Power System State Recovery using Local and Global Smoothness of its Graph Signals

Md Abul Hasnat and Mahshid Rahnamay-Naeini
Electrical Engineering Department, University of South Florida, Tampa, Florida, USA
hasnat@usf.edu, mahshidr@usf.edu

2022 IEEE Power & Energy Society General Meeting (PESGM) | 978-1-6654-0823-3/22/$31.00 ©2022 IEEE | DOI: 10.1109/PESGM48719.2022.9917018

***Abstract*—Recovering the state of unobservable power system components due to cyber attacks or limited meter availability is a crucial problem to address to enable efficient monitoring and operation of power systems. The graph signal processing (GSP) framework provides new opportunities to improve power system data analysis by capturing the topological information of the system. In this paper, the recovery of the unobservable states in power systems is formulated as a graph signal reconstruction problem in a GSP framework. Specifically, a novel reconstruction technique based on the statistics of the local smoothness of the graph signals along with the global smoothness of the graph signals is casted into an optimization framework. In contrast to many graph signal reconstruction techniques, which assume band-limited signals to be recovered, the proposed technique is applicable to general graph signals irrespective of their bandwidth. The performance evaluation of the proposed method using simulated graph signals for the IEEE 118 bus system show promising reconstruction accuracy.**

***Index Terms*—Graph Signal Processing, cyber attack, smart grid, local smoothness, graph signal reconstruction.**

I. Introduction

The state estimation in smart grids [1] is an essential function, which enables their secure and reliable monitoring and operation. However, this critical function is vulnerable to various forms of cyber attacks (e.g., Denial of Service-DoS and false data injection attacks). These attacks can hamper the availability and integrity of the system state information. Once a cyber attack is detected and located in the system, the recovery of the state information at the attacked locations becomes crucial to mitigate their effects.

The state recovery problem in smart grids can be modeled through a graph signal reconstruction framework [2], [3]. Graph signal reconstruction has been an active research area in the graph signal processing (GSP) [4] domain with vast potential applications. The goal of the graph signal reconstruction is to estimate the signal values corresponding to a subset of the vertices, which are unavailable due to down-sampling of the original signal or missing measurements (for instance, due to cyber attacks). In this paper, the power system states are considered as graph signals and a state recovery technique based on the local smoothness of the graph signals has been proposed to recover the state of the attacked components.

Many of the existing graph signal reconstruction techniques are built analogous to those in classical signal processing and sampling theory with the assumption of the band-limited graph signals [5], [6]. In our previous work [7], a graph signal sampling-reconstruction framework for band-limited power system graph signals was proposed to identify an efficient set of samples (representing the locations to mount the phasor measurement units-PMUs) for tracking the power system's state. However, the proposed method has limitations when the sampling-set is not chosen beforehand as in the case of cyber attacks. Moreover, the matrix operation-based reconstruction method used in [7] is applicable strictly to band-limited graph signals within the frequency equal to the number of sampling vertices.

Some reconstruction techniques are also formulated in optimization frameworks with the goal of minimizing the global smoothness of the recovered graph signals [8]–[10]. However, the global smoothness and bandwidth of a graph signal are global measures and cannot capture the localized dynamics of measurements. Due to the structural topology and the physics of the electricity, the electrical attributes of the power system's components (e.g., voltage angle) vary over the system, and as a result, the vertices have notable differences in their local smoothness values even during normal operations. In our previous work [11], the statistics of the local smoothness of the power system graph signals has been utilized to detect and locate stresses in the grid.

In this work, a novel graph signal reconstruction technique is proposed, which allows capturing the local dynamics by considering the local smoothness of the graph signal. Particularly, the probability distributions of the local smoothness values are exploited to identify the values for the missing signal measurements that maximize the likelihood of the local smoothness values. By utilizing the local smoothness of the graph signal, the localized information about the dynamics of component interactions is incorporated into the estimation of the missing states information, which improves the state estimation performance. The proposed reconstruction technique is also bandwidth-agnostic in the sense that the applicability of the technique is not restricted to band-limited graph signal.

