High Energy Physics – Theory

Axion-polaritons in the magnetic dual chiral density wave phase of dense QCD

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Abstract

We investigate the propagation of electromagnetic radiation in the magnetic dual chiral density wave (MDCDW) phase of dense quark matter. Considering the theory of low-energy fluctuations in this phase, we show how linearly polarized photons reaching the MDCDW medium couple to the fluctuation field to produce two hybridized modes of propagation that we call in analogy with similar phenomenon in condensed matter physics axion polaritons, one of them being gapless and the other gapped. The gapped mode's gap is proportional to the background magnetic field and inversely proportional to the amplitude of the inhomogeneous condensate. The generation of axion polaritons can be traced back to the presence of the chiral anomaly in the low-energy theory of the fluctuations. Considering the Primakoff effect in the MDCDW medium, we argued that axion polaritons can be generated inside quark stars bombarded by energetic photons coming from gamma-ray bursts and point out that this mechanism could serve to explain the missing pulsar paradox in the galaxy center.

1. Introduction

In recent years, many efforts have been dedicated to completing the temperature-density phase map of QCD. The regions of extremely high temperature or density are better understood thanks

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to the weakening of the strong coupling by the phenomenon of asymptotic freedom. They are described by the quark-gluon plasma (QGP) phase at high temperature and low density or by the color superconducting color-flavor locked (CFL) phase at asymptotically large density and low-temperature [1]. More challenging, nonetheless, is to determine the phases in the intermediate density-temperature regions, where lattice QCD is not applicable due to the sign problem, so one has to rely on nonperturbative methods and effective theories.

It has long been argued that the region of intermediate density and relatively low temperature may feature inhomogeneous phases, many of which have spatially inhomogeneous chiral condensates favored over the homogeneous ones. Such spatially inhomogeneous phases have been found in the large-N limit of QCD [2,3], in NJL models [4–7], and in quarkyonic matter [8,9]. In all the cases, chiral condensates with single-modulation are energetically favored over higher-dimensional modulations. However, single-modulated phases in three spatial dimensions are known to be unstable against thermal fluctuations, a phenomenon known in the literature as Landau-Peierls instability [10]. In dense QCD models, the Landau-Peierls instability occurs in the periodic real kink crystal phase [11]; in the Dual Chiral Density Wave (DCDW) phase [12], and in the quarkyonic phase [13]. The instability signals the lack of long-range correlations at any finite temperature, hence the absence of a true order parameter. Only a quasi long-range order remains in all these cases, a situation that resembles what happens in smectic liquid crystals [14].

Furthermore, magnetic fields are a common feature in the scenarios where quark matter phases under extreme conditions are realizable (neutron stars (NS) [15,16] and heavy-ion collisions [17]). Hence, investigating the field effects on quark matter phases has become a hot topic of research (see [18,19] and references therein), thereby adding an extra dimension to the QCD phase map. The effect of a magnetic field in inhomogeneous phases has been explored in quarkyonic matter [9] and in the magnetic dual chiral density wave (MDCDW) phase [20–23]. Among other things, a magnetic field may decrease the symmetry of the original theory and activate new channels of interaction, which in turn can generate additional condensates. For instance, in the quarkyonic phase, a magnetic field is responsible for the appearance of a new chiral spiral between the pion and magnetic moment condensates [9]. Additional condensates also emerge in the homogeneous chiral phase [24] and in color superconductivity [25,26].

In the case of the MDCDW phase, the external magnetic field decreases the original global symmetries of the theory and contributes to critical topological effects. First of all, it induces an asymmetry in the spectrum of the lowest Landau level (LLL) [20]. The asymmetry, in turn, gives rise to a topological term in the thermodynamic potential that significantly enhances the window of inhomogeneity [20,27]. Furthermore, in the presence of an electric field with a nonzero component in the direction of the background magnetic field, new topological effects emerge due to the lack of invariance of the path-integral fermion measure under the local chiral transformation. This lack of invariance gives rise to an ill-defined Jacobian that requires a proper regularization [21,22]. The required regularization procedure was carried out in [22] by using Fujikawa's method [28] and a representation of the fermion Jacobian in terms of a set of complete and orthogonal eigenfunctions of the Dirac operator that diagonalize the fermion action and ensure unitarity. Using this approach [22], it was extracted, in a gauge-invariant way, the regularized contribution to the effective action, which turned out to be the chiral anomaly in the electromagnetic sector $(\kappa/8)\theta F_{\mu\nu}^{*} F^{\mu\nu}$. This interaction couples the electromagnetic strength tensor and its dual to $\theta = qz$, with q the condensate modulation and z the spatial coordinate in the direction of the modulation. This chiral-anomalous term leads to anomalous electric transport properties. The difference in the symmetry group between the DCDW and the MDCDW phases and the

topological nature of the latter make them very different and physically distinguishable, despite both phases having the same type of inhomogeneous condensate.

