

# $E_6$ models in light of precision $M_W$ measurements

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We propose a solution to the recent  $W$  mass measurement by embedding the Standard Model within  $E_6$  models. The presence of a new  $U\delta 1P$  group shifts the  $W$  boson mass at the tree level and introduces a new gauge boson  $Z^0$  which has been searched for at collider experiments. In this article, we identify the parameter space that explains the new  $W$  mass measurement and is consistent with current experimental  $Z^0$  searches. As  $U\delta 1P$  extensions can be accommodated in supersymmetric models, we also consider the supersymmetric scenario of  $E_6$  models, and show that a 125 GeV Higgs may be easily achieved in such settings.

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## I. INTRODUCTION

Precision measurements have been crucial in testing physics beyond the Standard Model (SM). In recent years, tensions between theory and experiment have been building with the muon  $g - 2$  measurement [1,2], flavor anomalies [3–8], and most recently the  $W$  boson mass measurement by the CDF collaboration [9]. The CDF II experiment measured the  $W$  boson mass to be

$$M_W^{\text{CDF}} \approx 80.4335 \pm 0.0094 \text{ GeV}; \quad \delta 1P$$

which deviates from the SM prediction [10] by about  $7\sigma$ ,

$$\delta M_W \equiv M_W^{\text{CDF}} - M_W^{\text{SM}} \approx 76 \pm 11 \text{ MeV}; \quad \delta 2P$$

This measurement has increased the tension between the SM and previous Tevatron measurements [11,12], but is also in tension with the previous world average by more than  $2\sigma$  [10]. The tension between various experiments can be from unknown systematic uncertainties, which is beyond the scope of this study.

In this article, we focus on the compelling possibility that the deviation of results between the new CDF experiment, along with previous Tevatron experiments,

and the SM predictions is a hint of new physics beyond the SM [13–23,23–34,34–101]. In particular, we focus on a possible tree-level modification to the  $W$  boson mass coming from an extension the SM gauge group. The simplest extension is to include a new  $U\delta 1P$  gauge group, which we call  $U\delta 1P^0$ . This results in two electrically neutral gauge bosons,  $Z$  and  $Z^0$ , that are linear combinations of the SM  $Z^0$  boson and the gauge boson of the new  $U\delta 1P^0$  group. Due to the interconnectedness of the electroweak sector, these additions alter the  $W$  boson mass at the tree level which can explain the CDF II measurement.

There are many well-motivated theories beyond the SM that feature at least one extra  $U\delta 1P$  group [102,103], such as grand unified theories (GUT) [104–107], superstrings [108–111], extra dimensions [112], little Higgs [113–115], dynamical symmetry breaking [116,116], and the Stueckelberg mechanism [117–122]. Among the GUT models, the ones based on rank-6 gauge groups, known as  $E_6$  have been extensively studied for phenomenological interests [123]. The  $E_6$  models can be considered in both supersymmetric and nonsupersymmetric scenarios. Extending the minimal supersymmetric Standard Model (MSSM) with an extra  $U\delta 1P$  group also has numerous advantages. For example, similar to the next-to-minimal supersymmetric Standard Model (NMSSM), the tree-level Higgs mass in the  $U\delta 1P$ -supersymmetric model (UMSSM) is increased, and a 125 GeV Higgs can be obtained without the need of large radiative corrections [124]. Furthermore, UMSSM scenarios embed the discrete  $Z_3$  symmetry of the NMSSM into a continuous one, and therefore, do not suffer from the cosmological domain walls problems in the NMSSM [125].

In this article, we discuss supersymmetric  $E_6$  models in light of the CDF II  $M_W$  measurement. We note that

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although our analysis is based on  $E_6$ , it can easily be generalized to any new physics scenario, supersymmetric or not, with at least one additional  $U(1)^0$  gauge group. This article is structured as follows. In Sec. II, we show the contribution to the  $W$  mass from the  $U(1)^0$  group. In Sec. III we review the experimental constraints, especially the direct  $Z^0$  searches. These constraints are then applied to  $E_6$  models containing the  $U(1)^0$  group. In Sec. IV, we discuss the predictions of the Higgs mass within supersymmetric  $E_6$  models. Section V is reserved for conclusions.

## II. CONTRIBUTION TO $M_W$

Models that extend the SM by an extra  $U(1)^0$  gauge group introduce a new gauge boson  $Z^0$ . The Cartan subalgebra of  $E_6$  models contains two additional  $U(1)^0$  generators. We consider the following breakdown of  $E_6$

$$\begin{aligned} E_6 &\rightarrow SO(10) \times U(1)_\psi \\ &\rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi \\ &\rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_\chi \times U(1)_\psi : \end{aligned}$$

The two extra  $U(1)^0$  groups yield two additional gauge bosons,  $Z_\psi$  and  $Z_\chi$ . Upon electroweak symmetry breaking, they mix to form two gauge bosons  $Z^0$  and  $Z'^0$ , with the mixing parametrized by the  $E_6$  mixing angle  $\theta_{E_6}$ ,

$$\begin{aligned} Z^0 &\propto Z_\chi \cos \theta_{E_6} + Z_\psi \sin \theta_{E_6} \\ Z'^0 &\propto -Z_\chi \sin \theta_{E_6} + Z_\psi \cos \theta_{E_6} : \end{aligned} \quad (83)$$

Often, only one of the new gauge bosons is assumed to be around the TeV scale, leading to an effective rank-5 group. In this analysis, we will only consider the contributions from the lighter state of the two. We will also allow for a kinetic mixing term,  $\frac{\sin \chi}{2} B_\mu Z'^{\mu}$  [126–130], which has been studied in the context of a leptophobic  $Z^0$ . The relevant Lagrangian terms are given in the appendix.