II. Related Work

Over the last decade, with the emergence of graph signal processing as promising field of research, the reconstruction of graph signal is getting attention by the researchers. The topic is often studied in the context of the non-uniform sampling of the graph signal, [5], [10], and the reconstruction method involves the eigen-vector decomposition of the graph shift operators. In this connection, a large number of works on the graph signal reconstruction are dependent on the band-limited assumption of the graph signal. For example, Tanaka *et al.* [6] provides a detailed discussion on the theory and application of graph signal sampling from graph-frequency domain perspective in which the reconstruction process relies on the bandwidth of the graph signals. The concept of band-limited assumption is related to the global smoothness of the graph signal, and in the reconstruction literature minimizing the global smoothness to recover the missing signal values is widely known [8]–[10].

Among the other methods Isufi *et al.* [12] proposed reconstruction of the missing graph signal values using graph Wiener filter while the frequency response of the graph Wiener filter is approximated by ARMA graph filter and implemented distributively. Wang *et al.* [3] introduces the concepts of *local set* in connection with the frame theory and proposed two local set-based iterative graph signal reconstruction techniques. Mao and Gu [13] proposed a band-limited graph signal joint detection and reconstruction technique by using mixed integer linear programming. Romero *et al.* [14] proposes a kernel based method for band-limited graph signal reconstruction.

In our previous work [7], it has been shown that power system graph signals are approximately band-limited in normal conditions. We proposed a sampling-set selection method based on the error introduced in each vertex (bus) during the band-limitation process by an anti-aliasing filter. Although the proposed method was effective in enhancing the reconstruction performance of the graph-signal sampling and reconstruction process and applicable to certain power system applications such as optimum PMU placement, this method has some limitations in the context of a few other power grid applications. Firstly, during the cyber attacks, the grid operator does not have the opportunity to select the sampling-set i. e. compromised buses. Moreover, in some cases, removing the high-frequency components from a graph signal deters the quality of the signal for specific smart grid applications related to monitoring and operation. Since the reconstruction technique applied in the previous paper is strictly limited to the band-limitation assumption of graph signals, certain not-band-limited signals cannot be reconstructed using that technique. For overcoming these limitations, in this article, we propose a reconstruction technique that is not directly dependent on the band-limited graph signal assumption.

III. Graph Signal Recovery

A. Power system graph signal

In this work, the power grid is modeled as a weighted undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where, $\mathcal{V} = \{v_1, v_2, ..., v_N\}$ and $\mathcal{E} = \{e_{ij} : (i, j) \in \mathcal{V} \times \mathcal{V}\}$ are, respectively, the set of all vertices (buses) and the set of all edges (transmission lines). If there are N buses connected by M transmission lines in the grid, then $|\mathcal{V}| = N$ and $|\mathcal{E}| = M$, where $|.|$ denotes the cardinality of a set. The weight, w_{ij} of the edge $e_{i,j}$ is defined as the reciprocal of the geographic distance between the buses i and j, denoted by d_{ij}, if there exists a link between vertices i and j, otherwise $w_{ij} = 0$. The graph Laplacian matrix $\mathbf{L}$ containing the topological information of the graph $\mathcal{G}$, with elements l_{ij} is defined as: $l_{ij} = \sum_{j=1}^{N} w_{ij}$ if $i = j$ and $l_{ij} = -w_{ij}$, otherwise. The graph signal $x(n)$ representing the bus voltage angle at bus n, can be considered as a mapping of the vertices of the graph to real-number space, $x : \mathcal{V} \to \mathbb{R}$. All the values of $x(n)$ arranged in a vector form is denoted by $\underline{\mathbf{x}}$.

B. Problem Formulation

Let $\mathcal{A}$ be the set of all the buses for which the measurements and the state are unobservable either due to a cyber attack or the unavailability of the measurement devices or meters. The power system state recovery involves estimating the graph signal values in the unobservable buses, i.e., $x(n_{\mathcal{A}}), n_{\mathcal{A}} \in \mathcal{A}$ using the graph signal values in the observable buses, i.e., $x(n), n \in \mathcal{V} \setminus \mathcal{A}$.