In addition, as shown in Ref. [29], the magnetic field allows the formation of new structures in the generalized Ginzburg-Landau (GL) expansion of the MDCDW phase that are not present in the DCDW case. These structures are consistent with the symmetry group that remains after the explicit breaking of the rotational and isospin symmetries by the external magnetic field. When considering the phonon fluctuations about the inhomogeneous condensate, these new field-induced structural terms, whose nonzero coefficients can be traced back to the presence of the LLL asymmetric spectrum, give rise to a linear transverse mode in the spectrum of the fluctuations. The stiffness of the fluctuation's energy dispersion in the direction perpendicular to the modulation, prevents the existence of the Landau-Peierls (LP) instability in the MDCDW phase. This is in sharp contrast to the DCDW case where the fluctuation spectrum is soft in the transverse direction and, as a consequence, the system exhibits the LP instability.

The results discussed in [29] were found assuming that the only electromagnetic field in the system was the background magnetic field. However, as mentioned above, if there is an electric field with a component parallel to the background magnetic field, the free-energy will contain a chiral anomaly term. Under such circumstances, some important questions emerge: Would this anomaly affect the theory of the fluctuations in the MDCDW phase? if it does, what would be the physical consequences? The goal of the present paper is to investigate these questions. To do that, we will add electromagnetic waves to the mix to explore the consequences of matter-light interactions for the low-energy theory of fluctuations in the MDCDW phase.

As shown below, in the presence of the background magnetic field, the fluctuation of the axion field, which is proportional to the phonon, linearly couples to the electric field of the electromagnetic wave via the chiral anomaly. This coupling leads to two hybridized propagating modes of coupled axion and photon fields that we call, inspired by condensed matter analogies, axion polaritons (AP). One hybridized mode is gapped with a gap proportional to the magnetic field and inversely proportional to the magnitude of the inhomogeneous chiral condensate. A similar linear coupling between an axion field and a photon in the presence of a magnetic field has been found in topological magnetic insulators. There, the time-reversal symmetry is spontaneously broken by an antiferromagnetic order, and the magnetic fluctuations couple to an axion field that depends on the band structure [30,31].

On the other hand, the MDCDW phase exhibits three characteristics that are significant for the astrophysics of NS: First, the critical temperature needed to evaporate the inhomogeneous condensate for fields $\sim 10^{18}$ G is beyond the stellar temperatures for the whole range of densities characteristic of relatively old magnetars [32]; second, it was proved in [33] that the maximum stellar mass of a hybrid star with a quark-matter core in this phase satisfies the maximum mass observation constraints ($M \gtrsim 2M_\odot$) [34,35]; and third, in [36] it was shown that the heat capacity of a star with MDCDW matter in its core will be well above the lower limit expected for NS ($C_V \gtrsim 10^{36}(T/10^8)$ erg/K) [37].

Hence, the robustness of the MDCDW phase at finite temperature makes it a viable candidate for the interior phase of magnetars. Along this line, we shall discuss here a possible consequence for the astrophysics of magnetars based on the conversion of energetic γ-photons into gapped axion polaritons via the so-called Primakoff effect [38]. In particular, we will point out that the discussed polariton effect could serve as an explanation for the so-called missing pulsar problem in the galaxy center (GC).

The paper is organized as follows: In Sec. 2, we introduce the Lagrangian density that models the phonon-photon interaction in the MDCDW medium. In Sec. 3, we study the effect of the

phonon fluctuations on an electromagnetic wave propagating into the MDCDW medium. With this goal in mind, we solve the corresponding axion-electrodynamic equations of the neutral dense medium. As a consequence, we find two hybridized modes, one gapped and one gapless, that are identified with similar axion-polaritons modes that appear in condensed matter physics in the so-called topological magnetic insulators. In Sec. 4, diagonalizing the effective Lagrangian density that includes the photon-phonon interaction in the MDCDW medium, we obtain the axion-polariton eigenfields which are complex pseudo-scalar field. Furthermore, new conservation laws for the axion-polariton particle number and the corresponding Noether four-current in the weak-field approximation are obtained. Using these results, we point out a possible astrophysical application of the transmutation of photons into AP. Finally in Sec. 5, we present our concluding remarks, and in the Appendix, we discuss the properties of the gauge fixing conditions used in the calculations.

