Erratum: Non-linear sigma model with particle-hole asymmetry for the disordered two-dimensional electron gas [Phys. Rev. B 103, 125422 (2021)]

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In the original paper, we erroneously neglected the influence of massive modes on the derivation of the four-gradient term in the NLσM action. Explicit integration of the massive modes gives the following additional contribution

$$S_M = \frac{\pi\nu}{16} DD'_\varepsilon \mathrm{Tr}[\nabla^2 \hat{Q}(\nabla\hat{Q})^2]. \tag{Er1}$$

This term should be included on the right-hand side of Eq. (13). When combined with the contribution originating from the gradient expansion in the absence of massive modes, $S^{(2)}_{0,\eta,\varphi} = -\frac{\pi\nu}{8} DD'_\varepsilon \mathrm{Tr}[\nabla^2\hat{Q}(\nabla\hat{Q})^2]$, Eq. (18), the overall coefficient of the four-gradient term is halved, $S_M + S^{(2)}_{0,\eta,\varphi} = -\frac{\pi\nu}{16} DD'_\varepsilon \mathrm{Tr}[\nabla^2\hat{Q}(\nabla\hat{Q})^2]$. Except for the coefficient of the four-gradient term, the calculations and conclusions presented in the original paper remain unchanged. In particular, the interaction corrections to the static part of the correlation function in the one-loop approximation presented in Sec. IV do not depend on the four-gradient term. The mechanism for producing the four-gradient term S_M through the coupling of soft and massive modes had been noted previously in Ref. 1 in the context of the Quantum Hall Effect.

In order to understand the origin of S_M, it is sufficient to focus on the non-interacting case as described in Sec. V B. Retracing the steps outlined in Appendix B, the parametrization of the matrix $\hat{Q}$, Eq. (B2), should be generalized to include massive fluctuations[2] $\hat{Q} \to \hat{Q}_M = \hat{U}\hat{P}_M\hat{\bar{U}}$. Here, $\hat{P}_M$ is a Hermitian matrix that is block-diagonal in Keldysh space and $\delta\hat{P}_M = \hat{P}_M - \hat{\sigma}_3$ parametrizes massive fluctuations around the saddle point. Correspondingly, the Keldysh partition function is written as $Z = \int_{\Psi^\dagger,\Psi,\hat{P}_M,\hat{U}} I[\hat{P}_M]\exp(iS)$ with

$$S = \int \vec{\bar{\Psi}}\Big(\hat{G}_0^{-1} + \frac{i}{2\tau}\hat{P}_M + \hat{\bar{U}}[\hat{G}_0^{-1},\hat{U}]\Big)\vec{\Psi} + \frac{i\pi\nu}{4\tau}\mathrm{Tr}[\hat{P}_M^2]. \tag{Er2}$$

In the expression for the partition function, $I[\hat{P}_M]$ is the Jacobian arising due the parametrization of $\hat{Q}_M$. With the definition $\hat{G}_M^{-1} = \hat{G}^{-1} + \frac{i}{2\tau}\delta\hat{P}_M$, the partition function after integration over the fermionic fields can be presented as $Z = \int_{\hat{P}_M,\hat{U}} \mathrm{e}^{iS}$ with $S = S[\hat{U},\delta\hat{P}_M] + S[\delta\hat{P}_M]$ and

$$S[\hat{U},\delta\hat{P}_M] = -i\mathrm{tr}\ln[1 + \hat{G}_M\hat{\bar{U}}[\hat{G}_0^{-1},\hat{U}]], \tag{Er3}$$

$$S[\delta\hat{P}_M] = -i\mathrm{tr}\ln\left[1 + \hat{G}\frac{i}{2\tau}\delta\hat{P}_M\right] + \frac{i\pi\nu}{4\tau}\mathrm{tr}[\hat{P}_M^2] - i\ln I[\hat{P}_M]. \tag{Er4}$$

