum Unavailable Turbines	21.6	15.6	8.5	13.8
Average Unused Life (days)	66	24.2	26.1	25.9
Maintenance Cost	\$1.51 **M**	\$1.21 **M**	\$1.48 **M**	\$1.23 **M**
Operational Revenue	\$18.33 **M**	\$18.65 **M**	\$18.64 **M**	\$18.67 **M**
Net Profit	\$16.83 **M**	\$17.43 **M**	\$17.17 **M**	\$17.43 **M**

The impact of chance constraints in reducing unavailability can be clearly observed in Table 2. Compared to the TBO policy, SAOMOS-Safe and SAOMOS-Scenario decrease the average unavailability by 69.86% and 65.79%, respectively, whereas this decrease is 65.42% for SANO, which does not consider chance constraints. Perhaps more notably, compared to TBO, SAOMOS-Safe and SAOMOS-Scenario reduce the maximum number of unavailable turbines by 36.11% and 60.65%, respectively, while SANO could only reduce it by 27.78%.

All three sensor-driven maintenance models (SANO, SAOMOS-Safe, SAOMOS-Scenario) are able to significantly reduce the number of corrective maintenance actions, and at the same time, the average unused life of the turbines. By incorporating failure predictions into maintenance decisions, these models provide a good balance between conducting maintenance too soon (which results in waste in the form of unused life) or too late (which results in unavailability, corrective maintenance, and reduced production). The effectiveness of sensor-driven models in avoiding waste can also be observed in the number of maintenance actions: SANO results in 5.02% fewer preventive maintenance actions than TBO, whereas SAOMOS-Safe and SAOMOS-Scenario reduce the number of preventive maintenance actions by 6.24% and 5.77%, respectively. Since the chance-constrained models have a strong emphasis on turbine availability, they take into account that turbines become unavailable for production during maintenance and therefore schedule fewer preventive maintenance actions than SANO.

The sensor-driven models are also effective in reducing the number of crew visits. By making use of opportunistic preventive and corrective maintenance options, SANO and SAOMOS-Scenario are able to provide higher turbine availability and operational revenue than the TBO policy, with fewer crew visits. Differently than SANO and SAOMOS-Scenario, SAOMOS-Safe resulted in more crew visits than TBO, due to its high level of conservatism. It is important to recall that the safe approximation, unlike the scenario approximation, guarantees the satisfaction of the original chance constraints, and therefore is more conservative in limiting unavailability. The impact of this conservatism can also be observed in the total maintenance costs: While SANO and SAOMOS-Scenario result in 19.75% and 18.34% lower total maintenance costs than TBO, this reduction in maintenance cost is only 2.24% for SAOMOS-Safe.

All sensor-driven policies result in a higher net profit than the TBO policy. More interestingly, SANO and SAOMOS-Scenario generate the same net profit, whereas SAOMOS-Scenario has lower average and maximum unavailability. This means that the chance-constrained model with a scenario approximation can achieve higher availability compared to an availability-neutral model (with no chance constraints) without any significant additional costs.

5.2.2. Case 2: Effect of the Corrective Maintenance Cost

In this section, the effect of increasing corrective maintenance costs on maintenance schedules and resulting performance metrics is examined. To do so, the ratio of corrective maintenance cost to preventive maintenance cost, $\frac{C^f}{C^p}$, is varied between 3 and 5, and all other parameter values are kept constant. The results are presented in Table 3. Note that from this point onward, unless stated otherwise, the scenario approximation method is used for tractably approximating the chance constraints. For the sake of brevity, the label "SAOMOS" is used to indicate the SAOMOS model with scenario-approximated chance constraints.

Table 3: Impact of Corrective Maintenance Cost on Maintenance Policies

$\frac{C^f}{C^p}$	Policy	Preventive Actions	Corrective Actions (Planned/On-the-Spot)	Maximum Unavailable Turbines	Crew Visits	Average Curtailed Power (MW)	Average Unavailable Days	Average Unused Life (days)	Maintenance Cost	Net Profit
3	**TBO**	147.4	32.5/3.3	21.6	29	308.74	8.61	135.2	$1.44 M	$16.90 M
	SANO	138	4.9/12.7	15.5	25.4	110.57	3.1	23.8	$1.19 M	$17.44 M
	SAOMOS	139.7	4.8/13.5	10.7	28.7	109.29	3.03	26.5	$1.31 M	$17.33 M
4	**TBO**	147.4	32.5/3.3	21.6	29	308.74	8.61	135.2	$1.51 M	$16.83 M
	SANO	140	3.7/12.3	15.6	25.1	106.71	2.98	24.2	$1.21 M	$17.43 M
	SAOMOS	138.6	3.8/13.4	10.5	29.1	100.8	2.76	26.7	$1.35 M	$17.31 M
5	**TBO**	147.4	32.5/3.3	21.6	29	308.74	8.61	135.2	$1.58 M	$16.75 M
	SANO	141.6	3.1/12.6	15.4	25	103.96	2.91	24.7	$1.24 M	$17.41 M
	SAOMOS	141.6	2.6/12.1	10.4	29.3	92.19	2.55	26.3	$1.37 M	$17.30 M