2. Photon-phonon interaction in the MDCDW phase

In the absence of photons, the low-energy theory of fluctuations in the MDCDW phase is given by the phonon Lagrangian density [29]

$$\mathcal{L}_\theta = \frac{1}{2}[(\partial_0\theta)^2 - v_z^2(\partial_z\theta)^2 - v_\perp^2(\partial_\perp\theta)^2], \tag{1}$$

conveniently expressed in terms of the axion field $\theta = mqu(x)$ that is proportional to the phonon fluctuation $u(x)$ [29].

The coefficients

$$v_z^2 = a_{4,2} + m^2 a_{6,2} + 6q^2 a_{6,4} + 3q b_{5,3}, \tag{2}$$

and

$$v_\perp^2 = a_{4,2} + m^2 a_{6,2} + 2q^2 a_{6,4} + q b_{5,3}, \tag{3}$$

represent the group velocities (squares) in the directions parallel and transverse to the modulation respectively. They produce an anisotropy in the low-energy theory that reflects the breaking of the rotational symmetry in the MDCDW phase.

The dynamical parameters m and q in (2)-(3) are solutions of the stationary conditions

$$\partial\mathcal{F}/\partial m = 0, \quad \partial\mathcal{F}/\partial q = 0 \tag{4}$$

with $\mathcal{F}$ the free-energy of the MDCDW GL expansion [29]

$$\begin{aligned} \mathcal{F} = a_{2,0}m^2 + b_{3,1}qm^2 + a_{4,0}m^4 + a_{4,2}q^2m^2 + b_{5,1}qm^4 \\ + b_{5,3}q^3m^2 + a_{6,0}m^6 + a_{6,2}q^2m^4 + a_{6,4}q^4m^2, \end{aligned} \tag{5}$$

where the coefficients a and b are functions of temperature, T, baryonic chemical potential, μ, and magnetic field, B. They can be found from the thermodynamic potential of the theory [22]. In our notation, the first subindex indicates the total order of the term (power of the order parameter plus derivatives), and the second subindex denotes the number of derivatives in that term. An explicit calculation of the coefficients [32] showed that the power series in q effectively becomes an expansion in powers of $q/2\mu$.

Using (1), the low-energy spectrum for the axion fluctuation in the absence of photons is

$$\epsilon \simeq \sqrt{v_z^2 k_z^2 + v_\perp^2 k_\perp^2}, \tag{6}$$

with $k_\perp^2 = k_x^2 + k_y^2$. Noticeably, it is linear in both parallel and transverse momenta, a property that ensures the lack of Landau-Peierls instability [29] in the MDCDW phase. This is in sharp contrast to the DCDW case, which has the same type of density wave condensate Δe^{iqz}, but its transverse group velocity vanishes. This is due to the fact that in that case the b coefficients in (3) are zero, since they are only present at $B \neq 0$ and originate from the non-trivial topology of the LLL dynamics. We call attention that if $B = 0$ the expression at the rhs of (3) reduces to the stationary condition for q in (4) [29].

Let's consider now the propagation of light in the MDCDW medium. As shown in [21,22], this situation triggers the appearance of the chiral anomaly

$$\mathcal{L}_{\bar\theta} = \frac{\kappa}{8}\bar\theta F_{\mu\nu}\tilde{F}^{\mu\nu} \tag{7}$$

in the fermion effective action with $\bar\theta = mqz$ a background axion field linearly proportional to the chiral condensate parameters m and q.

Then, when light-matter interactions are present in this phase, the low-energy theory of the fluctuations in the MDCDW medium takes the form

$$\mathcal{L}_{\theta-A} = \mathcal{L}_\theta + +\mathcal{L}_A + \frac{\kappa}{8}\bar\theta F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\kappa}{8}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{8}$$

where

$$\mathcal{L}_A = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu \tag{9}$$

is the conventional electromagnetic action, with J^μ the (non-anomalous) electromagnetic four-current found after integrating out the fermions in the original MDCDW effective action [22]. The phonon fluctuation term $(\frac{\kappa}{8}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu})$ is introduced in the chiral anomaly through the shift $z \to z + u(x)$ in $\bar\theta$. The coupling constant $\kappa \equiv \frac{2\alpha}{\pi m}$ characterizes the interaction between the axion and the photon.

