

Near-optimal Online Algorithms for Joint Pricing and Scheduling in EV Charging Networks

Roozbeh Bostandoost University of Massachusetts Amherst United States of America rbostandoost@umass.edu

> Carlee Joe-Wong Carnegie Mellon University United States of America cjoewong@andrew.cmu.edu

ABSTRACT

With the rapid acceleration of transportation electrification, public charging stations are becoming vital infrastructure in smart sustainable cities to provide on-demand electric vehicle (EV) charging services. As more consumers seek to utilize public charging services, the pricing and scheduling of such services will become vital, complementary tools to mediate competition for charging resources. However, determining the right prices to charge is difficult due to the online nature of EV arrivals. This paper studies a joint pricing and scheduling problem for the operator of EV charging networks with limited charging capacity and time-varying energy costs. Upon receiving a charging request, the operator offers a price, and the EV decides whether to accept the offer based on its own value and the posted price. The operator then schedules the real-time charging process to satisfy the charging request if the EV admits the offer. We propose an online pricing algorithm that can determine the posted price and EV charging schedule to maximize social welfare, i.e., the total value of EVs minus the energy cost of charging stations. Theoretically, we prove the devised algorithm can achieve an order-optimal competitive ratio under the competitive analysis framework. Practically, we show the empirical performance of our algorithm outperforms other benchmark algorithms in experiments using real EV charging data.

CCS CONCEPTS

- Theory of computation \rightarrow Online algorithms; Networks \rightarrow Network economics.
- **KEYWORDS**

online algorithm, scheduling, optimization, pricing

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than the author(s) must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission

and/or a fee. Request permissions from permissions@acm.org. e-Energy '23, June 20–23, 2023, Orlando, FL, USA

© 2023 Copyright held by the owner/author(s). Publication rights licensed to ACM. ACM ISBN 979-8-4007-0032-3/23/06...\$15.00 https://doi.org/10.1145/3575813.3576878

Bo Sun
The Chinese University of Hong Kong
China
bsun@cse.cuhk.edu.hk

Mohammad Hajiesmaili University of Massachusetts Amherst United States of America hajiesmaili@cs.umass.edu

ACM Reference Format:

Roozbeh Bostandoost, Bo Sun, Carlee Joe-Wong, and Mohammad Hajiesmaili. 2023. Near-optimal Online Algorithms for Joint Pricing and Scheduling in EV Charging Networks. In *The 14th ACM International Conference on Future Energy Systems (e-Energy '23), June 20–23, 2023, Orlando, FL, USA*. ACM, New York, NY, USA, 12 pages. https://doi.org/10.1145/3575813.3576878

1 INTRODUCTION

Electric vehicles (EVs) have long promised to make transportation systems more efficient and sustainable [1]. Bloomberg predicts that by 2040, more than half of the new car sales will be EVs, accounting for 5% of total electricity usage [23]. According to the New York Times [11], California has already set a 2035 deadline for all new cars sold in California to be powered by electricity or hydrogen and be free of greenhouse gas emissions. This rule requires automakers to increase the building of zero-emission vehicles. The percentage of new electric and plug-in hybrid vehicles sales will be 35% in 2026, go up to 68% in 2030, and reach 100% in 2035. This predicted EV popularity follows a similarly rapid growth in the number of public charging stations. For example, the city of Los Angeles has 4,689 public charging stations and has added 1,185 new charging stations since mid-June 2022. ChargePoint, the world's largest EV charging network, plans to build out 2.5 million public charging ports by 2025 [18]. Despite the growing number of charging stations, however, each individual station has a fairly limited charging capacity that may not be enough to serve the growing demand for EV vehicles. The scale of this growth then raises a fundamental challenge: how should EV charging networks manage the ever-increasing demands for their services?

Answering this question naturally raises *dynamic pricing* as a solution. To regulate their users' demands, charging platforms may wish to charge users more when demand is high relative to their limited capacity and vice versa when demand is low. The advantages of dynamic pricing over static pricing (i.e., users are charged with the same price independent of user demand and station capacity) have been extensively discussed in theoretical studies [3, 8, 10]. In practice, ChargePoint is already piloting such dynamic pricing [2]. Dynamic pricing allows these platforms to adjust their charging demands to match available resources, i.e., the transformers' capacity, while carefully scheduling which users receive resources at which time can further improve revenue and efficiency. Yet despite considerable prior work on both pricing and scheduling problems for

	Integral/Fractional	Scheduling	Reusable Resource	Resource Cost	Optimality of CRs
Zhou et al. [38]	Integral	No	No	No	Yes
Zheng et al. [37]	Fractional	Yes	Yes	Yes	N/A
Sun et al. [29]	Fractional	Yes	Yes	No	Yes
Zhang et al. [34]	Integral	Yes	Yes	No	No
Tan et al. [30]	Integral	No	Yes	Yes	No
Sun et al. [28]	Integral	No	Yes	No	Yes
This Work	Integral	Yes	Yes	Yes	Yes

Table 1: Comparison of previous works and ours in terms of multiple criteria

adaptive EV charging networks [21, 35, 36], there is still little rigorous theoretical understanding of how they can be jointly optimized to improve social welfare and ensure platform profitability.

Optimizing these prices for charging, however, requires resolving *uncertainty challenges* of the future charging demand in the network. Moreover, the possibility of adaptive EV charging scheduling provides a new design space for further optimization. For example, the users may park their EVs at a charging platform while at work, and the vehicle can be charged at any time before the user leaves work. Thus, the EV's impact on the platform, and the price it should be charged, will depend on how congested the platform is at any time before it leaves. However, users would generally wish to know the prices and their guaranteed charging amounts upon their arrival in order to eliminate payment uncertainty (e.g., as offered by ridesharing services [25]), at which time future information on platform congestion is unknown.

In this paper, we tackle the problem of joint pricing and scheduling for EV charging from the perspective of a charging operator who manages a charging station. Our goal is to develop pricing algorithms for admitting EV users to the charging network and scheduling their charging rates such that their energy demand is fully satisfied during their window of availability, i.e., the time interval for which the EV is present at the charging station. A major challenge in the above pricing and scheduling problem is the uncertainty of the environment in terms of future EV charging demand. EVs arrive at the charging station in an online manner, and their availability windows might be different.

1.1 Contributions

In this paper, we formulate the joint problem of dynamic pricing and scheduling for EV charging in a station with multiple charging ports and limited capacity. In this scenario, some EVs with different private values for getting charged, energy demands, and availability windows arrive in *an online manner*, and the objective is to maximize the social welfare by offering a charging price with proper scheduling, given the capacity limit of the station.

We develop *online algorithms* for the above joint pricing and scheduling problem. Upon the arrival of a charging request from an EV, the algorithm calculates a charging schedule for the EV and posts a corresponding total charging price for the EV. The posted price is determined as a function of the current utilization of the charging station. The pricing function is designed by solving a differential equation induced by an online primal-dual approach. The calculation of the price is based on the charging request of the

EV, e.g., energy demand, the availability windows; the utilization of the station, and the time-vary energy cost of the station. The EV compares the posted price from the station, and if the price is less than its private value, i.e., the value that the EV is willing to pay, it accepts the offer and joins the station.

We then analyze the robustness of the proposed online algorithms using the competitive ratio (CR), a well-established performance metric for online algorithms. Our analysis in Theorem 3.1 shows that the competitive ratio of the proposed algorithm is $O(\ln\theta)$, where $\theta=UD^{\max}/(D^{\min}(L-p^{\max}))$, U and L are the maximum and minimum private values of per-unit energy requested by the EVs, D^{\max} and D^{\min} are the maximum and minimum availability window of EVs, and p^{\max} is the maximum energy cost. By establishing a connection between the problem of interest in this paper and two classical online problems in the prior literature, we then derive a lower bound of $\Omega(\ln(UD^{\max}/LD^{\min}))$ for the competitive ratio of any algorithm for the joint pricing and scheduling problem. This lower bound result shows that the competitive ratio of the proposed algorithm is order-optimal.

Last, we use the Caltech EV ACN-Data dataset [19] on over 700 EV charging sessions over 90 days to evaluate the performance of the proposed algorithm and compare it with the prior EV pricing and scheduling algorithms and baselines. The experimental results show that the empirical CR of the proposed algorithm is less than 2 in 80% of the experiments on average, which is much better than our theoretical bounds. Also, this empirical result shows that our algorithm outperforms other comparison algorithms substantially; the empirical CRs of others are less than 2 in only 65%, 25%, and 20% of the experiments on average.

The rest of the paper is organized as follows. Section 2 introduces the offline and online formulation of the joint pricing and scheduling problem. The proposed online solution algorithm and our main theoretical performance bounds are presented in Section 3. Then Section 4 provides proofs of the key theorems in Section 3. Section 5 validates the numerical performance of our proposed algorithm using real data. Finally, we discuss related work in Section 6 and conclude the paper in Section 7.

1.2 The Significance of the Theoretical Results

We note that although we present our joint pricing and scheduling algorithms in the context of EV charging, the underlying optimization problem is of independent interest and could be applicable to scenarios beyond EV charging. More specifically, the problem of interest in this paper is an extension of the basic online knap-sack problem and captures reusable resources with time-varying resource costs. Consider an online scenario where some items with different values and demands arrive one by one over time and stay in the system for a limited duration. Upon the arrival of a new item, an online decision maker should decide whether or not to admit the item given its value and demand, the limited capacity of the knapsack, and the time-varying cost of using the knapsack. Further, if the decision is to admit the item, what is the scheduling policy to fully satisfy the demand for the admitted item?

The online knapsack problem and its variants have been extensively studied in the literature [28–30, 34, 37, 38]. Nearly all online algorithms in this stream of works are based on a similar idea that estimates the price of admitting one item using a function of the knapsack utilization and admits the item if the item value is larger than the estimated price. The key to these algorithms lies in how to design the pricing function to guarantee the best possible CR. We review the literature in Section 6 extensively and in what follows we highlight the differences and significance of our results with the most relevant works.

