Electromagnetic Non-Contacting Position Estimation of a Centimetre-Scale Robot

N. Pushpalayam*, L. Alexander and R. Rajamani*****

** University of Minnesota, Twin Cities, Minneapolis, MN 55455, USA (e-mail: pushp008@umn.edu, PIN: 149442).*
*** University of Minnesota, Twin Cities, Minneapolis, MN 55455, USA (e-mail: alexa035@umn.edu, PIN: 12320).*
**** University of Minnesota, Twin Cities, Minneapolis, MN 55455, USA (e-mail: rajamani@umn.edu, PIN: 24626), Corresponding author*

Abstract: This paper develops an electromagnet-based position estimation system for a cm-scale robot with two degrees of freedom. The orientation of an external electromagnet is actively controlled in real-time to maximize the magnetic field magnitude at the robot. This results in a monotonic relationship between the magnetic field magnitude and radial distance leading to a simple and robust position estimation system. The radial distance is then estimated using an asymptotically stable nonlinear observer designed using a linear matrix inequality. The analytical principles of the estimation system are first presented using key technical lemmas and proofs. Experimental results are then presented on the verification of the analytical principles and on the performance of the position estimation system.

Keywords: position estimation, magnetic position sensing, nonlinear observer, robot position.

1. INTRODUCTION

This paper develops an electromagnet-based two-dimensional position estimation system. Key features of the proposed system are active rotational control of an electromagnet to highly simplify the position estimation problem and use of a nonlinear observer for position estimation. The key analytical principles of the position estimation system are presented and the performance of the developed system in finding the position of a mobile cm-scale robot is evaluated with experiments.

Humans have long used a magnetic compass to find the north pole during navigation for hundreds of years (Kreutz,1973). Based on the same principle, magnetic sensors are often used together with inertial measurement units (IMUs) to find "attitude" or 3-dimensional orientation of an object. The accelerometers on the IMU can measure the two static vertical angles based on the known downward direction of gravity, and the magnetic sensor can estimate the horizontal plane angle based on the known direction of earth's magnetic field, thus enabling estimation of all three angles (Markley & Crassidis, 2014). A large body of literature exists on estimation algorithms that utilize three-axes accelerometers, gyroscopes and magnetic sensors to find 3-dimensional orientation of a moving object in the presence of sensor bias errors, sensor noise and measurement disturbances from translational movements. (Crassidis et al, 2007; Zhang et al, 2012; Mahony et al, 2008; Wang & Rajamani, 2018; Wie, 2008; Choukroun, 2003; Wei & Wang, 2013).

In addition to attitude sensing, magnetic sensors have also been used for proximity sensing in many industrial and vehicle applications. For example, a permanent magnet installed on the piston is used to detect if a pneumatic actuator has reached its end of stroke (Nyce, 2003). Likewise, hall-effect magnetic sensors are used to measure the passing of a spoked wheel to measure the rotational speed of a wheel on all modern automobiles (Fraden, 2003). Here it should be noted that magnetic fields have mostly been utilized for linear position sensing only in the case where the movement between the magnet and magnetic sensor is very small (a few mm), using either hall-effect (Zhang et al,2016), or eddy current sensors (Sadler & Ahn, 2001). While some magnetic sensors, such as AMR and TMR sensors, are highly sensitive and can measure magnetic fields at large distances from the magnet, there is an inherent problem due to the nonlinear and non-monotonic nature of the magnetic field (Madson & Rajamani, 2018). The magnetic field of a magnet varies as a highly nonlinear function of position. Some papers have utilized nonlinear estimation techniques, such as the iterated extended Kalman Filter or the unscented Kalman filter for position estimation over large strokes of motion (Madson & Rajamani, 2018; Movahedi et al, 2021). However, most of these techniques have focused only on position measurement during one-dimensional motion.

This paper focuses on estimation of two-dimensional position of a robot or other object equipped with a magnetic sensor. The contributions of the paper include

1) The presentation of key principles enabling active orientation control of an electromagnet so as to maximize the real-time magnetic field magnitude at the sensor location.
2) Development of a position sensing system which does not rely on complex two-dimensional magnetic models but instead relies only on a simple one-variable model of magnetic field.