It can be observed in Table 3 that as the corrective maintenance cost increases, all three policies result in higher maintenance costs and lower profits. It is interesting to note that the rate at which net profit decreases with increasing corrective maintenance costs is smaller in sensor-driven policies (SANO and SAOMOS) compared to the TBO policy. When $\frac{C^f}{C^p} = 3$, SANO and SAOMOS bring in 3.20% and 2.54% more profit than TBO, respectively, whereas these figures become 3.94% and 3.28%, respectively, when $\frac{C^f}{C^p} = 5$. The sensor-driven policies result in increasingly more profitable schedules, because they are able to adapt their O&M schedules to the increasing corrective maintenance costs unlike the TBO policy, which schedules maintenance actions within fixed, industry-recommended time windows, regardless of costs. Note that due to the same reason, the number of preventive and corrective maintenance actions, crew visits, availability, and unused life remains the same for TBO, irrespective of the corrective maintenance cost ratio.

As the corrective maintenance cost increases, both SANO and SAOMOS increase the number of preventive maintenance actions in order to avoid failures and costly corrective maintenance actions, and therefore result in higher average availability, lower curtailment, and longer unused life. It can also be observed that SAOMOS has a lower average number of unavailable days than SANO, although the difference is small. Furthermore, the difference in average unavailability between SANO and SAOMOS increases with increasing corrective maintenance cost, which means that SAOMOS is better at lowering unavailability when corrective maintenance cost is high. The maximum number of unavailable turbines does not change significantly, since increased unavailability due to preventive maintenance actions balances out the decreased unavailability due to failures.

5.2.3. Case 3: Effect of the Crew Visit Cost

This section presents an investigation of how increasing crew visit costs affect the maintenance schedules and their corresponding performance metrics. In order to observe the effects of the crew visit cost, the ratio of crew visit cost to the preventive maintenance cost, $\frac{C^v}{C^p}$, is changed between 8 and 16, while all other parameter values are kept constant. The resulting performance metrics for TBO, SANO, and SAOMOS policies are given in Table 4.

Table 4: Impact of Crew Cost on Maintenance Policies

$\frac{C^v}{C^p}$	Policy	Preventive Actions	Corrective Actions (Planned/On-the-Spot)	Maximum Unavailable Turbines	Crew Visits	Average Curtailed Power (MW)	Average Unavailable Days	Average Unused Life (days)	Maintenance Cost	Net Profit
8	**TBO**	152.2	29.7/4.7	14.2	35.5	239.92	6.79	66.4	$1.15 **M**	$17.29 **M**
	SANO	135.9	4.7/12.9	10.1	35.3	93.05	2.66	18.66	$0.98 **M**	$17.69 **M**
	SAOMOS	137.3	4/14.1	9.1	38.8	90.28	2.56	18.67	$1.04 **M**	$17.63 **M**
12	**TBO**	151.1	33.1/4.5	14.9	32.9	297.04	8.21	65	$1.39 **M**	$16.96 **M**
	SANO	141.7	3.8/12.6	11.2	26.4	116.47	3.22	25.1	$1.05 **M**	$17.58 **M**
	SAOMOS	139.4	4/12.9	9.8	32	100.33	2.84	25.9	$1.18 **M**	$17.47 **M**
16	**TBO**	150.4	32.7/4.5	15.3	32.2	309.09	8.58	65.4	$1.63 **M**	$16.71 **M**
	SANO	145	3.4/12	12	23.7	100.98	2.86	27.7	$1.17 **M**	$17.48 **M**
	SAOMOS	145.6	3.1/13.2	10.8	28.9	95.48	2.56	28.2	$1.35 **M**	$17.32 **M**

It can be observed in Table 4 that all models respond to increasing crew visit costs by decreasing the number of crew visits. The adaptive formulations of SANO and SAOMOS casts a balance between: *(i)* reducing crew visits and the associated crew logistics expenditures, and *(ii)* controlling the opportunistic grouping of turbines (*e.g.*, expediting or delaying maintenances) in an effort to contain the risks associated with premature maintenances and turbine failures. The response of the SANO policy is the most prominent, as it reduces the number of crew visits by 32.86% when the crew visit cost ratio is increased from 8 to 16. This reduction in the number of visits is 25.52% in SAOMOS. The reduction in the number of crew visits is less in SAOMOS, because of its explicit focus on high turbine availability. In contrast to the sensor-driven policies, TBO reacts much less to increasing crew visit costs. This is due to the obligation of the TBO to adhere to fixed maintenance time windows.