3. Axion polaritons in the MDCDW medium

Let us explore the consequences of the photon-axion interactions. With that aim in mind, we assume that a linearly polarized electromagnetic wave with electric field parallel to the background magnetic field propagates in the MDCDW medium. Then, the corresponding field equations for the axion and electromagnetic fields are

$$\nabla \cdot \mathbf{E} = J^0 + \frac{\kappa}{2}\nabla\bar\theta \cdot \mathbf{B} + \frac{\kappa}{2}\nabla\theta \cdot \mathbf{B}, \tag{10}$$

$$\nabla \times \mathbf{B} - \partial\mathbf{E}/\partial t = \mathbf{J} - \frac{\kappa}{2}(\frac{\partial\theta}{\partial t}\mathbf{B} + \nabla\theta \times \mathbf{E}), \tag{11}$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} + \partial\mathbf{B}/\partial t = 0 \tag{12}$$

$$\partial_0^2\theta - v_z^2\partial_z^2\theta - v_\perp^2\partial_\perp^2\theta + \frac{\kappa}{2}\mathbf{B} \cdot \mathbf{E} = 0. \tag{13}$$

For application to NS, we should consider a neutral medium; hence we assume that J^0 contains an electron background charge that ensures overall neutrality

$$J^0 + \frac{\kappa}{2}\nabla\bar\theta \cdot \mathbf{B} + \frac{\kappa}{2}\nabla\theta \cdot \mathbf{B} = 0. \tag{14}$$

In the presence of a static and uniform background magnetic field $\mathbf{B}_0$, the coupling between the axion fluctuation and the photon is linear, so that the linearized field equations take the form

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = \nabla^2 \mathbf{E} + \frac{\kappa}{2} \frac{\partial^2 \theta}{\partial t^2} \mathbf{B}_0 \tag{15}$$

$$\frac{\partial^2 \theta}{\partial t^2} - v_z^2 \frac{\partial^2 \theta}{\partial z^2} - v_\perp^2 \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + \frac{\kappa}{2} \mathbf{B}_0 \cdot \mathbf{E} = 0. \tag{16}$$

In momentum space these field equations can be written as

$$\left(\omega^2 - p^2 \right) E - \left(\frac{\kappa}{2} \omega^2 B_0 \right) \theta = 0 \tag{17}$$

$$-\left(\frac{\kappa}{2} B_0 \right) E + \left(\omega^2 - P^2 \right) \theta = 0 \tag{18}$$

where

$$P^2 = v_z^2 p_z^2 + v_\perp^2 p_\perp^2. \tag{19}$$

From (17)-(18) we notice that there is a mix between the phonon and photon modes that gives rise to two hybridized propagating modes with eigenvalues obtained from the dispersion relation

$$\det \begin{bmatrix} \omega^2 - p^2 & -\kappa \omega^2 B_0/2 \\ -\kappa B_0/2 & \omega^2 - P^2 \end{bmatrix} = 0 \tag{20}$$

The solutions of (20) correspond to one gapless, ω_0, and one gapped, ω_δ, propagation modes, which are respectively given by

$$\omega_0^2 = \omega_1^2 - \omega_2^2, \quad \omega_\delta^2 = \omega_1^2 + \omega_2^2 \tag{21}$$

with

$$\omega_1^2 = \frac{1}{2} [p^2 + P^2 + (\frac{\kappa}{2} B_0)^2], \tag{22}$$

$$\omega_2^2 = \frac{1}{2} \sqrt{[p^2 + P^2 + (\frac{\kappa}{2} B_0)^2]^2 - 4p^2 P^2}. \tag{23}$$

The gap of the ω_δ mode is field-dependent and given by

$$\omega_\delta(\vec{p} \to 0) = \delta = \alpha B_0/\pi m \tag{24}$$

We call these hybridized modes axion polaritons inspired by condensed matter analogies. In condensed matter, polaritons often appear as couple modes of optical phonons and light, magnons and light, or, as in the case of topological magnetic insulators, as the coupled mode of light and the axionic mode of an antiferromagnet [39]. It is pretty remarkable that despite significant differences between the underlying physics of earth-bound topological materials and the MDCDW phase of dense quark matter, a phase that can only exist under the extreme conditions of NS interiors, their low-energy physics is described by similar propagating modes.

It is worth noticing that the AP gap (24) is proportional to the magnitude of the applied magnetic field. The gap δ is present as long as the MDCDW phase exists, i.e., as long as $m \neq 0$. On the other hand, the gap does not explicitly depend on the modulation q or the quark chemical potential. For a magnetic field value of $B \simeq 10^{17}$ G, δ is in the range [0.06, 0.5] MeV and $m \in$ [23.5, 2.8] MeV for intermediate baryonic densities $\rho \sim 3\rho_s$ [32], with ρ_s being the nuclear saturation density.

