Photon propagation in magnetized dense quark matter. A possible solution for the missing pulsar problem.

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Abstract. In this paper it is reviewed the topological properties and possible astrophysical consequences of a spatially inhomogeneous phase of quark matter, known as the Magnetic Dual Chiral Density Wave (MDCDW) phase, that can exist at intermediate baryon density in the presence of a magnetic field. Going beyond mean-field approximation, it is shown how linearly polarized electromagnetic waves penetrating the MDCDW medium can mix with the phonon fluctuations to give rise to two hybridized modes of propagation called as axion polaritons because of their similarity with certain modes found in condensed matter for topological magnetic insulators. The formation of axion polaritons in the MDCDW core of a neutron star can serve as a mechanism for the collapse of a neutron star under the bombardment of the gamma rays produced during gamma ray bursts. This mechanism can provide a possible solution to the missing pulsar problem in the galactic center.

1. Introduction

Determining the state of matter in the interior of neutron stars (NS) has attracted much attention recently. Possessing that knowledge allows to obtain the corresponding equation of state (EoS), which is instrumental in predicting together with the TOV equations the range of values of the stellar observational parameters as its mass, radius, tidal deformability, etc. On the other hand, this knowledge is also essential to know the properties of cold-dense nuclear matter under such extreme conditions that can only occur inside NS.

It is of a general consent that at asymptotically high densities nuclear matter is deconfined forming quark matter in the color superconducting color-flavor locked (CFL) phase, which is the one energetically preferred at those densities [1]-[3]. Nevertheless, such asymptotically high densities are probably not reached inside NS, but some intermediate densities where the CFL phase ceases to be the favorite.

It has long been argued that the region of intermediate densities and relatively low temperatures may feature inhomogeneous phases, many of which have spatially inhomogeneous chiral condensates favored over the homogeneous ones. Such spatially inhomogeneous phases have been found in the large-N limit of QCD [4]-[8], in NJL models [9]-[13], and in quarkyonic matter [14]-[18].

On the other hand, in determining the properties of dense quark matter in the interior of NS, it will be important to consider that magnetic fields permeate all the stellar compact objects ranging from $10^8 - 10^{12}$ G [19] at the surface of some radio pulsars up to $10^{14} - 10^{15}$ G for some special compact objects called magnetars [20]. Moreover, we should take into account that because the stellar medium has a very large electric conductivity, it follows that the magnetic flux is conserved and hence the inner core magnetic field can be even larger. They have been estimated to range from 10^{18} G for nuclear matter stars [21] to 10^{20} G for quark matter stars [22]. The fact that strong magnetic fields have significant consequences for several star properties has motivated many works focused on the study of the EOS of magnetized NS (see [23] and references there).

In particular, there is a magnetized inhomogeneous phase that is gaining interest as a possible candidate for NS interior. It is the Dual-Chiral-Density-Wave phase in the presence of a magnetic field [24]-[29], which was then called the MDCDW phase (M for magnetic). This inhomogeneous phase has the following interesting properties that can be of interest for the astrophysics of NS: *i*) Although it is a single modulated phase, it does not suffer from the Landau-Peierls instability [30]-[31]. This is so, because the asymmetry of the energy spectrum in the lowest Landau level (LLL) creates new non-trivial topological structures that lead to a linear, anisotropic spectrum of the thermal fluctuations, which lacks soft transverse modes [32]. Soft transverse modes are the essence of the Landau-Peierls instability because they produce infrared divergences in the mean square of the fluctuation field that in turn wipe out the average of the condensate at any low temperature. This result makes the phase stable at finite temperature. *ii*) The temperature needed to evaporate the inhomogeneous condensate for fields $\sim 10^{18}$ G is higher than the stellar temperatures for the whole range of densities characteristic of NS [33], *iii*) The maximum stellar mass of a hybrid star with a quark-matter core in this phase satisfies the maximum mass observation constraints ($M \sim 2M_\odot$) [34, 35], as obtained from the TOV equations in [36], and *iv*) In [37] it was shown that if the NS core is formed by quark matter in the MDCDW phase, the heat capacity will be well above the lower limit expected for NS ($C_V > 10^{36}(T/10^8)$ erg/K). This limit was established by long-term observations of NS temperatures in the range from months to years after accretion outburst together with continued observations on timescales of years.