The basic version of the online knapsack problem without reusable resources and resource cost was first studied in [38], and an online algorithm was designed to achieve the optimal CR. In the general setting, this stream of works can be distinguished based on three criteria: (i) fractional or integral admission decision, i.e., whether each item can be fractionally admitted or not; (ii) scheduling, i.e., whether the demand of the item is fixed or can be flexibly determined over time; and (iii) resource cost, i.e., whether using the knapsack incurs a cost. In terms of the significance of the result, the ultimate goal is to achieve online algorithms that are (order)-optimal, where optimality refers to an online algorithm with the best possible CR either exactly or order-wise. In Table 1, we compare the most related works based on these criteria.

This paper designs an order-optimal online algorithm for the most general integral setting. Compared to the other two works with optimal CRs (i.e., [29] and [28]), this work makes additional technical contributions in design and analysis of the algorithm in the general setting. In particular, we extend the analysis of the fractional admission in [29] to integral admission by considering two classes of problem instances and analyzing their corresponding worst-case CRs using different approaches (see Sections 4.2 and 4.3). We extend the fixed demand setting in [28] to include scheduling setting by formulating an auxiliary cost minimization (see problem (2)) for scheduling decisions and analyzing the CR by an online primal-dual approach. Further, our primal-dual analysis results in a natural design of the pricing function, which is in contrast to the ad-hoc design in [28]. In addition, this paper takes into account the resource cost, which adds an extra dimension in the algorithm analysis.

2 PROBLEM STATEMENT

We consider an online pricing and scheduling problem for EV charging management. A set $\mathcal{N} := \{1, \dots, N\}$ of EVs sequentially arrive over a time horizon $\mathcal{T} := \{1, \dots, T\}$. A system operator manages a charging station with multiple chargers and a fixed overall charging rate capacity C, and faces a time-varying electricity price $\{p_t\}_{t \in \mathcal{T}}$.

Note that we assume that there are sufficient chargers in the station to admit all EVs; however, the per-slot aggregate charging rate of all chargers in the charging station is limited to capacity C. Upon arrival, each EV n proposes a request $\gamma_n := \{a_n, d_n, E_n, R_n\}$, where a_n and d_n denote the arrival and departure time of EV n, E_n is the energy demand, and R_n is the rate limit of charging power in one time slot. For the charging request γ_n , EV n has a private value v_n (i.e., willingness-to-pay), which is unknown to the operator.

After receiving the request γ_n , the operator posts a price ξ_n for serving EV n. Then EV n itself decides to pay ξ_n for charging if and only if EV n has a positive utility surplus, i.e., $v_n - \xi_n \ge 0$; otherwise, EV n will leave the system without charging and seek for alternative refueling opportunities.

If EV n decides to join the station, the operator collects payment ξ_n and schedules its charging $\mathbf{y}_n := \{y_{nt}\}_{t \in \mathcal{T}_n}$, where $\mathcal{T}_n := \{a_n, a_n+1, \ldots, d_n-1\}$ is the set of time slots when EV n is available and y_{nt} is the delivered energy in slot t. The charging schedule incurs energy cost $\sum_{t \in \mathcal{T}_n} p_t y_{nt}$. For a quick reference, notations are summarized in Table 2.

Offline social welfare maximization problem. Let $x_n \in \{0, 1\}$ indicate whether EV n decides to charge at the station. The utility surplus of all EVs is $\sum_{n \in \mathcal{N}} (v_n - \xi_n) x_n$ and the surplus of the charging station operator is $\sum_{n \in \mathcal{N}} (\xi_n - \sum_{t \in \mathcal{T}_n} p_t y_{nt}) x_n$. Our objective is to maximize the social welfare of all EVs and the operator [27, 37]. Define an instance $I := \{\{\gamma_n; v_n\}_{n \in \mathcal{N}}\}$ as a sequence of the EV charging requests and the corresponding values. Given I from the start, the offline social welfare maximization problem can be formulated as follows.

$$\max_{x_n, y_{nt}} \sum_{n \in \mathcal{N}} v_n x_n - \sum_{t \in \mathcal{T}} p_t \sum_{n \in \mathcal{N}_t} y_{nt}$$
s.t.
$$\sum_{t \in \mathcal{T}_n} y_{nt} \ge E_n x_n, \quad \forall n \in \mathcal{N},$$
(1a)

$$\sum_{n \in \mathcal{N}_t} y_{nt} \le C, \quad \forall t \in \mathcal{T}, \tag{1b}$$

$$0 \le y_{nt} \le R_n x_n, \quad \forall n \in \mathcal{N}, t \in \mathcal{T}_n,$$
 (1c)

$$x_n \in \{0, 1\}, \quad \forall n \in \mathcal{N},$$
 (1d)

where \mathcal{N}_t denotes the set of EVs that are available at time t. Problem (1) captures multiple constraints. Constraint (1a) requires the charging station to *fully* charge the EV n up to its demand if it accepts the charging offer ($x_n = 1$). Constraint (1b) says the aggregate of scheduled energy for EVs that are available at each slot t must be smaller than the station capacity. Constraint (1c) requires the scheduled energy in each slot t is less than the EV's rate limit.

Online problem. We consider an online version of the social welfare maximization problem (1). Upon the arrival of each EV n, the operator must immediately decide the posted price ξ_n only based on the past and current EV requests, and without knowing the future EV requests. In this paper, we consider the electricity price $\{p_t\}_{t\in\mathcal{T}}$ is known to the charging station operator from the start, e.g., from predetermined time-of-use prices or estimated electricity prices based on historical data. The electricity price and charging station capacity C are defined as fixed setup information of the system. Let $\mathsf{OPT}(I)$ and $\mathsf{ALG}(I)$ denote the social welfare achieved by the optimal offline problem and an online algorithm under instance I, respectively. An online algorithm is called π -competitive

 R_n

 v_n

 \mathcal{T}_n

 y_{nt}

 x_n

Name	Description
N	The set of all EVs, indexed by <i>n</i>
T	Number of time slots
\mathcal{T}	The set of time slots, i.e., $\mathcal{T} \in \{1, 2,, T\}$, indexed by t
С	The capacity of the charging station
p_t	Electricity price at time <i>t</i>
a_n	Arrival time of EV n
d_n	Departure time of EV n
E_n	Energy demand of EV n

Indicator variable which indicates whether EV n has decided to be charged at the station or not

Rate limit of charging power in one time slot for EV *n*

Set of time slots that EV *n* is available at the station

The amount of energy that is being delivered to EV n at time t

Valuation of EV n for receiving its demand

Table 2: Summary of Notations

if $\mathsf{OPT}(I)/\mathsf{ALG}(I) \leq \pi$ holds for all instances, where $\pi \geq 1$ is called the competitive ratio (CR). We aim to design an online algorithm with a CR that is as small as possible.

Assumptions. We make following assumptions through the paper.

Assumption 1 (Infinitesimal schedule). The rate limit of individual EV charging rate is much smaller than the charging capacity of the station, i.e., $R_n \ll C \forall n \in \mathcal{N}$.

Practically, a charging station can charge tens of EVs simultaneously; therefore, its capacity is much larger than the charging rate of each EV. This assumption is also common in the literature with competitive analysis, e.g., [16, 29]. Additionally, assumption 1 allows us to focus on the nature of our problem with mathematical convenience.

Next, we define the value density of EV n as its value on per-unit energy, i.e., v_n/E_n , which is the payment amount that each EV is willing to pay per unit of energy.

Assumption 2 (Bounded value density). The value densities of EVs are bounded, i.e., $\frac{v_n}{E_n} \in [L, U], \forall n \in \mathcal{N}$, where L and U are the lower and upper bounds for the value density.

In our algorithm design and analysis, we assume that L and U are known in advance, e.g., from user surveys or historical behavior.

Assumption 3 (Bounded availability). The available window of EV charging is bounded, i.e., $d_n - a_n \in [D^{\min}, D^{\max}], \forall n \in \mathcal{N}$.

In practice, it is reasonable to have a lower bound for the EV available window since the EV cannot stay less than a minimum duration needed to get charged up to their demand given the charging rate limit. Also, each EV's availability time typically has an upper bound that is imposed by the station to ensure there are enough chargers available for newly arriving EVs.

Assumption 4 (Bounded electricity Price). The electricity price $\{p_t\}_{t\in\mathcal{T}}$ is bounded, i.e., $0\leq p_t\leq p^{\max}\leq L, \forall t\in\mathcal{T}.$

We assume the value density is greater than the maximum electricity price, i.e., $v_n/E_n>p^{\max}$, $\forall n\in \mathcal{N}$. Thus, EVs can be charged at the stations with non-negative utility surplus.

Algorithm 1 Online Posted-Pricing Algorithm (OPA(ϕ))

- 1: **Inputs:** setup information $\{C, \{p_t\}_{t \in \mathcal{T}}\}$, pricing function $\phi = \{\phi_t(\cdot)\}_{t \in \mathcal{T}}$;
- 2: **Initialization:** capacity utilization $w_t^{(0)} = 0, \forall t \in \mathcal{T};$
- 3: **while** a new EV n arrives with $\{a_n, d_n, E_n, R_n\}$ **do**
- 4: solve a candidate charging schedule $\{\hat{y}_{nt}\}_{t\in\mathcal{T}_n}$ from

$$\min_{y_{nt} \geq 0} \quad \sum\nolimits_{t \in \mathcal{T}_n} \int_{w_t^{(n-1)}}^{w_t^{(n-1)} + y_{nt}} \phi_t(u) du \tag{2a}$$

s.t.
$$\sum_{t \in \mathcal{T}_n} y_{nt} \ge E_n$$
, $(\hat{\mu}_n)$ (2b)

$$y_{nt} \le R_n, \forall t \in \mathcal{T}_n. \quad (\hat{\sigma}_{nt})$$
 (2c)

- set the posted price $\xi_n = \sum_{t \in \mathcal{T}_n} \phi_t \left(w_t^{(n-1)} + \hat{y}_{nt} \right) \hat{y}_{nt};$
- 6: **if** EV *n* accepts the charging **then**
- 7: set $\hat{x}_n = 1$ and charge EV n by $\{\hat{y}_{nt}\}_{t \in \mathcal{T}_n}$; 8: **else**
- 9: set $\hat{x}_n = 0$, $\hat{y}_{nt} = 0$, $\forall t \in \mathcal{T}_n$;
- 10: end if
- 11: update utilization $w_t^{(n)} = \begin{cases} w_t^{(n-1)} + \hat{y}_{nt} & t \in \mathcal{T}_n \\ w_t^{(n-1)} & t \in \mathcal{T} \setminus \mathcal{T}_n \end{cases}$
- 12: end while

3 ALGORITHM AND RESULTS

In this section, we propose an online posted-pricing algorithm (OPA) that can jointly determine the posted price and the corresponding charging schedule in EV charging networks.