The presence of a new  $Z^0$  boson contributes to the  $W$  boson mass at the tree level. The shift in the  $W$  boson mass from the SM prediction can be expressed in terms of the oblique parameters  $S$ ,  $T$ ,  $U$  [131]:

$$\begin{aligned} M_W^2 &\propto \delta M_W^{\text{SM}} + \\ &+ \frac{\alpha c_W^2}{c_W^2 - s_W^2} M_Z^2 - \frac{1}{2} S + c_W^2 T + \frac{c_W^2 - s_W^2}{4s_W^2} U \end{aligned} \quad (84)$$

where  $s_W$  and  $c_W$  are the sine and cosine of the weak mixing angle, and  $M_Z$  is the physical mass of the SM  $Z^0$  boson. The oblique parameters may be derived from the transformation matrix responsible for bringing  $L$  into a basis of fields with canonical kinetic mixing and diagonal mass matrices [132]. In the appendix we derive this matrix, and from that the oblique parameters. Here we express the

oblique parameters in terms of the mixing angle  $\xi$  between the new  $Z^0$  boson and the SM  $Z^0$  boson, and the kinetic mixing angle  $\chi$  between the  $U(1)_Y$  and  $U(1)^0$  gauge bosons. To first order in  $\xi$ ,

$$\alpha S \propto 4c_W^2 s_W \xi \tan \chi; \quad (85)$$

and  $U \propto 0$ .  $T$  is given by the wave function renormalization  $\Delta_Z$  of the  $Z$  boson (found in the transformation matrix) as well as the shift in the  $Z$  boson mass from its SM prediction. To first order in  $\xi$ , the wave function renormalization is

$$\Delta_Z \propto s_W \xi \tan \chi; \quad (86)$$

With  $Z - Z^0$  mass mixing, the tree-level  $Z$  boson mass  $M_Z$  is shifted from its SM value  $m_Z$  as

$$m_Z^2 \propto M_Z^2 + \frac{1}{2} M_{Z^0}^2 - M_Z^2 \sin^2 \xi; \quad (87)$$

which is an identical relation between the  $Z - Z^0$  mass matrix and its diagonalized form. For small  $Z - Z^0$  mixing angles of  $\xi^2 \ll M_Z^2/M_{Z^0}^2$  Eq. (7) is approximately

$$M_Z^2 \approx m_Z^2 \left( 1 - \xi^2 \frac{M_{Z^0}^2}{M_Z^2} - 1 \right); \quad (88)$$

These changes to the properties of the  $Z$  boson are combined to form the  $T$  parameter:

$$\begin{aligned} \alpha T &\propto 2\delta\Delta_Z - \Delta_Z + \\ &+ \xi^2 \frac{M_{Z^0}^2}{M_Z^2} - 1 \end{aligned} \quad (89)$$

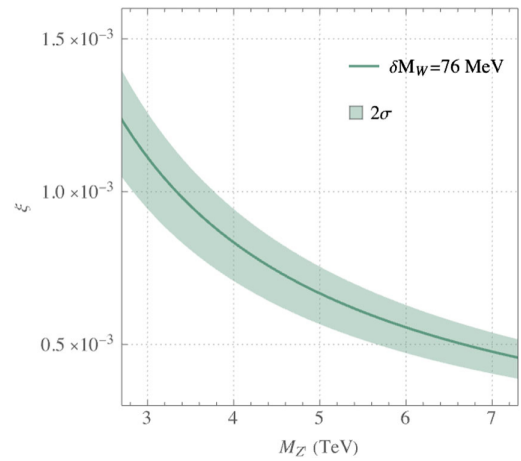


FIG. 1. The solid line shows the solution to Eq. (10) for  $\delta M_W \propto 76$  MeV, which is the central value of the deviation of the SM from the CDF II experiment. The shaded region contains solutions for  $54 \text{ MeV} \leq \delta M_W \leq 98 \text{ MeV}$ , corresponding to a  $2\sigma$  confidence level of the CDF II measurement.



sections in  $E_6$  models and in the SSM. We parametrize the ratio following Ref. [136],

$$\frac{\sigma \delta p p \rightarrow Z^0 \rightarrow l^- l^+ p}{\sigma \delta p p \rightarrow Z_{\text{SSM}}^0 \rightarrow l^- l^+ p} \approx \frac{p c_u}{p c_{u,\text{SSM}}} \frac{\delta 1 - p c_d}{\delta 1 - p c_{d,\text{SSM}}} : \delta 15p$$

The parameter  $p$  is a numerical fit depending on  $M_{Z^0} = \frac{p \sqrt{s}}{S}$  to account for the parton distribution function:

$$p \approx 0.77 - 0.17 \tan^{-1} \frac{2.6 - 9.5 \frac{M_{Z^0}}{p \sqrt{s}}}{S} \quad \delta 16p$$

with  $\frac{p \sqrt{s}}{S}$  being the center-of-momentum energy. The production cross section and branching ratios are written with quantities  $c_u, c_d$  where

$$c_q \approx \frac{M_{Z^0}^2 g_{Z^0}^4}{24\pi \Gamma_{Z^0}} \delta Q_{2qL}^2 p Q_{2qR}^2 p \delta Q_{2eL}^2 p Q_{2eR}^2 p \quad \delta 17p$$

$$c_{q,\text{SSM}} \approx \frac{M_{Z^0}^2 g_{Z^0}^4}{24\pi \Gamma_{Z^0}} \delta Q_{2qL}^2 p Q_{2qR}^2 p \delta Q_{2eL}^2 p Q_{2eR}^2 p : \delta 18p$$

Here,  $\Gamma_{Z^0}$  is the full width of the  $Z^0$  boson found by summing partial widths for decays into massless fermion-antifermion pairs. In  $E_6$  models,

$$\Gamma_{Z^0} \approx \sum_f g_{Z^0}^2 \frac{M_{Z^0}^2}{24\pi} \delta Q_{2fL}^2 p Q_{2fR}^2 p; \quad \delta 19p$$

assuming negligible fermion masses. In the SSM, the couplings  $g_{Z^0}$  and charges  $Q_{2i\delta L;R\delta}$  are replaced by  $g_Z \equiv g_2 = c_W$  and  $Q_{Zi\delta L;R\delta}$  respectively. When calculating the  $Z^0$  width, we assume the new  $E_6$  fermions are heavy enough to be ignored.

