

Probing the critical nucleus size in the metal-insulator phase transition of VO₂

Lei Jin,^{1,2} Yin Shi,³ Frances I. Allen,^{1,4} Long-Qing Chen,³ Junqiao Wu^{1,2,*}

¹*Department of Materials Science and Engineering, University of California, Berkeley, California 94720, USA*

²*Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

³*Department of Materials Science and Engineering, Pennsylvania State University, University Park, PA 16802, USA.*

⁴*National Center for Electron Microscopy, Molecular Foundry, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

*wuj@berkeley.edu

In a first-order phase transition, critical nucleus size governs nucleation kinetics, but the direct experimental test of the theory and determination of the critical nucleation size have been achieved only recently in the case of ice formation in supercooled water. The widely known metal-insulator phase transition (MIT) in strongly correlated VO₂ is a first-order electronic phase transition coupled with a solid-solid structural transformation. It is unclear whether classical nucleation theory applies in such a complex case. In this work, we directly measure the critical nucleus size of the MIT by introducing size-controlled nanoscale nucleation seeds with focused ion irradiation at the surface of a deeply supercooled metal phase of VO₂. The results compare favorably with classical nucleation theory and are further explained by phase-field modeling. This work validates the application of classical nucleation theory as a parametrizable model to describe phase transitions of strongly correlated electron materials.

Nucleation is the initial step in the formation of a new thermodynamic phase in a supersaturated parent phase (1, 2). For nearly a century, it has been assumed that a successful nucleation event requires the size of the nucleus to exceed a critical nucleation size in order to stabilize the new phase, a hypothesis that constitutes the basis of classical nucleation theory (CNT) (3, 4). Nevertheless, because of the nanoscale size and transient nature of the nucleation process, as well as the difficulty to achieve supercooled matter at ultrahigh purities, direct experimental demonstration of CNT has not been reported until very recently by Bai *et al.* (5). In that work, it was shown that graphene oxide nanosheets serve as seeds to trigger ice nucleation in supercooled water droplets, but they do so only if their diameters are above certain critical sizes. The existence of these critical nucleation sizes is well expected and understood for first-order transitions in conventional condensed matter (e.g., the water-ice transition).

In contrast, the metal-insulator transition (MIT) of vanadium dioxide (VO₂) is fundamentally different. VO₂ switches from an insulating (I) phase at temperatures (T) lower than $T_{\text{MIT}} = 67^\circ\text{C}$ to a metallic (M) phase at temperatures higher than T_{MIT} . The MIT is accompanied by a structural transformation from a monoclinic structure at $T < T_{\text{MIT}}$ to a tetragonal structure at $T > T_{\text{MIT}}$. The nature of the MIT in VO₂ is still under debate and the nucleation process of the MIT is poorly understood (6-10), with unanswered questions such as whether the MIT and the structural transformation are decoupled at the nucleation stage and follow different kinetic pathways afterwards. Inspired by the experimental demonstration of the applicability of CNT to water (5), in this work we experimentally tested CNT and probed the critical nucleation size of the I-phase nucleus in a supercooled M phase of VO₂. The experimental results are quantitatively explained by phase-field modeling of the energetics of the process.

To probe the critical nucleation size, one must tackle the first challenge that the system needs to be in a deeply supersaturated state to have a sufficiently high thermodynamic driving force for the phase transition. We achieve this in single-crystal VO₂ nanobeams (thickness $\sim 100\text{nm}$, width $\sim 250\text{ nm}$, length $\sim 50\text{ }\mu\text{m}$) by using “irradiation shielding” via helium ion (He⁺) irradiation. Achieving supercooled VO₂ has been reported previously by applying substrate strain (11) or using graded tungsten doping (12). However, the supercooled VO₂ obtained by substrate strain or graded W-doping exhibits a MIT deviating from the intrinsic behavior of VO₂. In contrast, irradiation shielding can be used to achieve deeply supercooled VO₂ specimens that show a “clean”, intrinsic MIT, as demonstrated below. In this approach, we employ the established MIT engineering technique using He⁺ irradiation, where the T_{MIT} of VO₂ is controllably reduced by He⁺ irradiation using a Zeiss Orion NanoFab He ion microscope (HIM) (13, 14). In the irradiation process, most of the energetic ($\sim 30\text{keV}$) He⁺ ions penetrate through the entire thickness of the VO₂ nanobeams, leaving behind uniformly distributed point defects (vacancies and interstitials). The overall effect of these defects is to donate free electrons, which reduces T_{MIT} and increases the electrical conductivity (σ) of I-phase VO₂ (13, 15-18). This effect of reducing T_{MIT} with He⁺ irradiation is shown in Fig. 1a. The T_{MIT} of the nanobeam can be tuned from the natural T_{MIT} ($\sim 341\text{K}$) to near room temperature (300K) by increasing the He⁺ irradiation dose up to 10^{16} ions/cm^2 . The irradiated VO₂ still maintains a high-quality MIT as manifested by the sharp jump in σ , and is fully crystalline in this dose range (13).