C. Reconstruction Method

In this paper, a graph signal reconstruction technique based on the smoothness property of the power systems' graph signals has been proposed. The global smoothness [15] of the graph signal $x(n)$ is defined as $s_{Global} = \frac{\underline{\mathbf{x}}^T \mathbf{L} \underline{\mathbf{x}}}{\underline{\mathbf{x}}^T \underline{\mathbf{x}}}$ and quantifies the overall amount of fluctuations form vertices to vertices (which also relates to the bandwidth or the amount of high-frequency components in the signals). Note that smaller values of global smoothness represent smoother signals. Under the assumptions of a smooth graph signal, the reconstruction of the graph signal can be formulated with the goal of identifying values that minimize the global smoothness of the recovered graph signal. Since the power system graph signals are generally smooth [11], [16], the global smoothness can be one of the criteria for power system's graph signal recovery. However, the global smoothness of a graph signal is a global parameter and therefore lacks the local information about how signal values vary within local neighborhoods of vertices. The local smoothness of a graph signal is described for each vertex of the graph signal by $s(n) = \frac{l_x(n)}{x(n)}, \quad x(n) \neq 0$, where $l_x(n)$ is the $n-$th element of the vector, $\mathbf{L}\underline{\mathbf{x}}$ and $\mathbf{L}$ is the Laplacian matrix. The local smoothness specifies the amount of fluctuation of the signal values from one vertex to its neighboring vertices. The concept of the local smoothness of a graph signal is an analogous concept to instantaneous frequency, which quantifies the rate of change in signals at each time instant [15]. By incorporating the local smoothness of the graph signal in the recovery process, the knowledge about the local dynamics in the grid can be utilized in addition to the global dynamics to achieve better recovery performance and a more robust estimation.

Our extensive simulations of power systems have shown that the local smoothness values of the power system's graph signal varies notably over vertices. By collecting and analyzing the measurement data for each vertex in the system, the probability distribution of the local smoothness values at each vertex, $p_{s_n}(\zeta)$, can be obtained. In this work, $p_{s_n}(\zeta)$ is characterized for bus voltage angle graph signals using data collected from our simulations.

To this end, the state information recovery technique for power system's graph signal is formulated as an optimization framework for maximizing the likelihood of the local smoothness values at all the vertices while minimizing the global smoothness of the graph signal. This optimization problem can be casted into the following formulation:

$$\max_{x(n_{\mathcal{A}}),n_{\mathcal{A}}\in\mathcal{A}} p_{s_1,s_2,\dots s_N}(s(n_1),s(n_2),\dots s(n_N)) - \lambda s_{Global}, \tag{1}$$

where λ is the Lagrange multiplier. The joint distribution of the local smoothness for all the buses, $p_{s_1,s_2,\dots s_N}(\zeta_1,\zeta_2,\dots\zeta_N)$ is computationally infeasible to compute from the marginal distributions, $p_{s_n}(\zeta)$. However, maximizing the likelihood of the local smoothness values at each bus would serve similar purpose. Therefore, we propose an alternative objective function by maximizing the minimum likelihood value of local smoothness from all the buses along with the minimizing the global smoothness.

$$\max_{x(n_{\mathcal{A}}),n_{\mathcal{A}}\in\mathcal{A}} [\min_n p_{s_n}(s(n))] - \lambda s_{Global}, \tag{2}$$

Our simulation data analyses have shown that the probability distribution of the local smoothness values at each bus does not follow any standard distribution. However, to simplify and solve this optimization problem, in this work the local smoothness values at bus n is assumed to follow a normal distribution with mean value μ_n and standard deviation σ_n. Due to this assumption, maximizing the likelihood of local smoothness values $p_{s_n}(s(n))$ at each bus in equation (2) takes the form of minimizing the absolute value of the normalized local smoothness, $z_n = \frac{s(n)-\mu_n}{\sigma_n}$ and the optimization problem can be expressed as:

$$\min_{x(n_{\mathcal{A}}),n_{\mathcal{A}}\in\mathcal{A}} [\max_n |\frac{s(n)-\mu_n}{\sigma_n}|] + \lambda s_{Global}. \tag{3}$$

The optimization in equation (3) is non-linear. We propose to solve this optimization problem using the surrogate optimization method [17] to obtain the global minimum.

IV. Simulation and Results

A. Experimental Setup

For validation of the proposed method and evaluating the state recovery performance, simulations have been done on the IEEE 118 bus system [18]. The power system graph signals, i.e., the bus voltage angle of each bus, have been obtained by simulating the power flow using MATPOWER [19]. A load pattern collected from the NYISO [20] is added to the default MATPOWER load to create variation of load in the system as described in [21]. For evaluating the state recovery performance, fifty random scenarios are considered for each fixed number of unobservable buses, which can represent the buses under the cyber attack. The unobservable buses are chosen randomly with a uniform distribution from all the buses of the system except the reference bus.