To establish a parameter space, we calculate the ratio of Eq. (15) and compare with its upper bound set by CMS searches at the LHC at  $\sqrt{s} \approx 13$  TeV, as well as CMS projections of high luminosity LHC (HL-LHC) at  $\sqrt{s} \approx 14$  TeV [133,137]. The production and decay of  $Z^0$  depends on its mass  $M_{Z^0}$  and the charges  $Q_{2i\delta L;R\delta}$  contributing to the neutral current  $J_{\frac{1}{2}}$ . As shown in Eq. (12),  $Q_{2i\delta L;R\delta}$  are determined by the  $Z - Z^0$  mixing  $\xi$ , kinetic mixing  $\chi$ , and the  $E_6$  mixing angle  $\theta_{E_6}$ . Additionally, Eq. (10) fixes  $\xi$  for a given  $M_{Z^0}$  to satisfy the CDF II result Eq. (2), and  $\xi$  is determined by  $\chi$  and  $M_{Z^0}$ , reducing the parameter space by two (using Eq. (A14) in the appendix and  $\tan \beta \approx 10$ ). The  $\sqrt{s} \approx 13$  TeV search excludes  $Z^0$  models that explain the CDF II result with

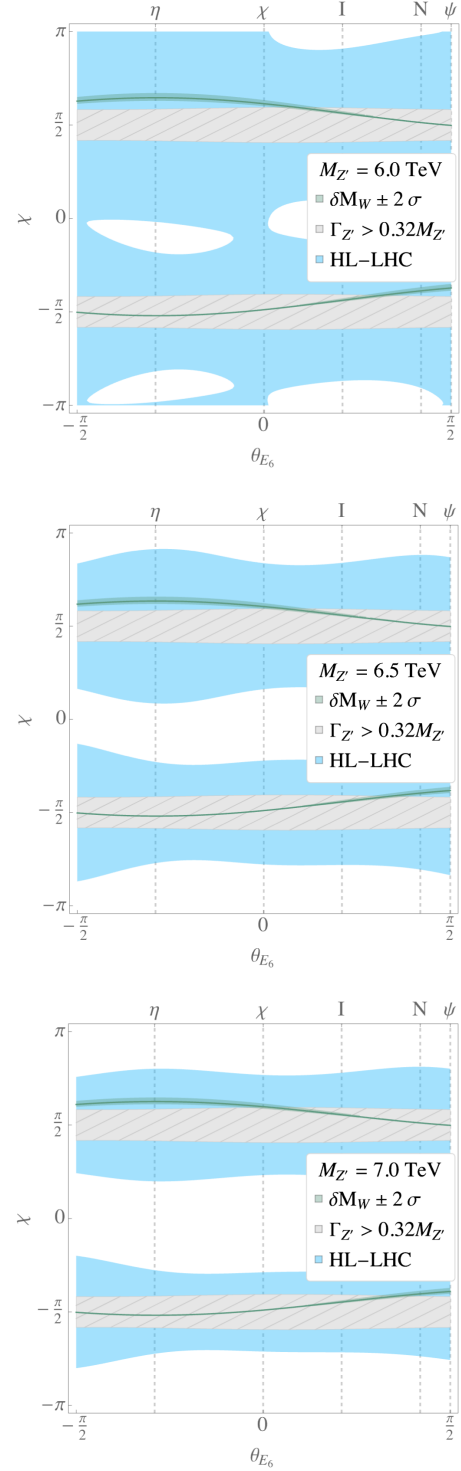


FIG. 2. Parameter space probeable by HL-LHC resonant dilepton searches at 14 TeV (blue) and a wide resonance region of  $\Gamma_{Z^0} = M_{Z^0} > 32\%$  (gray) for  $M_{Z^0} \approx 6$  TeV (top), 6.5 TeV (middle), and 7 TeV (bottom). Regions consistent with the CDF II  $W$  mass measurement are shown in green.

$M_{Z^0} \leq 5.5$  TeV. At higher masses, the resulting space which may be probed by the  $\sqrt{s} \approx 14$  TeV CMS search is shown in Fig. 2 by the blue region.

There exists open parameter space around  $\tan\beta \approx \pi/2$  due to the diverging behavior of both  $M_{Z^0}$  and the chiral couplings  $Q_{Z^0\ell\ell;RP}$  in this region. As the mass and couplings increase in magnitude, so does the full width  $\Gamma_{Z^0}$  of the  $Z^0$  boson. This widening of  $\Gamma_{Z^0}$  makes it easier for the  $Z^0$  to evade direct LHC searches. We close off these regions with gray hashing in Fig. 2 where the total  $Z^0$  width is more than 32% of the  $Z^0$  mass following the ATLAS study in [134]. This study finds that wide resonances with  $\Gamma_{Z^0}=M_{Z^0} \leq 32\%$  do not significantly affect search bounds utilizing the narrow width approximation (NWA) for  $Z^0$ s heavier than 4.5 TeV, and those effects become less significant as  $M_{Z^0}$  increases [133,134]. Even so, it should be noted that our use of the NWA introduces estimated errors of  $\mathcal{O}(\delta\Gamma_{Z^0}=M_{Z^0})$  [138]. Interference effects from SM gauge boson production also influence the sensitivity of collider searches. We do not consider these effects, however relative interference at the LHC can be as low as a few percent for searches that assume narrow widths [139].

For a sufficiently light  $Z^0$ , holes may be found in the probeable parameter space; as shown in the top panel of Fig. 2. There are two different charge suppressions responsible for the holes near the  $l$  and  $\eta$  models. For holes along  $\theta_{E_6}$  near the  $l$  model, these regions maintain small  $Z^0$  charges for up quarks which suppress  $Z^0$  production from proton collisions. On the other hand, the regions near holes along the  $\eta$  model maintain small  $Z^0$  charges for leptons which in turn yield small production cross sections in the lepton channels. These findings are consistent with leptophobic studies within the  $\eta$  model [126,127]. In either case, the  $pp \rightarrow Z^0 \rightarrow l^+l^-$  production inside these holes is small enough to evade the cross-sectional upper bounds found by CMS.

