Paper Title The Purposes of Preservice Teachers' Gestures During One-on-One Algebra Tutoring

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The Purposes of Pre-Service Teachers' Gestures During One-on-One Algebra Tutoring

Abstract

Gestures are an integral component of mathematics classroom discourse. There is a need to classify the types of gestures that teachers use according to their purposes towards supporting and extending students' mathematical thinking. We analyzed 16 algebra tutoring sessions between pre-service teachers (PSTs) and high school students to categorize the PSTs' gestures. We identified 10 categories of PST gestures that we roughly organized into three super-categories: gestures that facilitate shared attention and communication, gestures that emphasize written visual representations, and gestures that support verbal explanations. A taxonomy of gestures based on their purposes will enable further analyses of teacher gesturing and help pre-service and practicing teachers use gestures in more purposeful ways.

Background and Objective

Studies of classroom discourse have resulted in extensive documentation of the types of verbal contributions that teachers make to facilitate interaction with students, including moves such as prompting students for participation, revoicing, providing wait time, and asking questions (Chapin et al., 2009). These analyses of teachers' verbal moves during classroom discussions have helped explain how teacher comments and questions create opportunities for students to make sense of mathematics.

Gesture is an integral component of mathematics discourse, where "discourse" is taken to refer to verbal as well as non-verbal modes of participating in interaction (Arzarello et al., 2009; Morgan & Alshwaikh, 2012; Moschkovich, 2018; O'Halloran, 2015). Gestures can take different forms (Alibali & Nathan, 2012; McNeill, 1992), and they serve a variety of purposes. Some of these purposes are clearly mathematical, such as connecting different representations, linking ideas, and creating representations of abstract mathematical concepts (Author, Date; Bjuland et al., 2008; Nathan et al., 2019; Yu & Uttal, 2021), and sometimes gestures function more generally to support students' cognitive processes or intersubjectivity in interaction (Alibali & Nathan, 2012; Nathan & Alibali, 2011). Often, these purposes overlap.

The purpose of this study is to generate a catalogue of gesturing moves that support mathematical sense making in interaction. We posed the question, what functions did pre-service teachers' (PSTs) gestures serve during one-on-one tutoring sessions? Establishing a common language to document the range of gesturing purposes—similarly to how teachers' questions and other talk moves have been taxonomized according to their purposes—can make the study of gesture more accessible as a component of analyzing mathematics discourse.

Theoretical Framework

Our interest in the role of gestures as part of mathematics discourse is consistent with systemic functional linguistics (SFL; Halliday & Hasan, 1989; Halliday & Matthiessen, 2014) as a theory of language, which prioritizes the relationship between context and language. Communication—whether spoken, written, or otherwise—is influenced by three features of context, including the content being discussed (*what?*), the relationships between individuals involved in the discussion (*who?*) and the mode of communication (*how?*). SFL theorists have adopted the term "mode continuum" (e.g., Schleppegrell, 2012) to refer to the range of ways that ideas are constructed and shared, from close conversations between a small number of people, to verbal presentations, to standardized texts. Discourse in small-group interactions such as those in math classrooms often occurs in close quarters, where participants share a common artifact such as a worksheet. Verbal communication in this context is likely to involve substantial use of ambiguous nouns and adverbs (e.g., "we need to move *this*"; "distribute *this* factor") in combination with pointing or other gestures (Herbel-Eisenmann et al., 2013). Gestures are as integral to the construction of mathematical meanings as the words that speakers use.

McNeill's (1992) typology described four types of gestures, according to type of movement, and is still widely used. These gestures include pointing, iconic, metaphoric, and beat gestures. Studies of *how* gestures get used to support mathematics learning draw from a wider range of perspectives. Table 1 summarizes many of the purposes of teachers' gestures, in mathematics teaching contexts, that have been documented in prior literature. Our work used these categories as a starting point to develop a taxonomy of gestures used during one-on-one tutoring interactions, with a specific focus on the purposes gestures served.

Data Sources and Methods

This study comes from a larger effort to prepare pre-service special education teachers to tutor students with learning disabilities (LD) in Algebra 1. Over two years, we recruited eight special education majors, who had selected a focus in mathematics teaching, to provide tutoring to students in a local school and concurrently participate in a professional development experience. The professional development had three primary foci: to develop PSTs' algebra knowledge, to use gestures to support students with LD, and to pose questions to promote mathematical reasoning. Our discussions related to gesturing centered around the ways that gestures, in general, can help overcome working memory challenges and cognitive overload that students with LD often experience while doing mathematics (Author, Date). We did not discuss specific types of gestures with PSTs.