The increasing crew visit costs prompt the sensor-driven models to group maintenance actions more aggressively, and hence minimizing the number of crew visits. In sensor-driven models (i.e. SANO and SAOMOS), increasing crew visit cost leads to higher number of preventive maintenances, and lower number of failures & corrective maintenance actions. Specifically, the number of corrective maintenance actions decreases by 12.5% for SANO and 9.94% for SAOMOS, when the crew visit cost ratio increases from 8 to 16. It is noteworthy that the total number of corrective maintenance actions is larger in SAOMOS than in SANO, even though the number of planned corrective maintenance actions is comparable in both policies. The reason for this outcome is that the SAOMOS model places emphasis specifically on availability, and therefore takes better advantage of on-the-spot corrective maintenance opportunities. The impact of this behavior can be observed in average unavailable days and average curtailed power, which are always lower with SAOMOS than with the SANO policy.

All three policies respond to increasing crew visit costs with an increase in the maximum number of unavailable turbines. As crew costs increase, all three policies focus on grouping maintenance actions more aggressively. But regardless of the crew costs, SAOMOS always results in the smallest value for this metric. A similar behavior can be observed in the average unavailable days: Although the average unavailability increases in all policies with increasing crew costs, the minimum is always achieved with the SAOMOS policy. It is also noteworthy that under TBO, the increase in crew costs results in a significant increase in the average unavailability and power curtailment.

Similarly to the situation observed in Case 2 (Section 5.2.2), the increasing crew visit cost results in higher maintenance costs and lower profits in all three policies. However, the decrease in the profit is slower under SANO and SAOMOS than that under TBO. As the crew visit cost ratio, $\frac{C^v}{C^p}$, increases from 8 to 16, the net profit obtained with SANO and SAOMOS decrease by 1.19% and 1.76%, respectively, while the profit decrease under TBO is 3.35%.

5.2.4. Case 4: Effect of the Number of Wind Farms

This case study examines the impact of number of wind farm locations, $|\mathcal{L}|$, on the maintenance schedules and the resulting performance metrics of the three policies. In each experiment, a total of 100 wind turbines are assumed to be distributed among a number of wind farms as equally as possible. For observing the effects of the number of wind farm locations on the performance metrics, 3, 4, and 5 locations are considered.

The results of this case study are given in Table 5. It can be observed that as the number of locations increases, the number of crew visits increases for all three policies. However, this increase is the steepest under TBO: the number of crew visits increases by 31.43% under TBO when the number of locations increases from 3 to 5, whereas this increase amounts to only 17.48% and 24.08%, respectively, for SANO and SAOMOS.

When the TBO policy is adopted, the increasing number of locations results in an increase in unavailability and power curtailment. The larger number of locations forces the maintenance crew to visit each location less frequently, which results in longer waiting times before failed turbines are brought back to operational state. The situation, however, is quite different in the sensor-driven policies. By pursuing opportunistic maintenance more aggressively and scheduling more preventive maintenance actions with increasing number of locations, the SANO and SAOMOS policies are able to keep unavailability the same, or even decrease it. When the number of locations increase from 3 to 5, unavailability (curtailed power) increases by 19.32%

Table 5: Impact of Number of Wind Farm Locations on Maintenance Policies

$\lvert\mathcal{L}\rvert$	Policy	Preventive Actions	Corrective Actions (Planned/On-the-Spot)	Maximum Unavailable Turbines	Crew Visits	Average Curtailed Power (MW)	Average Unavailable Days	Average Unused Life (days)	Maintenance Cost	Net Profit
3	**TBO**	154.7	32.7/3.7	20.9	24.5	260.54	7.19	66.3	$1.38 **M**	$17.02 **M**
	SANO	138.3	4.4/11.4	15.6	19.1	126.23	3.55	23.8	$1.01 **M**	$17.6 **M**
	SAOMOS	142.1	3.9/11.8	10.8	24.6	96.3	2.63	25.5	$1.2 **M**	$17.46 **M**
4	**TBO**	148.1	32.8/4.6	21.5	27	294.52	8.2	66.6	$1.46 **M**	$16.9 **M**
	SANO	139.6	3.7/12.4	14.5	21.3	112.69	3.14	25.4	$1.09 **M**	$17.55 **M**
	SAOMOS	142.4	5/13.6	11.1	27.5	109.88	3	25.7	$1.31 **M**	$17.33 **M**
5	**TBO**	150.4	32.7/4.5	15.3	32.2	309.09	8.58	65.4	$1.63 **M**	$16.71 **M**
	SANO	145	3.4/12	12	23.7	100.98	2.86	27.7	$1.17 **M**	$17.48 **M**
	SAOMOS	145.6	3.1/13.2	10.8	28.9	95.48	2.56	28.2	$1.35 **M**	$17.32 **M**

(18.64%) under TBO, whereas a decrease of 19.48% (20%) and 2.7% (0.84%) is observed under SANO and SAOMOS, respectively.