4. Axion-polariton eigenfields and particle-number conservation

Our goal now is to find the propagating eigenfields inside the MDCDW dense medium under an incident electromagnetic wave. To find the corresponding eigenvectors we impose in (8) the temporal gauge $A_0 = 0$, together with the Feynman gauge (with gauge-fixing Lagrangian density $\mathcal{L}_g = (1/2)\partial_\mu A^\mu \partial_\nu A^\nu$). Notice that taken separately, none of them fix the gauge freedom completely, so one needs to impose them together (See Appendix A for a discussion of this point). After fixing the gauge, we obtain in the presence of the magnetic field the following quadratic Lagrangian density in Minkowskian momentum space,

$$\mathcal{L}_q = -\frac{1}{2}[\theta(p), A_3(p)] \begin{bmatrix} A & D \\ -D & C \end{bmatrix} \begin{bmatrix} \theta(-p) \\ A_3(-p) \end{bmatrix} \tag{25}$$

with

$$A = p_0^2 - P^2, \quad C = p_0^2 - (\vec{p})^2, \quad D = (2i\kappa)B_0 p_0 \tag{26}$$

and P^2 given in (19).

As can be seen from (25), the chiral anomaly term produces a mixture between the phonon field and the longitudinal electromagnetic field component (The transverse components of the electromagnetic field do not mix with the phonon). Thus, to find the system eigenvectors we need to rotate the fields by using the following unitary transformation

$$[\theta(p), A_3(p)] = [\Theta_0(p), \Theta_\delta(p)] \cdot S = [\Theta_0(p), \Theta_\delta(p)] \cdot \begin{bmatrix} \cos\beta & i\sin\beta \\ i\sin\beta & \cos\beta \end{bmatrix} \tag{27}$$

and

$$\begin{bmatrix} \theta(-p) \\ A_3(-p) \end{bmatrix} = S^{-1} \cdot \begin{bmatrix} \Theta_0'(-p) \\ \Theta_\delta'(-p) \end{bmatrix} == \begin{bmatrix} \cos\beta & -i\sin\beta \\ -i\sin\beta & \cos\beta \end{bmatrix} \cdot \begin{bmatrix} \Theta_0'(-p) \\ \Theta_\delta'(-p) \end{bmatrix} \tag{28}$$

From (27) and (28) we have that the rotated fields (i.e. the AP fields) are given in terms of the original fields by the linear combinations

$$\Theta_0(p) = \theta(p)\cos\beta - iA_3(p)\sin\beta \tag{29}$$

$$\Theta_\delta(p) = -i\theta(p)\sin\beta - A_3(p)\cos\beta \tag{30}$$

and

$$\Theta_0'(-p) = \Theta_0^*(-p) \quad \Theta_\delta'(-p) = \Theta_\delta^*(-p) \tag{31}$$

The fields Θ_δ and Θ_0 become the system eigenvectors when the rotation angle is given by the relations

$$\cos\beta = \frac{1}{\sqrt{2}}\left[1 + \frac{A-C}{\sqrt{(A-C)^2 - (2D)^2}}\right]^{1/2}, \quad \sin\beta = \frac{1}{\sqrt{2}}\left[1 - \frac{A-C}{\sqrt{(A-C)^2 - (2D)^2}}\right]^{1/2}. \tag{32}$$

It can be checked that the rotation matrix introduced in (27) with coefficients given in (32) diagonalizes, through a similarity transformation, the 2×2 matrix of the quadratic Lagrangian density (25) getting the form

$$\mathcal{L}_q = -\frac{1}{2}[\Theta_0(p), \Theta_\delta(p)] \cdot \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \Theta_0^*(-p) \\ \Theta_\delta^*(-p) \end{bmatrix} \tag{33}$$

with eigenvalues

$$\lambda_1 = \frac{1}{2}\left[(A + C) + \sqrt{(A - C)^2 - (2D)^2}\right],$$ (34)

$$\lambda_2 = \frac{1}{2}\left[(A + C) - \sqrt{(A - C)^2 - (2D)^2}\right]$$ (35)

When taking the dispersion relations $\lambda_1 = 0$ and $\lambda_2 = 0$, the modes ω_0 and ω_δ given in (21) are obtained, as it should be expected.