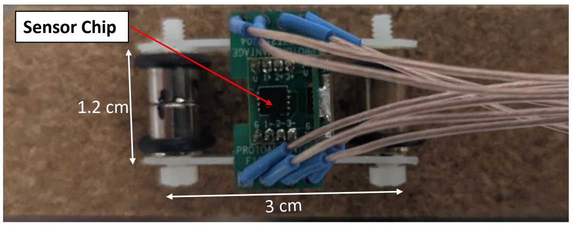

Fig.1. Centimeter-scale robot with the sensor chip

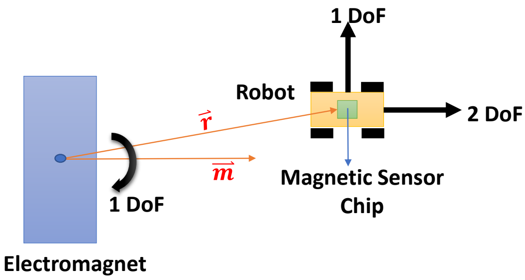

Fig.2.Schematic diagram of the 2D robot position estimation system

The outline of the paper is as follows. Section II presents the formulation of the position sensing problem and the configuration of the devices utilized in the position estimation system. Section III presents the fundamental principles and analytical results on which the magnetic position estimation system is based. Section IV presents the nonlinear observer design for the estimation process. Section V presents experimental verification of the analytical sensing principles and experimental results on position estimation of a moving cm-scale robot using the developed system. Finally, section VI presents a discussion of the results and conclusions of the paper.

2. DESCRIPTION OF MEASUREMENT SYSTEM

A photograph of the experimental robot for which the position estimation system is developed is shown in Figure 1. A schematic view of the variables in the 2D position estimation system is shown in Figure 2. The active measurement system consists of an electromagnet mounted on a motor and a small 2 axes magnetic sensor chip installed on the moving robot. For the purposes of this paper, the robot motion will be assumed to be limited to a 2D XY plane. The robot is assumed to have two degrees of freedom as shown in Figure 2. It can transverse along the longitudinal and lateral directions. The electromagnet generates an alternating magnetic field at 20 Hz, though any other frequency higher than the bandwidth of the robot movement can also be used.

The magnetic field vector, $\vec{B}$ of a dipole at a position vector $\vec{r}$ is given by:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{(3\vec{r}(\vec{m}.\vec{r}) - (\vec{r}.\vec{r})\vec{m})}{|r|^5} \quad (1)$$

where $\vec{m}$ is the magnetic dipole moment of the electromagnet and μ_0 is the relative magnetic permeability of the air. The variation of the amplitude of the magnetic field along the axial radial distance of the electromagnet is illustrated in Figure 3. The data in Figure 3 is experimentally obtained and the fitted curve is a 9th order polynomial function. As seen, the amplitude of the magnetic field is monotonically decreasing with radial distance.

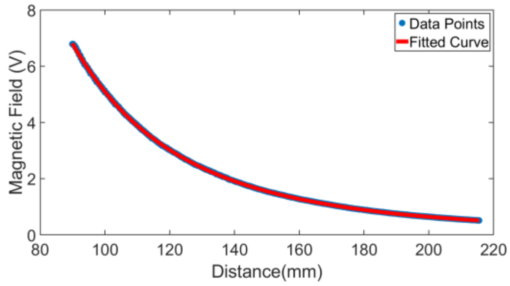

Fig.3 Experimental magnetic field measurement function showing magnetic field as a function of radial distance (1 V = 0.5 Gauss)

The 20 Hz alternating current is generated using a signal generator and then amplified using an audio amplifier before sending the signal to the electromagnet. The frequency of the alternating magnetic field should be sufficiently high compared to the frequency of motion of the robot. The generated magnetic field is sensed along two mutually perpendicular axes using two TMR sensors on the robot. The analog signal output from the magnetic sensor is amplified using an instrumental amplifier chip (INA2126) and the amplified signal is acquisitioned at a sampling frequency of 20 kHz.

