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Mixed-integer recourse in industrial demand response scheduling with interruptible load

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Abstract

While industrial demand response has become a prominent strategy for power-intensive chemical plants to remain cost-competitive, the financially incentivized provision of load reduction capacities, also called interruptible load, to the power grid is still a less explored topic. Here, a major challenge lies in dealing with the uncertainty that one does not know when load reduction will be requested. In this work, a scheduling model for a continuous industrial process providing interruptible load is developed, where we apply an adjustable robust optimization approach to address the uncertainty. The main difference to previous works is that we incorporate both continuous and binary recourse variables. When applied in our computational case study, the proposed model achieves significant cost savings when compared with a model that only considers continuous recourse.

Keywords: Interruptible load, multistage robust optimization, mixed-integer recourse

1. Introduction

In recent years, there have been significant advances in industrial demand response (DR) (Zhang and Grossmann, 2016), which has enabled large industrial consumers of electricity, such as power-intensive chemical plants, to reduce their operating costs by altering their power usage in response to varying electricity prices. DR is also beneficial to the power grid as it helps maintain grid stability, and a particularly effective way to do so is through the provision of interruptible load, which constitutes a form of ancillary services (Dowling et al., 2017). Here, economic incentives are offered to electricity consumers for committing load reduction capacities (that is, interruptible load) up to the agreed amount when requested by the grid operator.

The major challenge in providing interruptible load is that load reduction demand is not known in advance, but one must still guarantee dispatch upon request. Disregarding this uncertainty may jeopardize plant safety or lead to situations in which it is no longer possible to satisfy all product demand. Zhang et al. (2015) capture the uncertainty using a tailored uncertainty set and apply robust optimization (Ben-Tal et al., 2009) to this scheduling problem. However, they only model the static case where no recourse is considered, which leads to very conservative solutions. Zhang et al. (2016) address this shortcoming by incorporating continuous recourse decisions using affine decision rules to show significant increases in cost savings enabled by flexible recourse. Yet the proposed approach still cannot realize the full potential of interruptible load since it does not consider discrete recourse and hence does not allow, for example, full plant shutdowns when load reduction is required, which is what is often done in practice. In this work, we extend the previous framework to also include binary recourse decisions.

2. Problem statement

Consider a power-intensive continuously operated plant that manufactures a set of products. To satisfy demand, the products can be produced in the plant or purchased at a higher expense. The plant consumes electricity whose varying price is assumed to be known over the scheduling horizon. In addition, the plant can cut costs by providing interruptible load to the grid. To this end, the plant makes a commitment to reduce its electricity consumption at the grid operator's request. Load reduction may not always be requested but the plant earns revenue regardless. We assume that no additional payment is made to the plant when load reduction is requested; this assumption can be tweaked depending on the electricity market with no major changes to the model.

Given the plant model parameters (as discussed in Section 3) and a scheduling horizon, the goal is to determine the production schedule that is feasible for all possible realizations of load reduction while minimizing the total operating cost. Note that recourse variables, e.g., production rates and purchase amounts, depend on the realization of the uncertainty.

3. Deterministic model formulation

The model presented here is based on formulations developed in previous works (Mitra et al., 2012; Zhang et al., 2015) and is a direct extension of the model used in Zhang et al. 2016. Hence, only a brief description of the various constraints is provided.

3.1. High-level formulation

The structure of the deterministic model can be broadly expressed as follows:

$$\begin{aligned} \text{minimize} \quad & \textit{Net operating cost} && \text{(1a)} \\ \text{subject to} \quad & \textit{Feasible regions of operating modes} && \text{(1b)} \\ & \textit{Transition constraints} && \text{(1c)} \\ & \textit{Mass balance constraints} && \text{(1d)} \\ & \textit{Initial conditions} && \text{(1e)} \end{aligned}$$

The objective function is the cost of electricity plus the cost of additional products purchased minus the revenue from providing interruptible load. We assume that the plant can operate in different operating modes whose *feasible regions* are given in the form of polytopes. One such example of a plant with two products, P1 and P2, is shown in Fig. 1. Here, Region 1 could denote the off mode where no products are produced. *Transition constraints* are used to enforce feasible mode switching. Fig. 2 shows an example of requirements from the transition constraints. For instance, the process shown needs to stay in the off mode for at least 8 h before it can switch to the startup mode. *Mass balance constraints* ensure demand is satisfied through production or purchase; the remaining products are stored. *Initial conditions* are required for the problem to be well-defined.