In addition to direct searches, a  $Z^0$  gives rise to various corrections to the properties of the  $Z$  boson through  $Z - Z^0$  mixing parameter  $\xi$ , which is tightly constrained by precision  $Z$  boson measurements. As shown in Fig. 1, the  $Z - Z^0$  mixing required to explain the  $W$  mass measurement is well below  $10^{-3}$  for  $Z^0$  bosons heavier than 4 TeV. The combined fit for  $Z$ -pole observables put an upper bound on the  $Z - Z^0$  mixing parameter around  $3 \times 10^{-3}$  [140–143]. We have checked that the kinetic mixing introduced here did not lead to modifications beyond the current precision. For instance, a 6 TeV  $Z^0$  that explains the current  $W$  mass measurement in the  $\eta$  model has a deviation in the leptonic decay width of the  $Z$  boson of 0.0029%, which is within experimental uncertainties [10].

#### IV. HIGGS MASS

In SUSY models, the mass  $m_h$  of the Higgs boson is precisely predicted by a few relevant parameters, and can

be calculated through fixed-order calculations, effective field theory (EFT) calculations, and a hybrid calculation. Dominant three-loop contributions to  $m_h$  are known in the MSSM. (For  $m_h$  calculation in SUSY models, see [144] and references therein.) At the tree-level, the Higgs mass has an upper bound of  $M_Z \cos 2\beta$  in the MSSM. It receives substantial radiative corrections with the dominant contributions coming from loops involving the top, and its scalar partner, the stop, along with gluon and gluino exchanges. In particular, when the SUSY scale  $M_s$  is around 2 TeV and the stop mixing parameter is  $X_t \approx -\sqrt{6}M_s$ , a 125 GeV Higgs can be achieved with  $jH_u^0 j = jH_d^0 j \equiv \tan\beta \approx 10$  where  $jH_u^0 j$  is the vacuum expectation value (VEV) of the electrically neutral component of the doublet Higgs field  $H_u$ .  $M_s$  is predicted to be at least 10 TeV from  $m_h \approx 125$  GeV with a vanishing stop mixing parameter [144]. The current theoretical uncertainties in calculating  $m_h$  are around 2–3 GeV for the MSSM [144].

We propose extending the SM gauge group to explain the latest  $M_W$  measurement. Expanding into SUSY scenarios, when the MSSM is extended by an extra  $U(1)$  group there are additional contributions to the D-term [124],

$$\frac{g_{Z^0}^2}{2} \delta Q_{H_u}^0 jH_d^0 j^2 \mp Q_{H_u}^0 jH_d^0 j^2 \mp Q_S^0 jS j^2 \mp^2; \quad \delta 20 \mp$$

in which  $H_u$  and  $H_d$  are the Higgs doublets from the MSSM, and  $S$  is the singlet scalar field that breaks the new  $U(1)$  symmetry.  $Q_{H_u}^0$ ,  $Q_{H_d}^0$ , and  $Q_S^0$  are their corresponding charges under the  $U(1)$  group. We impose the charge-conserving relation  $Q_{H_u}^0 \mp Q_{H_d}^0 \mp Q_S^0 \approx 0$ , coming from the  $\lambda SH_u H_d$  term in the superpotential.  $E_6$  charges for the scalar fields are given in Table I. In addition to the new D-term contribution, the  $\lambda SH_u H_d$  term in the superpotential increases the upper bound of the Higgs mass as in the NMSSM [145,146]. Combining both contributions, the upper bound of the Higgs mass becomes [124]

$$m_h^2 \approx M_Z^2 \cos^2 2\beta \mp \lambda^2 v^2 \sin^2 2\beta \mp g_{Z^0}^2 v^2 \delta Q_{H_u}^0 \cos^2 \beta \mp Q_{H_u}^0 \sin^2 \beta \mp^2; \quad \delta 21 \mp$$

The increased tree-level Higgs mass implies that the stop sectors are much less constrained in the MSSM case.

To account for loop effects and the effects of the running couplings, we use FlexibleSUSY for our numerical analysis. FlexibleSUSY [147,148] is a Mathematica and C++ package for generating mass spectra of SUSY models. It includes 2-loop renormalization group equations (RGEs), it calculates  $m_h$  at the full 1-loop level, and it includes dominant corrections up to 3-loop, next to leading logarithms. The theoretical uncertainty in the UMSSM scenario calculated in FlexibleSUSY was estimated to be as large as 10 GeV [149]. The large uncertainty compared to the MSSM case is due to the altered RGEs in the  $E_6$  scenarios.

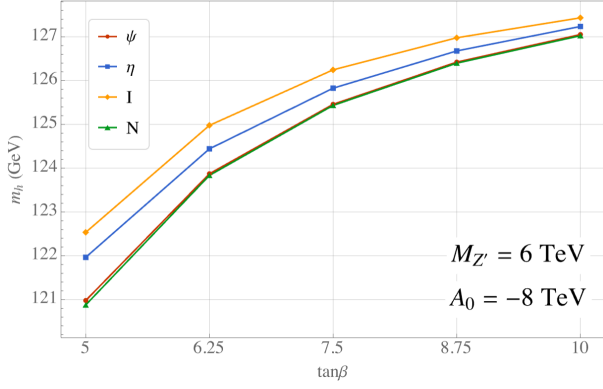


FIG. 3. Masses  $m_h$  of the lightest scalar mass eigenstate in different  $E_6$  models for the benchmark point in Section IV ( $m_0 \approx 7.9$  TeV,  $m_{1=2} \approx 1.2$  TeV,  $A_0 \approx -8$  TeV,  $\mu_{\text{eff}} \approx 200$  GeV,  $m_A \approx 2$  TeV) which satisfy the CDF II measurement.