The measured magnetic field at the robot location consists of the 20 Hz alternating magnetic field and additional low frequency magnetic fields generated by variety of other sources like the static earth's magnetic field, magnetic field produced by static ferromagnetic substances in the immediate surroundings etc. Hence, the sensed signal is filtered using a bandpass filter with cut off frequencies at 10 Hz and 30 Hz. Filtering enables the removal of both unwanted noise and bias from the measured magnetic field. The low pass corner frequency of the filter is high enough to be above of the bandwidth of the frequencies at which sensor or disturbing ferromagnetic objects move. A pure alternating signal with a zero-mean value at this pre-determined frequency is obtained after the band pass filtering.

A robust 1D radial position estimation system is developed using the dynamic amplitude of the alternating magnetic signal measured on the robot and a nonlinear observer-based position estimation algorithm. In polar coordinates, the magnetic field at the robot location (ρ, θ, z) is a function of ρ, θ and z (Figure 4). The vertical offset, z is a known constant parameter ($\approx$ 0 cm) in this 2 D estimation problem. The electromagnet is mounted on a NEMA 23 stepper motor so that its orientation can be controlled in real time. The electromagnet is continuously pointed at the magnetic sensor using an active control algorithm. Hence, θ of the position coordinate of the robot with respect to the electromagnet will be zero and helps to remove the θ dependence of the magnetic field at the robot location. Thus, the 2D position estimation problem is reduced to a 1 D position estimation problem by using active orientation control i.e., the magnetic field at the robot location will be a function of just the radial distance, ρ. The polar angle, θ is estimated from the encoder readings of the stepper motor and the radial distance of the robot from the electromagnet, ρ is estimated from the magnetic field measurements.

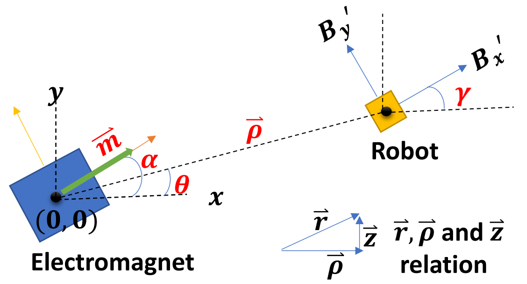

Fig.4. Schematic diagram of the electromagnet and the robot showing parameters that influence the magnetic field components

3. POSITION ESTIMATION PRINCIPLES

The components of the magnetic field due to the electromagnet, B_x' and $B_y{}'$, sensed by the magnetic sensor at a position vector $\vec{r}$ ($\vec{r} = \vec{\rho} + \vec{z}$) are determined by the parameters α, θ and γ shown in Figure 4. Here the variables in the figure can be described as follows:

- x, y: Global frame of reference for robot position
- x', y': Magnetic sensor frame of reference
- α: Angle subtended by the electromagnet to the X axis
- γ: Angle subtended by the magnetic sensor to the X axis
- $\vec{\rho}$: The position vector of the magnetic sensor with respect to the electromagnet in the XY plane
- θ: Angle subtended by the position vector, $\vec{\rho}$ to the x axis
- $\vec{m}$: Magnetic dipole moment vector of the electromagnet

The objective of the estimation problem is to estimate x, y (position coordinates)of the robot using the magnetic field sensor. This estimation problem is solved using the following lemmas. The lemmas are proved assuming that the electromagnet can be modelled using a dipole. However, the lemmas are also found to hold experimentally, when evaluated using an actual large electromagnet (not just a dipole).

Lemma 1: The magnitude of the magnetic field at the sensor location is maximum when the electromagnet is pointed exactly at the sensor.

Proof:

Rewrite $\vec{m}$, the dipole moment vector of the electromagnet, and $\vec{r}$, the position vector of the robot as a function of their components along the axes of the global frame of reference. Let $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ be the unit vectors along the x, y and z axes of the global frame of reference, respectively. Then

$$\vec{m} = |\vec{m}|(\cos(\alpha)\,\hat{\imath} + \sin(\alpha)\,\hat{\jmath}) \quad (2)$$

$$\vec{r} = |\vec{\rho}|(\cos(\theta)\,\hat{\imath} + \sin(\theta)\,\hat{\jmath}) + z\,\hat{k} \quad (3)$$

$$\vec{\rho} = x\hat{\imath} + y\hat{\jmath} \quad (4)$$

Here z is the constant offset distance ($\approx$ 0 cm) between the horizontal planes at which the electromagnet resides and the robot resides. Substituting (2) and (3) in equation (1) and rearranging,