This result implies that linearly polarized waves with their electric field along the background magnetic field propagate in the MDCDW medium via the Θ_δ and Θ_0 AP modes. A peculiarity of the phonon-photon mixing in this phase is that it is kinematic in the sense that the mixing angle explicitly depends on the momenta. This is in contrast to other mixing mechanisms, as for instance, the one driving the mixing between the W_μ^3 and B_μ bosons in the electroweak theory, which takes place with a constant mixing angle (the Weinberg angle) and that gives rise to the massive Z_μ boson plus the massless photon field A_μ.

From (29)-(31) we see that the AP fields are a mixture of a pseudoscalar field θ and the third component of a four-vector field A_μ. Notice, nonetheless, that each component of the linear combinations (29)-(31), transform identically under the discrete transformations C, P, T. To treat independently the different components of the electromagnetic field is feasible in this case because the Lorentz symmetry is broken at finite density, which differentiates A_0 from the rest, and the rotational symmetry is also broken due to the uniform magnetic field along the third-spatial direction, which makes special the longitudinal A_3 component. Similar characteristics for the mixing between photons and axion fields have been found in other contexts [40].

In addition to the free Lagrangian density (33), the effective Lagrangian density of the low-energy axion fluctuations (8) has an interaction term, which in momentum space and in terms of the AP fields is given by

$$\begin{aligned}
\mathcal{L}_{int} = \frac{i}{2}\kappa(p_0 + p_0')F_{12}(k')[&(-i\cos\beta\sin\beta)\Theta_0(k)\Theta_0^*(-k - k') \\
&+ (\cos^2\beta)\Theta_0(k)\Theta_\delta^*(-k - k') + (\sin^2\beta)\Theta_\delta(k)\Theta_0^*(-k - k') \\
&+ (i\cos\beta\sin\beta)\Theta_\delta(k)\Theta_\delta^*(-k - k')]
\end{aligned}$$ (36)

The Lagrangian density $\mathcal{L}_q + \mathcal{L}_{int}$, given in (33) and (36) is invariant under the continuous phase global transformation

$$\Theta_0 \to e^{i\alpha}\Theta_0, \quad \Theta_\delta \to e^{i\alpha}\Theta_\delta.$$ (37)

This means that the AP number is conserved, with a conserved four-current given in the weak-field approximation by

$$\begin{aligned}
J_\mu^{AP} = &i(\partial^\mu\Theta_0^*)\Theta_0 - i(\partial^\mu\Theta_0)\Theta_0^* \\
&+ iv_z^2(\partial^\|\Theta_\delta^*)\Theta_\delta - iv_\perp^2(\partial^\perp\Theta_\delta)\Theta_\delta^*
\end{aligned}$$ (38)

Here, we introduced the notations $\partial^\| = (\partial^0, \partial^3)$ and $\partial^\perp = (\partial^1, \partial^2)$ for the longitudinal and transverse partial derivatives respectively with respect to the direction of the magnetic field. In this approximation the gapless Θ_0 field has a free-field current, while the current associated to the gapped field Θ_δ exhibits a breaking in the rotational symmetry due to the presence of the magnetic field. Thus, the conserved current of the gapped field is more responsive to the magnetic field.

It is timely to discuss here an effect that is derived from the axion-photon vertex $\frac{\kappa}{8}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu}$ in (8). In the presence of an external magnetic field B one of the two A_μ fields entering in the vertex can be taken as the one associated to B. Then, through this vertex an incoming photon can be transformed into an axion field and vice versa. This is known as the Primakoff effect [38]. The Primakoff effect is a mechanism that can occur in theories with a vertex between a scalar or a pseudoscalar field and two photons so that via this vertex and in the presence of background electric or magnetic field, the photon is transformed into a spin-zero field. In the context of MD-CDW dense quark matter, the Primakoff effect allows incident photons to be transformed into AP, since the axion field in this medium is split into the two AP eigenfields. Since the numbers of AP's are conserved, the AP will accumulate within this high-density medium.

This result can be of interest for astrophysics, since if the interior of magnetars host quarks in the MDCDW phase, γ-photons that penetrate and reach the quark medium with sufficient energy can be converted into gapped AP's that will increase the star mass. We should call attention to the fact that although the AP mass for fields of order $\sim 10^{17}$ G is not too large (i.e. ~ 0.5 MeV), extragalactic sources of gamma ray bursts (GRB) show an isotropic distribution over the whole sky flashing with a rate of 1000/year. The energy output of these events is $\sim 10^{56} - 10^{59}$ MeV, with photon energies of order $0.1 - 1$ MeV [41]. Hence, each one of these events can produce at least $10^{56} - 10^{59}$ γ-photons that could be converted into AP's by the Primakoff effect. If we assume that about only 10% of these photons reach the star, which is a conservative estimate if the star is in the narrow cone of a GRB beam, then at least about $10^{55} - 10^{58}$ of those photons can reach the star per each GRB event. If the number of created gapped AP is enough to reach the Chandrasekhar limit, which determines the number of AP required to induce the star collapse, the pulsars in that region will be destroyed by the γ-ray bombardment. In a separate publication we will discuss how this scenario can serve to give an alternative explanation to the astronomical puzzle called the missing pulsar problem, which refers to the failed expectation to observe a large number of pulsars within the distance of 10 pc of the galactic center. Theoretical predictions have indicated that there should be more than 10^3 active radio pulsars in that region [42], but these numbers have not been observed.