Taking advantage of the localized irradiation capability of the HIM, we carry out patterned irradiation along the VO₂ nanobeam. The schematic in Fig. 1b depicts the idea of irradiation shielding. A pristine, unirradiated VO₂ segment (grey block) is sandwiched by two He⁺-irradiated segments (green blocks) along a single nanobeam. The platinum (Pt) bonding (gold

blocks), deposited using a gallium focused ion beam (FIB) with a platinum-based organometallic precursor, secures electrical and thermal contact between the nanobeam and the underlying electrodes. The nanobeam is suspended from the substrate. In the M to I phase transition during cooling, the He^+ -irradiated segments stay in the M phase until T is lower than their reduced T_{MIT} , following Fig.1a. Importantly, during this entire process, the central pristine segment also stays in the M phase, because it is shielded by the two neighboring, M-phase segments from sites that could nucleate the I phase in it, such as the substrate and the Pt contacts. It is obvious that the lowest temperature that the shielded VO_2 can be supercooled to (T_{sc}) is determined by the reduced T_{MIT} of the shields. For comparison, a similar VO_2 nanobeam without irradiation shielding is also shown schematically in Fig.1b. Figure 1c shows the measured $\sigma(T)$ of two VO_2 nanobeams, with and without irradiation shielding, respectively. The pristine VO_2 nanobeam without irradiation shielding shows a normal MIT at ~ 341 K with a small hysteresis of ~ 8 K. The $T_{\text{MIT}} = 333$ K observed upon cooling the unshielded nanobeam is named T_{td} , as it is the thermodynamically expected T_{MIT} during cooling. In contrast, σ of the shielded VO_2 device stays metallic during cooling until 308 K, showing a deep supercooling of 25 K below the T_{td} .

All VO_2 nanobeams were grown using the vapor transport method published previously (19). They are all single-crystalline with rectangular cross section and smooth surface (Fig.1e), maximally eliminating defects such as grain boundaries and domain walls that could nucleate the MIT. Therefore, such a shielded nanobeam, when suspended, provides an ideal supercooled platform for probing the critical nucleation size during the MIT. As shown in Fig.1d, two suspended micropads were used to support a nanobeam for electrical and optical measurements at variable temperatures (20). The micropads were suspended from the substrate with long, flexible arms to avoid strain accumulation along the nanobeam.

We use the visual color observed under an optical microscope to differentiate the M (dark) and I (bright) phases in the suspended VO_2 nanobeams. Figure 1f shows optical images of one such nanobeam during the cooling process. A 30- μm -long pristine segment is shielded by two irradiated segments (indicated by white boxes). The two irradiated segments (shields) switch to I-phase once the temperature drops below 308 K, consistent with Fig.1a. The two segments between each of the shields and the neighboring Pt contact are pristine, and switch to I-phase at 333 K, consistent with the unshielded, pristine MIT as shown in Fig.1c. In stark contrast, the VO_2 segment between the shields, although also pristine, remains in M-phase when T decreases passing T_{td} (333 K) until T_{sc} (308 K). Within this temperature window, this shielded VO_2 segment is the ideal supercooled platform where the following experiments are carried out.

The next step is to introduce nucleation seeds with controlled sizes into the supercooled VO_2 segment. As shown in Fig.2a, such seeds are created by irradiating the segment with focused Ga^+ ions (FEI Quanta FIB-SEM instrument). Ga^+ irradiation is typically used to mill surfaces or cut through samples, as Ga^+ ions are heavy and when energetic, can generate severe lattice damage (21), unlike the much lighter He^+ ions. In this work, the Ga^+ dose is limited to below 10^{16} ions/ cm^2 to avoid noticeable surface milling ($< a$ few nanometers as measured by an atomic force microscope). Special care was taken to focus the Ga^+ beam only into the targeted area. The penetration depth of 30 keV Ga^+ is simulated to be 15 nm in VO_2 using the Stopping and Range of Ions in Matter (SRIM) program (22), much smaller than the thickness of the VO_2 nanobeam. The disk-shaped nucleation seeds as defined by the Ga^+ irradiated zone can be controlled by their diameter (D) and the irradiation dose (n_{Ga^+}). The former is varied to probe the critical nucleation

sizes, while the latter varies the surface energy to tune the capability of the seed to nucleate the I-phase in supercooled, M-phase VO₂.