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k} \quad (5)$$

$$B_x = K\frac{|(3|\vec{\rho}|^2\cos(\alpha-\theta)(\cos(\theta) - |\vec{r}|^2\cos(\alpha))}{|\vec{r}|^5} \quad (6)$$

$$B_y = K\frac{(3|\vec{\rho}|^2\cos(\alpha-\theta)(\sin(\theta) - |\vec{r}|^2\sin(\alpha))}{|\vec{r}|^5} \quad (7)$$

$$B_z = K\frac{3|\vec{\rho}|^2\cos(\alpha-\theta)z}{|\vec{r}|^5} \quad (8)$$

where $K = \frac{\mu_0|\vec{m}|}{4\pi}$ and μ_0 is the relative magnetic permeability of the air. Rewrite the magnetic field at the robot location in the magnetic sensor frame of reference. Let $\hat{\imath}', \hat{\jmath}'$ and $\hat{k}'$ be the unit vectors along the x, y and z axes of the magnetic sensor frame of reference, respectively. Then

$$\vec{B} = B_{x'}\hat{\imath}' + B_{y'}\hat{\jmath}' + B_{z'}\hat{k}' \quad (9)$$

$$B_{x'} = B_x\cos(\gamma) + B_y\sin(\gamma) \quad (10)$$

$$B_{y'} = -B_x\sin(\gamma) + B_y cos(\gamma) \quad (11)$$

$$B_{z'} = B_z \quad (12)$$

Substituting (6) and (7) in the equations (10) and (11) and rearranging,

$$B_{x'} = K\frac{3|\vec{\rho}|^2\, cos(\alpha-\theta)\cos(\theta-\gamma) - |\vec{r}|^2 cos(\gamma-\alpha)}{|\vec{r}|^5} \quad (13)$$

$$B_{y'} = K\frac{3|\vec{\rho}|^2\, cos(\alpha-\theta)\sin(\theta-\gamma) - |\vec{r}|^2 sin(\gamma-\alpha)}{|\vec{r}|^5} \quad (14)$$

The horizontal component of the magnetic field, $\vec{B}_H$ is:

$$\overrightarrow{B_H} = B_{x'}\hat{\imath}' + B_{y'}\hat{\jmath}' \quad (15)$$

$$|\overrightarrow{B_H}| = \sqrt{(B_{x'})^2 + (B_{y'})^2} \quad (16)$$

Substituting (13) and (14) in the equation (15),

$$|\overrightarrow{B_H}| = \sqrt{\frac{K^2\left((9|\vec{\rho}|^4 + 6|\vec{\rho}|^2|\vec{r}|^2)\cos^2(\alpha-\theta) + |\vec{r}|^4\right)}{|\vec{r}|^{10}}} \quad (17)$$

Thus, $|\overrightarrow{B_H}|$is maximum for any given γ, when $\alpha = \theta$. When $\alpha = \theta$, the electromagnet is pointed exactly at the magnetic sensor. Hence, the magnitude of the magnetic field at the robot is maximum when the electromagnet is pointed to it, which implies the magnitude of the magnetic field at a point is maximum when the magnetic moment vector $\vec{m}$ is parallel to the position vector $\vec{\rho}$.

Lemma 2: When the electromagnet is pointed at the robot (sensor), the radial distance of the robot (sensor) from the electromagnet can be obtained using a monotonic algebraic relationship between the magnetic field and radial distance.

Proof:

When the electromagnet is pointed at the robot (sensor), $\alpha = \theta$. Substituting $\alpha = \theta$ and $|\vec{r}|^2 = |\vec{\rho}|^2 + z^2$ in the equation (17), gives $|\overrightarrow{B_H}|_{max}$ for a given $\vec{\rho}$ and z.

$$\left|\overrightarrow{B_H}\right|_{max} = \sqrt{\frac{K^2(9|\vec{\rho}|^4 + 6|\vec{\rho}|^2(|\vec{\rho}|^2 + z^2) + (|\vec{\rho}|^2 + z^2)^2)}{(|\vec{\rho}|^2 + z^2)^5}} \quad (18)$$

We assume z as a known constant vertical offset distance between the two horizontal planes as explained earlier, hence $\left|\overrightarrow{B_H}\right|_{max}$ is a univariate function of $|\vec{\rho}|$. Further analysis enables to identify that $\left|\overrightarrow{B_H}\right|_{max}$is a monotonically decreasing function of the radial distance $|\vec{\rho}|$ in the interval $|\vec{\rho}| = \left[\frac{z}{2}, \infty\right)$ ($z \approx 0\ cm$ in this estimation process).