5. Concluding remarks

This work explores the effect of the chiral anomaly in the low-energy theory of fluctuations about the inhomogeneous ground state of the MDCDW phase and how that affects the propagation of electromagnetic waves in that medium. Two main results came out of this investigation. First, we demonstrated that linear electromagnetic waves entering the MDCDW medium mix up with the axion field giving rise to two collective hybridized AP modes. Second, we showed that the number density of AP is a conserved quantity. Hence, the linearized photons that penetrate the MDCDW medium produce through the Primakoff effect [38] a number of AP's that will be conserved. If the number of photons is equally split into massless and massive AP's, a significant part of the photon energy will be converted to mass, what can have consequences for the physics of NS under γ-ray radiation as pointed out above.

As discussed in the paper, the fluctuation theory can be written in terms of a pseudoscalar field θ, which is on the other hand an axion field. In the absence of photons, the low-energy Lagrangian of θ reduces to an anisotropic but free theory. Thanks to the magnetic field, the dispersion modes of the axion field are stiff enough in the direction transverse to the modulation to prevent the destabilization of the ground state by thermal fluctuations at low temperatures. This property ensured the absence of the Landau-Peierls instabilities that usually affect single-

modulated condensates, a fact that underlines the robustness of the MDCDW inhomogeneous condensate at low temperatures [29].

Things become even more interesting when the MDCDW medium interacts with photons. When the medium interacts with linear electromagnetic waves whose electric field is parallel to the background magnetic field, the chiral anomaly is activated, and as a consequence, the Lagrangian of the fluctuations acquires a new term that couples the wave's electric field with θ. Such an anomaly-induced coupling gives rise to coupled equations of motions between the photon and the phonon, which once diagonalized give energy dispersions for two hybridized collective modes called AP, one gapped and one gapless, represented respectively by the complex-conjugate eigebfields, Θ_δ and Θ_0.

It is worth comparing the effects produced by the magnetic field and the chemical potential on homogenous and inhomogeneous chiral condensates. In a massless theory of fermions, an external magnetic field strengthens the condensate because the dimensional reduction that characterizes the dynamics of the LLL fermions implies that their lowest energy state has no separation from the energy of the LLL antiparticles in the Dirac sea, so pairing among them is enhanced forming a chiral condensate even at the subcritical coupling. This is the well-known phenomenon of magnetic catalysis of chiral symmetry breaking (MCχSB) [43], which takes place in any relativistic theory of interactive massless fermions in a magnetic field. On the other hand, a chemical potential μ increases the energy separation between particles and antiparticles, hence stressing the pairing up to the point that the homogeneous condensate is not energetically favored any longer. The picture gets more complicated when one considers the effect of the magnetic field on the coupling constant. In this case, the chiral condensate decreases with the magnetic field, a phenomenon known as inverse magnetic catalysis [44].

Spatially inhomogeneous chiral condensates typically emerge only when the chemical potential reaches a specific value, at which a Fermi surface is formed, and the inhomogeneous condensate is favored over the restored phase. In the case of the MDCDW condensate, however, the inhomogeneity is present even in the region of smaller chemical potentials because a small anomalous fermion number is generated by the anomalous term in the thermodynamic potential that is a direct consequence of the spectral asymmetry of the LLL. This result is so robust that it exists at both overcritical [20] and subcritical coupling [23]. When the chemical potential reaches certain value in the intermediate region where a homogeneous condensate would have normally been erased, the modulation of the inhomogeneous condensate of the MDCDW jumps to a larger value and continue increasing with μ. In this region, there is a competition between the particle-antiparticle pairing that tends to decrease the magnitude of the condensate and the particle-hole pairing that tends to increase its modulation [32]. The jump in the modulation indicates the formation of a Fermi surface and the triggering of the particle-hole pairing. The magnetic field, in turn, enhances the window of inhomogeneity [20] as compared to the case without magnetic field [4]. These results were all found assuming a fixed coupling constant. It remains an open question, however, what would happen in the MDCDW phase if one considers the magnetic field's effect on the strong coupling constant.