3.2. Interruptible load constraints

As mentioned previously, the grid operator can request load reduction (no greater than the agreed amount) from the IL provider and the plant needs to alter its production schedule to cater to that request. These alterations in production schedule are modeled by introducing variables of the form:

$$\overline{PD}_{mit} = \widehat{PD}_{mit} + \widetilde{PD}_{mit} \quad \forall\, m \in M, i \in I, t \in T, \qquad \text{(2a)}$$

$$\bar{y}_{mt} = \hat{y}_{mt} + \tilde{y}_{mt} \quad \forall\, m \in M, t \in T, \qquad \text{(2b)}$$

where T denotes the set of time periods and $\overline{PD}_{mit}$ is the production rate of product i in mode m in time period t. The nominal production rate is denoted by $\widehat{PD}_{mit}$, and $\widetilde{PD}_{mit}$ is the deviation from the nominal value when load reduction is requested. The binary variable $\bar{y}_{mt}$ equals 1 if the plant operates in mode m in time period t; $\hat{y}_{mt}$ is its nominal value while $\tilde{y}_{mt}$ is a discrete recourse variable that can take the values -1, 0, or 1, depending on the amount of load reduction requested. The decrease in power consumption associated with the decrease in production or, in some cases, a process shutdown must be at least as much as the amount of load reduction requested (LR_t). This is modeled as the following constraint:

$$\sum_m \delta_m \tilde{y}_{mt} + \sum_m \sum_i \gamma_{mi} \widetilde{PD}_{mit} \leq -LR_t \quad \forall\, t \in T, \tag{3}$$

where we assume that the electricity consumption is a linear function of production rates with a constant δ_m and coefficient γ_{mi} for the selected operating mode.

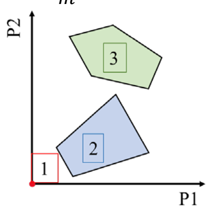

Fig. 1: Example for feasible operating regions of a process with three operating modes.

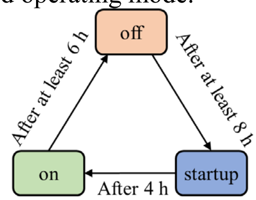

Fig. 2: Example for possible transitions between operating modes and corresponding operational constraints.

4. Multistage robust formulation with mixed-integer recourse

4.1. Uncertainty set

We adopt a "budget of uncertainty" approach (Bertsimas and Sim, 2004; Zhang et al., 2015) to formulate the following uncertainty set W for the load reduction demand:

$$W = \left\{ w \in \mathbb{R}^{|T|} : \left(0 \leq w_k \leq 1 \;\; \forall k = 1, \dots, t, \sum_{k=1}^{t} w_k = \Gamma_t \right) \;\; \forall\, t \in T \right\}, \tag{4}$$

where w_t is the normalized requested load reduction, i.e., $LR_t = IL_t w_t$, IL_t is the amount of interruptible load provided, and Γ_t is a budget parameter limiting the cumulative load reduction required up to time t. The choice of Γ_t can be based on historical data or the electricity market rules. Note that Γ_t must be a monotonically increasing parameter. We now write Eq. (3) in terms of the normalized load reduction:

$$\sum_m \delta_m \tilde{y}_{mt}(w) + \sum_m \sum_i \gamma_{mi} \widetilde{PD}_{mit}(w) \leq -w_t IL_t \quad \forall\, t \in T. \tag{5}$$

4.2. Adjustable robust formulation with mixed-integer decision rules

The overall adjustable robust optimization problem can be formulated as follows:

$$\begin{aligned} &\text{minimize} && \textit{Net operating cost at } w = 0 \textit{ or worst-case net operating cost} \\ &\text{subject to} && \text{Eqs. (1b)–(1e), (2), (5)} \;\; \forall\, w \in W, \end{aligned} \tag{6}$$

where we either minimize the net operating cost in the nominal or in the worst case. Here, the deviation variables $\widetilde{PD}_{mit}$, $\tilde{y}_{mt}$, etc., serve as the recourse variables and are hence functions of the uncertain parameters w. Following the concept of lifted uncertainty (Bertsimas and Georghiou, 2018) and the derivation for multistage problems in Feng et al. 2021, we implement (potentially discontinuous) piecewise linear decisions rules for the continuous variables and piecewise constant decision rules for the binary variables. As an example, the decision rules for $\widetilde{PD}_{mit}$ and $\tilde{y}_{mt}$ have the following form:

$$\widetilde{PD}_{mit} = \sum_{t'=t-\bar{\zeta}}^{t} \sum_{k=1}^{K_{t'}} \left(\bar{X}^t_{mit'k}\, \bar{w}_{t'k} + \hat{X}^t_{mit'k}\, \hat{w}_{t'k} \right) \quad \forall\, m \in M, i \in I, t \in T, \qquad (7a)$$

$$\tilde{y}_{mt} = \sum_{t'=t-\bar{\zeta}}^{t} \sum_{k=1}^{K_{t'}} \hat{Y}^t_{mt'k}\, \hat{w}_{t'k} \quad \forall\, m \in M, t \in T, \qquad (7b)$$

where $\bar{w}$ and $\hat{w}$ are the auxiliary lifted uncertain parameters, K_t is the number of breakpoints associated with w_t that define the piecewise structures of the decision rules, and $\bar{\zeta}$ determines how many uncertain parameters from previous time periods are considered in the decision rules. The coefficients $\bar{X}$, $\hat{X}$, and $\hat{Y}$ define the decision rules and are to be optimized. Problem (6) is a semi-infinite program, which we solve using the reformulation approach that leverages linear programming duality (Yanıkoğlu et al., 2019). For the sake of brevity, we refer the reader to Feng et al., 2021 for more details including the full reformulation, which results in a mixed-integer linear program (MILP).

5. Case study

In this section, the proposed robust model with mixed-integer recourse is applied to an illustrative example. All models were implemented in Julia v1.7 using the modeling environment JuMP v0.22.3 and solved to 1% optimality gap using Gurobi v9.5.1 on an Intel Core i7-8700 machine at 3.20 GHz with 8 GB RAM.

In this case study, a single-product plant is considered, and the scheduling problem is solved over a time horizon of 48 hours with hourly time discretization. The plant can operate in three different modes: off, startup, and on. Table 1 shows the details of the polytopes (which here are simple ranges) that characterize the operating modes and the electricity consumption in each operating mode. Table 2 shows the possible mode transitions and the respective minimum stay times. Fig. 3 shows the electricity prices and revenue from providing interruptible load over the time horizon. The plant is operating in the on mode at the start and it is assumed that no mode switching has occurred in the eight time periods prior to the beginning of the scheduling horizon.

The initial inventory is 1,000 kg. The minimum and maximum inventory levels are 0 and 5,000 kg, respectively, for all time points. At the end of the time horizon, the minimum inventory level is set to 1,000 kg. The cost of purchasing additional products is \$3/kg. If interruptible load is provided in a time period, then the provided amount must be between 200 and 5,000 kWh. We assume that the budget parameter Γ_t increases every 8 time periods by 1, i.e., maximum load reduction can only be requested once during the first 8 h, twice during the first 16 h, etc., and at most six times during all 48 h.

Table 1: Polytope equations, fixed (δ_m) and unit (γ_m) electricity consumption for each operating mode (product indices have been omitted).

Operating mode	**Polytope**	$\boldsymbol{\delta_m}$ [kWh]	$\boldsymbol{\gamma_m}$ [kWh/kg]
Off	$0 \leq PD_{mt} \leq 0$	0	0
Startup	$5 \leq PD_{mt} \leq 5$	0	60
On	$100 \leq PD_{mt} \leq 160$	1,200	20

5.1. Advantages of discrete recourse

We solve the problem for different $\bar{\zeta}$, which controls the amount of past information used in the decision rules. The flexibility in the solution increases with $\bar{\zeta}$, but it also increases the model size and hence the computational intensity. Table 3 compares the costs between the continuous recourse only case and the mixed-integer recourse case. The results show significant cost reductions, of up to 14%, for mixed-integer recourse case over continuous recourse case. In Figs. 4 and 5, we see that the nominal electricity consumption profiles in both cases are very similar. This implies that a major factor in the cost reduction is the additional interruptible load provided in the mixed-integer recourse case. Figs. 5 and 7 show that this additional interruptible load is provided in time periods 27 and 48, where process shutdown is a feasible recourse action. This is also reflected in the time period 48 of Fig. 7, where the nominal and recourse production amounts add up to zero, indicating a process shutdown in the worst-case uncertainty realization.

Table 2: Possible transitions between operating modes and minimum stay times.