For the MSSM-like parameters, we adopt a benchmark point motivated by the natural SUSY scenario [150], in which  $m_0 \approx 7.9$  TeV,  $m_{1=2} \approx 1.2$  TeV,  $\tan \beta \approx 10$ ,  $A_0 \approx -8$  TeV,  $\mu_{\text{eff}} \approx 200$  GeV, and  $m_A \approx 2$  TeV. With the new  $U(1)_{B-L}$  gauge group, there are two more free parameters: the gauge coupling  $g_{Z'}$  which we fix to be 0.46 from grand unification, and the VEV of the singlet field  $S$ ,  $|S| \approx \frac{2\mu_{\text{eff}}}{\sqrt{2}} = \lambda$ . For some  $\theta_E$ ,  $M_{Z'}$ , and kinetic mixing  $\chi$ ,  $|S|$  is specified, and therefore  $\lambda$  is specified. As seen in Fig. 2, for a given  $Z'$  mass and  $\theta_E$ , there are two solutions in the range  $-\pi \leq \chi \leq \pi$  that explain the new  $W$  mass measurement. We choose the solution for which  $|\chi| - \pi/2$  is maximized to avoid the diverging full width of the  $Z'$  boson at  $|\chi| \approx \pi/2$ .

Shown in Fig. 3 are predictions calculated by FlexibleSUSY for  $m_h$ , the mass of the lightest mass eigenstate within the model's scalar sector. We fix  $M_{Z'} \approx 6$  TeV and vary  $\tan \beta$ . As expected,  $m_h$  is increased compared to the MSSM case (125 GeV for the benchmark we chose), and it is reduced as  $\tan \beta$  decreases. In particular, for all  $E_6$  scenarios we consider,  $m_h$  is about 125 GeV for  $\tan \beta \approx 7$  in this benchmark. Conversely,  $m_h$  depends very weakly on  $M_{Z'}$  due to the following. The dependence of  $m_h$  on  $M_{Z'}$  is through  $\lambda \approx \frac{2\mu_{\text{eff}}}{\sqrt{2}} = |S|$ , which is suppressed by  $\sin^2 2\beta$ . Furthermore,  $M_{Z'}$  only depends on  $|S|$  weakly. The dominant contribution to the  $Z'$  mass is

$$M_{Z'}^2 \approx g_{Z'}^2 \delta Q_{H_u}^2 v^2 \sin^2 \beta + Q_{H_d}^2 v^2 \cos^2 \beta + Q_S^2 |S|^2 = \cos^2 \chi \delta 22p$$

(the full result is presented in the appendix). We found that heavier a  $Z'$  requires a larger kinetic mixing  $\chi$  to explain the CDF II  $W$  mass measurement. This increase in  $\chi$  yields a heavier  $Z'$  without requiring a large  $|S|$ .

Results for the  $\chi$  model are not shown in Fig. 3 because  $Q_S^2 \approx 0$  in this model. Equation (22) shows that a heavy  $Z'$

cannot be achieved with a vanishing  $Q_S^2$  unless the gauge coupling  $g_{Z'}$  is very large. In the above numerical analysis, we do not include the kinetic mixing contribution to the D-term in the  $m_h$  calculation. We have checked that it can lead to an up to 2 GeV shift in the Higgs mass at the tree-level. As discussed, when we adopt the benchmarks from the natural SUSY scenarios, in general, the predicted  $m_h$  is larger than 125 GeV. With the same set of parameters, we found the Higgs mass to be closer to 125 GeV with  $\tan \beta \approx 10$  and  $A_0 \approx 0$  across all  $E_6$  models discussed in this work. The possibility to accommodate a 125 GeV Higgs with small mixing in the stop sector is an encouraging feature, as the stop mixing is naturally small in minimal anomaly mediated SUSY models [151–161] and gauge mediated SUSY models [162–168].

In Table III, we present four benchmark points motivated by the natural SUSY scenario with various  $Z'$  masses. For each point, the  $W$  mass is at the central value of the CDF II measurement, and the  $Z'$  mass is allowed by current CMS results. We provide in Table III other SUSY parameters, the Higgs mass, and the particle spectrum of the benchmark points. For all four benchmarks, the gluino mass is 3.03 TeV, which is near the projected sensitivity of the HL-LHC [169,170]. In the first three benchmarks, a Higgs mass near 125 GeV is achieved by having a small  $A_0$ . In this case, BM1 shows that a reasonable Higgs mass can be achieved with large  $\tan \beta$ . BM4 represents the possibility of accommodating a 125 GeV Higgs by having a small  $\tan \beta$  and large mixing in the stop sector. For the electroweakino sector, as in the natural SUSY scenarios, the lightest chargino and the two lightest neutralinos are Higgsino-like. Those electroweakinos are produced with sizable rates at the LHC and can be searched for with a soft dimuon trigger, or a hard initial state radiation jet, or through the monojet channel [171–178]. The Higgsinos may also be

TABLE III. Four benchmark points with predicted  $M_W$  consistent with the CDF II measurement. For all four points,  $m_0 \approx 7.9$  TeV,  $m_{1=2} \approx 1.2$  TeV,  $\mu_{\text{eff}} \approx 200$  GeV, and  $m_A \approx 2$  TeV.