Here, $S[\hat{U},\delta\hat{P}_M]$ describes the coupling of soft and massive modes. The influence of the massive modes was entirely neglected in the expansion described in Sec. V B of the original paper, which was based on $S[\hat{U},\delta\hat{P}_M = 0]$. This expansion led to S_1, Eq. (58), and S_2, Eq. (59) (which equals $S^{(2)}_{0,\eta,\varphi}$ in the notation of Sec. III A). The integration of the massive modes produces a contribution to the NLσM with four gradients, S_M [Eq. (Er1)], of the same form as S_2. To obtain this term, it is sufficient to integrate $\delta\hat{P}_M$ in the Gaussian approximation. Therefore, $S[\delta\hat{P}_M]$ should be expanded up to second order in $\delta\hat{P}_M$. Upon substituting $\hat{P}_M = \sigma_3 + \delta\hat{P}_M$, linear terms in $\delta\hat{P}_M$ cancel between the first two terms in Eq. (Er4) by virtue of the saddle point approximation. Higher order terms in $\delta\hat{P}_M$ resulting from the expansion of the tr ln in Eq. (Er4) give subleading contributions (in the parameter $1/\varepsilon_F\tau$), since they involve a $\xi_{\mathbf{p}}$ integration over a product of only retarded (or only advanced) Green's functions. The Jacobian $I[\hat{P}_M]$ is not easily evaluated in a continuum model, as it requires a regularization. However, from diagrammatic considerations one expects deviations from the self-consistent Born approximation (which underlies the saddle point equation), to be suppressed by powers of $(\varepsilon_F\tau)^{-1}$. In effect, we may here approximate the quadratic form in $\delta\hat{P}_M$ by $S[\delta\hat{P}_M] \approx \frac{i\pi\nu}{4\tau}\mathrm{tr}[\delta\hat{P}_M^2]$.

Corrections to the NLσM originating from the coupling of soft and massive modes in $S[\hat{U},\delta\hat{P}_M]$ can be organized as a cumulant expansion in $\delta S = S[\hat{U},\delta\hat{P}_M] - S[\hat{U},\delta\hat{P}_M = 0]$. δS, in turn, is obtained by expanding G_M in powers of $\delta\hat{P}_M$. At first order, the cumulant expansion gives $\delta S^{(1)} = \langle\delta S\rangle$, where $\langle\dots\rangle$ stands for a Gaussian average with the action $S[\delta\hat{P}_M]$. Such terms can be checked to give small corrections only. The contribution of interest originates from the second cumulant $\delta S^{(2)} = \frac{i}{2}\langle\langle(\delta S)^2\rangle\rangle$ by replacing $\hat{\bar{U}}[\hat{G}_0^{-1},\hat{U}] \to \mathcal{O} = \frac{1}{2m}[\hat{\mathcal{V}}^i\overrightarrow{\nabla}^i - \overleftarrow{\nabla}^i\hat{\mathcal{V}}^i]$ in Eq. (Er3), expanding the logarithm to second order in $\mathcal{O}$, and further expanding one of the two Green's functions in the resulting expression for δS to first order in $\delta\hat{P}_M$ as $\hat{G}_M \approx \hat{G} - \frac{i}{2\tau}\hat{G}\delta\hat{P}_M\hat{G}$. After averaging with respect to $\delta\hat{P}_M$, one finds

$$S_M = \frac{i}{4\pi\nu\tau}\int d\mathbf{r}\ \mathrm{tr}[(\hat{G}\mathcal{O}\hat{G}\mathcal{O}\hat{G})^{\parallel}_{\mathbf{r},\mathbf{r}}(\hat{G}\mathcal{O}\hat{G}\mathcal{O}\hat{G})^{\parallel}_{\mathbf{r},\mathbf{r}}]. \tag{Er5}$$

Fig. 1 displays the corresponding diagram. Focusing only on the particle-hole asymmetric contribution, one obtains

$$S_M = -\pi\nu DD'_\varepsilon \mathrm{Tr}[\sigma_3\hat{\mathcal{V}}^{i\perp}\hat{\mathcal{V}}^{i\perp}\hat{\mathcal{V}}^{j\perp}\hat{\mathcal{V}}^{j\perp}], \tag{Er6}$$

which results in Eq. (Er1). For a comparison with Ref. 1, notice the relation $\mathrm{Tr}[\nabla^2\hat{Q}(\nabla\hat{Q})^2] = -\mathrm{Tr}[(\nabla\hat{Q})^2(\nabla\hat{Q})^2\hat{Q}]$.

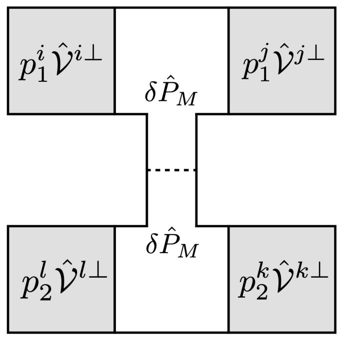

FIG. 1: Generation of the four-fermion term S_M through the coupling of soft and massive modes. The dashed line stands for an impurity line connecting two retarded or two advanced Green's functions.

Finally, we would like to note that after incorporating S_M into the derivation, Eq. (66) should include the term $-\frac{\pi\nu}{16}DD'_\varepsilon\mathrm{Tr}[\nabla^2\hat{Q}(\nabla\hat{Q})^2]$ on the right hand side.

[1] X.-F. Wang, Z. Wang, C. Castellani, M. Fabrizio, and G. Kotliar, Nuclear Physics B **415**, 589 (1994).
[2] A. M. Pruisken and L. Schäfer, Nuclear Physics B **200**, 20 (1982).