

Joint Coding of URLLC and eMBB in Wyner's Soft-Handoff Network in the Finite Blocklength Regime

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Abstract—Wyner's soft-handoff network is considered where transmitters simultaneously send messages of enhanced mobile broadband (eMBB) and ultra-reliable low-latency communication (URLLC) services. Due to the low-latency requirements, the URLLC messages are transmitted over fewer channel uses compared to the eMBB messages. To improve the reliability of the URLLC transmissions, we propose a coding scheme with finite blocklength codewords that exploits dirty-paper coding (DPC) to precancel the interference from eMBB transmissions. Rigorous bounds are derived for the error probabilities of eMBB and URLLC transmissions achieved by our scheme. Numerical results illustrate that they are lower than for standard time-sharing.

I. INTRODUCTION

The fifth and the forthcoming sixth generations of mobile communications have to accommodate both ultra-reliable and low-latency communication (URLLC) and enhanced mobile broadband (eMBB) services [1], [2]. URLLC services aim at guaranteeing high-reliability at a maximum end-to-end delay of 1ms and are used for delay-sensitive applications such as industrial control management as well as autonomous vehicle and remote surgery applications [2]. On the other hand, eMBB services aim to provide high data rates and are used for delay-tolerant applications such as video streaming, virtual and augmented reality applications [3], [4].

The different latency requirements of eMBB and URLLC services along with the fact that they are scheduled in the same frequency band make their coexistence challenging. Networks with such mixed-delay constraints have been studied recently. See [5]–[9] for a comprehensive review on related works. The previous studies are mostly focused on the performance of such networks in the asymptotic regime where the number of channel uses goes to infinity. Since the URLLC delay constraint limits the number of available channel uses, the problem of joint coding of messages with heterogeneous blocklengths is of an increasing interest. Notably, for the Gaussian point-to-point channel with messages of heterogeneous decoding deadlines, the work in [10], proposes a coding scheme which exploits dirty-paper coding (DPC) [11], [12]. Accounting for finite decoding deadline constraints, rigorous bounds are derived on the achievable error probabilities of the messages. Their numerical results illustrate that their proposed scheme outperforms time sharing for a wide range of blocklengths. For the Gaussian broadcast channel with heterogeneous blocklength constraints,

the work in [13], proposes a coding scheme which decodes the messages at time-instances that depend on the realizations of the random channel fading. The authors showed that significant improvements are possible over standard successive interference cancellation. In [14] achievable rates and latency of the early-decoding scheme in [13] are improved by introducing *concatenated shell codes*. Finally, [15] and [16] studied the uplink of the cloud radio access networks where URLLC messages are directly decoded at the base stations whereas decoding of eMBB messages can be delayed to the cloud center. In particular, [15] performs a hybrid analysis where URLLC transmissions are studied in the finite blocklength regime and eMBB transmissions in the asymptotic infinite blocklength regime.

In this paper, we consider Wyner's soft-handoff model with K interfering transmitters and receivers pairs. Each transmitter wishes to simultaneously transmit two messages of heterogeneous blocklengths: an URLLC message and an eMBB message. The URLLC message is transmitted over a shorter blocklength compared to the eMBB message. Tx s can hold a conferencing communication that depends only on the eMBB messages but not on the URLLC messages. By exploiting the DPC principle in [11], [17], we propose a coding scheme to jointly transmit the URLLC and eMBB messages. Unlike [10], [13], [15], we consider that codebooks are generated randomly according to independent uniform distributions on the power-shell. Rigorous bounds are derived for achievable error probabilities of eMBB and URLLC transmissions. To this end, Gel'fand-Pinsker analysis techniques for finite blocklengths in [12] are combined with the multiple parallel channels approach in [18]. Numerical results illustrate that our proposed scheme significantly outperforms standard time-sharing.

II. PROBLEM SETUP

Consider Wyner's soft-handoff network with K transmitters (Tx s) and K receivers (Rx s) that are aligned on two parallel lines so that each Tx k has two neighbours, Tx $k - 1$ and Tx $k + 1$, and each Rx k has two neighbours, Rx $k - 1$ and Rx $k + 1$. Define $\mathcal{K} := \{1, \dots, K\}$. The signal transmitted by Tx $k \in \mathcal{K}$ is observed by Rx k and the neighboring Rx $k + 1$. See Figure 1. Each Tx $k \in \mathcal{K}$ sends a so called *eMBB* type message $M_k^{(e)}$ to its corresponding Rx k , for $M_k^{(e)}$ uniformly

