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Superresolution in Interferometric Imaging of Thermal Sources of Arbitrary Strength

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ABSTRACT

It has been shown through quantum estimation theory that Rayleigh's limit can be avoided for single-lens imaging, referred to as superresolution. The quantum estimation approach has recently been used to show superresolution is also possible for imaging based on interferometer arrays in the weak source limit. Following this line of discussion, we consider the resolution limit of estimating the separation between two point sources of arbitrary strength using interferometer arrays. By carefully designing the measurement, we find it is possible to overcome the well-known resolution limit of interferometer arrays as determined by the longest baseline. We construct an optimal measurement to achieve superresolution using linear beam-splitters and photon-number-resolved detection.

Keywords: Interferometric imaging, superresolution, quantum sensing, quantum optics

1. INTRODUCTION

In the imaging problem, Rayleigh's limit quantifies the resolution limit of a finite-sized single lens under diffraction.[1] It was surprisingly discovered that we can avoid Rayleigh's limit for estimating the separation of two weak incoherent point sources using a single lens by employing a more suitable measurement strategy as determined by quantum estimation theory.[2] In this approach, imaging is modeled as a parameter estimation problem based on knowledge of the structure of the source. Then, quantum estimation theory is applied to optimize over measurements. This result has triggered much effort to model more imaging problems using parameter estimation. Several strategies for resolving two weak point sources have been discussed: the projection onto the Hermite-Gaussian spatial modes,[2] using an image-inversion interferometer,[3] adding a phase plate before half of the image,[4] exploiting Hong-Ou-Mandel interference using two copies of the incoming photonic state[5] and using an array of homodyne detectors.[6] Experimentally, superresolution has been demonstrated.[4,7] More types of sources have been discussed, including studying strong point sources[8,9] and those of unequal strength,[10,11] estimating point source locations in two and three dimensions,[12–15] and finding the sensitivity limit of imaging a more general extended source.[16–21]

Besides single lens imaging, interferometric imaging is also a widely used imaging method, which enables an array of lenses to work together as a much larger effective aperture. Interferometric imaging is based on the Van Cittert-Zernike theorem,[22] which shows the coherence function between distant telescopes gives information about the Fourier components of the intensity distribution in the source plane. This method has become a very powerful imaging method especially in the radio wavelength.[23,24] The first image of a supermassive black hole at the center of the Messier 87 Galaxy was obtained by the Event Horizon Telescope (EHT), a radio interferometer array.[25] The estimation of mutual coherence has been modeled as a parameter estimation problem and explored both theoretically and experimentally.[26,27] Methods to overcome the transmission loss of interferometric imaging systems while combining light from distant telescopes using quantum information techniques have been proposed.[28,29]

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Conventionally, astronomical interferometry involves measuring many coherence functions of different baseline. To improve the performance of interferometric imaging, most previous discussion has focused on how to estimate the mutual coherence function with better sensitivity, but the ability to resolve two point sources using interferometric telescope arrays is also limited by the longest baseline. This is because the reconstructed image based on the Van Cittert-Zernike theorem is the convolution of the original source with an effective point spread function (PSF) whose width is roughly determined by the longest baseline. This resolution limit holds even if we have infinite sensitivity of estimating the coherence function. The effective PSF will degrade the reconstructed image as long as the sampling of the image plane is incomplete, which is an analog of Rayleigh's limit for a finite-size single lens.

Inspired by the superresolution discussion for a single lens, we now ask the following question: can we achieve superresolution for interferometric imaging systems and avoid the resolution limit due to the effective PSF? This is first discussed in Ref.,[21] where they consider the fundamental limit of resolving positions of weak point sources using an interferometer array. Their result confirms that superresolution is also achievable for interferometric imaging of weak sources as long as the measurement is carefully designed. Here we present our recently published work[30] that extends the discussion of superresolution using interferometer arrays by removing the weak source limit used in Ref.[21] We focus on the resolution limit for measuring the positions of point sources of arbitrary strength using an interferometric imaging system. Strong thermal sources can have multiphoton coincidences and photon bunching that are ignored in discussions assuming the weak limit,[21] which has important consequences in some situations.[31] In addition, for astronomical interferometer arrays, heterodyne detection has very different sensitivity for strong versus weak thermal sources,[32] which is due to strong vacuum noise in the weak limit. These effects raise the question of whether we can achieve superresolution without invoking the weak limit. The consequence of strength has been discussed for imaging two incoherent point sources of arbitrary strength using a single lens.[8,9] We here present a similar discussion to show how superresolution will be affected by the strength of the source. We find that superresolution is still possible for incoherent thermal sources of arbitrary strength and give the construction of an optimal measurement to achieve superresolution. The measurement we find requires beam-splitters and photon-number-resolved detection.