As shown in Fig.2b, a 50-μm long single VO₂ nanobeam is patterned with eight periods of segments, where each segment consists of a 3-μm-long pristine zone as the supercooled “testbed”, as well as a 3-μm-long, He⁺-irradiated zone as the shield. Each shielded testbed is then implanted with one nucleation seed with specific values of D and n_{Ga^+} . One testbed is free of seed ($n_{\text{Ga}^+}=0$) to allow the measurement of T_{sc} , and another small segment outside the shields is also free of seed for measuring T_{td} , along the same nanobeam. During cooling, the temperature at which each supercooled testbed switches from M to I phase is named the nucleation temperature (T_{nuc}). By varying D and n_{Ga^+} , we measure the corresponding T_{nuc} for each supercooled testbed. If T_{nuc} is between T_{td} and T_{sc} , the I-phase nucleation in that testbed is triggered by the nucleation seed. Figure 2b shows the color change of each segment when the temperature gradually decreases from T_{td} (333 K) to T_{sc} (308 K). In this experiment, n_{Ga^+} is fixed at 2.2×10^{16} ions/cm² and D varies from 10 to 180 nm for the eight testbeds. For testbeds with D of 140 and 180 nm, the I phase was observed immediately at T_{td} . This indicates that the nucleation seed is large enough to fully suppress the supercooling. At smaller D values, for example, 100 nm, the I phase nucleates at $T_{\text{nuc}} = 317$ K, lower than T_{td} but higher than T_{sc} . When D is smaller than ~ 75nm, T_{nuc} becomes equal to T_{sc} , indicating that such nucleation seeds are too small to trigger the MIT.

We use classical nucleation theory to analyze the critical nucleation diameter ($2r_c$) as a function of T_{nuc} . According to CNT, the change in the Gibbs free energy ΔG for forming a new I phase in a supercooled M phase is expressed as (4)

$$\Delta G = \frac{2}{3}\pi r^3 \cdot \Delta g_v + 2\pi r^2 \cdot \beta + \pi r^2 \cdot \gamma, \quad [1]$$

where r is the radius of the I phase nucleus, Δg_v is the difference in volumetric Gibbs free energy, β is the interface energy between the I and M phases, and γ is the interface energy between the I phase and the nucleation seed. For simplicity, the I phase nucleus is assumed to be hemispherical, growing from the disk-shaped nucleation seed as shown in Fig.2a. From $d\Delta G/dr = 0$, the critical radius is found to be

$$r_c = \frac{2(\beta+\gamma/2)}{|\Delta g_v|}. \quad [2]$$

In addition (4),

$$\Delta g_v = \frac{\Delta h_f(T_{\text{td}}-T_{\text{nuc}})}{T_{\text{td}}}, \quad [3]$$

where Δh_f is the volumetric enthalpy of nucleus formation. Subsequently,

$$r_c = \left| \frac{2(\beta+\gamma/2)}{\Delta h_f} \right| \left(\frac{1}{1-T_{\text{nuc}}/T_{\text{td}}} \right), \quad [4]$$

and

$$T_{\text{nuc}} = T_{\text{td}} \cdot \left[1 - \left| \frac{2(\beta+\gamma/2)}{\Delta h_f} \right| \left(\frac{1}{r_c} \right) \right]. \quad [5]$$

The dependence of T_{nuc} on r_c is shown schematically in Fig.2c. T_{nuc} is bounded by the thermodynamic limit T_{td} and the supercooling limit T_{sc} . At these two limits, T_{nuc} no longer

depends on D . Between these two limits, the I-phase nucleation is triggered by the seed at T_{nuc} when D is comparable to the critical nucleation size $2r_c$. The I-phase is more likely to nucleate at seeds with larger D and higher n_{Ga^+} . Higher lattice disorder in the nucleation seed arising from higher n_{Ga^+} reduces the energy barrier to trigger the nucleation. This is depicted as a shift of the $T_{\text{nuc}}(D)$ curve toward smaller D with reduced γ , as shown in Fig.2c.