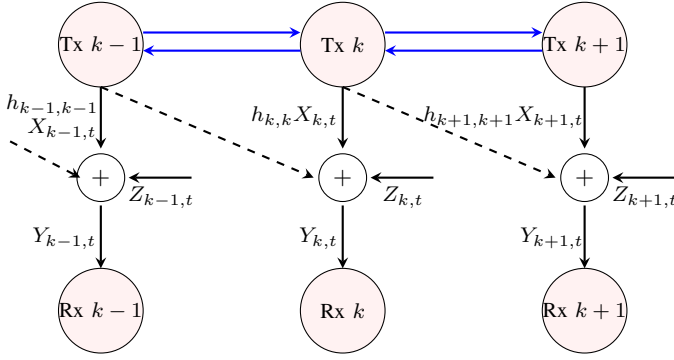


Fig. 1: System model.

distributed over $\mathcal{M}_k^{(e)} := \{1, \dots, L_e\}$. A subset of Txs $\mathcal{K}_U \subset \mathcal{K}$ also sends additional URLLC messages $M_k^{(U)}$, for $k \in \mathcal{K}_U$, for $M_k^{(U)}$ uniformly distributed over the set $\mathcal{M}^{(U)} := \{1, \dots, L_U\}$. We assume that K is even and

$$\mathcal{K}_U := \{1, 3, \dots, K-1\}, \quad (1)$$

so that URLLC transmissions are only interfered by the eMBB transmissions but not by other URLLC transmissions. (The study of sets $\mathcal{K}_U$ with interfering URLLC messages is left as a future research direction.)

Communication takes place in two phases.

Tx-cooperation phase:

The encoding starts with a first *Tx-cooperation phase* in which Txs not in $\mathcal{K}_U$ share their eMBB message with their left-neighbouring Txs in $\mathcal{K}_U$. (For example over high-rate optical fibers if the Txs are BSs.) The URLLC messages, which are subject to stringent delay constraints, are only generated after the Tx-cooperation phase, at the beginning of the subsequent *channel transmission phase*.

Channel transmission phase:

URLLC messages are transmitted over n_U channel uses and eMBB messages over $n_e > n_U$ channel uses. The blocklengths n_U and n_e are assumed to be fixed constants. Notice that while the transmission delay of URLLC messages is determined by the n_U channel uses, transmission delay of eMBB consists of both the delay of the Tx-cooperation phase as well as the delay induced by the n_e channel uses.

For each $k \in \mathcal{K}$, Tx k computes its time- t channel input $X_{k,t}$ with $t \in \{1, \dots, n_e\}$ as

$$X_{k,t} = \begin{cases} f_k^{(b)}(M_k^{(U)}, M_k^{(e)}, M_{k-1}^{(e)}), & k \in \mathcal{K}_U \text{ and } t \leq n_U \\ f_k^{(e)}(M_k^{(e)}), & k \notin \mathcal{K}_U \text{ or } n_U < t \leq n_e, \end{cases}$$

for some encoding functions $f_k^{(b)}$ and $f_k^{(e)}$ on appropriate domains satisfying the average block-power constraint

$$\frac{1}{n_e} \sum_{t=1}^{n_e} X_{k,t}^2 \leq P, \quad \forall k \in \mathcal{K}, \quad \text{almost surely.} \quad (2)$$

The input-output relation of the network is described as

$$Y_{k,t} = h_{k,k}X_{k,t} + h_{k-1,k}X_{k-1,t} + Z_{k,t}, \quad (3)$$

where $\{Z_{k,t}\}$ are independent and identically distributed (i.i.d.) standard Gaussian for all k and t and independent of all messages; $h_{k,\ell} > 0$ is the fixed channel coefficient between Tx k and Rx ℓ ; and we define $X_{0,t} = 0$ for all t .

After n_U channel uses, each Rx $k \in \mathcal{K}_U$ decodes the URLLC message $M_k^{(U)}$ based on its own channel outputs $\mathbf{Y}_k^{n_U} := \{Y_{k,1}, \dots, Y_{k,n_U}\}$. So, it produces:

$$\hat{M}_k^{(U)} = g_k^{(n_U)}(\mathbf{Y}_k^{n_U}), \quad (4)$$

for some decoding function $g_k^{(n_U)}$ on appropriate domains. The average error probability for each message $M_k^{(U)}$ is given by

$$\epsilon_{U,k} := \mathbb{P} \left\{ \hat{M}_k^{(U)} \neq M_k^{(U)} \right\}, \quad \text{for } k \in \mathcal{K}_U. \quad (5)$$

