

Subwavelength-Modulated Waveguides for Phase-matching Photons and Low-Energy Electrons

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Abstract: We investigate dielectric waveguides with subwavelength-scale modulation for applications in free-electron-photon interactions. We show that such waveguides are capable of supporting low-loss modes that can efficiently couple to co-propagating, <10-keV electrons. © 2023 The Author(s)
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Quantum coherent interactions between free electrons and photons offer opportunities for novel technology and basic science. For instance, these coherent interactions have been used for additional contrast in transmission electron microscopes (TEMs) [1]; and their potential for applications in electron-photon heralding, as well as entanglement, have recently been explored [2, 3]. For efficient interaction between free electrons and photons, phase-matching is essential. Specifically, the electron velocity must match the phase velocity of the interacting photons. Photons in transparent waveguides or glasses (with refractive indices > 1) can be slowed to phase-match with the high energy (>100 -keV) electrons available in TEMs [4]. However, phase-matching to slower, low-energy electrons presents challenges. These low-energy electrons are particularly attractive, as <30 -keV electrons are readily available from comparatively inexpensive and experimentally-friendly scanning electron microscopes (SEMs). Here, we present the possibility of using waveguides with subwavelength-scale modulation for this phase-matching. We present two candidate modulated waveguides and demonstrate their ability to phase-match <10 -keV electrons at efficiencies comparable to those achieved in TEM experiments.

The phase-matching mechanism and low loss in subwavelength-modulated waveguides follow from the basic properties of periodic waveguides. Consider a waveguide with periodicity Λ in the z direction. According to Floquet's theorem, the complex electric field of a mode in this waveguide can be written as $\vec{E}(x, y, z, t) = \sum_{n=-\infty}^{\infty} \vec{E}_n(x, y) e^{i(\omega t - k_n z)}$ where $k_n = k_0 + 2\pi n/\Lambda$. The sum in this expression is over spatial harmonics of the mode: k_0 corresponds to the fundamental harmonic, k_1 the first harmonic, and so on. Inspecting this sum, we see that the n th spatial harmonic has a phase velocity: $v_n = \omega/k_n$. For large n or small Λ , v_n can be much less than the speed of light. Such a slow harmonic can velocity match a low-energy electron and potentially enable efficient free-electron-photon coupling. Furthermore, it is well known that waveguides with periodic modulation at the subwavelength scale can exhibit effective guiding and low loss [5]; essentially, the spatial harmonics have wavenumbers high enough that they cannot radiate away from the guide. (For the waveguides we will discuss in Fig. 1, $\Lambda = 300$ nm < 1.5 $\mu\text{m} / n_{\text{Si}} = 430$ nm.)