In this paper, by going beyond the mean-field approximation, we will discuss how the thermal fluctuations of the inhomogeneous condensate of the MDCDW phase can affect the propagation of electromagnetic waves inside this medium. Thus, it will be shown in this context that the MDCDW medium is not transparent to photon propagation, but that the photons interact with the phonon fluctuations due to the chiral anomaly in the electromagnetic sector, which is activated in this medium in the presence of electromagnetic radiation [26]-[28]. Hence, the photon mixes with the phonon creating two hybridized modes called axion polaritons (AP) by analogy with similar phenomenology in condensed matter physics [38, 39]. One of the AP is gapless and the other gapped, with a gap proportional to the applied magnetic field and inversely proportional to the inhomogeneous condensate amplitude [40].

The AP becomes the eigenfields through which an incident electromagnetic field can propagate inside the MDCDW medium. Therefore, the existence of the chiral anomaly in the electromagnetic sector allows the transmutation of incident photons into gapped AP. This mechanism can be of relevance for the astrophysics of NS under intense γ-ray radiation as we will discuss below.

The plan of the paper is as follows. In Section 2, we will consider the MDCDW phase beyond mean-field approximation. Thus, we will introduce the phonon field characterizing the fluctuations of the inhomogeneous condensate of this phase. In this context, we will discuss how an electromagnetic field is affected in this medium by its interaction with the phonons through the chiral-anomaly vertex present in the medium. Then, we will show that as a consequence of that interaction the phonons and photons become mixed giving rise in the presence of a magnetic field to two hybridized fields called AP. One of them being gapped and the other gapless. In Section 3, we will discuss how the possibility to transfer photons into gapped AP can serve as a plausible explanation for the so called missing pulsar problem in astrophysics. Finally, in Section 4 we smmarize the main findings reported in the paper.

2. The MDCDW phase beyond the mean-field approximation

The MDCDW model describes a phase of two-flavor dense quark matter in the presence of a magnetic field [24]. The ground state of this phase is characterized by an inhomogeneous chiral condensate

$$\langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma^5\tau_3\psi\rangle = \Delta\exp(iqz) = -\frac{1}{2G}M_0(z), \tag{1}$$

Without loss of generality, we choose the magnetic field $\mathbf{B}$ in the z-direction and thus the condensate modulation is $\mathbf{q} = (0, 0, q)$, since in the MDCDW a modulation vector parallel to the magnetic field is energetically favored [24].

Then, assuming that the MDCDW order parameter is a single-modulated density wave $M(z) = me^{iqz}$, with $m \equiv -2G\Delta$, the free energy becomes [32]

$$\begin{aligned}\mathcal{F} &= a_{2,0}m^2 + b_{3,1}qm^2 + a_{4,0}m^4 + a_{4,2}q^2m^2 + b_{5,1}qm^4 \\ &+ b_{5,3}q^3m^2 + a_{6,0}m^6 + a_{6,2}q^2m^4 + a_{6,4}q^4m^2,\end{aligned} \tag{2}$$

where $a_{4,2} = a_{4,2}^{(0)} + a_{4,2}^{(1)}$, $a_{6,2} = a_{6,2}^{(0)} + a_{6,2}^{(1)}$, and we kept up to sixth order terms to ensure the stability of the MDCDW phase in the mean-field approximation. The coefficients a and b depend on T, μ and B. They are obtained from the MDCDW thermodynamic potential found in [28]. Their explicit expressions are not relevant for the present study.

In order to go beyond the mean-field approximation we need to consider the symmetry breaking pattern in the MDCDW phase. The ground state of the MDCDW system spontaneously breaks the chiral symmetry $U_A(1)$ and the translation along z, reducing the symmetry group to $U_V(1) \times SO(2) \times R^2$. Those symmetry breakings produce two Goldstone bosons: the neutral pion τ and the phonon ξ.

The corresponding global transformations on the order parameter of these broken groups are given by

$$M(x) \to e^{i\tau}M(z+\xi) = e^{i(\tau+q\xi)}M(z) \tag{3}$$

From Eq. (3), one clearly sees that there is a locking between the chiral rotation and the z-translation. Therefore, we can choose it as either the pion, the phonon, or a linear combination of them. Henceforth, without loss of generality, we consider it to be the phonon.