After n_e channel uses, each Rx k decodes its desired eMBB messages as

$$\hat{M}_k^{(e)} = b_k^{(n_e)}(\mathbf{Y}_k^{n_e}), \quad (6)$$

where $b_k^{(n_e)}$ is a decoding function on appropriate domains. The average error probability for message $M_k^{(e)}$ is given by

$$\epsilon_{e,k} := \mathbb{P} \left\{ \hat{M}_k^{(e)} \neq M_k^{(e)} \right\}, \quad \text{for } k \in \mathcal{K}. \quad (7)$$

We will be interested in the average URLLC and eMBB error probabilities

$$\epsilon_U := \frac{1}{|\mathcal{K}_U|} \sum_{k \in \mathcal{K}_U} \epsilon_{U,k}, \quad (8)$$

$$\epsilon_e := \frac{1}{|\mathcal{K}|} \sum_{k \in \mathcal{K}} \epsilon_{e,k}. \quad (9)$$

III. CODING SCHEME

Txs in $\mathcal{K}_U$ use DPC to precancel the interference of eMBB transmissions from their neighbouring transmissions and from their own eMBB transmissions on their URLLC transmissions. (Recall that during the Tx-cooperation rounds Txs in $\mathcal{K}_U$ learn the eMBB messages of their neighbouring Txs.)

A. Encoding at Txs in $\mathcal{K} \setminus \mathcal{K}_U$

Each Tx $k \in \mathcal{K} \setminus \mathcal{K}_U$ transmits only the eMBB message $M_k^{(e)}$ over the entire block of n_e channel uses. Over the first n_U channel uses, it transmits a codeword $\mathbf{X}_k^{(e,1)}(M_k^{(e)})$ that is uniformly distributed on the centered n_U -dimensional sphere of radius $\sqrt{n_U \beta_e P}$, for some $\beta_e \in [0, 1]$, independently of all other codewords. Tx k also describes its message $M_k^{(e)}$, and thus its input signal $\mathbf{X}_k^{(e,1)}$, to the neighbouring Tx to its right during the only Tx-cooperation round.

To encode $M_k^{(e)}$ over the following $(n_e - n_U)$ channel uses, Tx k employs a second codeword $\mathbf{X}_k^{(e,2)}(M_k^{(e)})$ that is uniformly distributed on the centered $(n_e - n_U)$ -dimensional sphere of radius $\sqrt{(n_e - n_U)(1 - \beta_e)P}$, independently of all other codewords.

B. Encoding at Tx in $\mathcal{K}_U$

Each Tx $k \in \mathcal{K}_U$ has both eMBB and URLLC messages to transmit. To transmit its URLLC message $M_k^{(U)}$, Tx k employs DPC encoding to precancel the interference of the eMBB transmission of the Tx to its left and its own eMBB transmission. Tx k transmits its URLLC message over only n_U channel uses whereas it sends its eMBB message over the entire block of n_e channel uses. To transmit both messages while satisfying (2), we divide the total transmit power P into three parts $\beta_U P$, $\beta_{e,1} P$, $\beta_{e,2} P$, where power $\beta_U P$ is used for URLLC transmission, power $\beta_{e,1} P$ for eMBB transmission during the first n_U channel uses, and power $\beta_{e,2} P$ for eMBB transmission during the last $n_e - n_U$ channel uses. The coefficients β_U , $\beta_{e,1}$, $\beta_{e,2} \in [0, 1]$ are chosen such that

$$\beta_U + \beta_{e,1} + \beta_{e,2} = 1. \quad (10)$$

Transmitting $M_k^{(e)}$ and $M_k^{(U)}$: Over the first n_U channel uses, Tx k sends its eMBB message $M_k^{(e)}$ jointly with its URLLC message $M_k^{(U)}$. To this end, it encodes $M_k^{(e)}$ using a codeword $\mathbf{X}_k^{(e,1)}(M_k^{(e)})$ that is uniformly distributed on the centered n_U -dimensional sphere of radius $\sqrt{n_U \beta_{e,1} P}$. To encode $M_k^{(U)}$, for each realization m of message $M_k^{(U)}$, $\lfloor 2^{n_U R_U} \rfloor$ codewords $\mathbf{V}_k(m, i)$, $i = 1, \dots, \lfloor 2^{n_U R_U} \rfloor$, are drawn uniformly from a centered n_U -dimensional sphere of radius $\sqrt{r_k n_U P}$ independently of each other and of all other codewords, where

$$r_k := \beta_U + \alpha_{k,1}^2 \beta_{e,1} + \alpha_{k,2}^2 \beta_{e,2}. \quad (11)$$