PSTs provided weekly tutoring throughout several months of a school year (and, in the case of the first cohort, across two school years), and therefore they addressed a range of topics within the Algebra 1 curriculum. Table 2 summarizes the tutoring sessions we included in our analysis, indicated by the tutors' initials and the topic of the session. We selected sessions to include at least one (and in many cases, 2-3) tutoring session from each PST and to include a variety of tutoring topics. Because our first cohort of PSTs tutored across two school years, we had more data from that cohort to draw from and, therefore, they represent the majority of sessions. The second cohort of PSTs had an abbreviated experience due to the Covid-19 pandemic. Systems of equations were a heavy content focus during the times of year that PSTs provided tutoring, which was primarily during the winter and spring, and therefore that topic is more heavily weighted in our sample of sessions.

PSTs recorded their tutoring sessions using document cameras that recorded video of the PSTs' and students' hands moving over a class handout on a table. We first transcribed each of the sixteen tutoring sessions with a column for verbal communication and a separate column to indicate written notation and gestures produced. A graduate assistant produced a first draft of each transcript, and the first author reviewed each transcript to make corrections and fill in details about gestures. After producing narrative descriptions of gestures in the transcript, we used the categories of gestures summarized in Table 1 as an initial set of codes to code our transcripts. Additionally, we referenced the categories of teacher questions described by Boaler and Brodie (2004) as potential codes for PST gestures, because in prior work we have observed that gestures sometimes serve analogous purposes to teacher questions, such as orienting students' attention to the important aspect of a task (Author, Date). In phase 1, each author coded three transcripts independently. We met to discuss overlaps in the existing codes, gesture purposes we noted that had not been captured by existing codes, and discrepancies in our coding. After this initial coding we put together a first-draft taxonomy of gesturing codes used by the PSTs and used this to re-code the first three transcripts plus one more. Our reliability at this stage was 91% on the transcripts we had previously coded, and 80% on the new transcript. We made final revisions to our taxonomy, and we coded the remaining transcripts.

Findings

Table 3 lists the categories of gestures that we observed in the PSTs' tutoring sessions. We divided these categories into three super-categories: gestures that facilitate shared attention and communication, gestures that emphasize written visual representations, and gestures that support verbal explanations. Gestures that facilitated shared attention were the most prevalent across the data. There were also many instances in which PST gestures met the technical definition of "facilitate shared attention" but which also served some superseding purpose, as will be illustrated in the example below. The gesturing categories that we developed characterized the range of gestures that PSTs performed across 16 tutoring sessions.

We share one excerpt from one of JB's (a PST from cohort 2) tutoring sessions to illustrate some of the categories of gestures that we observed in PSTs' work with students. This session came in year 2 of the project, several weeks after JB had begun tutoring and participating in the professional development. JB was working with a student, GB, on a handout from class on which the student needed to determine whether given functions were exponential or not. The task included in the following excerpt provided a graph of the function $f(x) = 2^x$. In turns 40 and 42, JB used the gesture, 'mimic a written visual representation' while using a sweeping gesture to indicate the general shape of an exponential graph, in coordination with the written graph that was on the page. This sweeping gesture provided a dynamic view of the static representation that had been provided.

Speaker	Turn	Gesture code
JB	40. When you have an exponential function it kind of looks like this (using pencil tip to follow the curve of an exponential graph drawn on the paper).	Mimic a written, visual representation
GB	41. Oh yeah, we did that.	
JB	42. You did that? So, instead of this one (pointing to the graph of a line from the previous task) being added each time, it's multiplied each time. That causes this sharp increase (using pencil tip to sweep along the fastest increasing part of the graph).	Contrast two representations Mimic a written, visual representation
	(3 second pause)	
JB	43. Because if you multiply by two each time you might go from 2 to 4 (using pencil tip to indicate points on the exponential graph corresponding to $y = 2$ and $y = 4$), then 4 to 8 (using pencil tip to indicate points on the exponential graph corresponding to $y = 4$ and $y = 8$), then 8 to 16	Introduce an alternative conception of an object (x3)

	(using pencil tip to indicate points on the exponential graph corresponding to $y = 8$ and $y = 16$), instead of going from 2 to 4, 4 to 6, 6 to 8. Does that make sense?

At the beginning of turn 42, JB used the gesture "contrast two representations" when she pointed back to a previous task that included a linear graph. We recognized that the way JB pointed to the graph while referring to it as "this one" could also have classified her gesture as "specifying a written representation". We assigned it the code of "contrasting" because of the way she specifically set up a contrast when she said "instead of this one" in turn 42, and the way she immediately shifted back to the exponential function.

In turn 43 JB used a gesture that we coded as "introduce an alternative conception of an object." Continuing to use the exponential graph, JB introduced a new pointing gesture to specify individual points on the graph. This new gesture, in coordination with her verbal explanation, was mathematically significant because it allowed for a more precise consideration of the concept of exponential change. While a general curve is often used to represent an exponential relationship, exponential functions are not the only graphs that curve, and exponential graphs curve in a specific way. JB pointed out individual points on the graph in a way that helped clarify her explanation of exponential functions.

