EPJ Web of Conferences **270**, 00022 (2022) https://doi.org/10.1051/epjconf/202227000022
QCD@Work 2022

Electromagnetism in quark matter at intermediate densities

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Abstract. Several anomalous electromagnetic effects that can take place in quark matter at low temperatures and intermediate densities will be discussed. The anomalous transport properties of the spatially inhomogeneous phase of quark matter known as the Magnetic Dual Chiral Density Wave (MDCDW) phase will be reviewed. Going beyond mean-field approximation, it will be shown how linearly polarized electromagnetic waves that penetrate the MDCDW medium mix with the phonon fluctuations to give rise to two hybridized modes of propagation called axion polaritons. Finally, some possible implications of these results for the astrophysics of neutron stars will be indicated.

1 Introduction

The QCD phase diagram has two regions that are very well understood: The region of extremely high temperature and the one with a very high density. Its feasibility is due to the weakening of the strong coupling by the phenomenon of asymptotic freedom. They are respectively described by the quark-gluon plasma (QGP) phase at high temperature and low density and by the color superconducting color-flavor locked (CFL) phase at asymptotically large density and low-temperature [1]. More challenging, nonetheless, is to determine the phases in the intermediate density-temperature regions, where lattice QCD is not applicable due to the sign problem, so one has to rely on nonperturbative methods and effective theories. Nevertheless, it is believed that precisely one of the denser objects that we have in nature, neutron stars (NS), should have densities which are not extremely high but in that intermediate region.

It has long been considered, on the other hand, that the region of intermediate density and relatively low temperature may feature inhomogeneous phases, many of which have spatially inhomogeneous chiral condensates favored over the homogeneous ones. Such phases have been found in the large-N limit of QCD [2, 3], in NJL models [4]-[7], and in quarkyonic matter [8, 9]. In all the cases, chiral condensates with single-modulation are energetically favored over higher-dimensional modulations. However, single-modulated phases in three spatial dimensions are known to be unstable against thermal fluctuations at any finite temperature, a phenomenon known in the literature as Landau-Peierls instability [10]. In dense QCD models, the Landau-Peierls instability occurs in the periodic real kink crystal phase [11]; in the Dual Chiral Density Wave (DCDW) phase [12], and in the quarkyonic phase [13].

Another factor that plays an important role in the physics of these inhomogeneous phases and which also has relevance in the astrophysics of compact objects, as well as in heavy-ion collisions (HIC), is the presence of a strong magnetic field. In off-central HIC, quark-matter degrees of freedom become relevant and are known to produce large magnetic fields ($eB \simeq$

10^{18} G at RHIC, $eB \simeq 10^{19}$ G, at the LHC [14, 15].). Likewise, NS can have strong inner magnetic fields. Estimates based on the scalar virial theorem give inner fields for magnetars of order 10^{18} G for nuclear matter [16] and up to 10^{20} G for quark matter [17]. Even inner fields, one to three orders of magnitude smaller, would be significant and should not be ignored in NS studies [18]. The magnetic field can noticeably enhance the window for inhomogeneous phases [9, 19, 20]; and activate attractive channels producing new condensates, as it occurs with chiral condensate [19], color superconductivity [20] and quarkyonic matter [9].

In the context of the DCDW phase, the presence of an external magnetic field favors the formation of a spatially inhomogeneous condensate with a modulation along the field direction [21]. This phase has a new symmetry pattern determined by the breaking of the rotational and isospin symmetries. This new magnetized phase is called the magnetic dual chiral density wave (MDCDW) phase [22, 23]. Another important result is that the presence of a magnetic field removes the Landau-Peierls instability in this phase [24]. Moreover, the constant external magnetic field produces Landau momentum quantization, giving rise to an asymmetric quark energy spectrum in the lowest Landau level (LLL). This asymmetry, together with the activation of the chiral anomaly in the presence of an electric field with a component along the magnetic field direction give the system a non-trivial topology. The topology of the MDCDW phase manifests in the effective electromagnetic action by the presence of a dynamical axion field coupled to the electromagnetic field. This coupling in turn leads to several topological effects [22, 23] that we will discuss later.