The sensor-driven policies respond to the increasing number of locations by conducting more opportunistic maintenance. Since being responsible for more wind farm locations means potentially having to wait longer before the maintenance crew can bring a failed turbine back to operational state, these policies act more proactively in conducting preventive maintenance actions when the number of locations increases. As a result, when the number of wind farm locations increases from 3 to 5, the SANO and SAOMOS policies conduct 4.84% and 2.46% more preventive maintenance actions, respectively. This increasingly opportunistic behavior also results in a 16.39% and 10.59% increase in average unused life, respectively, for SANO and SAOMOS.

Similar to Cases 2 and 3 (Sections 5.2.2 and 5.2.3), an overall increase in maintenance costs and a decrease in net profits is observed as the number of wind farm locations increases. Although the increased maintenance cost and decreased profit is observed under all policies, the sensor-driven policies are able to keep the rate of cost increase and profit decrease relatively low. As the number of locations increases from 3 to 5, the total maintenance cost increases by 17.64% and the net profit decreases by 1.84% under the TBO policy, while the maintenance cost decreases by 15.52% and 12.48%, and the net profit decreases by only 0.7% and 0.85%, respectively, for SANO and SAOMOS.

6. Conclusion

Due to ever-increasing energy demand and the uncertainties surrounding maintenance and its impact on turbine failure risks, *availability* is of increasing importance for wind farms. In this paper, we develop a risk-based maintenance and operations scheduling model, *SAOMOS*, that explicitly keeps track of availability and schedules maintenance actions for multiple wind farms by taking into consideration *(i)* preventive and corrective maintenance costs, *(ii)* a sensor-based dynamic maintenance cost that assesses the trade-off between maintaining too early and too late, *(iii)* uncertain electricity price, *(iv)* uncertain turbine failure scenarios, *(v)* crew travel time between wind farm locations, *(vi)* on-the-spot corrective maintenance actions in addition to planned ones, and *(vii)* chance constraints that limit the total unavailability at each time period. With the acknowledgment that the chance constraints given in (20) become intractable for wind farm systems of practically relevant size, we develop two methods, namely *safe approximation* (Section 4.1) and *scenario approximation* (Section 4.2), to approximate these chance constraints in a tractable manner.

The SAOMOS formulation is extensively tested on 100-turbine test instances with varying maintenance costs and number of wind farms, and the resulting maintenance schedules are compared with those resulting from *time-based opportunistic (TBO)* and *sensor-driven availability-neutral opportunistic (SANO)* policies, through simulations. All cases presented in Section 5 demonstrate the prominent effect of the chance constraints in significantly reducing the maximum number of turbines that are simultaneously unavailable. In all experiments, SAOMOS results in lower maximum unavailable turbines than the non-chance-constrained policies, TBO and SANO. It is noteworthy that SAOMOS also results in the lowest average unavailability and the lowest average curtailed power among all policies.

Sensor-based policies, SANO and SAOMOS, leverage sensor information for making accurate remaining life predictions, and use these predictions to decide when to conduct maintenance. It is observed in all experiments that the sensor-driven policies conduct fewer maintenance actions (both preventive and corrective) compared to the time-based strategy, and at the same time result in less average unused life. It is also noteworthy that when corrective maintenance cost (Section 5.2.2), crew visit cost (Section 5.2.3), or the number of wind farm locations (Section 5.2.4) increase, the net profit decreases for all policies (mostly due to increasing maintenance costs). However, this decrease occurs at a slower rate for the sensor-based policies than for the time-based policy. This observation demonstrates the strength of sensor-driven policies in adapting the maintenance schedules to changing conditions, such as increasing maintenance costs (and therefore an increasing emphasis on failure prevention and opportunistic maintenance) or increasing number of wind farm locations.

In Cases 2 and 3 (Sections 5.2.2 and 5.2.3), we demonstrate that not only the sensor-driven chance-constrained model (SAOMOS) results in maintenance schedules with a lower average and maximum unavailability compared to its non-chance-constrained counterpart (SANO), but also the difference in average availability between the two increases with increasing corrective maintenance and crew visit costs. This outcome provides evidence that SAOMOS is better than SANO at limiting unavailability when maintenance actions are increasingly costly and therefore timely and opportunistic maintenance is increasingly important.

The proposed chance-constrained model provides a general wind farm O&M framework that leverages sensor information to optimize condition based maintenance and operations. The proposed model can be adapted to an extensive set of wind farm operations (*ranging from onshore to offshore*) and demonstrates significant advantages in terms of improving operational revenue, reducing maintenance cost, while also mitigating availability risks. The framework also unlocks a number of interesting research directions in wind farm O&M. First research direction would be to augment the proposed model with spare part logistics. A second research direction would model turbines as multi-component systems, and develop chance constraints for turbine failures and farm-level availability risks. Finally, a third research direction relates to using risk based models to incorporate limited access of the maintenance crew to different wind farm locations. This application would be particularly important for offshore wind, where maintenance crew access may be blocked due to unfavorable weather or wave conditions.