T_{nuc} is experimentally measured by independently varying both n_{Ga^+} (from 1.1 to 8.8×10^{16} ions/cm²) and D (from 5 to 260 nm). Figure 2d plots T_{nuc} as a function of n_{Ga^+} for fixed ranges of D . A monotonic increase in T_{nuc} is observed with n_{Ga^+} , suggesting promoted nucleation with higher n_{Ga^+} . T_{nuc} is saturated at T_{td} for D larger than 120 nm and n_{Ga^+} higher than 3.3×10^{16} ions/cm². This is the condition of full suppression of the supercooling. If D is smaller than 20 nm or n_{Ga^+} is lower than 1.1×10^{16} ions/cm², T_{nuc} saturates at T_{sc} , and no I-phase nucleus can be stabilized at the implanted seeds. Figure 2e plots T_{nuc} as a function of D for fixed n_{Ga^+} , which shows good agreement with the dependence expected from CNT (Fig.2c). The shift of curves with higher n_{Ga^+} to smaller D indicates that the increase in n_{Ga^+} reduces the effective interface energy, presumably via a reduction in γ .

To quantitatively analyze the data, we use a previously developed phase-field model of VO₂ (23-26) to calculate the nucleation temperatures of the I phase in free-standing VO₂ nanobeams. This model describes the mesoscopic properties of VO₂ in terms of a structural order parameter field η (characterizing the lattice structural phases), an electronic order parameter field ψ (characterizing the insulating or metallic phases), the free carrier densities, and the elastic strain field. We simplify the model by setting the free carrier densities to be at equilibrium and the elastic energy to zero, because we are only concerned with equilibrium states and the strain in the free-standing nanobeams is fully relaxed. We then set up a VO₂ cube with a side length of 300 nm with a stress-free boundary condition imitating part of a free-standing VO₂ nanobeam. The initial state is set to that of a hemispherical I-phase nucleus with a given diameter embedded into the surface of the M-phase cube. We then tune the temperature and observe whether the nucleus grows or shrinks to find the nucleation temperature. We take into account of the interface energy between the I phase and the Ga⁺-irradiated VO₂, γ , by renormalizing the gradient energy coefficient (characterizing the domain wall energy) in the phase-field model. γ is related to the renormalized domain wall energy β_t and the true I-M domain wall energy β via $\gamma = 2(\beta_t - \beta)$, which simply results from the approximation that the shape of the I-phase nucleus is close to a hemisphere. We fit the calculated nucleation temperature as a function of the nucleus diameter to the experimentally measured relation by adjusting the renormalized gradient energy coefficient. The calculated results are shown as open symbols in Fig.2d. The yielded γ 's are 1.39 , 0.606 , and 0.282 J/m² for the Ga⁺ irradiation doses of 1.1 , 2.2 , and 4.4×10^{16} ions/cm², respectively. The good quantitative agreement between the measured and calculated results strongly supports that the CNT well describes the MIT in VO₂.

Furthermore, based on equation [4] or [5], the total interface energy can be expressed as

$$\beta + \gamma/2 = \frac{|\Delta h_f|}{2T_{\text{td}}} r_c (T_{\text{td}} - T_{\text{nuc}}). \quad [6]$$

Between the thermodynamic and supercooling limits, $D \approx 2r_c$. Given that Δh_f and T_{td} are constant, it is clear that $D\Delta T \propto \beta + \gamma/2$, where $\Delta T = T_{\text{td}} - T_{\text{nuc}}$. In Fig.3, the measured $D\Delta T$ is plotted as a function of $1/n_{\text{Ga}^+}$, where colored symbols represent data with I-phase nucleation occurring between (*i.e.*, not reaching) the thermodynamic and supercooling limits. These colored

data points can be fitted with a linear dependence on $1/n_{\text{Ga}^+}$. Such a dependence shows an empirical relationship between the Ga^+ irradiation dose (n_{Ga^+}) and the lowering of the interface energy γ .

In conclusion, we show that classical nucleation theory governs the kinetic nucleation process in the coupled structural-electronic phase transition in a strongly correlated electron material. The critical nucleation size in the transition is determined experimentally, and found to be as small as tens of nm depending on the interface energy of the heterogeneous nucleus. A deeply supercooled VO_2 testbed is created by shielding it from other nucleation sites using He^+ ion irradiation. A nucleation seed is introduced to the testbed by surface irradiation with energetic Ga^+ ions. The achieved deep supercooling state in the pristine, single-crystal and strain-free VO_2 may also serve as a clean platform for probing the intrinsic properties of its metal-insulator transition.

Acknowledgements: This work was supported by U.S. NSF Grant No. ECCS-1953803. The phase-field simulations done by Yin Shi and Long-Qing Chen was supported as part of the Computational Materials Sciences Program funded by the U.S. Department of Energy, Office of Science, Basic Energy Sciences, under Award No. DE-SC0020145. The helium ion irradiation was performed at the QB3 Biomolecular Nanotechnology Center at the University of California, Berkeley.