3.1 An Online Posted-Pricing Algorithm

When a new EV n arrives, $\mathsf{OPA}(\phi)$ uses a predetermined pricing function ϕ to estimate the charging cost of the EV and determines a candidate charging schedule $\{\hat{y}_{nt}\}_{t\in\mathcal{T}_n}$ by solving a (pseudo) costminimization problem (2). In this problem, $\phi_t(u)du$ estimates the (pseudo) cost of charging du unit of energy when the utilization of the station is u in time slot t. Then the objective of the problem (2) is to minimize the total (pseudo) cost of satisfying the EV's request.

Then algorithm (1) sets the posted price as the total estimated cost of charging EV n, i.e., $\xi_n = \sum_{t \in \mathcal{T}_n} \phi_t(w_t^{(n-1)} + \hat{y}_{nt})\hat{y}_{nt}$, where $w_t^{(n-1)}$ is the charging station's utilization in slot t when a new EV n arrives. If EV n's value is no smaller than the posted price, the offer will be accepted, and the station charges the EV based on the candidate schedule. Finally, the charging station's utilization is updated and used for estimating the charging cost for the next EV.

The performance of $\mathsf{OPA}(\phi)$ is determined by the pricing function. In this paper, we consider the pricing function in the following form:

$$\phi_t(w) = \begin{cases} L, & w \in [0, \beta) \\ \varphi(w), & w \in [\beta, C] \\ +\infty, & w \in (C, +\infty) \end{cases}, \forall t \in \mathcal{T}.$$
 (3)

This function consists of three segments. The first flat segment $(w \in [0,\beta))$ is set to the lower bound of EVs' value density. Then if EV n is among the first few EVs that come to the station, it will receive a posted price E_nL , which the lowest possible price, and thus will accept the offer. This design ensures that the station can at least admit some EVs to secure certain profits. The second segment $\varphi(w)$ is a non-decreasing function to realize an intuitive idea that the station becomes more selective to admit EVs as its utilization increases. The last segment sets the price to ∞ when the utilization exceeds the capacity. In this way, any charging schedule that exceeds the capacity constraint leads to an infinitely large posted price (which can be set to a cap price in practice), preventing capacity violations.

3.2 Main Results

Let $CR(\phi)$ denote the CR of OPA with pricing function ϕ . In this section, we present the main theoretical results of this paper, which show how to design the pricing function ϕ such that OPA can achieve the order-optimal CR for the joint pricing and scheduling problem.

Theorem 3.1. Under Assumptions 1-4, when the parameter satisfies $(U/L)(D^{\max}/D^{\min}) \ge 2$ and the pricing function is given by $\phi^* = \{\phi_t^*(\cdot)\}_{t \in \mathcal{T}}$, where for all $t \in \mathcal{T}$,

$$\phi_t(w) = \begin{cases} L, & w \in [0, C/\alpha) \\ \frac{L - p_t}{e} e^{\frac{\alpha}{C} w} + p_t, & w \in [C/\alpha, C], \\ +\infty, & w \in (C, +\infty) \end{cases}$$
(4)

with $\alpha=1+2\ln(\theta)$ and $\theta=\frac{UD^{\max}/D^{\min}}{L-p^{\max}}$, then the competitive ratio of OPA is $\text{CR}(\phi^*)=O(\ln(\theta))$.

This theorem provides the design of the pricing function for OPA and the corresponding CR. Note that the technical assumption $(U/L)(D^{\max}/D^{\min}) \geq 2$ states that multiplication of the value and duration fluctuation ratios should be larger than 2. We postpone the proof of Theorem 3.1 to Section 4.

Theorem 3.2. There is no online algorithm that can achieve a competitive ratio smaller than $\Omega(\ln(\theta))$ for our problem.

PROOF. To prove this result, we consider two special cases of our problem. First, the basic online knapsack problem [38] is a special case of our problem with no energy price, setting the departure time of each EV n to T, the demand of $(T - a_n)R_n$, which correspond to

 R_n charging demand in each slot. Define $\rho = \frac{U}{L}$ as the fluctuation ratio, and the lower bound of online knapsack is $\Omega(\ln(\rho))$ [38].

Secondly, with the special case of homogeneous values, i.e., $v_n=v$, $\forall n$, no energy cost, i.e., $p_t=0$, $\forall t$, and different duration of EVs, our problem degenerates to the online interval scheduling problem [15]. Let $\delta=D^{\max}/D^{\min}$ be the duration ratio. It is known that the lower bound on the competitive ratio for any online algorithm for the online interval scheduling is $\Omega(\ln(\delta))$ [15]. Now, with two special cases of the online knapsack problem and the online interval scheduling, the lower bound of any online algorithms for problem (1) is

$$\begin{split} \mathsf{CR}(\mathsf{OPA}) &\geq \max\{\Omega(\ln(\delta)), \Omega(\ln(\rho))\} \geq \frac{1}{2}\Omega(\ln(\delta)) + \frac{1}{2}\Omega(\ln(\rho)) \\ &\geq \frac{1}{2}\Omega(\ln(\delta\rho)) = \Omega(\ln(\theta)), \end{split}$$

where
$$\theta = \frac{UD^{\max}/D^{\min}}{L-p^{\max}}$$
 and with $p^{\max} = 0$, we have $\theta = \delta \rho$.

Theorem 3.2 gives a lower bound of the CR for the joint pricing and scheduling problem. Combining Theorem 3.1 and Theorem 3.2 concludes that our proposed OPA can achieve an order-optimal CR.

4 COMPETITIVE ANALYSIS

This section presents the proof of Theorem 3.1, and simultaneously shows how to design the threshold function ϕ^* for OPA to achieve the order-optimal CR.

4.1 Roadmap for the Proof

Recall that in OPA, the pricing function becomes infinitely large once the utilization exceeds the capacity of the charging station, i.e., $\phi_t(w_t^{(n-1)} + \hat{y}_{nt}) = +\infty, \forall t \in \mathcal{T}, n \in \mathcal{N} \text{ if } w_t^{(n-1)} + \hat{y}_{nt} > C. \text{ Thus, if }$ any EV's charging candidate schedule exceeds the station capacity, OPA sets the posted price to $+\infty$ to reject this request. However, the discontinuity of the pricing function, $\phi_t(w) : \mathbb{R}^+ \to \mathbb{R}^+$ at w = C, results in difficulties in the competitive analysis since the cost minimization problem (2) may not be a convex problem, prohibiting us from directly applying existing approaches (e.g., primal-dual based analysis). Thus, we divide the analysis of OPA into two cases. Let Ψ denote the set of instances that satisfy Assumptions 1-4 and divide Ψ into two disjoint subsets Ψ^1 and Ψ^2 . Ψ^1 includes the instances, under which OPA does not output a candidate schedule that violates the capacity. Then the utilization of the charging station under Ψ^1 is away from the capacity C and hence we call Ψ^1 capacity-free instances. $\Psi^2 = \Psi \setminus \Psi^1$ is then called capacity-limited instances because there exists at least one time slot when the capacity can be violated by one candidate schedule.

In the proof of Theorem 3.1, we first analyze the CR of $OPA(\phi^*)$ under capacity-free instances Ψ^1 in Section 4.2. In this case, the cost minimization problem (2) is a convex problem and we analyze the CR of OPA based on online primal-dual (OPD) framework [9, 30]. In fact, we can provide a constructive proof, which not only proves CR, but also shows how to design the pricing function ϕ^* .

Lemma 4.1. Given pricing function ϕ^* in Equation (4), the CR of OPA(ϕ^*) under capacity-free instances Ψ^1 is $\frac{\mathsf{OPT}(I)}{\mathsf{ALG}(I)} \leq \alpha = 1 + 2\ln(\theta), \forall I \in \Psi^1$.

Given the pricing function ϕ^* obtained in the capacity-free case, we then analyze the CR of OPA(ϕ^*) under the capacity-limited instances Ψ^2 . Since we cannot rely on the convexity of the problem (2) in this case, we derive the CR by figuring out the worst-case instances in different scenarios and finally obtain the following upper bound for the CR.

Lemma 4.2. Given pricing function ϕ^* in Equation (4), the competitive ratio of OPA(ϕ^*) under capacity-limited instances Ψ^2 is $\frac{OPT(I)}{ALG(I)} \le$ $3\max\left\{2\sqrt{e},\frac{\theta\alpha}{\exp(\alpha/2-1)}\right\}=3\sqrt{e}\max\left\{2,1+2\ln\theta\right\},\forall I\in\Psi^2.$

Combining Lemma 4.1 and Lemma 4.2 gives

$$\begin{split} \mathsf{CR}(\alpha^*) &\leq \max_{I \,\in\, \Psi^1 \cup \Psi^2} \frac{\mathsf{OPT}(I)}{\mathsf{ALG}(I)} \\ &= \max \left\{ \alpha, 6\sqrt{e}, \frac{3\theta\alpha}{\exp(\alpha/2 - 1)} \right\} = O(\ln \theta), \end{split}$$

which completes the proof of Theorem 3.1.