Tx k then chooses a codeword $\mathbf{V}_k(M_k^{(U)}, i)$ such that the sequence

$$\mathbf{X}_k^{(U)} := \mathbf{V}_k(M_k^{(U)}, i) - \alpha_{k,1} \mathbf{X}_k^{(e,1)} - \alpha_{k,2} \mathbf{X}_{k-1}^{(e,1)} \quad (12)$$

lies in the set

$$\mathcal{D}_k := \left\{ \mathbf{x}_k^{(U)} : n_U \beta_U P - \delta_k \leq \left\| \mathbf{x}_k^{(U)} \right\|^2 \leq n_U \beta_U P \right\} \quad (13)$$

for a given $\delta_k > 0$. If multiple such codewords exist, one of them is chosen at random, and if no appropriate codeword exists, an error is declared.

Over the first n_U channel uses, Tx k transmits

$$\mathbf{X}_k^{(U)} + \mathbf{X}_k^{(e,1)}. \quad (14)$$

Over the last $(n_e - n_U)$ channel uses, Tx k simply encodes $M_k^{(e)}$ using a codeword $\mathbf{X}_k^{(e,2)}(M_k^{(e)})$ that is uniformly distributed on the centered $(n_e - n_U)$ -dimensional sphere of radius $\sqrt{(n_e - n_U) \beta_{e,2} P}$.

C. Decoding at Rx in $\mathcal{K} \setminus \mathcal{K}_U$

Each Rx k in $\mathcal{K} \setminus \mathcal{K}_U$ only has an eMBB message to decode. Rx $k \in \mathcal{K} \setminus \mathcal{K}_U$ decomposes its channel outputs into two output blocks consisting of the first n_U and the last $(n_e - n_U)$ channel uses, respectively. These blocks are of the form:

$$\mathbf{Y}_{k,1} = h_{k,k} \mathbf{X}_k^{(e,1)} + h_{k-1,k} (\mathbf{X}_{k-1}^{(U)} + \mathbf{X}_{k-1}^{(e,1)}) + \mathbf{Z}_{k,1}, \quad (15a)$$

$$\mathbf{Y}_{k,2} = h_{k,k} \mathbf{X}_k^{(e,2)} + h_{k-1,k} \mathbf{X}_{k-1}^{(e,2)} + \mathbf{Z}_{k,2}, \quad (15b)$$

where $\mathbf{Z}_{k,1}$ and $\mathbf{Z}_{k,2}$ are independent i.i.d. standard Gaussian noise sequences. For $\mathbf{Y}_{k,1} = \mathbf{y}_{k,1}$ and $\mathbf{Y}_{k,2} = \mathbf{y}_{k,2}$, Rx k estimates $M_k^{(e)}$ as an index m for which the corresponding codewords $\mathbf{x}_k^{(e,1)}(m)$ and $\mathbf{x}_k^{(e,2)}(m)$ maximize the information density

$$\begin{aligned} & i_1(\mathbf{x}_k^{(e,1)}, \mathbf{x}_k^{(e,2)}; \mathbf{y}_{k,1}, \mathbf{y}_{k,2}) \\ & := \ln \frac{f_{\mathbf{Y}_{k,1}|\mathbf{X}_k^{(e,1)}}(\mathbf{y}_{k,1}|\mathbf{x}_k^{(e,1)}) f_{\mathbf{Y}_{k,2}|\mathbf{X}_k^{(e,2)}}(\mathbf{y}_{k,2}|\mathbf{x}_k^{(e,2)})}{f_{\mathbf{Y}_{k,1}}(\mathbf{y}_{k,1}) f_{\mathbf{Y}_{k,2}}(\mathbf{y}_{k,2})}, \end{aligned} \quad (16)$$

among all codeword pairs $\mathbf{x}_k^{(e,1)} = \mathbf{x}_k^{(e,1)}(m')$ and $\mathbf{x}_k^{(e,2)} = \mathbf{x}_k^{(e,2)}(m')$.