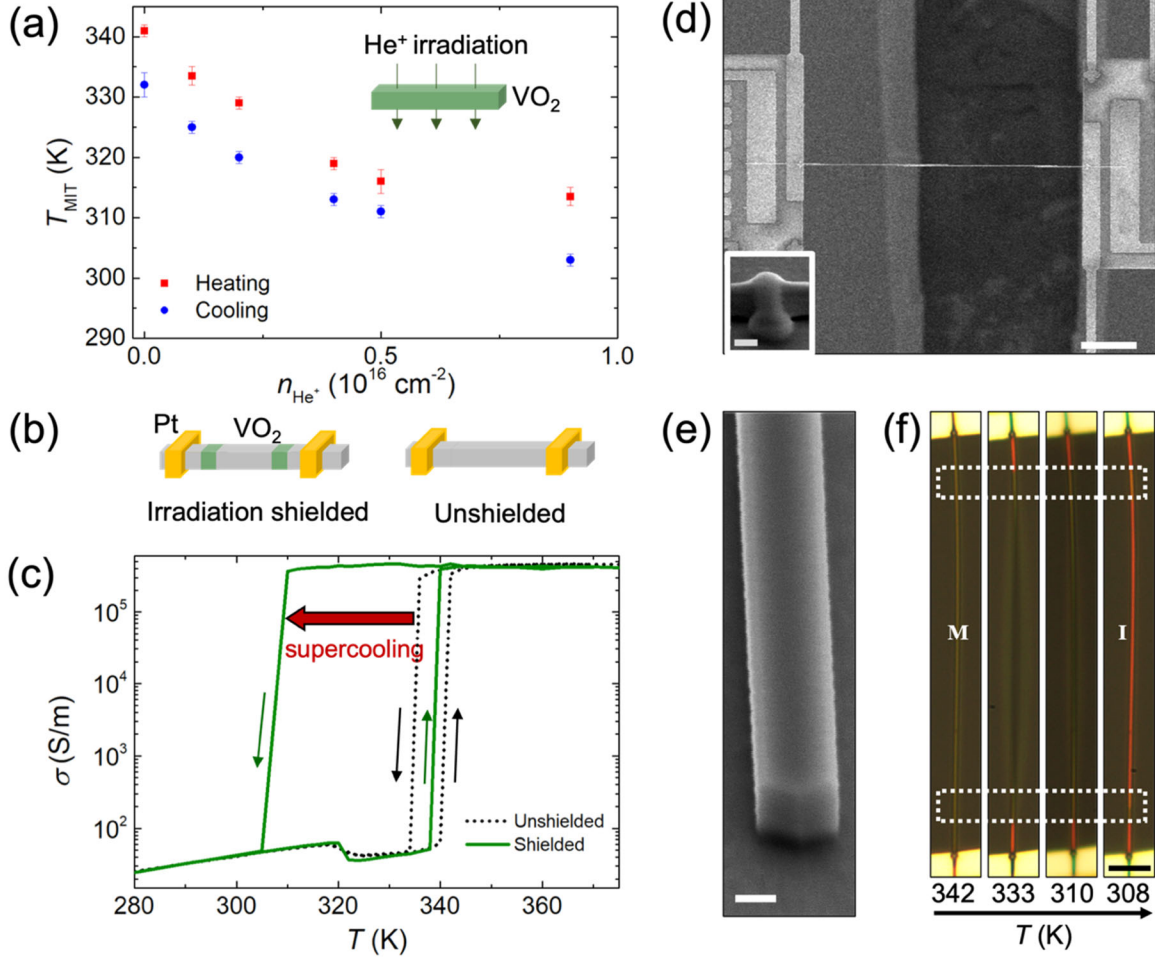


Figure 1: Creating a deep supercooled state in the MIT of VO₂. (a) The MIT temperature (T_{MIT}) of VO₂ nanobeams as a function of the dose of He⁺ ion irradiation. T_{MIT} is determined from electrical transport measurements. The inset shows schematically a VO₂ nanobeam uniformly and globally (*i.e.*, not locally) irradiated with He⁺ ions. (b) Schematics of He⁺ ion irradiation for nucleation shielding in a single VO₂ nanobeam. The middle, pristine VO₂ segment (grey) is shielded against influence from the Pt contacts (yellow blocks) by two end segments of VO₂ that are locally He⁺-irradiated (green). (c) Four-probe measured electrical conductivity of the same VO₂ nanobeams with or without irradiation shielding. Deep supercooling of the M phase down to 300 K is observed in the VO₂ end-shielded with 5×10^{15} ions/cm² He⁺ irradiation. (d) Scanning electron microscope (SEM) image of a VO₂ nanobeam supported by two suspended micropads. Scale bar is 10 μm . (Insert) SEM image of FIB-deposited Pt bonding of the nanobeam onto the underlying electrode to minimize electrical/thermal contact resistance. Scale bar is 100 nm. (e) SEM image of a VO₂ nanobeam showing a rectangular cross-section. Scale bar is 100 nm. (f) Optical images of a deeply supercooled, shielded VO₂ nanobeam as the temperature drops. A 30 μm -long VO₂ segment is shielded by two 4 μm -long He⁺-irradiated segments at the two ends (indicated by the two white dashed boxes). Despite the fact that the pristine segments between the shields and the electrodes transition to I phase at normal T_{MIT} (333K), the shielded, pristine VO₂ segment stays in the M phase (dark) and does not transition to the I phase (bright) until 308 K, which is close to the natural T_{MIT} ((a)) of the irradiated shield segments. Scale bar is 5 μm .