2. FUNDAMENTAL LIMIT OF RESOLVING TWO POINT SOURCES

We consider the estimation of the separation and centroid of two incoherent point sources of equal strength in one dimension using a linear interferometer consisting of two telescopes as shown in Fig. 1(a). The two sources at positions X_1 and X_2 are assumed to be monochromatic and described by canonical annihilation and creation operators c_1, $c_1^\dagger$ and c_2, $c_2^\dagger$. The sources c_1, c_2 are in a thermal state described by

$$\rho = \rho^{th}(\bar{N}) \otimes \rho^{th}(\bar{N}) = \frac{1}{(\pi\bar{N})^2}\int_{C^2} d^2\alpha_1 d^2\alpha_2 \exp\left(-\frac{|\alpha_1|^2+|\alpha_2|^2}{\bar{N}}\right) |\alpha_1\rangle\langle\alpha_1|_{c_1} \otimes |\alpha_2\rangle\langle\alpha_2|_{c_2}, \tag{1}$$

where $\bar{N}$ determines the strength of each source.

We now derive the state received at the two telescopes, described by a_1, $a_1^\dagger$ and a_2, $a_2^\dagger$. The evolution of mode $c_{1,2}$ to $a_{1,2}$ is given by

$$c_i \to \sqrt{\eta}a_1 + \sqrt{\eta}e^{i\phi_i}a_2 + \sqrt{1-2\eta}\, v_i, \quad i = 1, 2, \tag{2}$$

where v_i are auxiliary environmental modes and η is the attenuation ratio. Besides attenuation, the state acquires phases $\phi_1 = kB\frac{X_1}{s_0}$ and $\phi_2 = kB\frac{X_2}{s_0}$ due to differences in light path lengths,[21,33] where B is the length of the baseline, k is the wavevector of the light and s_0 is the longitudinal distance to the source plane. From this evolution, we derive the states received by the interferometer $a_{1,2}$ as

$$\begin{aligned}\rho = &\frac{1}{(\pi\eta\bar{N})^2}\int_{C^2} d^2\alpha_1 d^2\alpha_2 \exp\left(-\frac{|\alpha_1|^2+|\alpha_2|^2}{\eta\bar{N}}\right) \\ &\times \left[|\alpha_1+\alpha_2\rangle\langle\alpha_1+\alpha_2|_{a_1} \otimes |\alpha_1 e^{-i\phi_1}+\alpha_2 e^{-i\phi_2}\rangle\langle\alpha_1 e^{-i\phi_1}+\alpha_2 e^{-i\phi_2}|_{a_2}\right],\end{aligned} \tag{3}$$

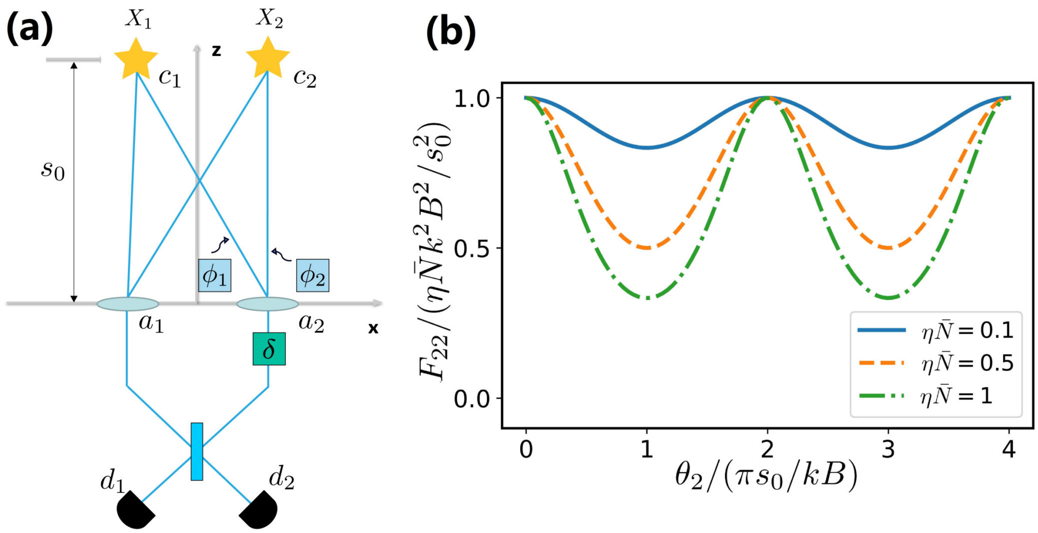

Figure 1. (a) Set up for estimating the position of two incoherent thermal point sources c_1 and c_2 at positions X_1 and X_2 in one dimension. Telescopes collect the light from the two sources, comprising a two-mode interferometer. The difference in path length introduces phases ϕ_1 and ϕ_2 to the received state by the two interferometer modes. (b) The quantum Fisher information F_{22} for estimating the separation θ_2, in units of $\eta\bar{N}k^2B^2/s_0^2$, as a function of the separation θ_2, for different source strengths $\eta\bar{N}$.