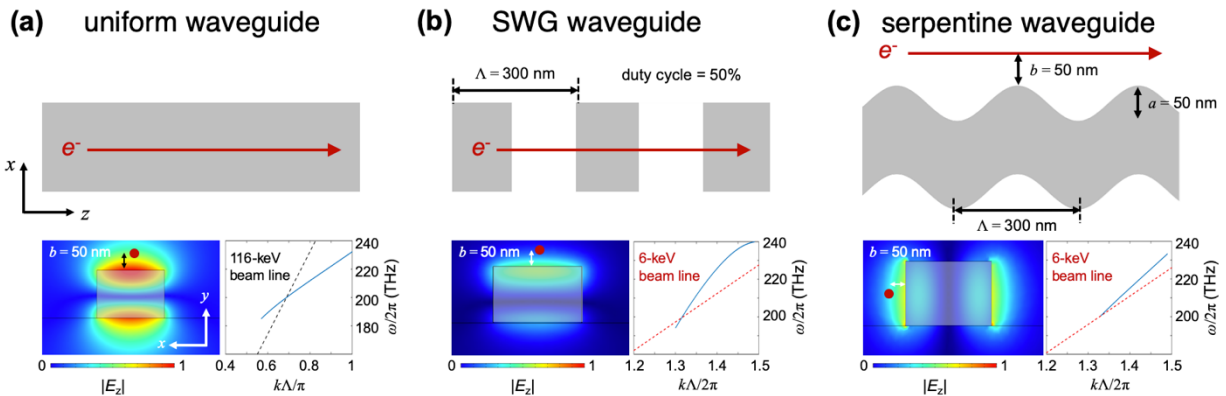


Figure 1: Waveguides for free-electron-photon interactions. Each panel contains: an illustration of the guide and interacting beam (top); a cross-section of the waveguide with a normalized plot of $|E_z|$ and the beam position (bottom left); and a sample of the waveguide dispersion diagram with the matching electron beam line at ≈ 200 THz (bottom right). All waveguides are Si on an SiO_2 substrate, and the impact parameter (b) is 50 nm. (a) Uniform waveguide with cross-section of $0.35 \mu\text{m}$ by $0.25 \mu\text{m}$. (b) Subwavelength grating (SWG) waveguide with cross-section of $0.55 \mu\text{m}$ by $0.45 \mu\text{m}$. The SWG waveguide resembles the uniform guide with sections periodically removed. The guide has a duty cycle of 50% and a periodicity of 300 nm. (c) Serpentine waveguide with cross-section of $0.32 \mu\text{m}$ by $0.22 \mu\text{m}$, periodicity of 300 nm, and sinusoidal edges with 50-nm peak-to-peak amplitude. For the SWG and serpentine waveguides, the dispersion diagram shows the first spatial harmonic (from $k\Lambda/2\pi = 1$ to 1.5).

In Fig. 1, we illustrate the waveguides that we investigate in this work. In Fig. 1a, we show a uniform silicon waveguide. For efficient coupling, the electron velocity must match the phase velocity of a photon in the guide. As shown in the dispersion diagram, for a 200-THz (1.5- μm -wavelength) photon in the uniform guide, the electron velocity (v_e) must be $0.58c$; the electron energy (E_e) must be 116 keV. In Fig. 1b, we illustrate a subwavelength-grating (SWG) waveguide. Due to the periodicity of the SWG waveguide, the dispersion becomes periodic. The first harmonic can velocity match an electron with $v_e = 0.15c$ ($E_e = 6$ keV). Finally, in Fig. 1c, we sketch a “serpentine” waveguide. The electron beam couples to the side of this guide, and we see that this guide also matches at $v_e = 0.15c$ ($E_e = 6$ keV).

In Fig. 2, we present an analysis of the coupling of free electrons to our waveguides illustrated in Fig. 1. To quantify the coupling, we use the coupling coefficient: $g_{\text{Qu}} = e/(2\hbar\omega\sqrt{n_{\text{ph}}}) \int_0^{45\text{ }\mu\text{m}} \tilde{E}_z(x_0, y_0, z) e^{i\omega z/v_e} dz$, where $\tilde{E}_z(x_0, y_0, z)$ is the phasor representation of the z -component of the electric field of a waveguide mode at the beam position (x_0, y_0), and n_{ph} is the number of photons in our 45- μm interaction length [2,3]. g_{Qu} can be thought of as the normalized, electron-velocity-matched field associated with a waveguide mode. In Fig. 2a, we see the uniform waveguide has large g_{Qu} but only couples to electrons with $v_e > 0.4c$ ($E_e > 46$ keV). In Fig. 2b, we see that our SWG waveguide can couple to low-energy electrons. At low velocities, clear peaks in the coupling coefficient occur. These peaks are associated with the first and second spatial harmonics. Finally, in Fig. 2c, we show the coupling for the serpentine waveguide. Again, the spatial harmonics enable coupling to low-energy electrons, and we find $g_{\text{Qu}} \approx 0.02$ with 6-keV electrons. Recent experiments with free-electron-photon interactions in TEMs have used similar impact parameters (50 nm) and interaction lengths (40 μm) and found coupling coefficients with 120-keV electrons of $g_{\text{Qu}} \approx 0.03$ [3]. Our results imply that similar coupling may be possible with subwavelength-modulated waveguides and <10 -keV electrons in SEMs.