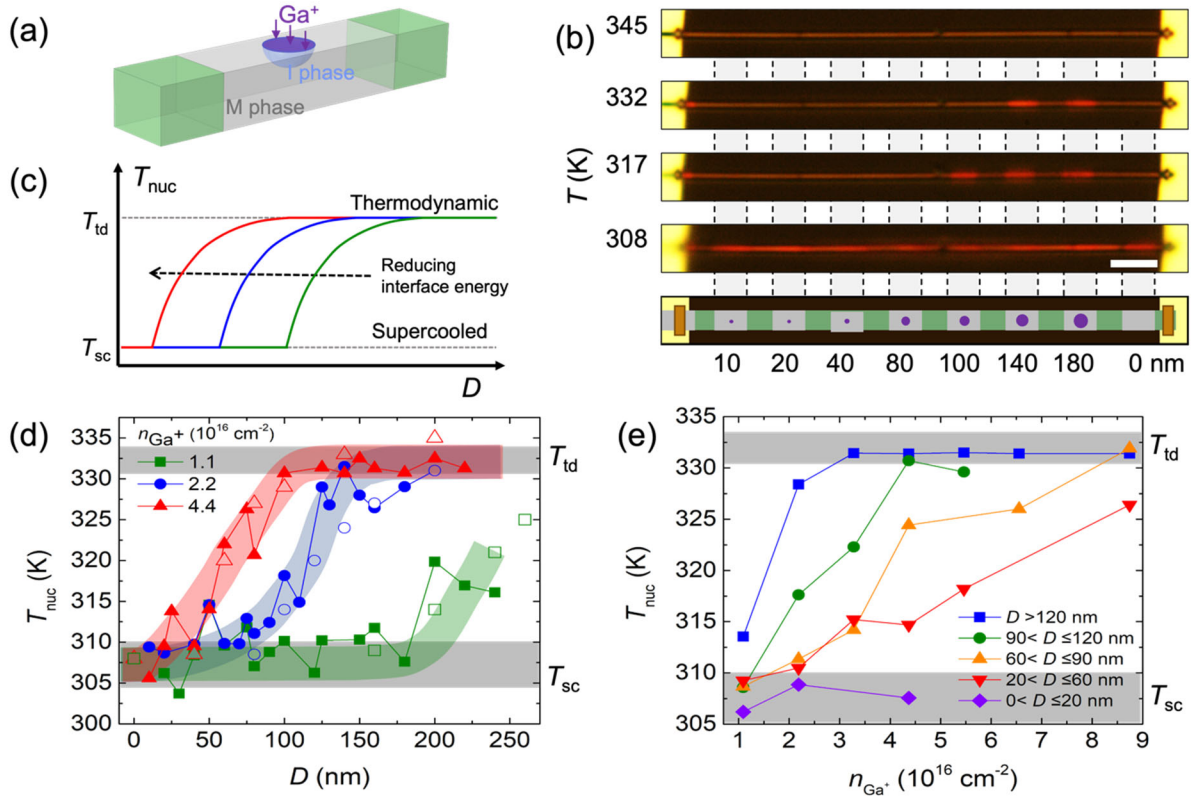


Figure 2: I-phase nucleation in supercooled VO₂. (a) Schematic showing a disk-shaped nucleation seed (purple) created by Ga⁺ ion irradiation on the surface of a shielded pristine VO₂ segment. The Ga⁺ ion penetration depth is $\sim 15 \text{ nm}$, very shallow compared to the diameter of the irradiated area, hence the nucleation can be approximated as a disk shape with zero thickness. (b) Optical image of the I-phase nucleation process along a long, suspended VO₂ nanobeam. The nanobeam is divided into eight pristine segments, each shielded by two He⁺-irradiated segments; then a nucleation seed is introduced onto the surface of each of the pristine segments by Ga⁺ irradiation at a dose of $2.2 \times 10^{16} \text{ ions/cm}^2$, but with different diameters. The lowest panel shows schematically the case where the diameter of the disk-shaped nucleation seed (D) increases from 10 to 180 nm. Scale bar is 5 μm . (c) Schematic dependence of I-phase nucleation temperature (T_{nuc}) on D , as predicted by classical nucleation theory. T_{nuc} is upper-bounded by the thermodynamic MIT temperature (T_{td}) and lower-bounded by the supercooled temperature (T_{sc}) of the shielded VO₂. With reduced interface energy, stable nuclei with smaller D are able to form at a given