4. NONLINEAR OBSERVER DESIGN

The 2D position estimation problem consists of:

- Active rotation control of the electromagnet to maximize the magnetic field at the robot location
- Estimation of the radial distance of the robot from the electromagnet using a nonlinear observer

This section discusses the nonlinear observer design for estimation of the kinematic states of the robot radial distance. The kinematics of the robot can be modelled as a constant radial acceleration motion with unknown jerk input in a polar coordinate system with the electromagnet at the center. The motor with the electromagnet rotates actively to compensate for the motion of the robot, hence from the electromagnet frame of reference, the electromagnet is always pointing at the robot and the robot moves only in the radial direction. The system model for radial motion is:

$$\dot{x} = Ax + Bu \quad (19)$$

where x is the vector of kinematic states of the robot i.e $x = [\rho; \dot{\rho}, \ddot{\rho}]^T$ where ρ is the radial distance of the of the robot from the electromagnet and u is the unknown jerk noise input, A and B are defined as:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \; B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (20)$$

The unknown jerk could be considered to be zero in a deterministic estimation framework, or as zero mean Gaussian noise in a stochastic framework. It should be noted that the use of a zero radial jerk model underlies an assumption that the acceleration changes slowly (i.e., its derivative is small).

The measurement function y is the magnitude of the magnetic field generated by the electromagnet at the robot position. It is a function of radial distance of the robot position from the electromagnet. Hence the measurement function can be represented as:

$$y = h(Cx) \quad (21)$$

where $C = [1\ 0\ 0]$ and $h(\rho)$ is a monotonic nonlinear model of the magnitude of the magnetic field. $h(\rho)$ is a function of radial distance ρ of the robot from the electromagnet and $\rho = Cx$. The plot in Figure 3 shows the measurement function $h(\rho)$, the magnitude of the magnetic field as a function of radial distance ρ from the electromagnet. It can be seen that, the measurement function is monotonically decreasing in our region of interest (ρ = 90 mm to ρ = 215 mm).

Further, let M be the lower bound of the partial derivative of $h(Cx)$ with respect to radial distance and N be the upper bound of the same partial derivative:

$$M \leq \frac{\partial h(Cx)}{\partial (Cx)} \leq N \quad (22)$$

Let the observer be given by:

$$\dot{\hat{x}} = A\hat{x} + Bu + L\big(h(Cx) - h(C\hat{x})\big) \quad (23)$$

where L is the observer gain. Let the estimation error be $\tilde{x} = x - \hat{x}$. Then the estimation error dynamics will be:

$$\dot{\tilde{x}} = A\tilde{x} - L\big(h(Cx) - h(C\hat{x})\big) \quad (24)$$

Note that the estimation error dynamics (24) are nonlinear. Theorem 1 below describes how to choose L to stabilize this nonlinear system.

Theorem 1. If an observer gain L, a diagonal matrix $\Gamma > 0$ and a symmetric positive definite matrix $P > 0$ that satisfy inequality (25) can be obtained, then the observer (23) with this observer gain is globally exponentially stable.

$$\begin{bmatrix} A^TP + PA - \frac{C^TM^T\Gamma NC + C^TN^T\Gamma MC}{2} + \sigma P & -PL + \frac{C^T(M^T + N^T)\Gamma}{2} \\ -L^TP + \frac{\Gamma(MC + NC)}{2} & -\Gamma \end{bmatrix} \leq 0 \quad (25)$$

The proof of theorem 1 can be obtained by modification of the Corollary 2.1 from the reference (Rajamani, et al, 2020). Hence solving the LMI in the equation (25), provides a suitable observer gain L, which ensures the exponential stability of the observer given in (23).