In summary, we have shown that subwavelength-modulated, dielectric waveguides may provide a path towards phase-matching free-electron-photon interactions. In particular, these waveguides support low-loss modes capable of efficient coupling to low-energy electrons, <10 -keV electrons.

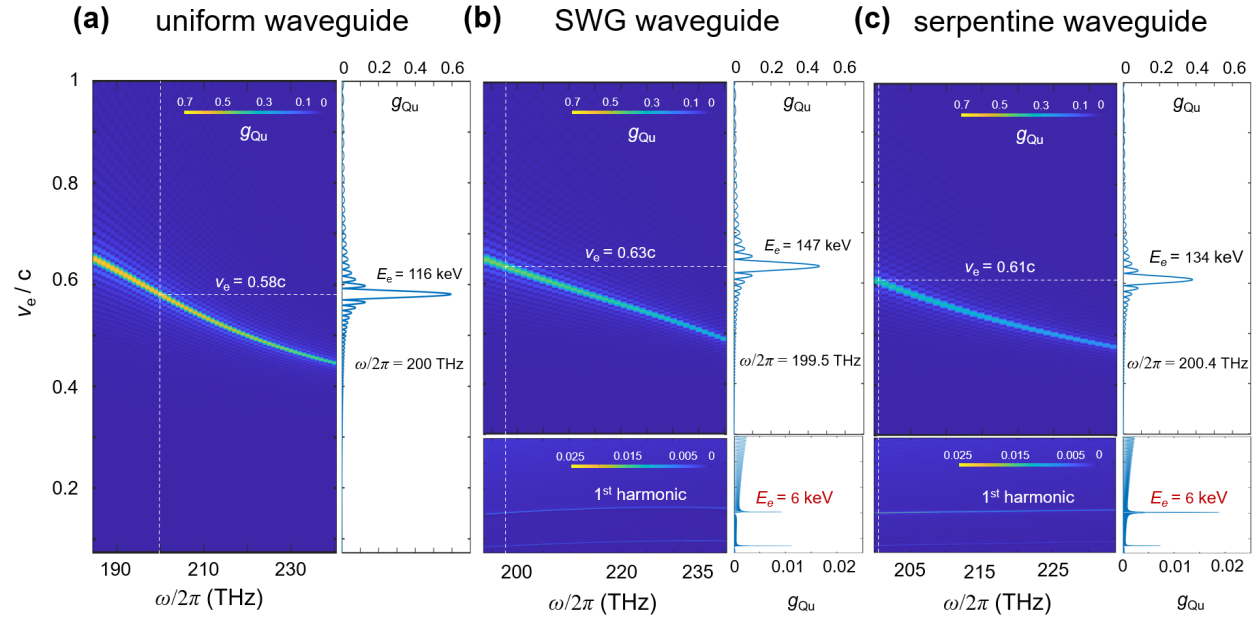


Figure 2: Calculations of free-electron-photon coupling coefficient (g_{Qu}) versus electron velocity (v_e) and mode frequency (ω) for the lowest-order modes in the waveguides in Fig. 1: (a) uniform waveguide, (b) SWG waveguide, and (c) serpentine waveguide. The coupling coefficient exhibits peaks at low electron velocities for the periodic SWG and serpentine waveguides. These peaks correspond to the first and second spatial harmonics. The total interaction length in these calculations is 45 μm . Additionally, the waveguide cross-sections (from Fig. 1) have been selected to tune our guides for operation near cutoff at 200 THz (1.5- μm wavelength). Near-cutoff operation ensures large fields outside the guide and, accordingly, large coupling. The data shown were calculated via finite element method simulations. The mode fields were calculated and used to generate the plots. The modes of the subwavelength-modulated guides used here exhibited the same low-loss behavior as the uniform guide.

References

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