Scholarly Significance

The meaning of "discourse" is often taken for granted, and it often gets used to refer only to verbal interactions, when in fact discourse can be taken to include a range of communication efforts including posture, gaze, and gesture (Moschkovich, 2018). Sustained efforts have contributed to some consensus on the types of talk moves mathematics teachers use and the types of talk moves that best support student learning (e.g., NCTM, 2014). Gesturing as a component of classroom discourse is less well understood, in part because it can be difficult to infer the significance of non-verbal communication. An initial taxonomy of teacher gesturing moves should be a resource to be used and refined to better understand how teachers use gestures when teaching mathematics.

Prior research has documented the critical role of gestures to support the working memory of students with LD, so that these students have opportunities to engage in mathematical reasoning and problem solving (Author, Date). Although our focus is on PSTs' use of gestures, this work is significant for students' opportunities to learn mathematics. Thus, our analysis of PSTs' gestures as a part of their mathematics discourse is part of a broader effort to document the ways that teachers and students construct meaning together, through verbal and non-verbal communication.

Table 1

Purposes of Teachers' Gestures in Mathematics Classrooms Documented in Prior Studies

Gesturing Purposes	Source
• Communicate links between mathematical ideas	Alibali et al. (2014)
• Show conceptual connections (cachment)	Nathan & Alibali (2011)

• Show links between new and familiar representations (linking)	
• Grounding cognition in the physical environment • Simulate action and perception • Make conceptual metaphors	Alibali & Nathan (2012) Alibali et al. (2013) Alibali et al. (2014)
• Make connections between representations and ideas • Make abstract concepts more concrete • Direct learners' attention	Nathan et al. (2019)
• Contrasting representations to make a counterargument • Illustrating the structure of a reasoning action	Krause (2015)

Table 2

Tutoring Sessions Included in Analysis of PSTs' Gestures

Tutor Initials and Date	Session Topic
Cohort 1	
LF 12/7/18	Graphing linear functions
AC 12/7/18	Graphing linear functions
VV 2/22/19	Systems of equations
AR 4/5/19	Graphing linear functions
SK 4/5/19	Graphing linear functions
LF 4/5/19	Graphing linear functions
AC 4/5/19	Graphing linear functions
VV 4/26/19	Modeling word problems with linear equations
AC 10/16/19	Modeling word problems with linear equations
LF 10/16/19	Modeling word problems with linear equations
AC 1/15/20	Systems of equations
Cohort 2	

SK 1/22/20	Systems of equations
AM 2/12/20	Systems of equations
JB 2/12/20	Simplifying expressions with exponents
JB 2/19/20	Exponential functions
JB, 3/4/20	Modeling word problems with quadratic equations

Table 3
Categories of Gestures

Gesture Type	Description	Example
Gestures that facilitate communication and joint attention		
Establish joint attention	Points to a shared handout or board work to establish shared attention on a task.	Pointing to a specific exercise while saying, "let's focus on this one."
Clarify symbolic notation and vocabulary	Points to a written representation to clarify the meaning of a term or phrase.	Pointing at the notation *f(x)* in a table, to note that it indicates a function.
Specify written representations	Gestures to specify the meaning of non-specific nouns and adverbs such as "this," "these," and "that one" in a question or explanation.	Indicating two separate tables while saying, "let's compare these two."
Focus a student's attention to an important part of a task	Within a given task, draws a student's attention to an important mathematical relationship that is described visually or in writing.	Points to the part of a word problem where the relationship between quantities is specified.
Gestures that emphasize written representations		
Coordinate multiple representations of a single concept or object	Indicates a connection between components of two or more representations (symbols, table, graph, context) of a single concept or task.	Pointing to a linear equation with a coefficient of 4, and then indicating how the *y*-values of a table increase by 4.

Mimic a written, visual representation	Enacts a dynamic representation of a static, visual representation, with the purpose of emphasizing some aspect of that representation.	Sweeping downward left to right to emphasize the direction of a line with negative slope.
Contrast two representations	Indicates a discrepancy or point of difference between two representations, to highlight a counterexample, error, or counterargument.	Gesturing along a pair of parallel lines to indicate they do not intersect, then pointing to a solved equation indicating a point of intersection.
Gestures that supplement verbal explanations		
Visualize technical mathematical language	Gestures to represent the shape or embodiment of a concept or idea while verbally providing a formal mathematical definition or explanation.	While defining an exponential function in terms of multiplication, making a sweeping gesture to indicate an exponential curve.
Introduce an alternative conception of an object	Shifts between two different perspectives of single concept or object, often to connect a more abstract perspective to a more concrete one.	Using the tip of a pencil to indicate a sweeping motion in a straight line, and then indicating horizontal and vertical change between a pair of points.
Connect two concepts	Enacts dynamic, visual depictions of two concepts or objects, either to illustrate similarities or points of contrast (often in absence of written visual representations).	While discussing the difference between exponential and linear functions, gesturing the shape of each.

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