5. EXPERIMENTAL VERIFICATION OF ESTIMATION PRINCIPLES

This section provides the experimental results for verifying Lemma 1 and Lemma 2. Lemma 1 states that the magnetic field at the robot is maximum when the electromagnet is pointed at the robot. To verify this lemma, the electromagnet and the robot are arranged as show in the schematic diagram in Figure 5 and the electromagnet angle (α) is changed from 90 degree to -90 degrees. The electromagnet is positioned at an angle of 90 degrees from the magnetic sensor at time, t=0 as shown in Figure 5.

The magnitude of the magnetic field at the robot location varies with the electromagnet angle, α as shown in Figure 6. From the graph in Figure 6, it can be concluded that $\left|\overrightarrow{B_H}\right|$ is maximum when α reaches zero. At $\alpha = 0$, $\alpha = \theta$ and the electromagnet points at the robot. Thus, the fact that the magnetic field is maximum when the electromagnet is pointed at the robot is found to be true for a real-world electromagnet, not just a theoretical dipole. This property aids in designing a control algorithm which controls the electromagnet angle in real-time to point at the magnetic sensor.

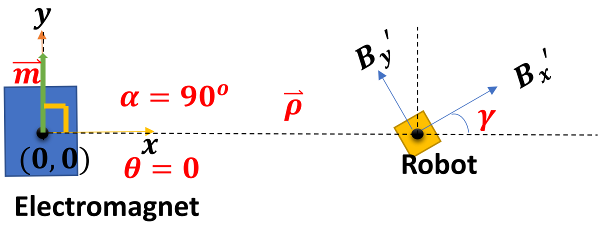

Fig.5.Schematic diagram of the electromagnet and the robot showing initial electromagnet angle for the electromagnet angular variation experiment

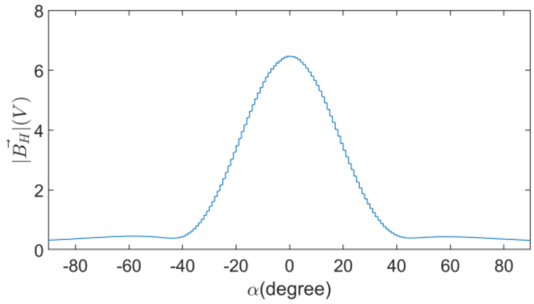

Fig.6. Angular relationship between magnetic field at robot and pointing angle of the electromagnet

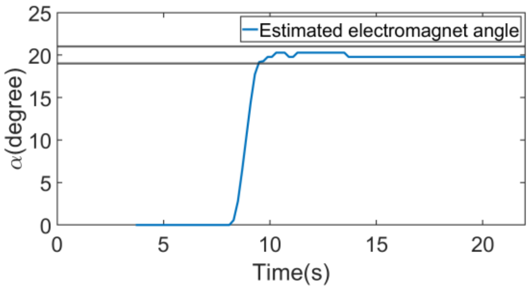

Fig.7. Plot of the α(electromagnet angle) control: horizontal lines are 19^0 and 21^0

The control algorithm involves a search of the α at which the $|\overrightarrow{B_H}|$ is maximum, say α^* and the α is controlled for the setpoint α^*. The plot in Figure 7 shows the control of the α for pointing the electromagnet at the robot for a random α and θ initialization. The electromagnet in the experiment is aligned such that $\alpha = 0$ and the robot position sensor $\vec{\rho}$, subtends an angle $\theta = 20^0$ with the x axis of the global frame of reference. A proportional controller is designed which enables the electromagnet angle α to converge to the α^* value of 20^0. The feedback error term for the proportional controller is the rate of change in the magnitude of the magnetic field with respect to the change in the electromagnet angle $\left(\frac{d|\overrightarrow{B_H}|}{d\alpha}\right)$. The feedback of $\frac{d|\overrightarrow{B_H}|}{d\alpha}$ helps to find the direction of change in α at which $|\overrightarrow{B_H}|$ increases by driving $|\overrightarrow{B_H}|$ to its maximum value so that $\frac{d|\overrightarrow{B_H}|}{d\alpha} \rightarrow 0$. Thus, the proportional controller manipulates α and converges it to α^*, the electromagnet angle at which the magnitude of the magnetic field $|\overrightarrow{B_H}|$ is maximized. This is the principle behind the control algorithm. The plot in Figure 7 shows that the control algorithm is able to control the electromagnet angle α, and converges it to α^* with an error bound of +/- 1^0. Thus, the angle at which the robot is located with respect to the electromagnet position (polar angle) can be estimated.