B. Estimating the Probability Distributions of the Local Smoothness

The local smoothness values of the buses, i.e., $s(n)$ values, associated with the voltage angle graph signals are calculated for a large number of simulated graph signals for the IEEE 118 bus system. Once the local smoothness values are calculated using simulated graph signals, the probability distribution of the local smoothness values at each bus, $p_{s_n}(\zeta)$, are estimated empirically from the calculated local smoothness values. The actual distribution of the local smoothness values is intractable; however, our experiments have shown that assuming normal distribution for local smoothness values at bus n with mean μ_n and standard deviation σ_n provides reasonable accuracy for the state recovery. Nonetheless, the parameters of the distributions need to be updated regularly to avoid the effect of data-drift [22] that can deteriorate the reconstruction performance.

C. Solving the Optimization Problem

In this work, the optimization problem in equation (3) has been solved by surrogate optimization method using MATLAB optimization toolbox [23]. The lower bounds and upper bounds of the values of $x(n_{\mathcal{A}})$ are considered as $\mu_{x_{n_{\mathcal{A}}}} - 3\sigma_{x_{n_{\mathcal{A}}}}$ and $\mu_{x_{n_{\mathcal{A}}}} + 3\sigma_{x_{n_{\mathcal{A}}}}$, respectively, where $\mu_{x_{n_{\mathcal{A}}}}$ and $\sigma_{x_{n_{\mathcal{A}}}}$ are the mean value and the standard deviation of the graph signal values at vertices $n_{\mathcal{A}}$, estimated from the past measurement data (in this paper, simulated data). The value of the Lagrange polynomial, λ decides the relative importance of the global smoothness and the local dynamics to reconstruct graph signals values. For all the simulations in this work, the value of λ is considered to be $5,000$. Depending on the system, the value of λ can be tuned to obtain the desirable performance.

D. State Recovery Performance Analysis

The performance of the proposed method has been illustrated in Fig. 1 in terms of the absolute error against the total number of unobservable buses (i.e., number of buses under cyber attack). The mean absolute error and the maximum absolute error are the average value and the maximum value of recovery error over all the unobservable buses, respectively. The first metric is important for evaluating the general performance of the recovery method and the second one is important for evaluating the suitability of the proposed recovery method in certain power system applications demanding a standard state estimation accuracy at each bus. As can be observed from the figure, the proposed method provides promising performance for recovering the missing states. The results also confirm that the error in recovery grows with the number of unobservable buses in the system.

E. Comparison with Other Reconstruction Methods

As discussed earlier, the reconstruction method applied in our previous work [7], which is based on matrix operation (including matrix inversion), is only applicable for graph signals that are perfectly band-limited to N_s frequency components, where N_s is the cardinality of the sampling set. For this reason, in [7], an anti-aliasing filter is applied to the

978-1-6654-0823-3/22/$31.00 ©2022 IEEE

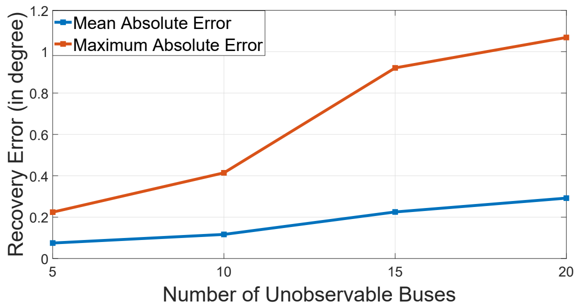

Fig. 1. State recovery error using the proposed method for different number of unobservable buses in the system.

originally approximately band-limited graph signals to discard the insignificant frequency components beyond N_s frequency components to make perfectly band-limited signals. The presence of components beyond N_s frequency components (even very small) leads to computation of ill-conditioned matrices resulting in total failure to estimate the missing states. Fig. 2(a) illustrates an example of a voltage angle graph signal, which is not band-limited (as ground truth for the experiment). In this example, the buses 59 to 64 in the IEEE 118 bus system are considered unobservable buses. In Fig. 2(b) the missing signal values at unobservable buses are recovered using the direct matrix operation method discussed in [7], which fails to estimate the missing states in comparison to ground-truth states in Fig. 2(a). However, the proposed method in the current paper, which incorporates the global and local dynamics of the grid, is capable of estimating the missing signal values with notable accuracy as illustrated in 2(c). This example confirms that the applicability of the proposed method does not rely on the band-limited assumption of the graph signal to be recovered. The relaxation of the band-limited assumption makes this method applicable to many scenarios in power grids; particularly in cases where the resulted graph signals are not band-limited.