where $\bar{N}$ represents the strength of each source and $|\alpha_1 + \alpha_2\rangle$ and $\left|\alpha_1 e^{-i\phi_1} + \alpha_2 e^{-i\phi_2}\right\rangle$ are the coherent states of the two interferometer modes a_1 and a_2.

The imaging problem is reduced to the estimation of centroid $\theta_1 = \frac{1}{2}(X_1 + X_2)$ and the separation between the two sources $\theta_2 = X_1 - X_2$. As a parameter estimation problem, we now consider the fundamental limit of estimating $\theta_{1,2}$ with two telescopes. The sensitivity of estimating θ_1 and θ_2 is bounded by the Fisher information (FI) F: $\Sigma_{\vec{\theta}} \geq F^{-1}$, with its (μ,ν) element $[\Sigma_{\vec{\theta}}]_{\mu\nu} = \mathbb{E}\left[(\theta_\mu - \check{\theta}_\mu)(\theta_\nu - \check{\theta}_\nu)\right]$, where $\check{\theta}_\mu$ is the unbiased estimator of the μ-th unknown parameter. This sensitivity limit given by the FI is the Cramér-Rao bound (CRB).[34] The FI depends on the positive operator-valued measure (POVM) performed on the states. The optimal FI optimized for all POVM performed on a state is the quantum Fisher information (QFI) which gives a fundamental limit for the sensitivity, which is usually called the quantum Cramér-Rao bound (QCRB).[35–38]

The QFI for the estimation of the centroid θ_1 is

$$F_{11} = -\frac{2k^2B^2}{s_0^2}\frac{\eta\bar{N}(1+\cos(\phi_1-\phi_2))}{-1-\eta\bar{N}+\eta\bar{N}\cos(\phi_1-\phi_2)} \xlongequal{\theta_2\to 0} 4\frac{k^2B^2}{s_0^2}\eta\bar{N}. \tag{4}$$

The QFI for the separation θ_2 is given by

$$F_{22} = -\frac{k^2B^2}{s_0^2}\frac{\eta\bar{N}(1+3\eta\bar{N}+\eta\bar{N}\cos(\phi_1-\phi_2))}{-1-2\eta\bar{N}(2+\eta N)+2\eta^2\bar{N}^2\cos(\phi_1-\phi_2)} \xlongequal{\theta_2\to 0} \frac{k^2B^2}{s_0^2}\eta\bar{N}. \tag{5}$$

We have checked that the off-diagonal elements of the QFI vanish; i.e., $F_{12} = F_{21} = 0$. We plot the values of F_{22}, which is the QFI per photon of estimating separation θ_2, in Fig. 1(b). We can see that when the separation between the two point sources tends to zero, i.e. $\theta_2 \to 0$, the quantum Fisher information approaches a constant. This means fundamentally we are allowed to estimate the separation with a finite sensitivity and will not encounter a resolution limit similar to Rayleigh's limit. For the intermediate values of θ_2, the QFI decreases with $\eta\bar{N}$, which is a net effect of multiphoton events, as argued for the case of a single lens.[9] F_{22} is periodic over θ_2. This is because two telescopes in an interferometer array can only distinguish the separation with a sensitivity that depends on $\phi_1 - \phi_2$ and that thus cycles with integer 2π. To address this, we can add more telescopes in the interferometer array.

3. OPTIMAL MEASUREMENT AND OTHER PRACTICAL ISSUES

We now construct a measurement strategy that can achieve the sensitivity limit derived in the previous section. According to quantum estimation theory,[35–38] the optimal POVM can be found from the eigenbasis of the symmetric logarithmic derivative (SLD). We find the SLD for estimating the separation θ_2 is

$$\mathcal{L}_{\theta_2} = 2l_1 a_1^\dagger a_1 + 2l_1 a_2^\dagger a_2 + 2l_2 a_1 a_2^\dagger + 2l_2^* a_1^\dagger a_2 + C_{\theta_2}, \tag{6}$$