Proof of Lemma 4.1

Based on OPD, we can derive the following technical lemma that provides a sufficient condition on the pricing function to ensure the competitiveness of OPA.

Lemma 4.3. Under capacity-free instances, $OPA(\phi)$ is α -competitive if the pricing function $\phi = {\phi_t}_{t \in \mathcal{T}}$ is given by, for all $t \in \mathcal{T}$,

$$\phi_t(w) = \begin{cases} L, & w \in [0, \beta), \\ \varphi_t(w), & w \in [\beta, C], \end{cases}$$
 (5)

where $\beta \geq C/\alpha$ is a utilization threshold, and φ_t is a non-decreasing function that satisfies:

$$\begin{cases} \alpha \varphi_t(w) - C \varphi_t'(w) \ge \alpha p_t, w \in [\beta, C] \\ \varphi_t(\beta) = L, \varphi_t(C) \ge \frac{UD^{\max}}{D^{\min}} \end{cases}$$
 (6)

Based on Lemma 4.3, we can design the pricing function ϕ that satisfies the differential equation (6) and minimize the competitive ratio α . To do so, we bind all inequalities in (6) and solve this boundary value problem. Then we can obtain that $\beta = C/\alpha$ and the pricing function ϕ^* is given by equation (4), which proves Lemma 4.1. In what follows, we prove the technical Lemma 4.3 based on OPD.

Consider the relaxed primal problem (1) as follows

$$\max_{x_n \ge 0, y_{nt} \ge 0} \quad \sum_{n \in \mathcal{N}} v_n x_n - \sum_{t \in \mathcal{T}} p_t \sum_{n \in \mathcal{N}_t} y_{nt}$$
 (7a)

s.t.
$$\sum_{t \in \mathcal{T}_n} y_{nt} \ge E_n x_n$$
, $\forall n \in \mathcal{N}$, (μ_n) (7b)

$$\sum_{n \in \mathcal{N}_t} y_{nt} \le C, \quad \forall t \in \mathcal{T}, \quad (\lambda_t)$$
 (7c)

$$y_{nt} \le R_n x_n, \quad \forall n \in \mathcal{N}, t \in \mathcal{T}_n, \quad (\sigma_{nt})$$
 (7d)

$$x_n \le 1, \quad \forall n \in \mathcal{N}, \quad (\eta_n),$$
 (7e)

where we relax the binary variable $x_n \in \{0, 1\}$ to be continuous, i.e., $x_n \in [0,1]$, and $\boldsymbol{\mu} := \{\mu_n\}_{n \in \mathcal{N}}, \boldsymbol{\lambda} := \{\lambda_t\}_{t \in \mathcal{T}}, \boldsymbol{\sigma} := \{\sigma_{nt}\}_{n \in \mathcal{N}, t \in \mathcal{T}_n},$ $\eta := \{\eta_n\}_{n \in \mathcal{N}}$ are the Lagrange multipliers associated with the

corresponding constraints. The dual of the problem (7) is:

$$\min_{\mu,\lambda,\sigma,\eta\geq 0} \quad \sum_{t\in\mathcal{T}} \lambda_t C + \sum_{n\in\mathcal{N}} \eta_n \tag{8a}$$

s.t.
$$v_n - \mu_n E_n + \sum_{t \in \mathcal{T}} \sigma_{nt} R_n - \eta_n \le 0, \forall n \in \mathcal{N},$$
 (8b)

$$\mu_n - \lambda_t - p_t - \sigma_{nt} \le 0, \forall n \in \mathcal{N}, t \in \mathcal{T}_n.$$
 (8c)

The key idea of the OPD approach is to construct a feasible dual solution based on the online solution produced by an online algorithm, and then build the upper bound of the offline optimum using the feasible dual objective based on weak duality. Given an arrival instance \mathcal{I} , we denote the online solution by $\hat{X} := (\hat{x}, \hat{y})$ and the constructed dual variable by $\bar{\Lambda} := (\bar{\mu}, \bar{\lambda}, \bar{\sigma}, \bar{\eta})$. Also let P_n and D_n denote the objective values of the primal problem (7) and dual problem (8) after the n-th EV is processed, respectively. Then the OPD can be summarized as the following proposition.

Proposition 4.1 (Proposition 3.1 [30]). An online algorithm is α -competitive if the following conditions hold:

- (i) \hat{X} and $\bar{\Lambda}$ are feasible solutions of the primal problem (7) and the dual problem (8), respectively;
- (ii) There exists an index $k \in N$ such that the following sufficient inequality holds:

 - (initial inequality) $P_k \geq \frac{1}{\alpha}D_k$, (incremental inequality) $P_n P_{n-1} \geq \frac{1}{\alpha}(D_n D_{n-1}), \forall n =$

Next we prove Lemma 4.3 based on Proposition 4.1.

Constructing feasible solutions. Given the feasible primal solution $\hat{X} := (\hat{x}, \hat{y})$ produced by OPA(ϕ), we construct the dual solutions $\bar{\Lambda} := (\bar{\mu}, \bar{\lambda}, \bar{\sigma}, \bar{\eta})$ as follows:

$$\bar{\lambda}_t = [\phi_t(w_t^{(N)}) - p_t] \cdot \mathbb{I}\{w_t^{(N)} \ge \beta\}, \forall t \in \mathcal{T},$$
 (9a)

$$\bar{\mu}_n = \hat{\mu}_n = \max_{t \in \mathcal{T}_n: \hat{\mathcal{Y}}_{nt} > 0} \phi_t(w_t^{(n)}), \tag{9b}$$

$$\bar{\sigma}_{nt} = \hat{\sigma}_{nt} = \left[\hat{\mu}_n - \phi_t(w_t^{(n)})\right] \cdot \mathbb{I}\{\hat{y}_{nt} > 0\},\tag{9c}$$

$$\bar{\eta}_{n} = \begin{cases} v_{n} - \bar{\mu}_{n} E_{n} + \sum_{t \in \mathcal{T}_{n}} \bar{\sigma}_{nt} R_{n}, & \hat{x}_{n} = 1, \\ 0, & \hat{x}_{n} = 0, \end{cases}$$
(9d)

where $\hat{\mu}_n$ and $\{\hat{\sigma}_{nt}\}_{t\in\mathcal{T}_n}$ are the optimal dual variable of the cost minimization problem (2), and $w_t^{(n)}$ is the utilization of the charging station at slot t after processing EV n.

LEMMA 4.4. The constructed dual variables in Equation (9) are feasible for the problem (8).

PROOF. It can be easily checked that all constructed dual variables are non-negative. We aim to show the constructed dual variables satisfy dual constraints (8b) and (8c). Since the cost minimization problem (2) is convex under the capacity-free instances, by checking its KKT conditions, we have the equations as follows:

$$\phi_t(w_t^{(n)}) - \hat{\mu}_n + \hat{\sigma}_{nt} - \hat{\gamma}_{nt} = 0, \quad \forall t \in \mathcal{T}_n$$
 (10a)

$$\hat{\mu}_n E_n - \hat{\mu}_n \sum_{t \in \mathcal{T}_n} \hat{y}_{nt} = 0 \tag{10b}$$

$$\hat{\sigma}_{nt}\hat{y}_{nt} - \hat{\sigma}_{nt}R_n = 0, \quad \forall t \in \mathcal{T}_n$$
 (10c)

$$\hat{\gamma}_{nt}\hat{y}_{nt} = 0, \forall t \in \mathcal{T}_n, \tag{10d}$$

where the last equation is from the constraint $y_{nt} \ge 0$ and $\hat{\gamma}_{nt}$ is the corresponding dual variable.

By multiplying \hat{y}_{nt} on both sides of Equation (10a), summing over \mathcal{T}_n , and substituting equations (10b)-(10d), we can finally have

$$\sum_{t \in \mathcal{T}_n} \hat{y}_{nt} \phi_t(w_t^{(n)}) = \hat{\mu}_t E_n - \sum_{t \in \mathcal{T}_n} \hat{\sigma}_{nt} R_n. \tag{11}$$

Note that the constructed dual variables $\bar{\mu}_n$ and $\bar{\sigma}_{nt}$ in Equations (9b) and (9c) are exactly the same as $\hat{\mu}_n$ and $\hat{\sigma}_{nt}$. Then we can check the dual constraints (8b). When $\hat{x}_n=1$, we have $v_n-\bar{\mu}_nE_n+\sum_{t\in\mathcal{T}_n}\bar{\sigma}_{nt}R_n-\bar{\eta}_n=0$. When $\hat{x}_n=0$, we have $\bar{\eta}_n=0$ and $v_n<\sum_{t\in\mathcal{T}_n}\hat{\eta}_{nt}\phi_t(w_t^{(n)})=\bar{\mu}_tE_n-\sum_{t\in\mathcal{T}_n}\bar{\sigma}_{nt}R_n$. Thus, $\bar{\Lambda}$ satisfies constraint (8b).

Next, we check the dual constraint (8c).

When $\hat{y}_{nt} > 0$, we have $\bar{\sigma}_{nt} = \bar{\mu}_n - \phi_t(w_t^{(n)})$. Thus, $\bar{\mu}_n - \bar{\lambda}_t - p_t - \bar{\sigma}_{nt} = \bar{\mu}_n - \phi_t(w_t^{(n)}) - \bar{\sigma}_{nt} = 0$. When $\hat{y}_{nt} = 0$, $\bar{\mu}_n < \phi_t(w_t^{(n)})$ and $\bar{\sigma}_{nt} = 0$. This gives $\bar{\mu}_n - \bar{\lambda}_t - p_t - \bar{\sigma}_{nt} = \bar{\mu}_n - \phi_t(w_t^{(n)}) < 0$. Thus, $\bar{\Lambda}$ satisfies constraint (8c) and this completes the proof.

Guaranteeing the sufficient inequality: We aim to find a set of pricing functions ϕ and the corresponding CR α such that the sufficient inequality holds.