A relevant question that naturally emerges is the compatibility of this quark matter phase with known NS astrophysical observations. In this sense, it is quite encouraging to know that the MDCDW is energetically favored over the symmetric ground state at all the densities of relevance for compact stars, and for magnetic fields and temperatures compatible with NS conditions [32]. That is, the critical temperature to erase the inhomogeneous condensate for inner magnetic fields $\sim 10^{17}$ G is several orders above the characteristic temperatures of old magnetars in the whole range of expected intermediate baryonic densities. Moreover, the MDCDW phase has already

been shown to be compatible with the observed $\sim 2M_\odot$ mass of NS [33]. On the other hand, long-term observations of NS temperatures during time intervals from months to years after accretion outburst, together with continued observations on timescales of years, have shed some light on the limit value of the heat capacity of NS cores. Considering these observations, it has been argued [37] that the lower-limit value $(C_V \gtrsim 10^{36}(T/10^8)$ erg/K) puts out of the game matter structures that exhibit superfluidity or superconductivity of any kind, since as showed in [36] all these cases are exponentially damped. This will lead to the striking conclusion that, if the only quark matter state to be realized in NS interior is the color superconducting CFL/MCFL phases, quarks will be ruled out from being the matter structure of NS cores [37]. Nevertheless, it has been recently proved [36] that if the NS core is formed by quarks in the MDCDW phase the heat capacity will be well above the lower limit expected for NS [37]. Still, other astrophysical observations, like tidal deformations and any new, more precise determination of NS mass/radius ratios [45] have yet to be considered to confirm a full compatibility of the MDCDW phase with all the relevant observations.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: No other activity that can conflict.

Data availability

No data was used for the research described in the article.

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Appendix A. Annotation on gauge fixing sufficiency

To understand and clarify the necessity to fix the Feynman gauge together with the temporal gauge, we summarize as follows some known results from Quantum Gauge Field Theory.

As known, the Maxwell Lagrangian density in momentum space is given by

$$\mathcal{L}_A = \frac{1}{2}A^\mu(\partial^2 g_{\mu\nu} - \partial_\mu \partial_\nu)A^\nu \tag{A.1}$$

Let's start by imposing a general covariant gauge given by the Lagrangian density

$$\mathcal{L}_g = -(1/2\xi)\partial_\mu A^\mu \partial_\nu A^\nu \tag{A.2}$$

where ξ is the gauge fixing parameter.

Adding (A.2) to (A.1) we have

$$\mathcal{L}_{A,g} = \mathcal{L}_A + \mathcal{L}_g = \frac{1}{2}A^\mu[\partial^2 g_{\mu\nu} - (1 - \frac{1}{\xi})\partial_\mu \partial_\nu]A^\nu \tag{A.3}$$

Fixing the gauge parameter as $\xi = 1$, we have the so-called Feynman's gauge, and the Lagrangian density reduces to

$$\mathcal{L}_{A,g_F} = \mathcal{L}_A + \mathcal{L}_{g_F} = \frac{1}{2}A^\mu \partial^2 g_{\mu\nu} A^\nu \tag{A.4}$$

Now, it is easy to check that the Lagrangian density (A.4) still has the gauge freedom

$$A^\mu \to A'^\mu = A^\mu + \partial^\mu \alpha, \quad \text{with} \quad \partial_\mu \partial^\mu \alpha = 0 \qquad (A.5)$$

Clearly, this gauge freedom can be eliminated by imposing to (A.4), an additional gauge constraint, as for instance, the temporal gauge condition, $A_0 = 0$, to get

$$\mathcal{L}_{A,g_{F,T}} = \frac{1}{2} A^i \partial^2 g_{ij} A^j, \quad i, j = 1, 2, 3 \qquad (A.6)$$

In this way, the Lagrangian density only depends on the two transverse physical modes.

On the other hand, the temporal gauge alone eliminates only one degree of freedom, leaving a Hilbert space larger than the set of physical states. Although the temporal gauge has been used since the early history of QFT [46], the question of how to eliminate the extra unphysical longitudinal degree of freedom, in this case, opens the possibility to treat the problem by introducing different constraints [47]. Another well-known problem that arises when only considering the temporal gauge is that the photon propagator in that gauge exhibits a pole at the zero-momentum component, $q_0 = 0$. Integrals that present spurious singularities as this are infrared divergent and demand special prescriptions to provide consistent calculation procedures [48].

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