We now subject the order parameter to a small phonon fluctuation $u(x)$ and expand it about the condensate solution up to quadratic order in the fluctuation,

$$M(x) = M(z+u(x)) \simeq M_0(z) + M_0'(z)u(x) + \frac{1}{2}M_0''(z)u^2(x), \tag{4}$$

where $M_0(z) = \bar{m}e^{i\bar{q}z}$ is the ground state (1) with $\bar{m}$ and $\bar{q}$ the solutions of $\partial\mathcal{F}/\partial m = 0, \partial\mathcal{F}/\partial q = 0$.

Inserting (4) into the GL expansion of the free energy, and keeping terms up to quadratic order in $u(x)$, we arrive at the phonon free energy [32]

$$\mathcal{F}[M(x)] = \mathcal{F}_0 + v_z^2(\partial_z\theta)^2 + v_\perp^2(\partial_\perp\theta)^2 + \zeta^2(\partial_z^2\theta + \partial_\perp^2\theta)^2, \tag{5}$$

where we expressed the free energy in terms of the pseudo boson field $\theta = qmu$, and introduced the notation $\mathcal{F}_0 = \mathcal{F}(M_0)$, $(\partial_\perp\theta)^2 = (\partial_x\theta)^2 + (\partial_y\theta)^2$ and $\zeta^2 = a_{6.4}$.

The coefficients v_z^2, $v_\perp^2$ are the squares of the parallel and transverse group velocities respectively given by

$$v_z^2 = a_{4.2} + m^2 a_{6.2} + 6q^2 a_{6.4} + 3qb_{5,3} \tag{6}$$

$$v_\perp^2 = a_{4.2} + m^2 a_{6.2} + 2q^2 a_{6.4} + qb_{5,3} - a_{4.2}^{(1)} - m^2 a_{6.2}^{(1)} \tag{7}$$

Here, the dynamical parameters m and q in (6) and (7) must be understood as the solutions of the stationary conditions.

Thus, the corresponding low-energy phonon Lagrangian density is

$$\mathcal{L}_\theta = \frac{1}{2}[(\partial_0\theta)^2 - v_z^2(\partial_z\theta)^2 - v_\perp^2(\partial_\perp\theta)^2 - \zeta^2(\partial_z^2\theta + \partial_\perp^2\theta)^2], \tag{8}$$

with energy spectrum anisotropic and linear in the longitudinal and transverse momenta

$$E \simeq \sqrt{v_z^2 k_z^2 + v_\perp^2 k_\perp^2}, \tag{9}$$

where $k_\perp^2 = k_x^2 + k_y^2$.

We should point out that the energy spectrum (9) is quadratic in both parallel and transverse momenta, a property that ensures the lack of Landau-Peierls instability [32] in the MDCDW phase. To accomplish this result the magnetic field plays a crucial role. This is due to the fact that when evaluated the dynamical parameters in the stationary condition the transverse group velocity (7) will be different from zero if the b coefficients are different from zero, what only happens when $B \neq 0$. We call attention that the b coefficients originate from the non-trivial topology of the LLL dynamics.

Now we are ready to switch on an electromagnetic field in the system. As shown in [26, 28], this situation triggers the appearance of the chiral anomaly

$$\mathcal{L}_{\bar{\theta}} = \frac{\kappa}{8}\bar{\theta}F_{\mu\nu}\tilde{F}^{\mu\nu} \tag{10}$$

in the fermion effective action with $\bar{\theta} = mqz$ a background axion field linearly proportional to the chiral condensate parameters m and q. In (10) the coupling constant is given by $\kappa \equiv \frac{2\alpha}{\pi m}$.

Then, when light-matter interactions are taken into account, the low-energy theory of the fluctuations in the MDCDW medium takes the form

$$\mathcal{L}_{\theta-A} = \mathcal{L}_\theta + \mathcal{L}_A + \frac{\kappa}{8}\bar{\theta}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\kappa}{8}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{11}$$

where

$$\mathcal{L}_A = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu \tag{12}$$

is the conventional Maxwell action, with J^μ the (non-anomalous) electromagnetic four-current found after integrating out the fermions in the original MDCDW effective action [28]. The phonon fluctuation term ($\frac{\kappa}{8}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu}$) is introduced in the chiral anomaly through the shift $z \to z + u(x)$ in $\bar{\theta}$.

Then, the Lagrangian density $\mathcal{L}_{A-\theta}$ describes the low-energy theory of an axion field $\theta(x)$ interacting nonlinearly with the photon via the chiral anomaly in the MDCDW medium. The peculiarities of the medium are given through the group velocities (6)-(7) and the values of the dynamical parameters m and q satisfying the stationary conditions.