We first show there exists $k \in \mathcal{N}$ such that the initial inequality holds. Let k be the index of the first EV, after processing which the station utilization reaches β in at least one slot. Under the infinitesimal scheduling assumption 1, we can assume there exists a set of time slots $\bar{\mathcal{T}}_k \subseteq \mathcal{T}_k$ such that $w_t^{(k)} = \beta, \forall t \in \bar{\mathcal{T}}_k$ and $w_t^{(k)} < \beta, \forall t \in \mathcal{T} \setminus \bar{\mathcal{T}}_k$. Note that the first k EVs are all admitted since the posted price $\xi_n = \sum_{t \in \mathcal{T}_n} \hat{y}_{nt} \phi_t(w_t^{(n)}) = LE_n \leq v_n, \forall n \in [k]$. Then we can have

$$P_k = \sum_{n \in [k]} v_n - \sum_{n \in [k]} \sum_{t \in \mathcal{T}_n} p_t \hat{y}_{nt}, \tag{12a}$$

$$= \sum_{n \in [k]} [\bar{\eta}_n + \sum_{t \in \mathcal{T}_n} \hat{y}_{nt} \phi_t(w_t^{(n)})] - \sum_{n \in [k]} \sum_{t \in \mathcal{T}_n} p_t \hat{y}_{nt}$$
 (12b)

$$= \sum_{n \in [k]} \bar{\eta}_n + \sum_{t \in \mathcal{T}} (L - p_t) \sum_{n \in [k]} \hat{y}_{nt}$$
(12c)

$$\geq \sum_{n \in [k]} \bar{\eta}_n + \sum_{t \in \bar{\mathcal{T}}_k} (L - p_t) \beta \tag{12d}$$

$$\geq \frac{1}{\alpha} \sum_{n \in [k]} \bar{\eta}_n + \sum_{t \in \bar{\mathcal{T}}_k} (L - p_t) \frac{C}{\alpha} = \frac{1}{\alpha} D_k, \tag{12e}$$

where (12b) is obtained by substituting the definition of $\bar{\eta}$ and Equation (11), (12c) holds since $w_t^{(k)} \leq \beta$, $\forall t \in \mathcal{T}$, and (12e) holds since $\beta \geq C/\alpha$. Thus, given the sufficient condition in Lemma 4.3, the initial inequality holds.

The increments of primal and dual objectives can be cast as

$$\begin{split} P_{n} - P_{n-1} &= v_{n} - \sum_{t \in \mathcal{T}_{n}} p_{t} \hat{y}_{nt}, \\ D_{n} - D_{n-1} &= \sum_{t \in \mathcal{T}_{n}} C(\phi_{t}(w_{t}^{(n)}) - \phi_{t}(w_{t}^{(n-1)})) + \bar{\eta}, \end{split}$$

where we use $\phi_t(w_t^{(N)}) = \sum_{n \in \mathcal{N}} [\phi_t(w_t^{(n)}) - \phi_t(w_t^{(n-1)})]$. When $\hat{x}_n = 0$, we have $P_n - P_{n-1} = D_n - D_{n-1} = 0$ and thus the incremental inequality holds. When $\hat{x}_n = 1$, we can show that the incremental

inequality holds if the pricing function and the competitive ratio α satisfy the sufficient condition (6) in Lemma 4.3. To see this, note

$$D_{n} - D_{n-1}$$

$$= \sum_{t \in \mathcal{T}_{n}} C(\phi_{t}(w_{t}^{(n)}) - \phi_{t}(w_{t}^{(n-1)})) + v_{n} - \bar{\mu}_{n} E_{n} + \sum_{t \in \mathcal{T}_{n}} \bar{\sigma}_{nt} R_{n}$$

$$= \sum_{t \in \mathcal{T}_{n}} C(\phi_{t}(w_{t}^{(n)}) - \phi_{t}(w_{t}^{(n-1)})) + v_{n} - \sum_{t \in \mathcal{T}_{n}} \hat{y}_{nt} \phi_{t}(w_{t}^{(n)})$$
(13a)

$$= \sum_{t \in \mathcal{T}_{t}} \left[C(\phi_{t}(w_{t}^{(n)}) - \phi_{t}(w_{t}^{(n-1)})) - \hat{y}_{nt}\phi_{t}(w_{t}^{(n)}) \right] + v_{n} \quad (13b)$$

$$\leq \sum_{t \in \mathcal{T}_n} \hat{y}_{nt} \left[C \phi_t'(w_t^{(n)}) - \phi_t(w_t^{(n)}) \right] + v_n \tag{13c}$$

$$\leq \sum_{t \in \mathcal{T}_{-}} \hat{y}_{nt} \left[(\alpha - 1)\phi_t(w_t^{(n)}) - \alpha p_t \right] + v_n \tag{13d}$$

$$\leq (\alpha - 1)v_n - \alpha \sum_{t \in \mathcal{T}_n} \hat{y}_{nt} p_t + v_n = \alpha (P_n - P_{n-1}), \tag{13e}$$

where we substitute equation (11) to obtain (13a), use the inequality $\phi_t(w_t^{(n)}) - \phi_t(w_t^{(n-1)}) \leq \hat{y}_{nt}\phi_t'(w_t^{(n)})$ based on assumption 1 in (13c), apply the sufficient condition (6) in (13d), and use inequality $v_n \geq \sum_{t \in \mathcal{T}_n} \hat{y}_{nt}\phi_t(w_t^{(n)})$ (since $\hat{x}_n = 1$ in OPA(ϕ)) in (13e).

Therefore, $OPA(\phi)$ is α -competitive if the sufficient condition (6) holds in Lemma 4.3. This completes the proof.

4.3 Proof of Lemma 4.2

Let I be an instance under the capacity-limited cases Ψ^2 and $\{w_t^{(N)}\}_{t\in\mathcal{T}}$ be the final utilization of all time slots after OPA runs for all EVs in I. Assume the time horizon T is long and is an integer multiple of D^{\max} . We divide the time horizon into $H=T/D^{\max}$ partitions. We define $\mathcal{T}^h:=\{t\in\mathcal{T}:(h-1)D^{\max}+1\leq t\leq hD^{\max}\}$ as the set of time slots in the h-th time partition, and $\hat{\mathcal{T}}^h=\mathcal{T}^h\cup\mathcal{T}^{h+1}$. I^h is the sub-instance of I which includes EVs whose arrival times are in \mathcal{T}^h . Furthermore, we define \tilde{I}^h as a 3-partition sub-instance $\tilde{I}^h=I^{h-1}\cup I^h\cup I^{h+1},h\in[H]$, where $I^0=I^{H+1}=0$. The competitive ratio of OPA under instance I is:

$$\operatorname{CR}(\phi^*) = \frac{\operatorname{OPT}(I)}{\operatorname{ALG}(I)} = \frac{\sum_{h \in [H]} \operatorname{OPT}(I^h)}{\sum_{h \in [H]} \operatorname{ALG}(I^h)}$$
(14)

$$= \frac{3\sum_{h \in [H]} \mathsf{OPT}(\boldsymbol{I}^h)}{\mathsf{ALG}(\boldsymbol{I}^1) + \sum_{h \in [H]} \mathsf{ALG}(\tilde{\boldsymbol{I}}^h) + \mathsf{ALG}(\boldsymbol{I}^H)} \leq 3\max_{h \in [H]} \frac{\mathsf{OPT}(\boldsymbol{I}^h)}{\mathsf{ALG}(\tilde{\boldsymbol{I}}^h)}$$

Therefore, for calculating CR, we need to calculate $\max_{h \in [H]} \frac{\mathsf{OPT}(I^h)}{\mathsf{ALG}(\tilde{I}^h)}$

Proposition 4.2. The total surplus of EVs that arrive in \tilde{I}^h and accept the posted prices from OPA(ϕ^*) is lower bounded by

$$\mathsf{ALG}(\tilde{I}^h) \ge \frac{1}{\alpha} \sum_{t \in \hat{\mathcal{T}}^h} \phi_t(w_t^{(N)}) C. \tag{15}$$

PROOF. We have

$$\begin{split} \sum_{t \in \hat{\mathcal{T}}^h} \phi_t(w_t^{(N)}) C &= \sum_{t \in \hat{\mathcal{T}}^h} \sum_{n \in I} C[\phi_t(w_t^{(n)}) - \phi_t(w_t^{(n-1)})] \\ &= \sum_{t \in \hat{\mathcal{T}}^h} \sum_{n \in \tilde{I}^h: t \in \mathcal{T}_n} C[\phi_t(w_t^{(n)}) - \phi_t(w_t^{(n-1)})] \\ &\leq \sum_{n \in \tilde{I}^h} \sum_{t \in \mathcal{T}_n} C[\phi_t(w_t^{(n)}) - \phi_t(w_t^{(n-1)})], \end{split}$$

where \mathcal{T}_n is the availability window of EV n. The second equality holds because the maximum duration of each EV is D^{\max} ; hence, the EVs that can stay in $\hat{\mathcal{T}}^h$ must be from $\tilde{\mathcal{I}}^h$. The last inequality holds since the EVs in $\tilde{\mathcal{I}}^h$ can stay up to partition h+2.

We define $\triangle ALG_n = (v_n - \sum_{t \in \mathcal{T}_n} p_t y_{nt}) \hat{x}_n$ as the surplus increment of $OPA(\phi^*)$ after EV n is processed. Next we aim to show

$$\sum_{t \in \mathcal{T}_n} C[\phi_t(w_t^{(n)}) - \phi_t(w_t^{(n-1)})] \le \alpha \Delta \mathsf{ALG}_n. \tag{17}$$

Case(i). When EV n rejects the offer, both sides of (17) are zero; thus, it stands correct.

Case(ii). When EV n is admitted, we have $w_t^{(n)} = w_t^{(n-1)} + \hat{y}_{nt}$. Because the pricing function (4) has three segments; thus, we have three scenarios based on which segment $w_t^{(n)}$ and $w_t^{(n-1)}$ lie in.