where

$$\begin{aligned} C_{\theta_2} &= -\eta\bar{N}[8l_1 + 2l_2(e^{i\phi_1} + e^{i\phi_2}) + 2l_2^*(e^{-i\phi_1} + e^{-i\phi_2})], \\ l_1 &= \frac{kB}{s_0}\frac{(1+4\eta\bar{N})\cot\frac{\phi_1-\phi_2}{2}}{-4[1+2\eta\bar{N}(2+\eta\bar{N})] + 8\eta^2\bar{N}^2\cos(\phi_1-\phi_2)}, \\ l_2 &= -\frac{kB}{s_0}\frac{e^{-\frac{1}{2}i(\phi_1+\phi_2)}(1+3\eta\bar{N}+\eta\bar{N}\cos(\phi_1-\phi_2))\csc\frac{\phi_1-\phi_2}{2}}{4[-1-2\eta\bar{N}(2+\eta\bar{N}) + 2\eta^2\bar{N}^2\cos(\phi_1-\phi_2)]}. \end{aligned} \tag{7}$$

In order to find the eigenbasis of the SLD, we diagonalize $\mathcal{L}_{\theta_2}$ by assuming $d_1 = \frac{1}{\sqrt{2}}(a_1 + e^{i\delta}a_2)$, $d_2 = \frac{1}{\sqrt{2}}(a_1 - e^{i\delta}a_2)$ and drop the constant terms to find

$$\mathcal{L}_{\theta_2} =(2l_1 + l_2 e^{i\delta} + l_2^* e^{-i\delta})d_1^\dagger d_1 + (2l_1 - l_2 e^{i\delta} - l_2^* e^{-i\delta})d_2^\dagger d_2 + (l_2 e^{i\delta} - l_2^* e^{-i\delta})d_1^\dagger d_2 - (l_2 e^{i\delta} - l_2^* e^{-i\delta})d_2^\dagger d_1. \tag{8}$$

When we choose $l_2 e^{i\delta} - l_2^* e^{-i\delta} = 0$ or equivalently $\delta = \frac{1}{2}(\phi_1 + \phi_2)$, the eigenbasis of the SLD are the Fock basis of d_1, d_2. Thus, the optimal POVM for estimating θ_2 is $\{|m,n\rangle_d \langle m,n|_d\}_{\{m,n\}}$, with $d_1^\dagger d_1 |m,n\rangle_d = m|m,n\rangle_d$ and $d_2^\dagger d_2 |m,n\rangle_d = n|m,n\rangle_d$.

The SLD for estimating the centroid θ_1 is

$$\mathcal{L}_{\theta_1} = 2l_3 a_1 a_2^\dagger + 2l_3^* a_1^\dagger a_2 + C_{\theta_1}, \tag{9}$$

where

$$\begin{aligned} C_{\theta_1} &= -\eta\bar{N}[2l_3(e^{i\phi_1} + e^{i\phi_2}) + 2l_3^*(e^{-i\phi_1} + e^{-i\phi_2})], \\ l_3 &= i\frac{kB}{s_0}\frac{e^{-i\phi_1} + e^{-i\phi_2}}{-4 - 4\eta\bar{N} + 4\eta\bar{N}\cos(\phi_1-\phi_2)}. \end{aligned} \tag{10}$$

We also diagonalize $\mathcal{L}_{\theta_1}$ by assuming $d_1 = \frac{1}{\sqrt{2}}(a_1 + e^{i\delta}a_2)$, $d_2 = \frac{1}{\sqrt{2}}(a_1 - e^{i\delta}a_2)$ and dropping the constant terms:

$$\mathcal{L}_{\theta_1} = (l_3 e^{i\delta} + l_3^* e^{-i\delta})d_1^\dagger d_1 - (l_3 e^{i\delta} + l_3^* e^{-i\delta})d_2^\dagger d_2 + (l_3 e^{i\delta} - l_3^* e^{-i\delta})d_1^\dagger d_2 - (l_3 e^{i\delta} - l_3^* e^{-i\delta})d_2^\dagger d_1. \tag{11}$$

When we choose $l_3 e^{i\delta} - l_3^* e^{-i\delta} = 0$ or equivalently $\delta = \frac{1}{2}(\phi_1 + \phi_2) - \frac{\pi}{2}$, the eigenbasis of the SLD is the Fock basis of d_1, d_2. Thus, the optimal POVM for estimating θ_1 is $\{|m,n\rangle \langle m,n|_d\}_{\{m,n\}}$, with $d_1^\dagger d_1 |m,n\rangle_d = m|m,n\rangle_d$ and $d_2^\dagger d_2 |m,n\rangle_d = n|m,n\rangle_d$.