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Appendices

Appendix A. Preliminaries and Proof of Proposition 1

A.1. Preliminaries and Definitions

Recall that the non-availability random variable, χ_t^i is defined as follows:

$$\chi_t^i = \left(1-\eta_t^i\right)z_t^i + \eta_t^i\left(1-u_{\tau_i,t-1}^{\ell}\right)$$

For notational convenience, we define two random variables X_t^i and Y_t^i, to denote the unavailability of turbine i at period t due to preventive maintenance being conducted and being at a failed (and not yet maintained) state, respectively.

$$X_t^i = \left(1-\eta_t^i\right)z_t^i$$
$$Y_t^i = \eta_t^i\left(1-u_{\tau_i,t-1}^{\ell}\right)$$

Given that η_t^i denotes whether time period t is on or after the failure time of turbine i, τ_i, X_t^i and Y_t^i are Bernoulli random variables where

$$X_t^i = \begin{cases} 1, & \text{with probability } p_{it}^X(\mathbf{z}) = P\left(t < \tau_i\right) z_t^i \\ 0, & \text{with probability } 1 - p_{it}^X(\mathbf{z}) \end{cases}, \quad \text{and}$$
$$Y_t^i = \begin{cases} 1, & \text{with probability } p_{it}^Y(\mathbf{u}) = P\left(t \geq \tau_i\right)\left(1-u_{\tau^i,t-1}^{\ell}\right) \\ 0, & \text{with probability } 1 - p_{it}^Y(\mathbf{u}) \end{cases}$$

Since the failure time period of wind turbine i, τ_i, is uncertain, we obtain $p_{it}^Y(\mathbf{u})$ by conditioning on possible failure periods of turbine i:

$$p_{it}^Y(\mathbf{u}) = \sum_{s=1}^{t-1} P\left(s-1 \leq \tau_i < s\right)\left(1-u_{s,t-1}^{\ell}\right)$$

Unavailability random variable χ_t^i can be defined as $\chi_t^i = X_t^i + Y_t^i$, and then chance constraints (20) can be rewritten as follows:

$$P\left(\sum_{i\in\mathcal{G}^o}\left(X_t^i + Y_t^i\right) \geq \mathcal{N}\right) \leq \epsilon, \qquad \forall t \in \mathcal{T}$$

Lemma 1. *Random variables $e^{\alpha\sum_{i\in\mathcal{G}^o} X_t^i}$ and $e^{\alpha\sum_{i\in\mathcal{G}^o} Y_t^i}$ are negatively associated, i.e.,*

$$\mathbb{E}\left[e^{\alpha\sum_{i\in\mathcal{G}^o}\left(X_t^i+Y_t^i\right)}\right] \leq \mathbb{E}\left[e^{\alpha\sum_{i\in\mathcal{G}^o} X_t^i}\right]\mathbb{E}\left[e^{\alpha\sum_{i\in\mathcal{G}^o} Y_t^i}\right].$$

Proof. The covariance of random variables $e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}$ and $e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}$ can be computed as follows:

$$\begin{aligned}
\sigma\left(e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}, e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}\right) &= \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i} e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}\right] - \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}\right] \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}\right] \\
&= \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} \left(X_t^i + Y_t^i\right)}\right] - \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}\right] \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}\right]
\end{aligned}$$

So, proving that the covariance of $e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}$ and $e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}$ is non-positive would suffice to show the desired result.

Recall that $\chi_t^i = X_t^i + Y_t^i$. Thus,

$$\sigma\left(e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}, e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}\right) = \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} \chi_t^i}\right] - \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}\right] \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}\right] \tag{A.1}$$

Also recall that χ_t^i is a Bernoulli random variable which represents the non-availability of wind turbine i in period t.

$$\chi_t^i = \begin{cases} 1, & \text{with probability } p_{it}^X(\mathbf{z}) + p_{it}^Y(\mathbf{u}) \\ 0, & \text{with probability } 1 - \left(p_{it}^X(\mathbf{z}) + p_{it}^Y(\mathbf{u})\right) \end{cases}$$

The availability (and therefore non-availability) of a turbine at a certain time period is independent of another turbine's availability at the same time period. More specifically, $\chi_t^{i'}$ and $\chi_t^{i''}$ are independent for each $i', i'' \in \mathcal{G}^o$. Using this independence, we get:

$$\begin{aligned}
\mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} \chi_t^i}\right] &= \prod_{i \in \mathcal{G}^o} \mathbb{E}\left[e^{\alpha \chi_t^i}\right] = \prod_{i \in \mathcal{G}^o} \left(P\left(\chi_t^i = 1\right) e^{\alpha} + \left(1 - P\left(\chi_t^i = 1\right)\right)\right) \\
&= \prod_{i \in \mathcal{G}^o} \left(\left(p_{it}^X(\mathbf{z}) + p_{it}^Y(\mathbf{u})\right) e^{\alpha} + \left(1 - p_{it}^X(\mathbf{z}) - p_{it}^Y(\mathbf{u})\right)\right) \\
&= \prod_{i \in \mathcal{G}^o} \left(\underbrace{p_{it}^X(\mathbf{z})\left(e^{\alpha} - 1\right)}_{\beta_{it}^X} + \underbrace{p_{it}^Y(\mathbf{u})\left(e^{\alpha} - 1\right)}_{\beta_{it}^Y} + 1\right) \\
&= \prod_{i \in \mathcal{G}^o} \left(\beta_{it}^X + \beta_{it}^Y + 1\right)
\end{aligned} \tag{A.2}$$

Note that for any $\alpha > 0$, β_{it}^X and β_{it}^Y are non-negative. So, $\beta_{it}^X + \beta_{it}^Y + 1 \geq 1$.