Let us now consider that a linearly polarized electromagnetic wave, with its electric field $\mathbf{E}$ parallel to the background magnetic field $\mathbf{B}_0$, propagates in the MDCDW medium [40]. The field equations of this theory are:

$$\nabla \cdot \mathbf{E} = J^0 + \tfrac{\kappa}{2}\nabla\theta_0 \cdot \mathbf{B} + \tfrac{\kappa}{2}\nabla\theta \cdot \mathbf{B}, \tag{13}$$

$$\nabla \times \mathbf{B} - \partial\mathbf{E}/\partial t = \mathbf{J} - \tfrac{\kappa}{2}(\tfrac{\partial\theta}{\partial t}\mathbf{B} + \nabla\theta \times \mathbf{E}), \tag{14}$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} + \partial\mathbf{B}/\partial t = 0 \tag{15}$$

$$\partial_0^2\theta - v_z^2\partial_z^2\theta - v_\perp^2\partial_\perp^2\theta + \tfrac{\kappa}{2}\mathbf{B}\cdot\mathbf{E} = 0. \tag{16}$$

We can notice that in these equations there are terms coupling the axion field with the photon. In (13)-(16), $\mathbf{B}$ is the total magnetic field, meaning the background field plus the wave magnetic field.

We should consider a neutral medium for applications to NS. Hence, we assume that J^0 contains an electron background charge that ensures overall neutrality

$$J^0 + \frac{\kappa}{2}\nabla\theta_0 \cdot \mathbf{B} + \frac{\kappa}{2}\nabla\theta \cdot \mathbf{B} = 0. \tag{17}$$

The linearized field equations in the presence of the background magnetic field $\mathbf{B}_0$ and an electric field propagating along $\mathbf{B}_0$ can then be written as

$$\partial^2\mathbf{E}/\partial t^2 = \nabla^2\mathbf{E} + \tfrac{\kappa}{2}(\partial^2\theta/\partial t^2)\mathbf{B}_0 \tag{18}$$

$$\partial^2\theta/\partial t^2 - v_z^2(\partial^2\theta/\partial z^2) - v_\perp^2(\partial^2\theta/\partial x^2 + \partial^2\theta/\partial y^2) + \tfrac{\kappa}{2}\mathbf{B}_0 \cdot \mathbf{E} = 0. \tag{19}$$

Their solutions describe two hybridized propagating modes of coupled axion and photon fields that we call AP, borrowing the term from condensed matter [40]. Polaritons are hybridized propagating modes that emerge when a collective mode like phonons, magnons, etc., couples linearly to light [38, 39].

The energy spectrum of the hybrid modes are

$$\omega_0^2 = A - B, \tag{20}$$

$$\omega_\delta^2 = A + B \tag{21}$$

with

$$A = \frac{1}{2}[p^2 + q^2 + (\frac{\kappa}{2}B_0)^2], \tag{22}$$

$$B = \frac{1}{2}\sqrt{[p^2 + q^2 + (\frac{\kappa}{2}B_0)^2]^2 - 4p^2q^2}, \tag{23}$$

and $q^2 = v_z^2 p_z^2 + v_\perp^2 p_\perp^2$.

From (20)-(23) we identify ω_0 as the gapless mode and ω_δ as the gapped mode with field-dependent gap

$$\omega_\delta(\vec{p} \to 0) = \delta = \alpha B_0/\pi m \tag{24}$$

The gap δ exists as long as the inhomogeneous condensate of the MDCDW phase is not erased (i.e., as long as $m \neq 0$). Notice that the gap does not explicitly depend on the modulation q or the quark chemical potential. For a magnetic field value of $B \simeq 10^{17}$ G, δ is in the range $[0.06, 0.5]$ MeV and $m \in [23.5, 2.8]$ MeV for intermediate baryonic densities $\rho \sim 3\rho_s$ [33], with ρ_s being the nuclear saturation density.

Similarly, coupled modes of axion and photon have been found in topological magnetic insulators [39]. This underlines the striking connection between matter in two very different regimes: MDCDW dense quark matter and topological materials in condensed matter. A fact whose origin lies in the similarity of their topological properties.