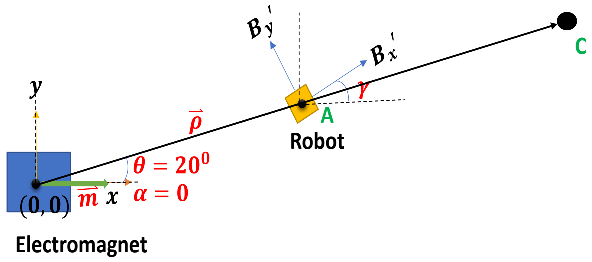

Fig.8. Radial distance estimation experiment schematic diagram

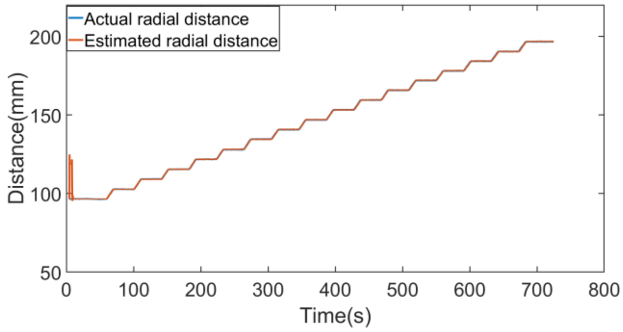

Fig.9. Radial distance estimation results

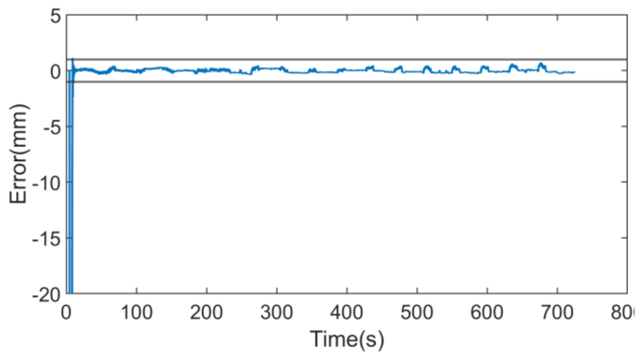

Fig.10. Error in the radial distance estimation of the robot: horizontal lines are +/- 1 mm

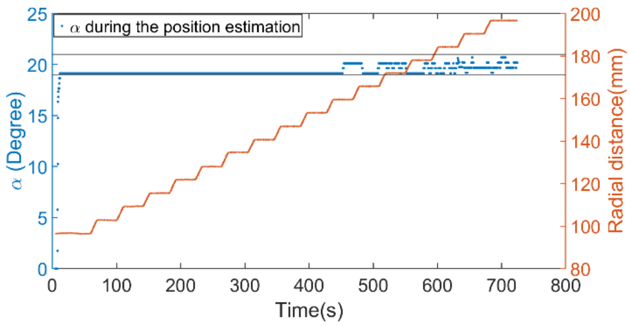

Fig.11. Electromagnet angle estimation during the radial distance estimation of the robot: horizontal lines are 19^0 and 21^0

Lemma 2 states that the radial distance of the robot from the electromagnet can be estimated from the magnetic field using a nonlinear observer. Experimental results of the radial distance estimation of the robot from the electromagnet is provided in Figure 9. In this experiment, the robot is initialized at a random position and the robot is moved along the radial direction as shown in Figure 8 (the robot is moved from point A to point C along the radial direction). The active control algorithm enables the electromagnet to continuously point at the robot and estimate the radial distance of the robot from the electromagnet. The error in the radius estimation is provided in Figure 10. The error is computed using a reference distance measurement measured by a laser displacement measurement sensor (Banner Engineering laser displacement sensor). The electromagnet angle (α) is estimated from the encoder readings of the motor on which the electromagnet is mounted. The electromagnet angle estimation results during the above experiment are provided in Figure 11.