Similarly to χ_t^i, X_t^i for each turbine $i \in \mathcal{G}^o$ and Y_t^i for each turbine $i \in \mathcal{G}^o$ are also independent. Thus, we have:

$$\begin{aligned}
\mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}\right] \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}\right] &= \prod_{i \in \mathcal{G}^o} \mathbb{E}\left[e^{\alpha X_t^i}\right] \mathbb{E}\left[e^{\alpha Y_t^i}\right] \\
&= \prod_{i \in \mathcal{G}^o} \left(p_{it}^X(\mathbf{z}) e^{\alpha} + \left(1 - p_{it}^X(\mathbf{z})\right)\right) \left(p_{it}^Y(\mathbf{u}) e^{\alpha} + \left(1 - p_{it}^Y(\mathbf{u})\right)\right) \\
&= \prod_{i \in \mathcal{G}^o} \left(p_{it}^X(\mathbf{z})\left(e^{\alpha} - 1\right) + 1\right) \left(p_{it}^Y(\mathbf{u})\left(e^{\alpha} - 1\right) + 1\right) \\
&= \prod_{i \in \mathcal{G}^o} \left(\beta_{it}^X + 1\right) \left(\beta_{it}^Y + 1\right)
\end{aligned}$$

$$= \prod_{i \in \mathcal{G}^o} \left(\beta_{it}^X + \beta_{it}^Y + \beta_{it}^X \beta_{it}^Y + 1\right) \tag{A.3}$$

By substituting (A.2) and (A.3) in the covariance formula (A.1) we obtain:

$$\sigma\left(e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}, e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}\right) = \prod_{i \in \mathcal{G}^o} \left(\underbrace{\beta_{it}^X + \beta_{it}^Y + 1}_{\geq 1}\right) - \prod_{i \in \mathcal{G}^o} \left(\underbrace{\beta_{it}^X + \beta_{it}^Y + \beta_{it}^X \beta_{it}^Y + 1}_{\geq 1}\right)$$

Since all β_{it}^X and β_{it}^Y values are non-negative, and therefore $\beta_{it}^X + \beta_{it}^Y + 1 \geq 1$ and $\beta_{it}^X + \beta_{it}^Y + \beta_{it}^X \beta_{it}^Y + 1 \geq 1$, $\beta_{it}^X + \beta_{it}^Y + 1 \leq \beta_{it}^X + \beta_{it}^Y + \beta_{it}^X \beta_{it}^Y + 1$ holds for all $i \in \mathcal{G}$ and $t \in \mathcal{T}$. Thus,

$$\sigma\left(e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}, e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}\right) = \prod_{i \in \mathcal{G}^o} \left(\beta_{it}^X + \beta_{it}^Y + 1\right) - \prod_{i \in \mathcal{G}^o} \left(\beta_{it}^X + \beta_{it}^Y + \beta_{it}^X \beta_{it}^Y + 1\right) \leq 0$$

□

A.2. Proof of Proposition 1

First, using Markov's inequality, we have:

$$P\left(\sum_{i \in \mathcal{G}^o} \chi_t^i \geq \mathcal{N}\right) \leq \frac{\mathbb{E}\left[\sum_{i \in \mathcal{G}^o} \chi_t^i\right]}{\mathcal{N}}, \qquad \forall t \in \mathcal{T}$$

Thus, whenever (A.4) holds, the chance constraints (20) will also hold. So, (A.4) provides a safe approximation to chance constraints (20).

$$\mathbb{E}\left[\sum_{i \in \mathcal{G}^o} \chi_t^i\right] \leq \mathcal{N}\epsilon, \qquad \forall t \in \mathcal{T} \tag{A.4}$$

In what follows, we obtain the Chernoff bound for the random variable $\sum_{i \in \mathcal{G}^o} \chi_t^i$ by applying Markov's inequality to the random variable $e^{\alpha \sum_{i \in \mathcal{G}^o} \chi_t^i}$.