It is timely to comment that in addition to the properties of the MDCDW phase that were already pointed out, this phase exhibits other important characteristics that make it more attractive as a possible candidate for the inner matter phase of NS. First, it has been shown in [25] that the temperature needed to evaporate the inhomogeneous condensate for fields $\sim 10^{18}$ G is higher than the characteristic stellar temperatures for the whole range of densities characteristic of NS; second, it was proved in [26] that the maximum stellar mass of a hybrid star with a quark-matter core in the MDCDW phase satisfies the maximum mass observation constraints [27, 28]; and finally, in [29] it was shown that if the NS core is formed by quarks in the MDCDW phase, the heat capacity will be well above the lower limit expected for NS ($C_V \gtrsim 10^{36}(T/10^8)$ erg/K) [30]. This limit was established by long-term observations of NS temperatures in the range from months to years after accretion outburst together with continued observations on timescales of years. On the other hand, this lower-limit value put out of the game the matter components that exhibit superfluidity and/or superconductivity of any kind, since as showed in [29] all these cases are exponentially damped. This will lead to the striking conclusion that, if the only quark matter state to be realized in NS interior is the color superconducting phases, quark will be ruled out from forming part of the NS core [30].

The rest of the paper is organized in two sections. In Section 2, it is expose how the presence of the chiral anomaly in the MDCDW phase produces a peculiar electromagnetism with field equations know as axion electrodynamics, which exhibit magnetoelectricity. In Section 3, by going beyond mean-field approximation, we show that the photon-phonon interaction in this medium produces a couple of hybridized modes, one of those being gapped, and which are known as axion polaritons (AP). Possible astrophysical applications are indicated.

2 Magnetoelectricity in the MDCDW phase

The action of the MDCDW phase is [22, 23]

$$\begin{aligned} S_{eff} &= \int d^4x\{\bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu + i\tau_3\gamma_5\partial_\mu\theta) + \gamma_0\mu - m]\psi - \frac{m^2}{4G} \\ &+ \frac{\kappa}{4}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\}, \end{aligned} \tag{1}$$

Here m is the condensate magnitude, b its modulation and $\theta(x)$ is the axion field whose coupling with the electromagnetic tensor is given by $\frac{\kappa}{4} = \frac{3(e_u^2-e_d^2)}{8\pi^2} = \frac{e^2}{8\pi^2} = \frac{\alpha}{2\pi}$. In (1), $Q = \mathrm{diag}(e_u, e_d) = \mathrm{diag}(\frac{2}{3}e, -\frac{1}{3}e)$ denotes the electric-charge matrix in flavor.

The electromagnetic effective action $\Gamma(A)$ in the MDCDW phase is obtained as usual from

$$\Gamma = -i \log Z, \tag{2}$$

where the partition function Z is

$$Z = e^{i\Gamma} = \int \mathcal{D}\bar{\psi}(x)\mathcal{D}\psi(x)e^{iS_{eff}} \tag{3}$$

Integrating in the fermion fields, performing the Matsubara sum, taking the zero-temperature limit, and expanding Γ in powers of the fluctuation field A_μ we obtain

$$\begin{aligned}\Gamma(A) &= -V\Omega + \int d^4x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \kappa\int d^4x\epsilon^{\mu\alpha\nu\beta}A_\alpha\partial_\nu A_\beta\partial_\mu\theta\right] \\ &- \int d^4x A_\mu(x)J^\mu(x),\end{aligned} \tag{4}$$

where we integrated by parts the axion term and cut the series in power of A_μ in the linear term. The four-current $J^\mu(x) = (J^0, \mathbf{J})$ represents the contribution of the ordinary (non-anomalous) electric four-current, obtained from the one-loop tadpole diagrams [22, 23]. In (4), V is the system volume and Ω the thermodynamic potential which does not depend on A_μ.

From (4) we can obtain the source depending equations of axion electrodynamics, which in terms of the D and H fields, are given by

$$\nabla\cdot\mathbf{D} = J^0, \quad \nabla\times\mathbf{H} - \frac{\partial\mathbf{D}}{\partial t} = \mathbf{J} \tag{5}$$

where

$$\mathbf{D} = \mathbf{E} - \kappa\theta\mathbf{B}, \quad \mathbf{H} = \mathbf{B} + \kappa\theta\mathbf{E} \tag{6}$$