D. Decoding at Rx in $\mathcal{K}_U$

Similarly to the previous subsection, also Rx in $\mathcal{K}_U$ decompose their channel outputs into two output blocks consisting of the first n_U and the last $(n_e - n_U)$ channel uses, respectively. For a Rx $k \in \mathcal{K}_U$, these blocks are of the form:

$$\mathbf{Y}_{k,1} = h_{k,k} (\mathbf{X}_k^{(U)} + \mathbf{X}_k^{(e,1)}) + h_{k-1,k} \mathbf{X}_{k-1}^{(e,1)} + \mathbf{Z}_{k,1}, \quad (17a)$$

$$\mathbf{Y}_{k,2} = h_{k,k} \mathbf{X}_k^{(e,2)} + h_{k-1,k} \mathbf{X}_{k-1}^{(e,2)} + \mathbf{Z}_{k,2}, \quad (17b)$$

where $\mathbf{Z}_{k,1}$ and $\mathbf{Z}_{k,2}$ are independent i.i.d. standard Gaussian noise sequences.

1) Decoding $M_k^{(U)}$: Rx k decodes $M_k^{(U)}$ based on the outputs of the first channel inputs $\mathbf{Y}_{k,1}$ defined in (17a). Rx k estimates $M_k^{(U)}$ as an index m for which the corresponding codeword $\mathbf{v}_k(m, i)$ maximizes the information density

$$i(\mathbf{v}_k; \mathbf{y}_{k,1}) := \ln \frac{f_{\mathbf{Y}_{k,1}|\mathbf{V}_k}(\mathbf{y}_{k,1}|\mathbf{v}_k)}{f_{\mathbf{Y}_{k,1}}(\mathbf{y}_{k,1})}, \quad (18)$$

among all codewords $\mathbf{v}_k = \mathbf{v}_k(m', j)$.

2) Decoding $M_k^{(e)}$: Rx k decodes $M_k^{(e)}$ based on the channel outputs of the first and second channels $\mathbf{Y}_{k,1}$ and $\mathbf{Y}_{k,2}$ by looking for the index m for which the corresponding codewords $\mathbf{x}_k^{(e,1)}(m)$ and $\mathbf{x}_k^{(e,2)}(m)$ maximize the information density

$$\begin{aligned} & i_2(\mathbf{x}_k^{(e,1)}, \mathbf{x}_k^{(e,2)}; \mathbf{y}_{k,1}, \mathbf{y}_{k,2}) \\ & := \ln \frac{f_{\mathbf{Y}_{k,1}|\mathbf{X}_k^{(e,1)}}(\mathbf{y}_{k,1}|\mathbf{x}_k^{(e,1)}) f_{\mathbf{Y}_{k,2}|\mathbf{X}_k^{(e,2)}}(\mathbf{y}_{k,2}|\mathbf{x}_k^{(e,2)})}{f_{\mathbf{Y}_{k,1}}(\mathbf{y}_{k,1}) f_{\mathbf{Y}_{k,2}}(\mathbf{y}_{k,2})} \end{aligned} \quad (19)$$

among all codeword pairs $\mathbf{x}_k^{(e,1)}(m')$ and $\mathbf{x}_k^{(e,2)}(m')$.

IV. MAIN RESULT

Fix $\beta_e, \beta_{e,1}, \beta_{e,2}, \beta_U \in [0, 1]$ such that (10) is satisfied. Define

$$\begin{aligned} \sigma_1^2 &:= h_{k,k}^2 (r_k + (1 - \alpha_{k,1})^2 \beta_{e,1}) P \\ &\quad + (h_{k-1,k} - h_{k,k} \alpha_{k,2})^2 \beta_e P + 1, \end{aligned} \quad (24a)$$

$$\begin{aligned} \sigma_2^2 &:= h_{k-1,k}^2 (r_k + (1 - \alpha_{k-1})^2 \beta_{e,1}) P \\ &\quad + h_{k-1,k}^2 \alpha_{k-1,2}^2 \beta_e P + h_{k,k}^2 \beta_e P + 1, \end{aligned} \quad (24b)$$

$$\sigma_3^2 := (h_{k,k}^2 (1 - \beta_e) + h_{k-1,k}^2 \beta_{e,2}) P + 1, \quad (24c)$$

$$\sigma_4^2 := (h_{k,k}^2 \beta_{e,2} + h_{k-1,k}^2 (1 - \beta_e)) P + 1, \quad (24d)$$

$$c_1 := \left(h_{k,k} \sqrt{\beta_e} + h_{k-1,k} (\sqrt{\beta_U} + \sqrt{\beta_{e,1}}) \right), \quad (24e)$$

$$c_2 := \left(h_{k,k}(\sqrt{\beta_U} + \sqrt{\beta_{e,1}}) + h_{k-1,k}\sqrt{\beta_e} \right), \quad (24f)$$

$$c_3 := \left(h_{k,k}\sqrt{1 - \beta_e} + h_{k-1,k}\sqrt{\beta_{e,2}} \right), \quad (24g)$$

$$c_4 := \left(h_{k,k}\sqrt{\beta_{e,2}} + h_{k-1,k}\sqrt{(1 - \beta_e)} \right). \quad (24h)$$

By employing the scheme proposed in Section III, we have the following theorem on the upper bounds on the average URLLC and eMBB error probabilities ϵ_U and ϵ_e .