In the implementation of the optimal POVM constructed above, the light received at the two telescopes is simply combined and photon-number-resolving detection is performed after adding phase delays, as shown in Fig. 1(a). Then, each temporal mode of the received state of the interferometer is projected onto the Fock bases $|m,n\rangle_d$ of the $d_{1,2}$ modes. The number of outcomes (m,n) are counted and the corresponding probability is calculated to find the estimated $P(m,n)$. The probability distribution $P(m,n)$ for different $\theta_{1,2}$ can be theoretically calculated. We show some of these $P(m,n)$ in Fig. 2(a) as examples. The theoretical and estimated $P(m,n)$ are fitted to obtain an estimated separation θ_2. In practice, the detectors may only be able to resolve Fock states $|m,n\rangle_d$ with $m \leq M$, $n \leq N$. We calculate the FI for different M,N as shown in Fig. 2(b). We see that even if we are unable to implement the ideal measurement with $M,N \to \infty$, we can still get reasonable sensitivity when compared to the ideal case. In particular, as the separation $\theta_2 \to 0$, the FI with small M,N also approaches a nonvanishing constant.

Now that we have determined the fundamental sensitivity limit and the optimal measurement to saturate the bound, an important question is: how does this compare with the conventional interferometric imaging

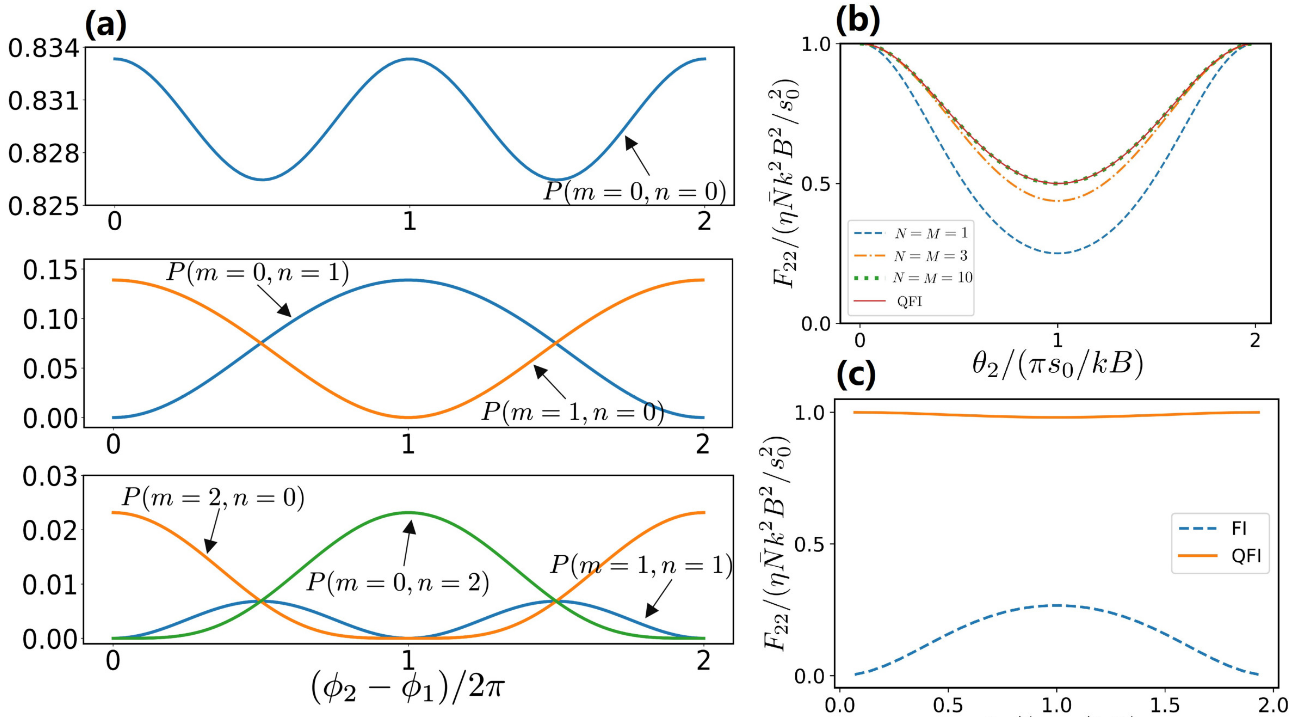

Figure 2. (a) The probability $P(m,n)$ of projecting the state onto $|m,n\rangle_d$ with $\eta\bar{N}=0.1$ as a function of $\phi_2-\phi_1$. (b) Fisher information for photon-number-resolving detection that can only distinguish the Fock state $|m,n\rangle$ for $m\leq M$, $n\leq N$. Events with greater photon number are ignored. Other parameters are chosen as $\eta\bar{N}=1$. (c) Approximate Fisher information (blue dashed curve) and the quantum Fisher information (solid orange curve) as a function of separation θ_2. Other parameters are chosen as $\bar{N}=0.01$; $m,n\leq 3$.