It can be seen that radius estimation is possible with sub millimeter accuracy (Figure 10) using the magnetic field and a nonlinear observer. As discussed earlier, an active control algorithm enables the continuous pointing of the magnet at the robot. In the above experiment, the robot is travelling along a straight line at an angle (θ) of 20^0 from the origin. The results in Figure 11 shows that the active control algorithm is able to control the electromagnet angle (α) to continuously point at the robot ($\alpha = \theta$) with an accuracy of +/-1^0.

6. CONCLUSIONS

This paper developed an electromagnet based 2 D position estimation system for a robot travelling in a 2 D plane. The advantages of this estimation technique include low-cost, non-contacting operation, easy installation, robustness to magnetic disturbances and the use of a simple 1 D magnetic field map for position estimation instead of a complex 2D magnetic field map. In this estimation technique, the electromagnet is constantly pointed at the robot using a proportional controller. Then the radial position of the robot is estimated using a nonlinear observer and the electromagnet angle is estimated from the encoder data of the motor on which the electromagnet is installed. The radial distance of the robot from the electromagnet is shown to be estimated with a submillimeter accuracy and the angle at which the robot is positioned with respect to the origin is estimated with an accuracy of 1^0.

ACKNOWLEDGMENTS

This work was funded in part by a research grant from the National Science Foundation (NSF Grant EFMA 1830958).

REFERENCES

D. Choukroun, “Novel methods for attitude determination using vector observations,” Technion - Israel Institute of Technology, 2003.

J. L. Crassidis; F. L. Markley and Y. Cheng (2007). A survey of nonlinear attitude estimation methods. Journal of Guidance, Control, and Dynamics, 30(1):12{28, 2007.

J. Fraden, Handbook of Modern Sensors, Physics, Design and Applications, 3rd Edition, Springer, ISBN 0-387-00750-4, 2003.

B. M. Kreutz, “Mediterranean Contributions to the Medieval Mariner’s Compass,” Technology and Culture, vol. 14, no. 3, pp. 367–383, 1973.

R. Madson and R. Rajamani, “Magnetic Position Estimation in Ferromagnetic Systems Involving Significant Hysteresis,” IEEE/ASME Transactions on Mechatronics, Vol. 23, No. 4, pp. 1555-1563, August 2018.

R. Mahony; T. Hamel and J. M. Pimlin (2008). Nonlinear complementary filters on the special orthogonal group. IEEE Transactions on Automatic Control, 53(5):1203{1218, 2008.

F. L. Markley and J. L. Crassidis, Fundamentals of Spacecraft Attitude Determination and Control, vol. 33. New York: Springer, Springer New York, 2014.

H. Movahedi, A. Zemouche and R. Rajamani, “Magnetic Position Estimation Using Optimal Sensor Placement and Nonlinear Observer for Smart Actuators,” Control Engineering Practice, Vol. 112, Article 104817, July 2021.

D. S. Nyce, Linear Position Sensors, John Wiley & Sons, ISBN 9780471474289, 2003.

R. Rajamani, W. Jeon, H. Movahedi and A. Zemouche, 2020. On the need for switched-gain observers for non-monotonic nonlinear systems. Automatica, 114, p.108814.

D. J. Sadler and C. H. Ahn, “On-chip eddy current sensor for proximity sensing and crack detection,” Sensors and Actuators A: Physical, vol. 91, no. 3, pp. 340–345, Jul. 2001.

Y. Wang and R. Rajamani, “Direction cosine matrix estimation with an inertial measurement unit,” Mech. Syst. Signal Process., vol. 109, pp. 268–284, Sep. 2018.

B. Wie, Space Vehicle Dynamics and Control, Second Edition. Reston, VA: American Institute of Aeronautics and Astronautics, 2008.

L. Wei and J. Wang (2013). E_ective adaptive kalman _lter for mems-imu/magnetometers integrated attitude and heading reference systems. Journal of Navigation, 66:99{113, 2013.

Z. Q. Zhang; X. L. Meng and J. K. Wu (2012). Quaternion-based Kalman filter with vector selection for accurate orientation tracking. IEEE Transactions on Instrumentation and Measurement, 61(10):2817{2824, 2012.