Case(iia): when $w_t^{(n)} \ge \beta$ and $w_t^{(n-1)} \ge \beta$, we have

$$\sum_{t \in \mathcal{T}_n} C[\phi_t(w_t^{(n)}) - \phi_t(w_t^{(n-1)})]$$
 (18a)

$$= C \sum_{t \in \mathcal{T}_{-}} G \exp(w_t^{(n)} \alpha / C) \left[1 - \exp(-\hat{y}_{nt} \alpha / C) \right]$$
 (18b)

$$\leq C \sum_{t \in \mathcal{T}_n} G \exp(w_t^{(n)} \alpha / C) \hat{y}_{nt} \alpha / C \tag{18c}$$

$$= \alpha \sum_{t \in \mathcal{T}} \hat{y}_{nt} [G \exp(w_t^{(n)} \alpha / C) + p_t] - \alpha \sum_{t \in \mathcal{T}} p_t \hat{y}_{nt}$$
 (18d)

$$= \alpha \left[\sum_{t \in \mathcal{T}_n} \hat{y}_{nt} \phi_t(w_t^{(n)}) - \sum_{t \in \mathcal{T}_n} p_t \hat{y}_{nt} \right]$$
 (18e)

$$\leq \alpha [v_n - \sum_{t \in \mathcal{T}_n} p_t \hat{y}_{nt}]] \leq \alpha \Delta \mathsf{ALG}_n, \tag{18f}$$

where $G:=\frac{L-p_t}{e}$, Equation (18b) is obtained by substituting pricing function to (4), and Inequality (18f) holds because when EV n admits the offer, we have $v_n \geq \sum_{t \in \mathcal{T}_n} \hat{y}_{nt} \phi_t(w_t^{(n-1)})$.

Case(iib): when $w_t^{(n)} > \beta$ and $w_t^{(n-1)} < \beta$, we have

$$\sum_{t \in \mathcal{T}_n} C[\phi_t(w_t^{(n)}) - \phi_t(w_t^{(n-1)})]$$
(19a)

$$=C\sum_{t\in\mathcal{T}}\left[\phi_t(w_t^{(n)})-L\right] \tag{19b}$$

$$\leq C \sum_{t \in \mathcal{T}_n} [\phi_t(w_t^{(n)}) - (G \exp(w_t^{(n-1)} \alpha/C) + p_t)]$$
 (19c)

$$= C \sum_{t \in \mathcal{T}_{-}} G \exp(w_t^{(n)} \alpha / C) [1 - \exp(-\hat{y}_{nt} \alpha / C)], \tag{19d}$$

where Inequality (19c) holds by noting that $G \exp(w_t^{(n-1)}\alpha/C) + p_t \le \phi_t(w_t^{(n-1)}) = L$. Then, we can continue the proof in the same procedure from Inequality (18c) to (18f).

Case(iic): when $w_t^{(n)} < \beta$ and $w_t^{(n-1)} < \beta$, we have $\phi_t(w_t^{(n)}) = \phi_t(w_t^{(n)}) = L$ and thus $\sum_{t \in \mathcal{T}_n} C[\phi_t(w_t^{(n)}) - \phi_t(w_t^{(n-1)})] = 0 \le \alpha \Delta ALG_n$. Therefore, in all cases, Inequality (17) stands correct, and by combining inequalities (16) and (17), we have Inequality (15). \square

To calculate the upper bound of the competitive ratio under capacity-limited instances Ψ^2 , we aim to lower bound the objective value of OPA(ϕ^*) and upper bound offline optimum. In Figure 1, we depict an instance that can lead to the least possible utilization in Ψ^2 . In this instance, there are two groups of EVs that follow three rules: (a) group 1 leaves the charging station at time slot t' and group 2 arrives at the charging station at time slot t'; (b) each EV in each group is available at the charging station for D^{\min} slots; and (c) the energy demand of each EV equals $D^{\min}R_n$, which means each EV must be charged up to its rate limit in each time slot during its available window. We define k_1 (k_2) as the final utilization from group 1 (group 2) and k_1 and k_2 can be visualized by the heights of the red and blue rectangles in Figure 1.

Before we proceed, we note that the scheduling part of the proposed algorithm as a solution to problem (2) is a water-filling solution, which can be shown as

$$\hat{y}_{nt} = \min \left\{ \left[\phi_t^{-1}(\hat{\mu}_n) - w_t^{(n-1)} \right]^+, R_n \right\}, \tag{20}$$

where $\hat{\mu}_n$ can be determined by solving $\sum_{t\in\mathcal{T}_n}\hat{y}_{nt}=E_n$. Thus, the per-slot charging rate scheduling starts with the least utilized slot and fills it up to either the water level or charging rate. The same process then will be applied to the next least utilized slot until the charging demand is satisfied. This water-filling solution is derived based on the KKT condition (10). Consider the following three cases: (i) if $\phi_t(w_t^{(n)}) > \hat{\mu}_n$, we have $\hat{\gamma}_{nt} > \hat{\sigma}_{nt} \geq 0$ and this gives $\hat{y}_{nt} = 0$ from the complementary slackness; (ii) if $\phi_t(w_t^{(n)}) < \hat{\mu}_n$, we have $\hat{\sigma}_{nt} > \hat{\gamma}_{nt} \geq 0$, and thus $\hat{y}_{nt} = R_n$; (iii) if $\phi_t(w_t^{(n)}) = \hat{\mu}_n$, we have $\hat{y}_{nt} = \phi_t^{-1}(\hat{\mu}_n) - w_t^{(n-1)}$. Combining these three cases gives the water-filling solution (20), and the determination of $\hat{\mu}_n$ is based on Equation (10b).

In the following, we can argue that the final utilization obtained by $\mathsf{OPA}(\phi^*)$ under any capacity-limited instance is no less than that depicted in Figure 1. First, there exists at least one time slot (t') in Figure 1) whose utilization is at its full capacity based on the definition of the capacity-limited instance. To ensure the utilization of the overlapping slot reaches the capacity, each EV cannot have flexibility to shift its charging demand over time (each EV has D_{\min} slots of available window and $r_n D_{\min}$ energy demand in Figure 1) since otherwise the water-filling solution leads to higher utilization for the slots excluding t'.

We define $\psi(w)$ as

$$\psi(w) = \left\{ \begin{array}{ll} L, & w \in [0, C/\alpha) \\ \frac{L - p^{\max}}{e} e^{\frac{\alpha}{C} w} + p^{\max}, & w \in [C/\alpha, C] \end{array} \right.,$$

and it can be easily verified that

$$\psi(w) \le \phi_t^*(w), \quad \forall t \in \hat{\mathcal{T}}^h, w \in [0, C]. \tag{21}$$

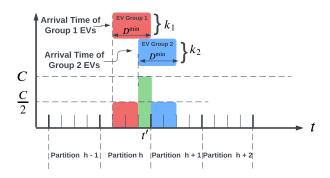


Figure 1: Illustration of the worst-case final utilization.

Based on Proposition 4.2 and Inequality (21), we have:

$$ALG(\tilde{I}^h) \ge \frac{C}{\alpha} [(D^{\min} - 1)\psi(k_1) + (D^{\min} - 1)\psi(k_2) + \psi(C)]. \tag{22}$$

According to Jensen's inequality and convexity of the pricing function, we additionally have $\psi(k_1) + \psi(k_2) \ge \psi(\frac{C}{2}) + \psi(\frac{C}{2})$ when $k_1 + k_2 = C$. Because our pricing function (4) is a two segment function, we have two cases:

Case(i): if $\alpha \le 2$, then we have $k_1 = k_2 = \frac{C}{2} \le \beta = \frac{C}{\alpha}$ and

$$\mathsf{ALG}(\tilde{I}^h) \ge \frac{C}{\alpha} [(2D^{\min} - 2)L + \psi(C)] \tag{23a}$$

$$\geq \frac{2D^{\min}LC}{\alpha}$$
 (23b)

$$\geq \frac{2D^{\min}C(L - p^{\max})}{\alpha},\tag{23c}$$

where Inequality (23a) stands correct because $\psi(k_1) = \psi(k_2) = L$. Inequality (23b) is correct since $(U/L)(D^{\max}/D^{\min}) \ge 2$, and $\psi(C) \ge UD^{\max}/D^{\min}$ so we have: $\psi(C) \ge 2L$.

Case(ii): if $\alpha > 2$, then $k_1 = k_2 = \frac{C}{2} > \beta = \frac{C}{\alpha}$, and

$$\mathsf{ALG}(\tilde{I}^h) \ge \frac{C}{\alpha} \left((2D^{\min} - 2) (\frac{L - p^{\max}}{e}) \exp(\frac{\alpha}{2}) \right)$$

$$+ (\frac{L - p^{\max}}{e}) \exp(\alpha)$$

$$\ge \frac{C}{\alpha} \left(2D^{\min} (\frac{L - p^{\max}}{e}) \exp(\frac{\alpha}{2}) \right),$$
(24a)

Inequality (24b) also stands because when $\alpha \geq 2$, we have

$$(\frac{L-p^{\max}}{e})\exp(\alpha) > 2(\frac{L-p^{\max}}{e})\exp(\frac{\alpha}{2}).$$

The upper bound of offline optimum is $\mathrm{OPT}(I^h) \leq 2D^{\max}CU$, which is the maximum value the offline algorithm could achieve in $2D^{\max}$ time slots, and the competitive ratio is as follows. When $\alpha \leq 2$, according to Inequality (23), we have

$$\frac{\mathsf{OPT}(\boldsymbol{I}^h)}{\mathsf{ALG}(\tilde{\boldsymbol{I}}^h)} \leq \frac{D^{\max}U\alpha}{D^{\min}(L-p^{\max})} \leq 2\theta = 2\sqrt{e},$$

where we apply $\alpha = 1 + 2 \ln(\theta) = 2$.