temperature. (d) Measured (solid symbols) and calculated (open symbols) T_{nuc} as a function of D for different Ga⁺ irradiation doses. The thick curved bands are a guide for the eye. The critical sizes of the nucleus can be determined from the onset of the rise above the supercooled limit. (e) Measured dependence of T_{nuc} on Ga⁺ irradiation dose (n_{Ga^+}) in the seed at different D intervals. A heavier dose in the seed promotes I-phase nucleation as shown by the increased T_{nuc} . T_{nuc} for nucleation seeds with large D and high doses is found to reach the thermodynamic limit. T_{td} and T_{sc} are marked by the grey shaded areas in (d) and (e).

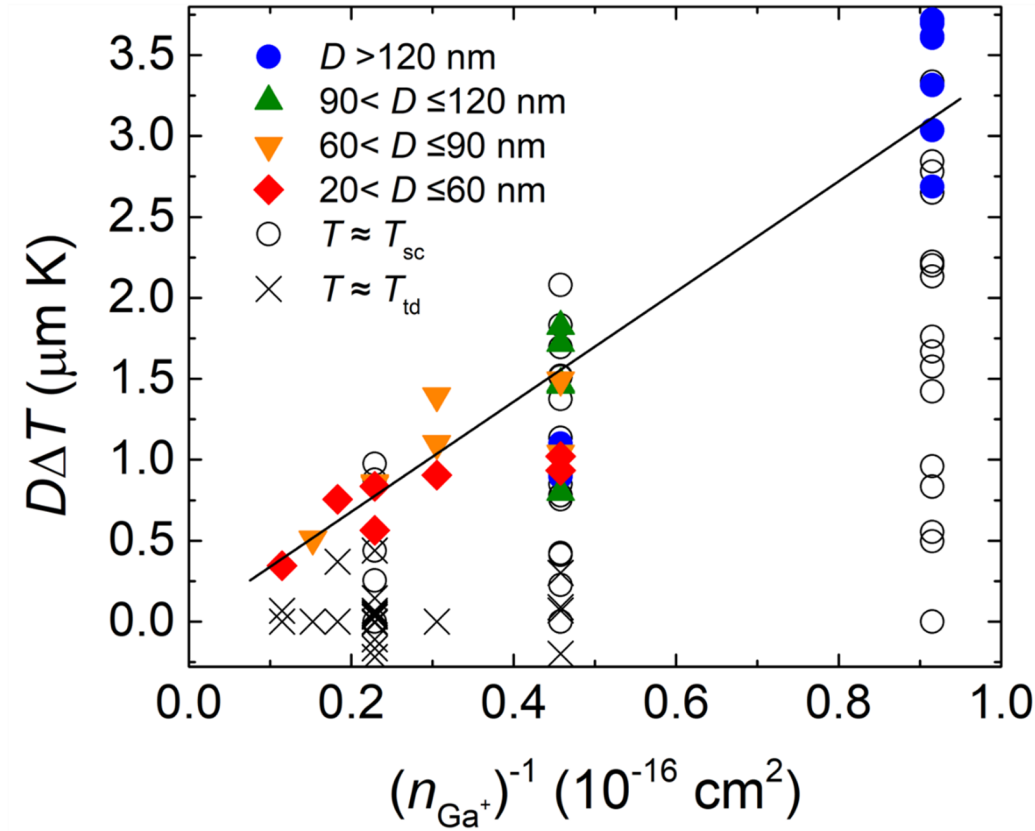


Figure 3: Measured $D\Delta T$ of all nucleation seeds with different diameters as a function of the reciprocal Ga^+ dose density. The black cross hairs and black open circles are $D\Delta T$ data measured from the nucleation seeds that drive the VO_2 to the thermodynamic and supercooled limit, respectively. Between these two limits (solid and colored symbols), T_{nuc} depends on D and the dose of the nucleation seeds. A linear fit of all colored symbols (black solid line) indicates that the interface energy ($\propto D\Delta T$) of the nucleation seeds is inversely proportional to their Ga^+ irradiation dose density n_{Ga^+} .