3. A possible solution for the missing pulsar problem

We should start by discussing an effect that is present due to the presence of the axion-photon vertex $\frac{\kappa}{8}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu}$ in (11). In an external magnetic field, B, one of the two A_μ fields entering in the vertex can be taken as the one giving rise to the external field B. Then, through this vertex in the presence of a magnetic field an incoming photon can be transformed in an axion field and vice versa. This is the well known Primakoff effect [41], which is a mechanism that can occur in theories with vertices between a scalar or a pseudoscalar field and two photons so that via this vertex and in the presence of background electric or magnetic field, the photon is transformed into a spin-zero field. In the case of the MDCDW phase under electromagnetic radiation, the Primakoff effect allows an incident photon to be transformed into AP inside the medium, since the axion field in this medium is split into the two AP eigenfields.

The fact that the MDCDW medium can create massive AP's when it is bombarded with energetic photons may have important implications for the physics of NS as we will discuss as follows. In this sense, the first step will be to see how the incoming photons can reach the quarks forming the inner region of NS. For this, we should take into account that γ-rays at moderate energies (i.e. energies smaller than $2m_e = 1.02$ Mev) are mainly attenuated by their interaction with electrons through Compton scattering. Now, electrons are distributed in NS in two different ways depending of the kind of NS we are considering. In hybrid stars, where the quarks are confined into the star core, the electrons are mixed along several kilometers with the hadronic matter that is surrounding the core; while in the so-called quark stars, where the star contains only quarks, the electrons are forming a very thin electron cloud of approximately 300 fm depth around the surface [42].

Taking into account the high electron density in the outer core of hybrid stars, it can be proved that the γ radiation will be absorbed in a distance of less than a hundred of fm into the mantle [43]. On the contrary, for quark stars it can be shown [43] that the ratio of intensities is $I/I_0 \approx 0.983$ after crossing the entire electron cloud, which shows that for 1MeV γ rays the attenuation is negligible. A similar calculation for the least energetic incident γ-ray, with 0.1 MeV, still shows small attenuation $I/I_0 \approx 0.64$. Thus, we conclude that in the case of quark stars the incident γ-rays can be converted into AP inside the star.

Now a question that follows is: Can the gapped AP be trapped inside the star? To answer it, we need to show that the velocity of the created gapped AP is smaller than the star's escape velocity $v_e/c = \sqrt{2GM_{star}/c^2R_{star}}$. For a star with $M_{star} = 2M_\odot$ and $R_{star} = 10$ km, we have $v_e = 0.8c$. The velocity v_{AP} that an AP of mass δ can reach depends on the energy E it acquires from the incident γ-rays

$$v_{AP}/c = \sqrt{1 - \left(\frac{c^2\delta}{E}\right)^2}. \tag{25}$$

For instance, for $c^2\delta = 0.3$ MeV, all the AP with energy $E < 0.5$ MeV cannot escape. Similarly, if $c^2\delta = 0.06$ MeV, the AP's with energies $E < 0.1$ MeV will be gravitationally trapped. This implies that gapped AP in the energy interval $(0.1, 0.5)$ MeV will always be trapped. The use of $2M_\odot$ stars in the escape velocity formula is motivated by recent indications [44] that the heaviest neutron stars, with masses $\sim 2M_\odot$, should have deconfined quark-matter inside. Another important factor to consider is that the number density of AP is conserved as it was demonstrated in [40]. Thus, if they cannot scape their number will be conserved.

On the other hand, if the number of gapped AP's that accumulates in the star's center is higher than the Chandrasekhar limit for these bosons, the AP's will create a mini black hole in the star center that will destroy the host NS, leaving a remnant black hole. The Chandrasekhar limit that determines the number of AP's required to induce the collapse is given by [45, 46]

$$N_{AP}^{Ch} = \left(\frac{M_{pl}}{\delta}\right)^2 = 1.5 \times 10^{44} \left(\frac{MeV}{\delta}\right)^2 \tag{26}$$

where $M_{pl} = 1.22 \times 10^{19}$ GeV is the Planck mass. Then, for the AP mass, $\delta = 0.3$ MeV, corresponding to the magnetic field $B = 10^{17}$ G field, we find $N_{AP}^{Ch} = 1.7 \times 10^{45}$.