Theorem 1: For fixed message set sizes L_U and L_e , the average error probabilities ϵ_U and ϵ_e are bounded by

$$\epsilon_U \leq \frac{2}{K} \sum_{k \in \mathcal{K}_U} \left(1 - F(u_{k,2} - u_{k,1}) + F(-u_{k,2} - u_{k,1}) \right. \\ \left. + (1 - \max\{\mathcal{L}_{k,1}, \mathcal{L}_{k,2}\})^{[2^{n_U R_U}]} \right) + L_U [2^{n_U R_U}] e^{-\gamma_U}, \quad (25)$$

$$\epsilon_e \leq \frac{1}{K} \sum_{k \in \mathcal{K} \setminus \mathcal{K}_U} \frac{1}{\bar{\gamma}_{e,1}} (\zeta_1 l_{k,1} + \zeta_2 l_{k,2} + n_U l_3 + (n_e - n_U) l_4) \\ + \frac{1}{K} \sum_{k \in \mathcal{K}_U} \frac{1}{\bar{\gamma}_{e,2}} (\zeta_1 d_{k,1} + \zeta_2 d_{k,2} + n_U d_3 + (n_e - n_U) d_4) \\ + L_e (e^{-\gamma_{e,1}} + e^{-\gamma_{e,2}}), \quad (26)$$

for any γ_U , $\gamma_{e,1}$ and $\gamma_{e,2}$, and where

$$u_{k,1} := \sqrt{n_U P} \left(c_2 + \frac{h_{k,k} \sqrt{r_k} \sigma_1^2}{\sigma_1^2 - 1} \right), \quad (27)$$

$$u_{k,2} := \sqrt{\frac{\sigma_1^2}{\sigma_1^2 - 1} \left(n_U \ln(\sigma_1^2) - \frac{2\gamma_1}{J_U} + \frac{n_U P r_k h_{k,k}^2}{\sigma_1^2} \right)}, \quad (28)$$

$$l_{k,1} := 2\sqrt{n_U P} \left(c_1 + h_{k,k} \sqrt{\beta_e} - \frac{c_1}{\sigma_2^2} \right), \quad (29)$$

$$l_{k,2} := 2\sqrt{(n_e - n_U) P} \left(c_3 + h_{k,k} \sqrt{1 - \beta_e} - \frac{c_3}{\sigma_3^2} \right), \quad (30)$$

$$d_{k,1} := 2\sqrt{n_U P} \left(c_2 + h_{k,k} \sqrt{\beta_{e,1}} - \frac{c_2}{\sigma_1^2} \right), \quad (31)$$

$$d_{k,2} := 2\sqrt{(n_e - n_U) P} \left(c_4 + h_{k,k} \sqrt{\beta_{e,2}} - \frac{c_4}{\sigma_4^2} \right), \quad (32)$$

and $l_3 := (\sigma_2^2 - 1)/\sigma_2^2$, $l_4 := (\sigma_3^2 - 1)/\sigma_3^2$, $d_3 := (\sigma_1^2 - 1)/\sigma_1^2$, $d_4 := (\sigma_4^2 - 1)/\sigma_4^2$, $\zeta_1 := \sqrt{2}\Gamma(\frac{n_U+1}{2})/\Gamma(\frac{n_U}{2})$, $\zeta_2 := \sqrt{2}\Gamma(\frac{n_e-n_U+1}{2})/\Gamma(\frac{n_e-n_U}{2})$, $\mathcal{L}_{k,1}$, $\mathcal{L}_{k,2}$, $\bar{\gamma}_{e,1}$, and $\bar{\gamma}_{e,2}$ are defined in (20) to (23), J_U is defined in (46), $J_{e,1}$ and $J_{e,2}$ are defined