Equations (6) show that a magnetic field induces an electric polarization $\mathbf{P} = -\kappa\theta\mathbf{B}$ and an electric field induces a magnetization $\mathbf{M} = -\kappa\theta\mathbf{E}$, a phenomenon known as magnetoelectricity. The linear magnetoelectricity of the MDCDW medium is a direct consequence of the chiral anomaly. It reflects the fact that the ground state of the MDCDW medium breaks parity and time inversion symmetries. The magnetoelectricity in the MDCDW phase is different from the one found in the magnetic-CFL phase of color superconductivity, where parity was not broken and the effect was a consequence of an anisotropic electric susceptibility [31], thus it was not linear. It also follows from Eq. (5) that there is an anomalous Hall current $\mathbf{J}_{anom} = -\frac{e^2}{4\pi^2}\mathbf{q}\times\mathbf{E}$, which is a medium-induced magnetic current density $\nabla\times\mathbf{M}$. This medium-induced magnetic current density is produced by the space-dependent anomalous magnetization coming from the axion term and it indicates that if there exists an electric field perpendicular to the magnetic field it will produce a dissipationless current in the medium. Such a current could act as a propellant of the large magnetic fields existing in magnetars in case that their core are formed by quark matter in the MDCDW phase.

3 Axion-Polariton modes in the MDCDW phase

Considering a small phonon fluctuation $\theta(x)$ on the order parameter, which is associated with the breaking of the translational symmetry by the single-modulated inhomogeneous condensate, and expanding it about the condensate solution up to quadratic order in the fluctuation, it can be obtained the fluctuation low-energy Lagrangian density as [24, 32]

$$\mathcal{L}_\theta = \frac{1}{2}[(\partial_0\theta)^2 - v_z^2(\partial_z\theta)^2 - v_\perp^2(\partial_\perp\theta)^2], \tag{7}$$

with coefficients given by

$$v_z^2 = a_{4.2} + \bar{m}^2 a_{6.2} + 6\bar{q}^2 a_{6.4} + 3\bar{q}b_{5,3} \tag{8}$$

$$v_\perp^2 = a_{4.2} + \bar{m}^2 a_{6.2} + 2\bar{q}^2 a_{6.4} + \bar{q}b_{5,3} - a_{4.2}^{(1)} - \bar{m}^2 a_{6.2}^{(1)} \tag{9}$$

These coefficients represent the squares of the parallel and transverse group velocities respectively.

Let's consider now the propagation of electromagnetic waves in the MDCDW phase by going beyond the mean-field approximation to study the effects of the phonon-photon interactions in the MDCDW medium.

When an electromagnetic wave penetrates the MDCDW medium, the low-energy theory of the fluctuations acquires the following additional contributions

$$\mathcal{L}_{A-\theta} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu + \frac{\kappa}{8}\theta_0(x)F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\kappa}{8}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{10}$$

The first two terms are the conventional Maxwell and ordinary 4-current contributions respectively, the latter obtained after integrating out the fermions in the original MDCDW effective action [22, 23], as it was explained above. The last two terms are the axial anomaly with background axion field $\theta_0(x) = mqz$ and its (phonon-induced) fluctuation $\theta(x)$. Here $\kappa = 2\alpha/\pi m$.

The combined Lagrangian $\mathcal{L} = \mathcal{L}_\theta + \mathcal{L}_{A-\theta}$ effectively describes the low-energy theory of an axion field $\theta(x)$ interacting nonlinearly with the photon via the chiral anomaly. Let us now assume that a linearly polarized electromagnetic wave, with its electric field $\mathbf{E}$ parallel to the background magnetic field $\mathbf{B}_0$, propagates in the MDCDW medium [33]. From (7) and (10) we obtain the field equations

$$\nabla\cdot\mathbf{E} = J^0 + \frac{\kappa}{2}\nabla\theta_0\cdot\mathbf{B} + \frac{\kappa}{2}\nabla\theta\cdot\mathbf{B}, \tag{11}$$

$$\nabla\times\mathbf{B} - \partial\mathbf{E}/\partial t = \mathbf{J} - \frac{\kappa}{2}(\frac{\partial\theta}{\partial t}\mathbf{B} + \nabla\theta\times\mathbf{E}), \tag{12}$$

$$\nabla\cdot\mathbf{B} = 0, \quad \nabla\times\mathbf{E} + \partial\mathbf{B}/\partial t = 0 \tag{13}$$

$$\partial_0^2\theta - v_z^2\partial_z^2\theta - v_\perp^2\partial_\perp^2\theta + \frac{\kappa}{2}\mathbf{B}\cdot\mathbf{E} = 0, \tag{14}$$

which contains terms where the axion field and the photon are coupled. In (11)-(14), $\mathbf{B}$ is the total magnetic field, meaning the background field plus the wave magnetic field.