method? The key difference is that in the conventional method, although we also interfere the light from the two telescopes on a beam splitter, we do not choose the same phase delay as constructed above. We now compare the performance of estimating the separation θ_2 between the conventional method and our method by calculating the FI. Assume the centroid is known and gives $\frac{1}{2}(\phi_1+\phi_2)=2\pi/3$. For the conventional method, we choose the phase delay $\delta=0,\frac{\pi}{2}$, which is conventionally used to extract information about the coherence function. The FI of the conventional method and the QFI are shown in Fig. 2(c) as a function of separation. We can see from Fig. 2(c) that for the conventional method, the separation must be larger than s_0/kB to get reasonable sensitivity, which is consistent with the angular resolution of an interferometric array λ/B, where λ is the wavelength. And when the separation θ_2 tends to zero, the FI vanishes, which implies the resolution limit, as argued before based on an effective PSF. But the QFI remains a constant. This shows that a better POVM can help avoid this limit, which is thus referred to as superresolution. For a more intuitive comparison of the performance between the conventional method and our method, we consider some practical examples. Assume the observation is made with wavelength $\lambda=5$ mm, longest baseline $B=10$ km and $\eta\bar{N}=0.01$. Then the resolution of the conventional method is $\lambda/B=5\times10^{-7}$ radians $\approx 0.1''$. For the case when the angular separation of the two point sources is $\theta_2/s_0=0.05'',0.01'',0.005''$, the FI of our scheme is larger than the conventional method by a factor of roughly 4, 30, 100, respectively. With the assumption that mean square error scales with the inverse of number of samples n as $\Delta(\theta_2/s_0)^2\propto 1/n$, our scheme outperforms the conventional method by a factor of 4, 30, 100 in terms of observation.

The optimal POVM constructed above for the estimation of the centroid and the separation requires prior knowledge about the centroid because the phase delay δ depends on the centroid $\theta_1\propto\phi_1+\phi_2$ of the two point sources. Thus, it is not possible to perfectly implement the optimal measurement if the estimation of the centroid is not infinitely accurate, and misalignment of the centroid must be taken into account in practice. We now show how the superresolution predicted by the QFI is affected by a deviation δ from $\frac{1}{2}(\phi_1+\phi_2)$. Define the deviation as $c=\frac{1}{2}(\phi_1+\phi_2)-\delta$. We only keep the contribution of $|m,n\rangle_d \langle m,n|_d$ with $m \leq 3$, $n \leq 3$, which is a reasonable assumption because even for small M, N we will still have nonvanishing FI as $\theta_2 \to 0$, as discussed previously. We first fix the misalignment c and plot the FI as a function of separation θ_2 in Fig. 3(a). With a nonzero misalignment, when the separation tends to zero, the FI will vanish, which means the deviation can destroy the superresolution. We then fix the separation θ_2 and plot the FI as a function of the misalignment c in Fig. 3(b). We observe that as misalignment increases, the FI decreases, which indicates poorer sensitivity. The deviation c must be smaller than the separation $\theta_2/(s_0/kB)$ in order to get reasonable sensitivity. However, we emphasize that there is no fundamental limit, such as Rayleigh's limit, to prevent us from improving the accuracy of estimating the centroid. So, in principle, it is always possible to make sure the deviation c is smaller than the threshold to have good sensitivity.