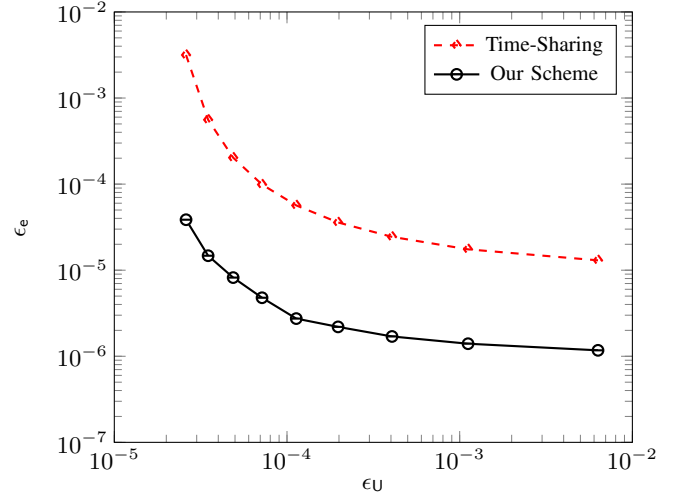


Fig. 2: ϵ_e vs ϵ_U for $P = 10$, $n_e = 100$ and n_U decreases from 90 to 10 with steps of 10.

in equations (79) and (96) of [21], and $F(\cdot)$ represents the cumulative distribution function (CDF) of a chi distribution of degree n_U .

Proof: See Appendix A for the proof of the bound in (25) and [21] for the proof of the bound in (26). ■

In Figure 2, we numerically compare the bounds in Theorem 1 with the time-sharing scheme where only TxS in $\mathcal{K}_U$ send URLLC messages over n_U channel uses whereas all the TxS, including TxS in $\mathcal{K}_U$, send eMBB messages but over only the remaining $n_e - n_U$ channel uses. In this plot, the value of n_U varies from 90 to 10 with step size 10, while the value of n_e is fixed at 100. In our simulations, the values of the parameters β_e , β_U , $\beta_{e,1}$, $\beta_{e,2}$, $\alpha_{k,1}$ and $\alpha_{k,2}$ are optimized to minimize ϵ_e for a given ϵ_U . As can be seen from this figure, our scheme outperforms the time-sharing scheme.

V. CONCLUSIONS

We considered Wyner's soft-handoff model where transmitters simultaneously send eMBB and URLLC messages of heterogeneous blocklengths. We proposed a coding scheme to jointly transmit URLLC and eMBB messages in such a network. We derived rigorous upper bounds on the error probability of eMBB and URLLC transmissions. Our numerical analysis

$$\mathcal{L}_{k,1} := \frac{\delta_k}{2n_U P \sqrt{\pi r_k}} \frac{\Gamma(\frac{n_U}{2})}{\Gamma(\frac{n_U-1}{2})} \frac{1}{\alpha_{k,1} \sqrt{\beta_{e,1}}} \left(1 - \left(r_k - \beta_U + \alpha_{k,2} \sqrt{\beta_e} (\alpha_{k,1} \sqrt{\beta_{e,1}} + \sqrt{r_k}) + \frac{\delta_k}{2n_U P} \right)^2 / (\alpha_{k,1}^2 r_k \beta_{e,1}) \right)^{\frac{n_U-3}{2}}, \quad (20)$$

$$\mathcal{L}_{k,2} := \frac{\delta_k}{2n_U P \sqrt{\pi r_k}} \frac{\Gamma(\frac{n_U}{2})}{\Gamma(\frac{n_U-1}{2})} \frac{1}{\alpha_{k,2} \sqrt{\beta_e}} \left(1 - \left(r_k - \beta_U + \alpha_{k,1} \sqrt{\beta_{e,1}} (\alpha_{k,2} \sqrt{\beta_e} + \sqrt{r_k}) + \frac{\delta_k}{2n_U P} \right)^2 / (\alpha_{k,2}^2 r_k \beta_e) \right)^{\frac{n_U-3}{2}}, \quad (21)$$

$$\bar{\gamma}_{e,1} := n_U \left(\ln(\sigma_2^2) - P \left(\left(c_1 + h_{k,k} \sqrt{\beta_e} \right)^2 - \frac{c_1^2}{\sigma_2^2} \right) \right) + (n_e - n_U) \left(\ln(\sigma_3^2) - P \left(\left(c_3 + h_{k,k} \sqrt{1 - \beta_e} \right)^2 - \frac{c_3^2}{\sigma_3^2} \right) \right) - \frac{2\gamma_{e,1}}{J_{e,1}}, \quad (22)$$

$$\bar{\gamma}_{e,2} := n_U \left(\ln(\sigma_1^2) - P \left(\left(c_2 + h_{k,k} \sqrt{\beta_{e,1}} \right)^2 - \frac{c_2^2}{\sigma_1^2} \right) \right) + (n_e - n_U) \left(\ln(\sigma_4^2) - P \left(\left(c_4 + h_{k,k} \sqrt{\beta_{e,2}} \right)^2 - \frac{c_4^2}{\sigma_4^2} \right) \right) - \frac{2\gamma_{e,2}}{J_{e,2}}. \quad (23)$$

showed that the proposed scheme significantly improves over the standard time-sharing.