Since we are interested in applications to NS, we should consider a neutral medium, hence we assume that J^0 contains an electron background charge that ensures overall neutrality

$$J^0 + \frac{\kappa}{2}\nabla\theta_0\cdot\mathbf{B} + \frac{\kappa}{2}\nabla\theta\cdot\mathbf{B} = 0. \tag{15}$$

Hence, from (11)-(14) and (15), the linearized field equations can be given as

$$\partial^2\mathbf{E}/\partial t^2 = \nabla^2\mathbf{E} + \frac{\kappa}{2}(\partial^2\theta/\partial t^2)\mathbf{B}_0 \tag{16}$$

$$\partial^2\theta/\partial t^2 - v_z^2(\partial^2\theta/\partial z^2) - v_\perp^2(\partial^2\theta/\partial x^2 + \partial^2\theta/\partial y^2) + \frac{\kappa}{2}\mathbf{B}_0\cdot\mathbf{E} = 0. \tag{17}$$

The solutions of (16)-(17) describe two hybridized propagating modes of coupled axion and photon fields that we call AP [33], borrowing the term from condensed matter. In general, polaritons are hybridized propagating modes that emerge when a collective mode like phonons, magnons, etc., couples linearly with light.

The energy spectrum of the hybrid modes are

$$\omega_0^2 = A - B, \tag{18}$$

$$\omega_m^2 = A + B \tag{19}$$

with

$$A = \frac{1}{2}[p^2 + q^2 + (\frac{\kappa}{2}B_0)^2], \tag{20}$$

$$B = \frac{1}{2}\sqrt{[p^2 + q^2 + (\frac{\kappa}{2}B_0)^2]^2 - 4p^2q^2}, \tag{21}$$

and

$$q^2 = v_z^2 p_z^2 + v_\perp^2 p_\perp^2 \tag{22}$$

where we used the notation $p_\perp^2 = p_1^2 + p_2^2$.

From (18)-(21) we identify ω_0 as the gapless mode and ω_m as the gapped mode with field-dependent gap

$$\omega_m(\vec{p} \to 0) = m_{AP} = \alpha B_0/\pi m, \tag{23}$$

which is proportional to the external applied magnetic field and inversely proportional to the inhomogeneous condensate amplitude.

Similarly coupled modes of axion and photon have been found in topological magnetic insulators [34], underlining once again the striking similarities between MDCDW quark matter and topological materials in condensed matter.

This result implies that if a linearized electromagnetic wave can reach the MDCDW medium, it will split into the two AP modes, which are the real eigenmodes of this medium. Hence, since one of the modes is massive, this can serve as a mechanism to transfer photon energy into rest mass.

This mechanism can be of interest for astrophysics, since if the interior of magnetars host quarks in the MDCDW phase, γ-photons that penetrate and reach the quark medium with sufficient energy can be converted into gapped AP's that will increase the star mass. We should call attention to the fact that although the AP mass for fields of order $\sim 10^{17}$ G is not too large (i. e. ~ 0.5 MeV), extragalactic sources of gamma ray bursts (GRB) show an isotropic distribution over the whole sky flashing with a rate of 1000/year. The energy output of these events is $\sim 10^{56} - 10^{59}$ MeV, with photon energies of order $0.1 - 1$ MeV [35]. Hence, each one of these events can produce at least $10^{56} - 10^{59}$ γ-photons that could be converted into AP's, with half of them being gapped. If we assume that about only 10% of these photons reach the star, which is a conservative estimate if the star is in the narrow cone of a GRB beam, then at least about $10^{55} - 10^{58}$ of those photons can reach the star per each GRB event. This scenario can serve to give an alternative explanation [32] to the astronomical puzzle called the missing pulsar problem, which refers to the failed expectation to observe a large number of pulsars within the distance of 10 pc of the galactic center. Theoretical predictions have indicated that there should be more than 10^3 active radio pulsars in that region [36], but these numbers have not been observed.

Acknowledgments: This work was supported in part by NSF grant PHY-2013222.

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