For any $\alpha > 0$ we have (by Markov's inequality):

$$P\left(\sum_{i \in \mathcal{G}^o} \left(X_t^i + Y_t^i\right) \geq \mathcal{N}\right) = P\left(e^{\alpha \sum_{i \in \mathcal{G}^o} \left(X_t^i + Y_t^i\right)} \geq e^{\alpha \mathcal{N}}\right) \leq e^{-\alpha \mathcal{N}} \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} \left(X_t^i + Y_t^i\right)}\right] \tag{$\star$}$$

By Lemma 1, $e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}$ and $e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}$ have a negative association. Given this, and the observation that X_t^i and Y_t^i random variables are independent for all $i \in \mathcal{G}^o$, an upper bound on expression ($\star$) can be computed as follows:

$$\begin{aligned} (\star) &\leq e^{-\alpha \mathcal{N}} \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} X_t^i}\right] \mathbb{E}\left[e^{\alpha \sum_{i \in \mathcal{G}^o} Y_t^i}\right] \\ &= e^{-\alpha \mathcal{N}} \prod_{i \in \mathcal{G}^o} \mathbb{E}\left[e^{\alpha X_t^i}\right] \mathbb{E}\left[e^{\alpha Y_t^i}\right] \end{aligned}$$

$$= e^{-\alpha\mathcal{N}} \prod_{i\in\mathcal{G}^o} \left(p_{it}^X(\mathbf{z})e^{\alpha} + \left(1 - p_{it}^X(\mathbf{z})\right)\right)\left(p_{it}^Y(\mathbf{u})e^{\alpha} + \left(1 - p_{it}^Y(\mathbf{u})\right)\right) \tag{$\star\star$}$$

Then, by using the geometric-arithmetic means inequality and defining $\mathbf{p}_t^X(\mathbf{z}) = \frac{1}{|\mathcal{G}^o|}\sum_{i\in\mathcal{G}^o} p_{it}^X(\mathbf{z})$ and $\mathbf{p}_t^Y(\mathbf{u}) = \frac{1}{|\mathcal{G}^o|}\sum_{i\in\mathcal{G}^o} p_{it}^Y(\mathbf{u})$, we have:

$$\begin{aligned}(\star\star)\ &\leq e^{-\alpha\mathcal{N}} \left(\mathbf{p}_t^X(\mathbf{z})e^{\alpha} + \left(1 - \mathbf{p}_t^X(\mathbf{z})\right)\right)^{|\mathcal{G}^o|}\left(\mathbf{p}_t^Y(\mathbf{u})e^{\alpha} + \left(1 - \mathbf{p}_t^Y(\mathbf{u})\right)\right)^{|\mathcal{G}^o|} \\ &= e^{-\alpha\mathcal{N}} \left(\mathbf{p}_t^X(\mathbf{z})\left(e^{\alpha} - 1\right) + 1\right)^{|\mathcal{G}^o|}\left(\mathbf{p}_t^Y(\mathbf{u})\left(e^{\alpha} - 1\right) + 1\right)^{|\mathcal{G}^o|} \qquad (\star\star\star)\end{aligned}$$

For any $\alpha > 0$, we have $e^{\alpha} - 1 > 0$. Then, using the geometric-arithmetic means inequality again, we obtain:

$$(\star\star\star)\ \leq e^{-\alpha\mathcal{N}} \left(\frac{\mathbf{p}_t^X(\mathbf{z})\left(e^{\alpha} - 1\right) + \mathbf{p}_t^Y(\mathbf{u})\left(e^{\alpha} - 1\right) + 2}{2}\right)^{2|\mathcal{G}^o|}$$

By upper bounding the final expression by ϵ, we get:

$$\begin{aligned}&e^{-\alpha\mathcal{N}} \left(\frac{\mathbf{p}_t^X(\mathbf{z})\left(e^{\alpha} - 1\right) + \mathbf{p}_t^Y(\mathbf{u})\left(e^{\alpha} - 1\right) + 2}{2}\right)^{2|\mathcal{G}^o|} \leq \epsilon \\ &\Longrightarrow \mathbf{p}_t^X(\mathbf{z})\left(e^{\alpha} - 1\right) + \mathbf{p}_t^Y(\mathbf{u})\left(e^{\alpha} - 1\right) \leq 2\left(\epsilon e^{\alpha\mathcal{N}}\right)^{\frac{1}{2|\mathcal{G}^o|}} - 2 \\ &\Longrightarrow \mathbf{p}_t^X(\mathbf{z}) + \mathbf{p}_t^Y(\mathbf{u}) \leq \frac{2\left(\epsilon e^{\alpha\mathcal{N}}\right)^{\frac{1}{2|\mathcal{G}^o|}} - 2}{e^{\alpha} - 1}\end{aligned}$$

Substituting $\mathbf{p}_t^X(\mathbf{z})$ and $\mathbf{p}_t^Y(\mathbf{u})$, we get:

$$\sum_{i\in\mathcal{G}^o} p_{it}^X(\mathbf{z}) + \sum_{i\in\mathcal{G}^o} p_{it}^Y(\mathbf{u}) \leq \frac{2|\mathcal{G}^o|\left(\left(\epsilon e^{\alpha\mathcal{N}}\right)^{\frac{1}{2|\mathcal{G}^o|}} - 1\right)}{e^{\alpha} - 1} \tag{A.5}$$