When $\alpha > 2$, according to Inequality (24) we have

$$\frac{\mathsf{OPT}(I^h)}{\mathsf{ALG}(\tilde{I}^h)} \leq \frac{D^{\max}U\alpha}{D^{\min}(\frac{L-p^{\max}}{e})\exp(\frac{\alpha}{2})} = \frac{\theta\alpha}{\exp(\alpha/2-1)}.$$

Combining the above two cases gives

$$\frac{\mathsf{OPT}(I^h)}{\mathsf{ALG}(\tilde{I}^h)} \leq \max \left\{ 2\sqrt{e}, \frac{\theta\alpha}{\exp(\alpha/2 - 1)} \right\}.$$

Therefore, according to Inequality (14), we have

$$\begin{split} \frac{\mathsf{OPT}(I)}{\mathsf{ALG}(I)} &\leq 3 \max_{h \in [H]} \frac{\mathsf{OPT}(I^h)}{\mathsf{ALG}(\tilde{I}^h)} \\ &\leq 3 \max \left\{ 2 \sqrt{e}, \frac{\theta \alpha}{\exp(\alpha/2 - 1)} \right\}. \end{split}$$

This completes the proof.

5 EXPERIMENTAL RESULTS

In this section, we report the experimental results using EV charging data traces. The goal is to evaluate the performance of the proposed algorithms as compared to alternatives and investigate the impact of the parameters on the performance of the algorithms.

5.1 Experimental Setup

Dataset. We use the Caltech ACN dataset [19], which includes more than 50K EV charging sessions from more than 50 charging stations. The dataset includes the arrival and departure time, the charging demand, and the charging rate for each EV.

Parameter settings and metrics. To capture the scenario of limited capacity, we consider a charging station with an aggregate capacity smaller than the total charging demand of EVs. We conduct a 1-day trial using 90 consecutive days in which we generate 20 trials randomly and calculate the average of the 20 profit ratios for each day. Each EV n has a different valuation (v_n) in each trial in a day, making $90 \times 20 = 1800$ trials in total. The ACN dataset does not include the private value (v_n) for EVs; hence, we follow the approach in [33] to estimate the value for EVs by modeling the distribution of historical arrivals. Each EV's value is a randomized function of the availability window, which means the higher the window, the higher the EV's value. As the counterpart of the competitive ratio in the empirical setting, we use *empirical profit ratio* as the performance metric, which is the ratio of the profit gained by the optimal solution to the offline problem.

Comparison algorithms. We compare our algorithm OPA with three other online algorithms: (1) A Utility-Based Online Algorithm (UBOA), where the admission control is simply based on a first-come-first-served policy. For the scheduling, UBOA adapts a water-filling policy and starts with the slots with lower utilization up to either the capacity or EV charging rate. (2) Price Based Online Algorithm (PBOA), where the admission policy is first-come first-served, but the scheduling is a water-filling approach based on the time-varying unit energy cost of each time slot, i.e., the scheduler starts with the time slot with lowest unit energy cost. (iii) Online Mechanism with Myopic Price (OMMP) [27]: The online algorithm proposed in [27], which calculates the charging schedule by solving a cost minimization based on the per-slot energy price. This per-slot energy price is

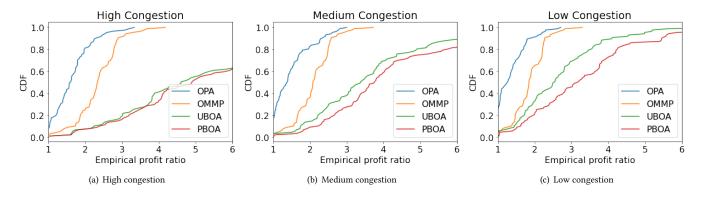


Figure 2: Variation of the profit ratio in different congestion scenarios.

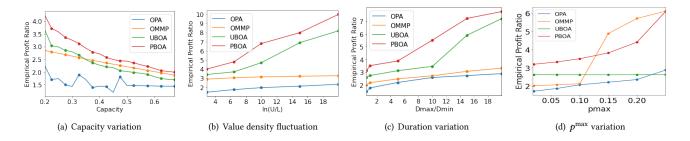


Figure 3: Impact of parameters.

a quadratic pricing function that we are utilizing to evaluate the algorithm's performance. The original maximization problem of ours (1) and [27] are quite similar; therefore, we modify the cost function in [27] into a linear one to compare the profit ratio fairly. Therefore, OMMP is an online algorithm that uses a pricing function based on the linear cost function to solve a cost minimization problem to calculate the candidate charging schedule and the corresponding marginal price for each unit of energy according to the pricing function. Then OMMP decides to admit EV n if its surplus (value - total scheduled energy price) is positive.

5.2 Experimental Results

Comparison results. In this experiment, we compare the performance of different algorithms in three different congestion levels: low, medium, and high, where the capacity in the station is approximately up to 60%, 30%, and 15% of the aggregate EV demand, respectively. Figure 2 depicts the cumulative distribution function (CDF) for empirical profit ratios of 1800 trials of experiments for all four algorithms in low, medium, and high congestion. The results show that in all congestion settings, OPA outperforms the alternatives, and the lower the congestion is, the better the OPA result gets as compared to other algorithms. A notable observation is that nearly 90%, 80%, and 70% of the OPA's profit ratios are less than 2 in low, medium, and high congestion scenarios, while these are substantially lower for all three alternative algorithms.

The impact of parameters. In this experiment, we evaluate the impact of variation of the parameters on the empirical profit ratios.

In particular, we vary the charging station capacity, fluctuation ratio ($\rho = U/L$), duration ratio ($\delta = D^{\text{max}}/D^{\text{min}}$), and the maximum electricity price (p^{max}). Figure 3(a) demonstrates that the empirical profit ratio decreased as the capacity increased. This observation makes sense since with more capacity, there is more room for EV admission and is aligned with the observation in three different congestion regimes in Figure 2. Figure 3(b) shows that the fluctuation ratio has a direct relation to the profit ratio, and the higher the fluctuation ratio, the higher the competitive ratio, which also approves the fact that in OPA and OMMP the theoretical competitive ratio increases logarithmically with ρ . Figure 3(c) shows when δ increases, there is a slight increase in the profit ratios of OPA and OMMP; however, profit ratios of UBOA and PBOA increase drastically. This observation is due to the fact that different from UBOA and PBOA, both OPA and OMMP take into account the duration of each EV request in their algorithmic decision making. Finally, Figure 3(d) shows that as the p^{max} increases, there's a slight increase in the profit ratio of OPA, but the slope of the increase is much smaller than those of OMMP and PBOA. The profit ratio of UBOA does not change because it is independent of electricity prices.

6 RELATED WORKS

EV charging management has become a central topic of transportation electrification in recent decades. The problem is to decide where to charge, when to charge, and what rate to charge given the profile of EV requests (e.g., energy demand, departure time, etc.). There exists a large body of literature that schedules EV charging

in offline settings [17, 22]. This paper focuses on the online setting where EVs arrive one by one and one must decide the scheduling of each EV upon its arrival without knowing the future information. In online EV charging, one stream of works studies how to charge EVs under unknown future electricity prices [7, 12–14, 21, 24, 32]. In [12], the goal is to minimize the EV charging cost considering real-time pricing. Furthermore, in [7, 13, 14, 24, 32], the focus is on EV charging scheduling under forecasting the future real-time prices or based on stochastic information of the real-time prices. The authors in [21] study a multi-objective optimization to balance best the cost of charging and dissatisfaction of the EVs. Different from these works, we focus on EV charging for a group of online arriving EVs from a theoretical approach of competitive analysis.

The online EV charging problem allocates limited charging capacity to a group of EVs under a capacity limit where it is impossible to admit all requests EVs and satisfy their requests. Therefore, there exist two strategies for designing online algorithms. The first type of strategy provides no guarantees on the received energy of EVs and makes best efforts to maximize the social welfare (utility of all EVs minus charging cost) [20, 29, 37]. Although [37] works on maximizing social welfare, they do not guarantee to fully charge each admitted EV. The work in [20] has the charging station's charging capacity constraint. But, it does not also guarantee to charge the admitted EV up to its charging demand. In [29], the authors propose an algorithm to maximize social welfare considering the charging capacity. However, in [29], energy cost is not considered and they only try to maximize social welfare regardless of trying to minimize the cost EVs pay to the charging station. Our work is also more challenging than [29] because we guarantee that if an EV is admitted, it will be fully charged to its demand; however, in [29] the EV could be partially charged based on how much the charging station is crowded and its power capacity constraint.

The second stream makes two decisions for each EV: admission control (whether to accept the EV for charging) and scheduling (how to charge the EV if it is admitted). This stream of work provides guaranteed service for all admitted EVs [4-6, 26, 27, 31]. Our work lies in this stream of works. In [4], the authors propose an algorithm to maximize the social welfare considering the charging capacity with on-arrival commitment, which guarantees a charging amount upon arrival of each EV; however, they only consider the sum of all EVs' utilities, disregard the electricity cost, and only guarantee to charge the EV more than a fraction of its demand, not to charge up to its full demand. In [26], whenever an agent is selected, their mechanism pre-commits to charging the vehicle by its reported departure time but maintains flexibility about when the charging takes place and at what rate. The online mechanism in [26] does not have the on-arrival commitment and commits to charge the EV at least to a fraction of its demand in arbitrary time after its arrival.

In terms of considering the resource cost, our work is most relevant to [27] because it considers the charging cost and also guarantees to charge an EV up to its demand if it is admitted. In [27], the authors propose a cost function and design a pricing function heuristically and then analyze its competitive ratio. The derivation of the pricing function in our work is based on the well-established online primal-dual framework and solving partial differential equations. This approach leads to a smaller competitive ratio and better empirical profit ratio compared to [27]. Last, [31] also guarantees

to charge of admitted EVs up to their demand and its objective function is minimizing the total energy cost, but ignores the capacity constraints of EV charging stations.

7 CONCLUSION

In this paper, we developed a competitive algorithm for joint pricing and scheduling of EV charging where EVs arrive at the station in an online manner. Our theoretical results are of independent interest since they provide the first order-optimal algorithm for the integral version of the online knapsack problem with reusable resources and time-varying resource costs. Using real data traces, we demonstrated the proposed algorithms outperform prior alternatives.