|                                                             | BM1 ( $\eta$ ) | BM2 ( $\eta$ ) | BM3 (N) | BM4 ( $\psi$ ) |
|-------------------------------------------------------------|----------------|----------------|---------|----------------|
| $\theta_E$ (rad)                                            | -0.91          | -0.91          | 1.32    | 1.57           |
| $M_{Z'}$ (TeV)                                              | 6              | 6.5            | 6.5     | 7              |
| $A_0$ (TeV)                                                 | 0              | -2             | 0       | -8             |
| $\tan \beta$                                                | 50             | 8              | 10      | 7              |
| $\chi$ (rad)                                                | 2.03           | 1.99           | -1.24   | -1.23          |
| $m_{\tilde{t}_1}$ (TeV)                                     | 4.86           | 4.76           | 4.65    | 3.39           |
| $m_{\tilde{\chi}_1^-}$ (GeV)                                | 193            | 244            | 253     | 254            |
| $m_{\tilde{\chi}_1^0}$ (GeV)                                | 187            | 236            | 245     | 247            |
| $m_{\tilde{\chi}_2^0}$ (GeV)                                | 198            | 250            | 259     | 261            |
| $m_h$ (GeV)                                                 | 126.51         | 125.20         | 126.03  | 124.95         |
| $\text{BR}(\tilde{Z}' \rightarrow  \tilde{b}  - \tilde{b})$ | 16.0%          | 16.1%          | 10.5%   | 12.5%          |
| $\Gamma_{Z'} = M_{Z'}$                                      | 10.0%          | 12.1%          | 20.7%   | 18.1%          |

accessible at lepton colliders [179]. Additionally, widths and leptonic branching ratios for the Z<sup>0</sup> are listed at the bottom of Table III with l being an electron or a muon.

## V. CONCLUSION

In summary, we point out a tree-level contribution to the W mass in supersymmetric E<sub>6</sub> models with kinetic mixing. The precision of the latest CDF II mass measurement of the W boson tightly restricts Z – Z<sup>0</sup> mixing to be  $\xi \lesssim \mathcal{O}(10^{-3})$  for Z<sup>0</sup> masses in the TeV range. When combined with the direct dilepton resonance searches at the LHC, further constraints are placed on the E<sub>6</sub> models. For example, we have checked that for M<sub>Z<sup>0</sup></sub> ≤ 5.5 TeV, θ<sub>E<sub>6</sub></sub> models are excluded at the 2σ level by the CDF II measurement and CMS searches at  $\sqrt{s} \approx 13$  TeV [133]. Moreover, we show how the HL-LHC run at 14 TeV is projected to further probe E<sub>6</sub> models for more massive Z<sup>0</sup> bosons. Additional calculations are made for the Higgs boson mass within a UMSSM. We find that a 125 GeV Higgs is possible within the reasonable parameter space allowed by experiments.

As for future directions, it will be interesting to study the associated phenomenology of the SUSY particles. With precision W boson and Higgs mass measurements, the scale and mixing of the stop sector can be predicted. Motivated by the natural SUSY scenario (in which the Higgsinos are light) we also expect rich phenomenology in the electroweakino sector. Thus, there is a complementarity between direct Z<sup>0</sup> searches, direct stop and electroweakino searches, and precision M<sub>W</sub> and m<sub>h</sub> measurements as they work together toward revealing new physics beyond the SM.

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## APPENDIX

### 1. Lagrangian

There are three scalar fields in our model: two Higgs doublets from the MSSM, and a singlet scalar field that breaks the new U(1)<sup>0</sup> symmetry. Their SU(2)<sub>L</sub> × U(1)<sub>Y</sub> × U(1)<sup>0</sup> group representations are

$$\begin{aligned} H_u &\sim (\mathbf{2}; 1/2; Q_{H_u}) \\ H_d &\sim (\mathbf{2}; -1/2; Q_{H_d}^0) \\ S &\sim (\mathbf{1}; 0; Q_S^0) \end{aligned}$$

The covariant derivative is

$$D_\mu \sim \partial_\mu - i\frac{1}{2}g_2 W_\mu^a T^a - ig_1 B_\mu Y - ig_{Z^0} Q_0$$

where: W<sub>μ</sub><sup>a</sup>, B<sub>μ</sub>, Z<sub>μ</sub><sup>0</sup> are the SU(2)<sub>L</sub>, U(1)<sub>Y</sub>, U(1)<sup>0</sup> gauge bosons; T<sup>a</sup>, Y, Q<sup>c</sup> are the group generators; a runs from 1 to 3.

After the electrically neutral components of the scalar fields acquire VEVs, their kinetic terms  $\bar{\psi} D_\mu \psi$  in the Lagrangian will yield mass terms for vector fields Z<sub>μ</sub>  $\sim$  c<sub>W</sub>W<sub>μ</sub><sup>3</sup> – s<sub>W</sub>B<sub>μ</sub> and Z<sub>μ</sub><sup>0</sup> shown below. The weak mixing angle θ<sub>W</sub> is defined by s<sub>W</sub>=c<sub>W</sub>  $\sim$  sin θ<sub>W</sub>=cos θ<sub>W</sub>  $\equiv$  g<sub>1</sub>=g<sub>2</sub>. The SM values for these parameters [10] are used throughout this study.

The relevant Lagrangian terms are given by

$$\begin{aligned} \mathcal{L} &\supset \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{current}}; \\ \mathcal{L}_{\text{mass}} &\sim -\frac{1}{2}m_Z^2 Z_\mu Z^\mu - \frac{1}{2}m_{Z^0}^2 Z_\mu^0 Z^{0\mu} - \Delta^2 Z_\mu Z^{0\mu} \\ \mathcal{L}_{\text{kinetic}} &\sim -\frac{1}{4}W_\mu^3 W^{3\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}Z_\mu^0 Z^{0\mu\nu} \\ &\quad - \frac{\sin \chi}{2} B_{\mu\nu} Z^{0\mu\nu} \\ \mathcal{L}_{\text{current}} &\sim -J_3^\mu W_\mu^3 - J_Y^\mu B_\mu - J^{0\mu} Z_\mu^0 \end{aligned}$$

where F<sub>μν</sub> is the field strength tensor for a gauge field F<sub>μ</sub>. The fermion currents are

$$\begin{aligned} J_3^\mu &\sim \frac{1}{2}g_2 \sum_i \bar{\psi}_i^\mu \frac{1}{2}T_{iL}^3 P_L + T_{iR}^3 P_R \psi_i \quad \delta A1 \\ J_Y^\mu &\sim \frac{1}{2}g_1 \sum_i \bar{\psi}_i^\mu \frac{1}{2}Y_{iL} P_L + Y_{iR} P_R \psi_i \quad \delta A2 \\ J^{0\mu} &\sim \frac{1}{2}g_{Z^0} \sum_i \bar{\psi}_i^\mu \frac{1}{2}Q_{iL}^0 P_L + Q_{iR}^0 P_R \psi_i \quad \delta A3 \end{aligned}$$

A group generator indexed by iL (iR) gives the corresponding charge of the left-handed (right-handed) component of fermion field ψ<sub>i</sub>. P<sub>L</sub> and P<sub>R</sub> are the left- and right-projection operators.