In [7], it has been shown that the major part of the error in the sampling-reconstruction process is introduced in the band-limiting process by the anti-aliasing graph filter. By avoiding the anti-aliasing filter error, the reconstruction error can be further reduced by the proposed method.

Acknowledgement

This material is based upon work supported by the National Science Foundation under Grant No. 2118510.

V. Conclusion

In this paper, a novel technique for reconstructing graph signals has been proposed for the power system's state recovery problem. The proposed technique specifically utilizes the local dynamics of the system through the local smoothness of the system's graph signals. The statistics of the local smoothness measures along with the assumption of globally smooth graph signals are used to formulate an optimization problem for the state recovery problem. One of the key advantages of the proposed technique is that it relaxes the band-limited signal assumption for reconstructing graph signals. Simulation results for the IEEE 118 bus system and various number of cyber attack scenarios show promising accuracy and performance in recovering the unobservable states.

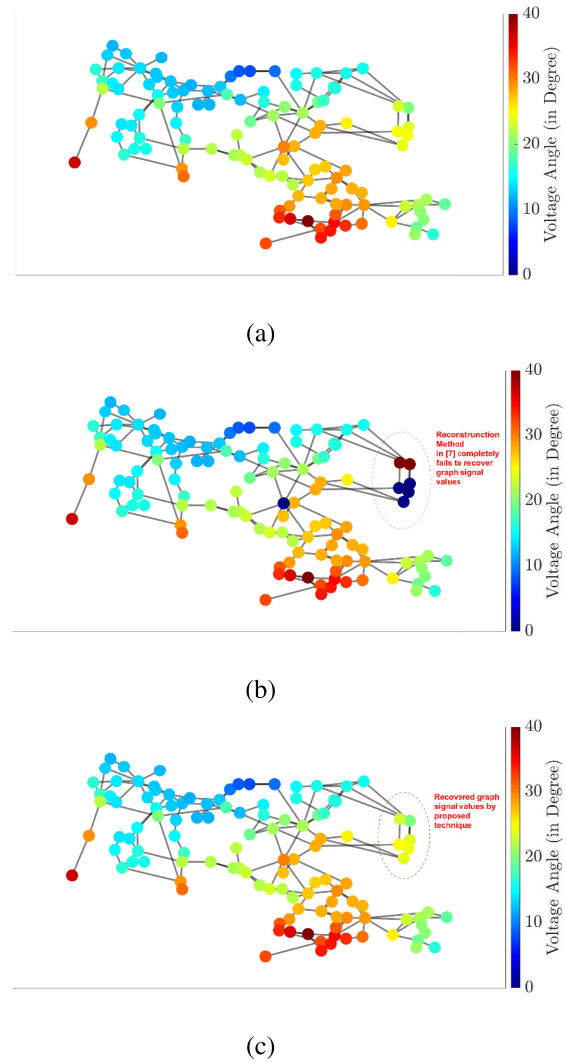

Fig. 2. An example of missing voltage angle graph signal recovery by graph signal reconstruction: (a) the actual voltage angles measurements which is not band-limited, (b) recovered signal by the matrix operation based method in [7], (c) recovered signal using proposed method.

References

[1] A. Abur and A. G.Exposito, *Power system state estimation: theory and implementation.*, CRC press, 2004.

[2] F. Wang, G. Cheung and Y. Wang, "Low-complexity Graph Sampling With Noise and Signal Reconstruction via Neumann Series," in IEEE Transactions on Signal Processing, vol. 67, no. 21, pp. 5511-5526, 1 Nov.1, 2019.

[3] X. Wang, P. Liu and Y. Gu, "Local-Set-Based Graph Signal Reconstruction," in IEEE Transactions on Signal Processing, vol. 63, no. 9, pp. 2432-2444, May1, 2015.

[4] A. Ortega, P. Frossard, J. Kovačević, J. M. F. Moura and P. Vandergheynst, "Graph Signal Processing: Overview, Challenges, and Applications," in Proceedings of the IEEE, vol. 106, no. 5, pp. 808-828, May 2018.