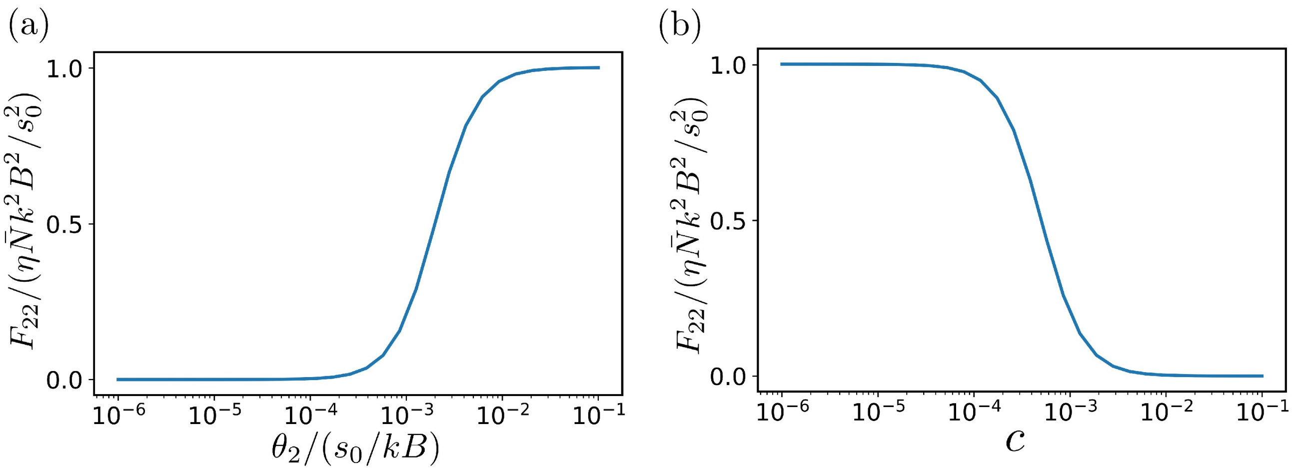

Figure 3. (a) Fisher information as a function of separation θ_2 for fixed misalignment $c=10^{-3}$. Other parameters are chosen as $\bar{N}=0.01$, $m,n \leq 3$. (c) Fisher information as a function of misalignment c for fixed separation $\theta_2/(s_0/kB)=10^{-3}$. Other parameters are chosen as $\bar{N}=0.01$; $m,n \leq 3$.

4. CONCLUSION

We have considered the imaging of two point sources of arbitrary strength using an interferometer array consisting of two telescopes. Although the resolution of conventional interferometric imaging for resolving the separation between two point sources is limited by an effective PSF just as in Rayleigh's limit, we showed that it is possible to avoid this limit by carefully designing the measurement. The optimal measurement that saturates the fundamental limit can be implemented with beam-splitters and photon-number-resolving detection. We hope our work can inspire more discussion about the fundamental imaging limit of interferometer arrays. Possible extensions include: resolving two point sources of unequal strength,[10,11] estimating separation in three dimensions similarly to Refs.,[13,21] and imaging a general extended source similarly to Refs.[17,18]

5. ACKNOWLEDGEMENTS

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REFERENCES

[1] Rayleigh, L., "Xxxi. investigations in optics, with special reference to the spectroscope," *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* **8**(49), 261–274 (1879).

[2] Tsang, M., Nair, R., and Lu, X.-M., "Quantum theory of superresolution for two incoherent optical point sources," *Physical Review X* **6**(3), 031033 (2016).

[3] Nair, R. and Tsang, M., "Interferometric superlocalization of two incoherent optical point sources," *Optics express* **24**(4), 3684–3701 (2016).

[4] Tham, W.-K., Ferretti, H., and Steinberg, A. M., "Beating rayleighâs curse by imaging using phase information," *Physical review letters* **118**(7), 070801 (2017).

[5] Parniak, M., Borówka, S., Boroszko, K., Wasilewski, W., Banaszek, K., and Demkowicz-Dobrzański, R., "Beating the rayleigh limit using two-photon interference," *Physical review letters* **121**(25), 250503 (2018).

[6] Datta, C., Jarzyna, M., Len, Y. L., Łukanowski, K., Kołodyński, J., and Banaszek, K., "Sub-rayleigh resolution of two incoherent sources by array homodyning," *Physical Review A* **102**(6), 063526 (2020).

[7] Zhou, Y., Yang, J., Hassett, J. D., Rafsanjani, S. M. H., Mirhosseini, M., Vamivakas, A. N., Jordan, A. N., Shi, Z., and Boyd, R. W., "Quantum-limited estimation of the axial separation of two incoherent point sources," *Optica* **6**(5), 534–541 (2019).

[8] Lupo, C. and Pirandola, S., "Ultimate precision bound of quantum and subwavelength imaging," *Physical review letters* **117**(19), 190802 (2016).

[9] Nair, R. and Tsang, M., "Far-field superresolution of thermal electromagnetic sources at the quantum limit," *Physical review letters* **117**(19), 190801 (2016).

[10] Řehaček, J., Hradil, Z., Stoklasa, B., Paúr, M., Grover, J., Krzic, A., and Sánchez-Soto, L., "Multiparameter quantum metrology of incoherent point sources: towards realistic superresolution," *Physical Review A* **96**(6), 062107 (2017).

[11] Řeháček, J., Hradil, Z., Koutnỳ, D., Grover, J., Krzic, A., and Sánchez-Soto, L. L., "Optimal measurements for quantum spatial superresolution," *Physical review A* **98**(1), 012103 (2018).