Transition from mode m to mode m'	Minimum stay time in m' [h]
Off → startup	4
Startup → on	6
On → off	8

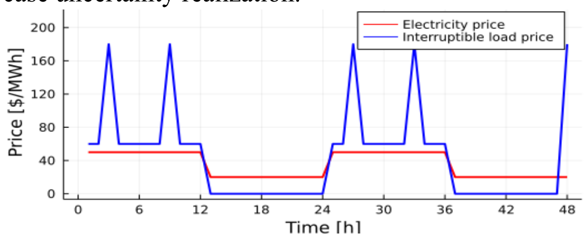

Fig. 3: Electricity and interruptible load prices for case study

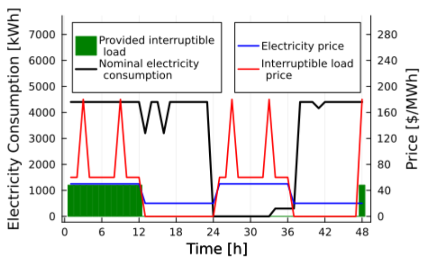

Fig. 4: Nominal electricity consumption and interruptible load provided with continuous recourse only and $\bar{\zeta} = 47$.

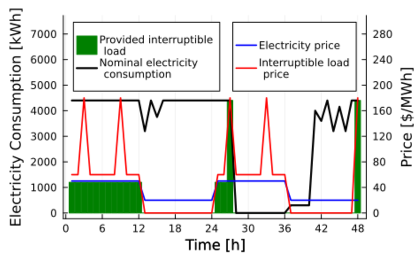

Fig. 5: Nominal electricity consumption and interruptible load provided with mixed-integer recourse and $\bar{\zeta} = 47$.

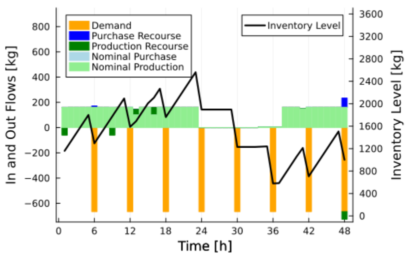

Fig. 6: Nominal and worst-case recourse product flows and nominal inventory profile for the case with continuous recourse only and $\bar{\zeta} = 47$.

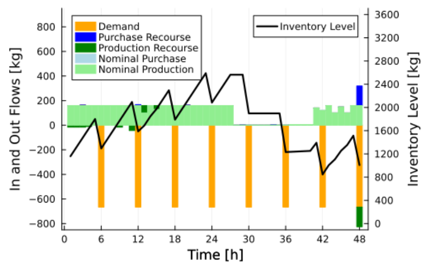

Fig. 7: Nominal and worst-case recourse product flows and nominal inventory profile for the case with mixed-integer recourse and $\bar{\zeta} = 47$.

Table 3: Net operating cost (C) in $ for cases with different $\bar{\zeta}$. Here, C^{nominal} is the cost when no load reduction is requested and C^{wc} is the worst-case cost when load reduction is requested.

$\bar{\zeta}$	Continuous recourse only (C^{nominal}, $)	Mixed-integer recourse (C^{nominal}, $)	Continuous recourse only (C^{wc}, $)	Mixed-integer recourse (C^{wc}, $)
47	3,214.1	2,755.3	3,266.3	2,917.8

23	3,214.1	2,755.3	3,266.3	2,917.8
11	3,214.1	2,810.4	3,282.7	2,999.8

5.2. Computation time

The computation times required to solve the models are shown in Table 4. As expected, considering mixed-integer recourse is computationally significantly more expensive. Adjusting the parameter $\bar{\zeta}$ can help reduce the solution time but may give a solution with higher objective value. Clearly, a trade-off exists between computational performance and solution quality. In the example considered, setting $\bar{\zeta} = 23$ provides a solution of the same quality as $\bar{\zeta} = 47$ in much shorter time.

Table 4: Computation times for cases with different $\bar{\zeta}$.

$\bar{\zeta}$	Continuous recourse only	Mixed-integer recourse
47	90 s	3,626 s
23	32 s	1,228 s
11	10 s	253 s

6. Conclusion

In this work, we developed a multistage robust optimization model for the scheduling of power-intensive plants that also participate in the interruptible load market. Piecewise linear/constant decision rules are used to determine the recourse actions necessary in both continuous and discrete variables. When applied to an illustrative example, the proposed model achieves significant cost savings compared to the formulation that only considers continuous recourse. The flexibility that mixed-integer recourse provides comes at a substantial computational cost. However, our model provides a way of exploring this trade-off by setting the amount of past information allowed to be considered in the decision rules. By doing so, we find that the problem can often be solved in much less time with little sacrifice on the solution quality.

7. Acknowledgements

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