References

1. D. Kashchiev, *Nucleation*. (Elsevier, 2000).
2. P. G. Vekilov, Nucleation. *Crystal growth & design* **10**, 5007-5019 (2010).
3. H. Vehkamäki, *Classical nucleation theory in multicomponent systems*. (Springer Science & Business Media, 2006).
4. V. I. Kalikmanov, in *Nucleation theory*. (Springer, 2013), pp. 17-41.
5. G. Bai, D. Gao, Z. Liu, X. Zhou, J. Wang, Probing the critical nucleus size for ice formation with graphene oxide nanosheets. *Nature* **576**, 437-441 (2019).
6. R. M. Wentzcovitch, W. W. Schulz, P. B. Allen, VO 2: Peierls or Mott-Hubbard? A view from band theory. *Physical review letters* **72**, 3389 (1994).
7. J. B. Goodenough, The two components of the crystallographic transition in VO₂. *Journal of Solid State Chemistry* **3**, 490-500 (1971).
8. S. Kim, K. Kim, C.-J. Kang, B. Min, Correlation-assisted phonon softening and the orbital-selective Peierls transition in VO 2. *Physical Review B* **87**, 195106 (2013).
9. J. D. Budai *et al.*, Metallization of vanadium dioxide driven by large phonon entropy. *Nature* **515**, 535-539 (2014).
10. T. Yao *et al.*, Understanding the nature of the kinetic process in a VO 2 metal-insulator transition. *Physical review letters* **105**, 226405 (2010).
11. W. Fan *et al.*, Large kinetic asymmetry in the metal-insulator transition nucleated at localized and extended defects. *Physical Review B* **83**, 235102 (2011).
12. S. Lee *et al.*, Axially engineered metal–insulator phase transition by graded doping VO₂ nanowires. *Journal of the American Chemical Society* **135**, 4850-4855 (2013).
13. L. Jin *et al.*, Disorder recovers the Wiedemann-Franz law in the metallic phase of VO 2. *Physical Review B* **102**, 041120 (2020).
14. F. I. Allen, A review of defect engineering, ion implantation, and nanofabrication using the helium ion microscope. *Beilstein Journal of Nanotechnology* **12**, 633-664 (2021).
15. J. Jeong *et al.*, Suppression of metal-insulator transition in VO₂ by electric field–induced oxygen vacancy formation. *Science* **339**, 1402-1405 (2013).
16. J. G. Ramirez *et al.*, Effect of disorder on the metal-insulator transition of vanadium oxides: Local versus global effects. *Physical Review B* **91**, 205123 (2015).
17. D. Lee *et al.*, Isostructural metal-insulator transition in VO₂. *Science* **362**, 1037-1040 (2018).
18. Z. Zhang *et al.*, Nanoscale Engineering in VO₂ Nanowires via Direct Electron Writing Process. *Nano letters* **17**, 851-855 (2017).
19. C. Cheng, K. Liu, B. Xiang, J. Suh, J. Wu, Ultra-long, free-standing, single-crystalline vanadium dioxide micro/nanowires grown by simple thermal evaporation. *Applied Physics Letters* **100**, 103111 (2012).
20. S. Lee *et al.*, Anomalously low electronic thermal conductivity in metallic vanadium dioxide. *Science* **355**, 371-374 (2017).
21. D. Drobne, M. Milani, V. Lešer, F. Tatti, Surface damage induced by FIB milling and imaging of biological samples is controllable. *Microscopy research and technique* **70**, 895-903 (2007).

22. J. F. Ziegler, M. D. Ziegler, J. P. Biersack, SRIM—The stopping and range of ions in matter (2010). *Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms* **268**, 1818-1823 (2010).
23. Y. Shi, F. Xue, L.-Q. Chen, Ginzburg-Landau theory of metal-insulator transition in VO₂: The electronic degrees of freedom. *EPL (Europhysics Letters)* **120**, 46003 (2018).
24. Y. Shi, L.-Q. Chen, Current-Driven Insulator-To-Metal Transition in Strongly Correlated VO₂. *Physical Review Applied* **11**, 014059 (2019).
25. Y. Shi *et al.*, Dynamics of voltage-driven oscillating insulator-metal transitions. *Physical Review B* **104**, 064308 (2021).
26. Y. Shi, L.-Q. Chen, Intrinsic Insulator-Metal Phase Oscillations. *Physical Review Applied* **17**, 014042 (2022).