Since $\mathbb{E}\left[\sum_{i\in\mathcal{G}^o} X_t^i\right] = \sum_{i\in\mathcal{G}^o} p_{it}^X(\mathbf{z})$ and $\mathbb{E}\left[\sum_{i\in\mathcal{G}^o} Y_t^i\right] = \sum_{i\in\mathcal{G}^o} p_{it}^Y(\mathbf{u})$, and therefore $\mathbb{E}\left[\sum_{i\in\mathcal{G}^o} \chi_t^i\right] = \mathbb{E}\left[\sum_{i\in\mathcal{G}^o} X_t^i\right] + \mathbb{E}\left[\sum_{i\in\mathcal{G}^o} Y_t^i\right] = \sum_{i\in\mathcal{G}^o} p_{it}^X(\mathbf{z}) + \sum_{i\in\mathcal{G}^o} p_{it}^Y(\mathbf{u})$, (A.5) is equivalent to

$$\mathbb{E}\left[\sum_{i\in\mathcal{G}^o} \chi_t^i\right] \leq \frac{2|\mathcal{G}^o|\left(\left(\epsilon e^{\alpha\mathcal{N}}\right)^{\frac{1}{2|\mathcal{G}^o|}} - 1\right)}{e^{\alpha} - 1}$$

for any $\alpha > 0$. To achieve the least conservative safe approximation, we select the α value that maximizes the right hand side of the constraint:

$$\mathbb{E}\left[\sum_{i\in\mathcal{G}^o} \chi_t^i\right] \leq \max_{\alpha>0} \frac{2|\mathcal{G}^o|\left(\left(\epsilon e^{\alpha\mathcal{N}}\right)^{\frac{1}{2|\mathcal{G}^o|}} - 1\right)}{e^{\alpha} - 1} \tag{A.6}$$

By combining the two safe approximations derived in (A.4) and (A.6) we obtain the desired result:

$$\mathbb{E}\left[\sum_{i\in\mathcal{G}^o}\chi_t^i\right] \leq \max\left\{\mathcal{N}\epsilon,\ \ \max_{\alpha>0}\frac{2|\mathcal{G}^o|\left(\left(\epsilon e^{\alpha\mathcal{N}}\right)^{\frac{1}{2|\mathcal{G}^o|}}-1\right)}{e^{\alpha}-1}\right\}$$

$\square$

Appendix B. Linearization of Nonlinear Constraints

The nonlinear part of constraints (18) can be linearized by introducing auxiliary binary variables $\delta^i_{t,\omega}$ for each operational turbine $i \in \mathcal{G}^o$ at every time period $t \in \mathcal{T}$ under each scenario $\omega \in \Omega$. This variable takes the value 1 if turbine i fails before its scheduled preventive maintenance under scenario ω, but experiences on-the-spot corrective maintenance by a maintenance crew visiting its wind farm location before time period t. This relationship is established by constraints (B.7) and (B.8).

$$\delta^i_{t,\omega} \leq u^{\ell}_{\tau^i_\omega, t-1}, \qquad \forall \ell \in \mathcal{L}, i \in \mathcal{G}^o_\ell, t \in \mathcal{T}, \omega \in \Omega \tag{B.7}$$

$$\delta^i_{t,\omega} \leq \sum_{s=\tau^i_\omega}^{T} z^i_s, \qquad \forall \ell \in \mathcal{L}, i \in \mathcal{G}^o_\ell, t \in \mathcal{T}, \omega \in \Omega \tag{B.8}$$

The limit on the number of on-the-spot corrective maintenance actions defined by the variables is enforced at every time period by constraints (B.9).

$$\sum_{\ell \in \mathcal{L}} \sum_{\substack{i \in \mathcal{G}^o_\ell: \\ \tau^i_\omega < t}} \delta^i_{t,\omega} \leq M^o_t, \qquad \forall t \in \mathcal{T}, \omega \in \Omega \tag{B.9}$$

Nonlinear constraints (18) are replaced with linear constraints (B.7)–(B.9) in order to solve SAOMOS as a Mixed-Integer Linear Programming (MILP) model.

The nonlinearity in the case III of constraints (16) is also addressed with the auxiliary binary variables $\delta^i_{t,\omega}$. Recall that the value of these variables are enforced by linear constraints (B.7) and (B.8). Thus, the linearization of constraints (16) becomes:

$$\zeta^i_{t,\omega} = \begin{cases} 1 - z^i_t & \text{if } t < \tau^i_\omega \\ \sum_{t'=1}^{t-1} z^i_{t'} & \text{if } t = \tau^i_\omega \\ \delta^i_{t,\omega} + \sum_{s=1}^{\tau^i_\omega - 1} z^i_s & \text{if } t > \tau^i_\omega \end{cases}, \qquad \forall \ell \in \mathcal{L}, i \in \mathcal{G}^o_\ell, t \in \mathcal{T}, \omega \in \Omega \tag{B.10}$$

Nonlinear constraints (16) are replaced with linear constraints (B.10) in order to solve SAOMOS as an MILP model.