The terms in  $\mathcal{L}$  which are absent from the SM all involve the new Z<sub>μ</sub><sup>0</sup> gauge field. In particular,  $\mathcal{L}_{\text{kinetic}}$  and  $\mathcal{L}_{\text{mass}}$  contain mixing terms which must be negotiated to find the proper mass eigenstates within  $\mathcal{L}_{\text{current}}$ .

### 2. Kinetic mixing

Before diagonalizing the mass matrix, we must diagonalize the kinetic mixing between the U(1)<sub>Y</sub> and U(1)<sup>0</sup> gauge bosons. This can be done through the following GL(3; R) transformation:

$$\begin{pmatrix} 2 & 2 & 2 \\ \hat{W}_\mu^3 & \hat{W}_\mu^3 & 1 \\ \hat{B}_\mu & \hat{B}_\mu & 0 \end{pmatrix} \sim \frac{1}{\sqrt{1-\tan^2 \chi}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\tan \chi \\ 0 & 0 & 1/\cos \chi \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \quad \delta A4$$

where the hats indicate fields with canonical kinetic mixing terms. We redefine fields in an analogous way to the (neutral) electroweak sector of the SM:

$$W \equiv \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{A}_\mu & \hat{Z}_\mu \\ \hat{Z}_\mu^c & \hat{Z}_\mu \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{W}_\mu^3 & \hat{W}_\mu^0 \\ \hat{W}_\mu^0 & \hat{W}_\mu^3 \end{pmatrix} \end{pmatrix} \quad \text{A5p}$$

Equation (A4) may now be rewritten with the redefined fields:

$$-L_{\text{mass}} = \frac{1}{2} \begin{pmatrix} \hat{A}_\mu & \hat{Z}_\mu \\ \hat{Z}_\mu^c & \hat{Z}_\mu \end{pmatrix} \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{21}^2 & M_{22}^2 \end{pmatrix} \begin{pmatrix} \hat{A}_\mu \\ \hat{Z}_\mu \end{pmatrix} \quad \text{A6p}$$

### 3. Mass mixing

$L_{\text{mass}}$  admits a  $Z_\mu - Z_\mu^c$  mass matrix of

$$m_z^2 = \frac{1}{2} \begin{pmatrix} \Delta^2 & \Delta^2 \\ \Delta^2 & m_{z^0}^2 \end{pmatrix} \quad \text{A7p}$$

$$m_{z^0}^2 = \frac{1}{2} \left( g_Z^2 v_u^2 + g_Z^2 v_d^2 \right) \quad \text{A8p}$$

$$\Delta^2 = \frac{1}{2} \left( g_Z^2 v_u^2 - g_Z^2 v_d^2 \right) \quad \text{A9p}$$

where  $g_Z^2 = g_1^2 + g_2^2$  and  $jH_u^0 j; jH_d^0 j; jSj$  are the VEVs of the electrically neutral components of the scalar fields, which we parametrize as  $jH_u^0 j = v \sin \beta$ ,  $jH_d^0 j = v \cos \beta$  with  $v = 246$  GeV. Anomaly cancellation requires  $Q_{H_u}^0 = Q_{H_d}^0 = Q_S^0 = 0$ .

In the new basis  $\hat{A}_\mu; \hat{Z}_\mu; \hat{Z}_\mu^c$  of canonical kinetic terms, the mass matrix  $m^2$  is transformed according to the  $\hat{Z}_\mu; \hat{Z}_\mu^c$  subspace in Eq. (A6). The transformation from this subspace to the  $\hat{Z}_\mu; \hat{Z}_\mu^c$  subspace is given by

$$R = \begin{pmatrix} 1 & s_W \tan \chi \\ 0 & 1 = \cos \chi \end{pmatrix} \quad \text{A10p}$$

so that in this basis,  $m^2$  becomes

$$-L_{\text{mass}} = \frac{1}{2} \begin{pmatrix} \hat{Z}_\mu & \hat{Z}_\mu^c \end{pmatrix} \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{21}^2 & M_{22}^2 \end{pmatrix} \begin{pmatrix} \hat{Z}_\mu \\ \hat{Z}_\mu^c \end{pmatrix} \quad \text{A11p}$$

where the elements of  $M^2$  are

$$M_{11}^2 = \frac{1}{2} m_Z^2$$

$$M_{12}^2 = \frac{1}{2} M_{21}^2 = \Delta^2 = \cos \chi \left( m_Z^2 s_W \tan \chi \right)$$

$$M_{22}^2 = \frac{1}{2} m_{Z^0}^2 = \cos^2 \chi \left( m_Z^2 s_W^2 \tan^2 \chi \right) + 2 \Delta^2 s_W \tan \chi = \cos \chi :$$