The Milky Way galactic center (GC) is, on the other hand, a very active astrophysical environment with numerous γ-ray emitting point sources [47]. Extragalactic sources of gamma-ray burst (GRB) display an isotropic distribution over the whole sky flashing with a rate of

1000/year. The energy output of these events is $\sim 10^{56}$ MeV, with photon energies of order $0.1 - 1$ MeV [48], meaning that each one of these events can produce 10^{56} or more photons. If we assume that only 10% of these photons reach the star, which is a conservative estimate if the star is in the narrow cone of a GRB beam, about 10^{55} of those photons can reach the NS. This implies that if just $10^{-8}\%$ of the 10^{55} photons reach the star with energies ~ 0.3 MeV or larger, they can in principle generate a large enough number of AP's to produce a mini black hole in the star's center and induce its collapse. Similarly, for an AP mass $\delta = 0.06$ MeV, we find $N_{AP}^{Ch} = 4.2 \times 10^{46}$, so in this case $10^{-7}\%$ of the total number of photons will have to reach the star to create the conditions for the collapse.

Let's see now how the mechanism to create AP inside quark stars can serve to give a plaussible solution to the so called missing pulsar problem, which is a long-standing puzzle in astrophysics. This problem refers to the failed expectation to observe a large number of pulsars within 10 pc of the galaxy center. There exist indications that there are more than 10^3 active radio pulsars in that region [49], but these numbers have not been observed. This paradox has been magnified by pulse observations of the magnetar SGR J1745-2900 detected by the NuSTAR and Swift satellites [50]-[52]. These observations revealed that the failures to detect ordinary pulsars at low frequencies cannot be simply due to strong interstellar scattering, but instead should be connected to an intrinsic deficit produced by other causes.

In addition, the detection of the young ($T \sim 10^4$ yr) magnetar SGR J1745-2900 indicates high efficiency for magnetars formation from massive stars in the GC, as it was pointed out in [53]. This is because it will be unlikely to see a magnetar unless magnetar formation is efficient there. In fact, it has been argued that the detection of SGR 1745-2900, with a projected offset of only 0.12 pc from the GC, should not have been expected unless magnetar formation is efficient in the GC with an order unity efficiency [53], and that the missing pulsar problem could be explained as a consequence of a tendency to create short-lived magnetars rather than long-lived ordinary pulsars. Furthemore, there is evidence that several magnetars are associated with massive stellar progenitors ($M > 40M_\odot$) [54], a fact that supports the idea that magnetars formed in the GC could be very massive compact objects made of quark matter.

Therefore, considering that the GC is mainly populated by massive magnetars, probably being quark stars because of their large masses, and the high GRB activity of the region, the possibility that the huge conversion of γ-rays into gapped AP can attain the Chandrasekhar limit to finally create a mini black hole in the star center that will devour the whole star. This could be the reason of the scarce number of pulsar found in that region.

It is important to attract attention to the fact that the presence of a magnetic field is crucial for the AP mechanism to work since the AP gap is proportional to the magnetic field value. However, it is worth to mention that the AP mechanism does not require unrealistically large magnetic fields to be viable. Fields of magnitude $10^{17} - 10^{18}$ G are enough to make the MDCDW phase energetically favored over the chirally restored one at intermediate densities and to produce the needed AP mass to make viable the formation of the mini black hole via the Chandrasekhar mechanism. These are, on the other hand, plausible fields commonly considered for the interior of magnetars, whose surface magnetic fields can be as high as 10^{15}G.

4. Concluding remarks

In this paper, it is considered the MDCDW phase of quark matter at intermediate density beyond the mean-field approximation. In particular, it is analyzed how the low-energy fluctuations about the inhomogeneous ground state of the MDCDW phase affect the propagation of electromagnetic waves in this medium. In this regard, it is shown how due to the chiral anomaly, which is a natural ingredient of the system, an incoming photon mixes with the phonon fluctuation to produce two hybridized fields called AP, by analogy with condensed matter similar phenomenology. The AP fields become the eigenfields in this medium, one with a gapless energy spectrum and the other with a gapped one. The gap being proportional to the magnetic field and inversely proportional to the inhomogeneous condensate amplitude. The number density of AP is a conserved quantity as it was shown in [40]. Hence, the linearized photons that penetrate the MDCDW medium produce, through the Primakoff effect [41], a number of AP's that will be conserved. If the number of photons is equally split into massless and massive AP's, a significant part of the photon energy will be converted to mass, what can have consequences for the physics of NS under γ-ray radiation.

Precisely, in the second part of this paper we discuss how the large number of AP that can be stored inside a NS under the bombardment of photons produced in GRB can produce a mini black hole, once its mass exceeds the Chandrasekhar limit, with the capacity to produce the star collapse. This scenario serves to give an alternative explanation to the so-called missing pulsar problem in the GC.

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