An interesting future direction is to extend the results into the multi-station setting, where the EVs submit their charging requests to multiple stations simultaneously, and the system operator should determine whether or not to accept the request and, if so, when (scheduling) and where (station assignment) to charge the vehicle.

ACKNOWLEDGMENTS

Mohammad Hajiesmaili acknowledges the partial support from the U.S. National Science Foundation (NSF) under grant numbers NSF CAREER-2045641, CNS-2102963, and CPS-2136199. Carlee Joe-Wong's research is supported in part by the NSF under grant number CNS-2103024.

REFERENCES

- [1] [n. d.]. Explaining Electric and Plug-In Hybrid Electric Vehicles. https://www.epa.gov/greenvehicles/explaining-electric-plug-hybrid-electric-vehicles.
- [2] [n. d.]. Smart Charging Stations. https://www.chargepoint.com/products/ commercial.
- [3] May Al-Roomi, Shaikha Al-Ebrahim, Sabika Buqrais, and Imtiaz Ahmad. 2013. Cloud computing pricing models: a survey. *International Journal of Grid and Distributed Computing* 6, 5 (2013), 93–106.
- [4] Bahram Alinia, Mohammad H Hajiesmaili, and Noël Crespi. 2019. Online EV charging scheduling with on-arrival commitment. IEEE Transactions on Intelligent Transportation Systems 20, 12 (2019), 4524–4537.
- [5] Bahram Alinia, Mohammad H. Hajiesmaili, Zachary J. Lee, Noel Crespi, and Enrique Mallada. 2022. Online EV Scheduling Algorithms for Adaptive Charging Networks with Global Peak Constraints. *IEEE Transactions on Sustainable Computing* 7, 3 (2022), 537–548. https://doi.org/10.1109/TSUSC.2020.2979854
- [6] Bahram Alinia, Mohammad Sadegh Talebi, Mohammad H Hajiesmaili, Ali Yekkehkhany, and Noel Crespi. 2018. Competitive online scheduling algorithms with applications in deadline-constrained EV charging. In 2018 IEEE/ACM 26th International Symposium on Quality of Service (IWQoS). IEEE, 1–10.
- [7] Eilyan Bitar and Yunjian Xu. 2016. Deadline differentiated pricing of deferrable electric loads. IEEE Transactions on Smart Grid 8, 1 (2016), 13–25.
- [8] Severin Borenstein, Michael Jaske, and Arthur Rosenfeld. 2002. Dynamic pricing, advanced metering, and demand response in electricity markets. (2002).
- [9] Niv Buchbinder, Joseph Seffi Naor, et al. 2009. The design of competitive online algorithms via a primal-dual approach. Foundations and Trends® in Theoretical Computer Science 3, 2-3 (2009), 93-263.
- [10] Tanmoy Chakraborty, Zhiyi Huang, and Sanjeev Khanna. 2013. Dynamic and nonuniform pricing strategies for revenue maximization. SIAM J. Comput. 42, 6 (2013), 2424–2451.
- [11] Coral Davenport, Lisa Friedman, and Brad Plumer. 2022. California to Ban the Sale of New Gasoline Cars. https://www.nytimes.com/2022/08/24/climate/californiagas-cars-emissions.html.
- [12] Ruilong Deng and Hao Liang. 2016. Whether to charge an electric vehicle or not? A near-optimal online approach. In 2016 IEEE Power and Energy Society General Meeting (PESGM). IEEE, 1-5.
- [13] Ruilong Deng and Hao Liang. 2017. Whether to charge or discharge an electric vehicle? An optimal approach in polynomial time. In 2017 IEEE 86th Vehicular Technology Conference (VTC-Fall). IEEE, 1–5.
- [14] Lingwen Gan, Ufuk Topcu, and Steven H Low. 2012. Optimal decentralized protocol for electric vehicle charging. *IEEE Transactions on Power Systems* 28, 2 (2012), 940–951.

- [15] Shashank Goyal and Diwakar Gupta. 2020. The online reservation problem. Algorithms 13, 10 (2020), 241.
- [16] Zhiyi Huang and Anthony Kim. 2019. Welfare maximization with production costs: A primal dual approach. Games and Economic Behavior 118 (2019), 648–667.
- [17] Alexandros-Michail Koufakis, Emmanouil S. Rigas, Nick Bassiliades, and Sarvapali D. Ramchurn. 2020. Offline and Online Electric Vehicle Charging Scheduling With V2V Energy Transfer. IEEE Transactions on Intelligent Transportation Systems 21, 5 (2020), 2128–2138. https://doi.org/10.1109/TITS.2019.2914087
- [18] Fred Lambert. 2018. ChargePoint pledges to deploy 2.5 million electric vehicle charge points by 2025. https://electrek.co/2018/09/14/chargepoint-millionelectric-vehicle-charge-points-2025/.
- [19] Zachary Lee, Tongxin Li, and SH Low. 2019. ACN-Data charging dataset: analysis and applications. Proc. of ACM eEnergy (2019).
- [20] Zachary J Lee, George Lee, Ted Lee, Cheng Jin, Rand Lee, Zhi Low, Daniel Chang, Christine Ortega, and Steven H Low. 2021. Adaptive charging networks: A framework for smart electric vehicle charging. IEEE Transactions on Smart Grid 12, 5 (2021), 4339–4350.
- [21] Qiulin Lin, Hanling Yi, and Minghua Chen. 2021. Minimizing Cost-Plus-Dissatisfaction in Online EV Charging Under Real-Time Pricing. IEEE Transactions on Intelligent Transportation Systems (2021).
- [22] Tian Liu, Bo Sun, Xiaoqi Tan, and Danny H. K. Tsang. 2017. Market for multidimensional flexibility with parametric demand response bidding. In 2017 North American Power Symposium (NAPS). 1–6. https://doi.org/10.1109/NAPS.2017. 8107290
- [23] Colin McKerracher and Siobhan Wagner. 2021. At Least Two-Thirds of Global Car Sales Will Be Electric by 2040. https://www.bloomberg.com/news/articles/2021-08-09/at-least-two-thirds-of-global-car-sales-will-be-electric-by-2040.
- [24] Niklas Rotering and Marija Ilic. 2010. Optimal charge control of plug-in hybrid electric vehicles in deregulated electricity markets. *IEEE Transactions on Power* Systems 26, 3 (2010), 1021–1029.
- [25] Marko Slavulj, K Kanizaj, and S Durdevic. 2016. The evolution of urban transport: Uber. Road and Rail Infrastructure IV (2016).
- [26] Sebastian Stein, Enrico Gerding, Valentin Robu, and Nick Jennings. 2012. A model-based online mechanism with pre-commitment and its application to electric vehicle charging. (2012).
- [27] Bo Sun, Xiaoqi Tan, and Danny HK Tsang. 2018. Eliciting multi-dimensional flexibilities from electric vehicles: A mechanism design approach. IEEE Transactions

- on Power Systems 34, 5 (2018), 4038-4047.
- [28] Bo Sun, Lin Yang, Mohammad Hajiesmaili, Adam Wierman, John CS Lui, Don Towsley, and Danny HK Tsang. 2022. The Online Knapsack Problem with Departures. Proceedings of the ACM on Measurement and Analysis of Computing Systems 6, 3 (2022), 1–32.
- [29] Bo Sun, Ali Zeynali, Tongxin Li, Mohammad Hajiesmaili, Adam Wierman, and Danny HK Tsang. 2020. Competitive algorithms for the online multiple knapsack problem with application to electric vehicle charging. Proceedings of the ACM on Measurement and Analysis of Computing Systems 4, 3 (2020), 1–32.
- [30] Xiaoqi Tan, Bo Sun, Alberto Leon-Garcia, Yuan Wu, and Danny HK Tsang. 2020. Mechanism design for online resource allocation: A unified approach. Proceedings of the ACM on Measurement and Analysis of Computing Systems 4, 2 (2020), 1–46.
- [31] Wanrong Tang, Suzhi Bi, and Ying Jun Zhang. 2014. Online coordinated charging decision algorithm for electric vehicles without future information. IEEE Transactions on Smart Grid 5, 6 (2014), 2810–2824.
- [32] Kai Man Tsui and Shing-Chow Chan. 2012. Demand response optimization for smart home scheduling under real-time pricing. IEEE Transactions on Smart Grid 3, 4 (2012), 1812–1821.
- [33] Ali Zeynali, Bo Sun, Mohammad Hajiesmaili, and Adam Wierman. 2021. Data-driven competitive algorithms for online knapsack and set cover. In Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 35. 10833–10841.
- [34] Zijun Zhang, Zongpeng Li, and Chuan Wu. 2017. Optimal posted prices for online cloud resource allocation. Proceedings of the ACM on Measurement and Analysis of Computing Systems 1, 1 (2017), 1–26.
- [35] Shizhen Zhao, Xiaojun Lin, and Minghua Chen. 2015. Peak-minimizing online EV charging: Price-of-uncertainty and algorithm robustification. In 2015 IEEE Conference on Computer Communications (INFOCOM). IEEE, 2335–2343.
- [36] Shizhen Zhao, Xiaojun Lin, and Minghua Chen. 2017. Robust Online Algorithms for Peak-Minimizing EV Charging Under Multistage Uncertainty. *IEEE Trans. Automat. Control* 62, 11 (2017), 5739–5754. https://doi.org/10.1109/TAC.2017. 2699290
- [37] Zizhan Zheng and Ness Shroff. 2014. Online welfare maximization for electric vehicle charging with electricity cost. In Proceedings of the 5th international conference on Future energy systems. 253–263.
- [38] Yunhong Zhou, Deeparnab Chakrabarty, and Rajan Lukose. 2008. Budget constrained bidding in keyword auctions and online knapsack problems. In International Workshop on Internet and Network Economics. Springer, 566–576.