This new mass matrix  $M^2$  may be diagonalized by an orthogonal matrix

$$O = \begin{pmatrix} \cos \xi & -\sin \xi \\ \sin \xi & \cos \xi \end{pmatrix} \quad \text{A12p}$$

and the  $Z - Z^0$  mixing angle  $\xi$  is given by

$$\tan 2\xi = \frac{2M_{12}^2}{M_{11}^2 - M_{22}^2} \quad \text{A13p}$$

or, in terms of the eigenvalues  $M_Z^2$  and  $M_{Z^0}^2$  of the matrix  $M^2$ ,

$$\sin 2\xi = \frac{2M_{12}^2}{M_Z^2 - M_{Z^0}^2} \quad \text{A14p}$$

The nonzero entries of the diagonal mass matrix  $O^T M^2 O$  are the tree-level squared masses of the observed  $Z$  boson and a new  $Z^0$ ,

$$M_Z^2 = \frac{1}{2} \left( M_{11}^2 + M_{22}^2 - \sqrt{(M_{11}^2 - M_{22}^2)^2 + 4M_{12}^2} \right)$$

$$M_{Z^0}^2 = \frac{1}{2} \left( M_{11}^2 + M_{22}^2 + \sqrt{(M_{11}^2 - M_{22}^2)^2 + 4M_{12}^2} \right) :$$

### 4. Mass eigenstates

After symmetry breaking,  $L_{\text{current}}$  becomes

$$L_{\text{current}} = -J_{\text{em}} A_\mu - J_Z^0 Z_\mu - J_{Z^0}^0 Z_\mu^0$$

where these new currents are given by

$$J_{\text{em}}^\mu \simeq \frac{g_1}{c_W} \sum_i c_W^X q_i \bar{f}_i \gamma^\mu f_i \quad \delta A15p$$

$$J_Z^\mu \simeq \frac{g_2}{c_W} \sum_i X_{iL} \bar{f}_i \gamma^\mu \frac{1}{2} Q_{ZiL} P_L + Q_{ZiR} P_R f_i \quad \delta A16p$$

with

$$Q_{Zi\delta L;R} \simeq T_{iL}^{B\delta L;R} - q_i s_W^2 \quad \delta A17p$$

where  $A_\mu \simeq s_W W_\mu^3 + c_W B_\mu$  and  $q_i \equiv T_{iL} + Y_{iL} \simeq T_{iR} + Y_{iR}$  is the electromagnetic charge of fermion  $f_i$  divided by the positron's charge.

Section A2 transforms  $L$  into a basis of fields with canonical kinetic mixing terms through the matrix  $WVW^{-1}$ . It is then illustrated in Sec. A3 how to transform  $L$  into a basis with a diagonal mass matrix using  $O$ . Here, we combine both of these transformations, allowing us to find the mass eigenstate basis:

$$\begin{aligned} -L_{\text{current}} &\simeq \frac{1}{2} J_{\text{em}}^\mu + J_Z^\mu + J_{Z^0}^\mu \begin{pmatrix} A_\mu \\ Z_\mu \\ Z_\mu^0 \end{pmatrix} \\ &\simeq \frac{1}{2} J_{\text{em}}^\mu + J_Z^\mu + J_{Z^0}^\mu WVW^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{A}_\mu \\ Z_{1\mu} \\ Z_{2\mu} \end{pmatrix} \\ &\equiv J_{\text{em}}^\mu \hat{A}_\mu + J_{Z1}^\mu Z_{1\mu} + J_{Z2}^\mu Z_{2\mu} \quad \delta A18p \end{aligned}$$

We identify the  $Z_{1\mu}$  eigenstate with the observed  $Z$  boson and the  $Z_{2\mu}$  eigenstate with a hypothetical  $Z^0$  boson. The currents coupled to these mass eigenstates are found through Eqs. (A15), (A16) as

$$J_H^\mu \simeq \sum_i f_i \bar{\psi}_i \gamma^\mu \frac{1}{2} Q_{niL} P_L + Q_{niR} P_R f_i \quad \delta A19p$$

with

$$\begin{aligned} Q_{1i\delta L;R} &\simeq g_1 c_W \frac{1}{2} - c_W \sin \xi \tan \chi q_i \\ &+ \frac{g_2}{c_W} \frac{1}{2} \cos \xi + s_W \sin \xi \tan \chi Q_{Zi\delta L;R} \\ &+ g_{Z^0} \frac{1}{2} \sin \xi = \cos \chi Q_{i\delta L;R}^{\ell} \quad \delta A20p \end{aligned}$$

$$\begin{aligned} Q_{2i\delta L;R} &\simeq g_1 c_W \frac{1}{2} - c_W \cos \xi \tan \chi q_i \\ &+ \frac{g_2}{c_W} \frac{1}{2} - \sin \xi + s_W \cos \xi \tan \chi Q_{Zi\delta L;R} \\ &+ g_{Z^0} \frac{1}{2} \cos \xi = \cos \chi Q_{i\delta L;R}^{B\delta L;R}; \quad \delta A21p \end{aligned}$$

The full transformation matrix of Eq. (A18) is

$$WVW^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & \Delta_A & \Delta_{AZ} & \Delta_{AZ^0} \\ \Delta_{ZA} & 1 & \Delta_Z & \Delta_{ZZ^0} \\ \Delta_{Z^0A} & \Delta_{Z^0Z} & 1 & \Delta_{Z^0} \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -c_W \sin \xi \tan \chi & -c_W \cos \xi \tan \chi \\ 0 & \cos \xi + s_W \sin \xi \tan \chi & -\sin \xi + s_W \cos \xi \tan \chi \\ 0 & \sin \xi = \cos \chi & \cos \xi = \cos \chi \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$$

and, following Ref. [132], the oblique parameters are given by

$$\alpha S \simeq 4c_W s_W \frac{1}{2} - \delta s_W - c_W \Delta_{AZ} - 2c_W s_W \Delta_A + 2c_W s_W \Delta_Z \quad \delta A22p$$

$$\alpha U \simeq -8s_W^2 \frac{1}{2} - c_W s_W \Delta_{AZ} + s_W^2 \Delta_A + c_W^2 \Delta_{Z^0} \quad \delta A23p$$

$$\alpha T \simeq 2\delta \Delta_Z - \hat{\Delta}_Z \quad \delta A24p$$

where  $\Delta_Z$  is the  $Z$  boson's fractional mass shift from its SM value, derived in Sec. II.

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