978-1-6654-0823-3/22/$31.00 ©2022 IEEE

[5] P. D. Lorenzo, S. Barbarossa and P. Banelli, “Sampling and recovery of graph signals” in *Cooperative and Graph Signal Processing*, 1st ed. Academic Press, 2018.
[6] Y. Tanaka, Y. C. Eldar, A. Ortega and G. Cheung, ”Sampling Signals on Graphs: From Theory to Applications,” in IEEE Signal Processing Magazine, vol. 37, no. 6, pp. 14-30, Nov. 2020.
[7] M. A. Hasnat and M. Rahnamay-Naeini, “Sampling of Power System Graph Signals,” IEEE PES ISGT-Europe, 2021.
[8] X. Mao, K. Qiu, T. Li and Y. Gu, “Spatio-Temporal Signal Recovery Based on Low Rank and Differential Smoothness,” in IEEE Transactions on Signal Processing, vol. 66, no. 23, pp. 6281-6296, 1 Dec.1, 2018.
[9] P. Berger, G. Hannak and G. Matz, “Graph Signal Recovery via Primal-Dual Algorithms for Total Variation Minimization,” in IEEE Journal of Selected Topics in Signal Processing, vol. 11, no. 6, pp. 842-855, Sept. 2017.
[10] S. Chen, A. Sandryhaila, J. M. F. Moura and J. Kovačević, “Signal Recovery on Graphs: Variation Minimization,” in IEEE Transactions on Signal Processing, vol. 63, no. 17, pp. 4609-4624, Sept.1, 2015.
[11] M. A. Hasnat and M. Rahnamay-Naeini, “Detection and locating cyber and physical stresses in smart grids using graph signal processing,” arXivpreprint arXiv:2006.06095, 2020.
[12] E. Isufi, P. Di Lorenzo, P. Banelli and G. Leus, “Distributed Wiener-Based Reconstruction of Graph Signals,” 2018 IEEE Statistical Signal Processing Workshop (SSP), 2018, pp. 21-25.
[13] X. Mao and Y. Gu, “A Joint Detection and Reconstruction Method for Blind Graph Signal Recovery,” 2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2018, pp. 4184-4188.
[14] D. Romero, M. Ma and G. B. Giannakis, “Kernel-Based Reconstruction of Graph Signals,” in IEEE Transactions on Signal Processing, vol. 65, no. 3, pp. 764-778, 1 Feb.1, 2017.
[15] M. Daković, L. Stanković, and E. Sejdić. “Local smoothness of graph signals,” Mathematical Problems in Engineering 2019.
[16] R. Ramakrishna and A. Scaglione, “Grid-Graph Signal Processing (Grid-GSP): A Graph Signal Processing Framework for the Power Grid,” in IEEE Transactions on Signal Processing, vol. 69, pp. 2725-2739, 2021.
[17] Z. H. Han and K. S. Zhang, “Surrogate-Based Optimization,” *Real-world applications of genetic algorithms*, Chapter 17, pp. 343, 2012.
[18] Electrical and Computer Engineering Department, IEEE 118-Bus,54 Unit, 24-Hour System Unit and Network Data, Illinois Institute of Tech.
[19] R. D. Zimmerman, C. E. Murillo-Sánchez and R. J. Thomas, “MAT-POWER: Steady-State Operations, Planning, and Analysis Tools for Power Systems Research and Education,” in IEEE Transactions on Power Systems, vol. 26, no. 1, pp. 12-19, Feb. 2011.
[20] The New York Independent System Operator, Inc[US], https://www.nyiso.com/.
[21] M. A. Hasnat and M. Rahnamay-Naeini, “A Data-Driven Dynamic State Estimation for Smart Grids under DoS Attack using State Correlations,” North American Power Symposium (NAPS), Wichita, KS, USA, 2019.
[22] A. Ahmed, K. S. Sajan, A. Srivastava and Y. Wu, “Anomaly Detection, Localization and Classification Using Drifting Synchrophasor Data Streams,” in IEEE Transactions on Smart Grid, vol. 12, no. 4, pp. 3570-3580, July 2021.
[23] Mathworks optimization Toolbox, https://www.mathworks.com/products/optimization.html.

978-1-6654-0823-3/22/$31.00 ©2022 IEEE