[12] Ang, S. Z., Nair, R., and Tsang, M., "Quantum limit for two-dimensional resolution of two incoherent optical point sources," *Physical Review A* **95**(6), 063847 (2017).

[13] Yu, Z. and Prasad, S., "Quantum limited superresolution of an incoherent source pair in three dimensions," *Physical review letters* **121**(18), 180504 (2018).

[14] Napoli, C., Piano, S., Leach, R., Adesso, G., and Tufarelli, T., "Towards superresolution surface metrology: Quantum estimation of angular and axial separations," *Physical review letters* **122**(14), 140505 (2019).

[15] Prasad, S. and Yu, Z., "Quantum-limited superlocalization and superresolution of a source pair in three dimensions," *Physical Review A* **99**(2), 022116 (2019).

[16] Dutton, Z., Kerviche, R., Ashok, A., and Guha, S., "Attaining the quantum limit of superresolution in imaging an object's length via predetection spatial-mode sorting," *Physical Review A* **99**(3), 033847 (2019).

[17] Zhou, S. and Jiang, L., "Modern description of rayleigh's criterion," *Physical Review A* **99**(1), 013808 (2019).

[18] Tsang, M., "Quantum limit to subdiffraction incoherent optical imaging," *Physical Review A* **99**(1), 012305 (2019).

[19] Tsang, M., "Semiparametric estimation for incoherent optical imaging," *Physical Review Research* **1**(3), 033006 (2019).

[20] Tsang, M., "Quantum semiparametric estimation," *arXiv preprint arXiv:1906.09871* (2019).

[21] Lupo, C., Huang, Z., and Kok, P., "Quantum limits to incoherent imaging are achieved by linear interferometry," *Physical Review Letters* **124**(8), 080503 (2020).

[22] Zernike, F., "The concept of degree of coherence and its application to optical problems," *Physica* **5**(8), 785–795 (1938).

[23] Wilson, T. L., Rohlfs, K., and Hüttemeister, S., [*Tools of radio astronomy*], vol. 5, Springer (2009).

[24] Kellermann, K. and Moran, J., "The development of high-resolution imaging in radio astronomy," *Annual Review of Astronomy and Astrophysics* **39**(1), 457–509 (2001).

[25] Collaboration, E. H. T., "First m87 event horizon telescope results. i. the shadow of the supermassive black hole," *Astrophysical Journal Letters* **875**(1), L1 (2019).

[26] Pearce, M. E., Campbell, E. T., and Kok, P., "Optimal quantum metrology of distant black bodies," *Quantum* **1**, 21 (2017).

[27] Howard, L., Gillett, G., Pearce, M., Abrahao, R., Weinhold, T., Kok, P., and White, A., "Optimal imaging of remote bodies using quantum detectors," *Physical review letters* **123**(14), 143604 (2019).

[28] Gottesman, D., Jennewein, T., and Croke, S., "Longer-baseline telescopes using quantum repeaters," *Physical review letters* **109**(7), 070503 (2012).

[29] Khabiboulline, E. T., Borregaard, J., De Greve, K., and Lukin, M. D., "Optical interferometry with quantum networks," *Physical review letters* **123**(7), 070504 (2019).

[30] Wang, Y., Zhang, Y., and Lorenz, V. O., "Superresolution in interferometric imaging of strong thermal sources," *Phys. Rev. A* **104**, 022613 (Aug 2021).

[31] Mandel, L. and Wolf, E., [*Optical coherence and quantum optics*], Cambridge university press (1995).

[32] Tsang, M., "Quantum nonlocality in weak-thermal-light interferometry," *Physical Review Letters* **107**(27), 270402 (2011).

[33] Wang, Y., Zhang, Y., and Lorenz, V. O., "Superresolution in interferometric imaging of strong thermal sources," *Physical Review A* **104**(2), 022613 (2021).

[34] Helstrom, C. W. and Helstrom, C. W., [*Quantum detection and estimation theory*], vol. 84, Academic press New York (1976).

[35] Braunstein, S. L. and Caves, C. M., "Statistical distance and the geometry of quantum states," *Physical Review Letters* **72**(22), 3439 (1994).

[36] Paris, M. G., "Quantum estimation for quantum technology," *International Journal of Quantum Information* **7**(supp01), 125–137 (2009).

[37] Monras, A., "Phase space formalism for quantum estimation of gaussian states," *arXiv preprint arXiv:1303.3682* (2013).

[38] Gao, Y. and Lee, H., "Bounds on quantum multiple-parameter estimation with gaussian state," *The European Physical Journal D* **68**(11), 347 (2014).