An interesting future line of work is to study this network under the assumption that n_e is much larger than n_U . This assumption allows the eMBB transmissions to benefit from their delay-tolerance feature. Another interesting scenario is to let all the TxS send URLLC messages which requires dealing with the interference from the URLLC messages on the URLLC transmissions as well. Finally, for more complex networks, employing rate-splitting multiple access based schemes [22] which allows suitable users to split messages depending on the service requirements is also of interest.

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APPENDIX A

PROOF OF THEOREM 1

In the following, we sketch the proof of the achievability bound (25). See [21] for full details and for the proof of the bound in (26).

A. Bounding $\epsilon_{U,k}$

We start by bounding the decoding error probability of a URLLC message at a given Rx k in $\mathcal{K}_U$. Define the decoding error event $\mathcal{E}_k^{(U)} := \{\hat{M}_k^{(U)} \neq M_k^{(U)}\}$ and $\mathcal{E}_{k,v}$ be the encoding error event that no appropriate codeword $\mathbf{V}_k(M_k^{(U)}, i)$ can be found so that $\mathbf{X}_k^{(U)}(M_k^{(U)}) \in \mathcal{D}_k$. Then we have

$$\epsilon_{U,k} \leq \mathbb{P}[\mathcal{E}_{k,v}] + \mathbb{P}[\mathcal{E}_k^{(U)} | \mathcal{E}_{k,v}]. \quad (33)$$

1) *Analyzing $\mathbb{P}[\mathcal{E}_{k,v}]$:* To calculate this probability, from (13) we notice that $\mathbf{V}_k - \alpha_{k,1}\mathbf{X}_k^{(e,1)} - \alpha_{k,2}\mathbf{X}_{k-1}^{(e,1)} \in \mathcal{D}_k$ if and only if

$$n_U\beta_U\mathbf{P} - \delta_k \leq \|\mathbf{V}_k - \alpha_{k,1}\mathbf{X}_k^{(e,1)} - \alpha_{k,2}\mathbf{X}_{k-1}^{(e,1)}\|^2 \leq n_U\beta_U\mathbf{P}.$$

Recall that $\|\mathbf{V}_k\|^2 = n_U r_k \mathbf{P}$ almost surely. Thus event $\mathcal{E}_{k,v}$ holds except when the following condition is satisfied.

$$\begin{aligned} & n_U(r_k - \beta_U)\mathbf{P} + \|\alpha_{k,1}\mathbf{X}_k^{(e,1)} + \alpha_{k,2}\mathbf{X}_{k-1}^{(e,1)}\|^2 \\ & \leq 2\alpha_{k,1}\langle \mathbf{V}_k, \mathbf{X}_k^{(e,1)} \rangle + 2\alpha_{k,2}\langle \mathbf{V}_k, \mathbf{X}_{k-1}^{(e,1)} \rangle \\ & \leq n_U(r_k - \beta_U)\mathbf{P} + \|\alpha_{k,1}\mathbf{X}_k^{(e,1)} + \alpha_{k,2}\mathbf{X}_{k-1}^{(e,1)}\|^2 + \delta_k. \end{aligned} \quad (34)$$

Equation (34) is equivalent to

$$C_k \leq \langle \mathbf{V}_k, \mathbf{X}_k^{(e,1)} \rangle \leq C_k + \frac{\delta_k}{2\alpha_{k,1}}. \quad (35)$$

where

$$\begin{aligned} C_k &:= \frac{n_U(r_k - \beta_U)\mathbf{P}}{2\alpha_{k,1}} \\ & - \frac{\alpha_{k,2}}{\alpha_{k,1}} \langle \mathbf{V}_k, \mathbf{X}_{k-1}^{(e,1)} \rangle \\ & + \frac{\|\alpha_{k,1}\mathbf{X}_k^{(e,1)} + \alpha_{k,2}\mathbf{X}_{k-1}^{(e,1)}\|^2}{2\alpha_{k,1}}. \end{aligned} \quad (36)$$

Since $\mathbf{X}_k^{(e,1)}$ is drawn uniformly from the sphere, the distribution of $\langle \mathbf{V}_k, \mathbf{X}_k^{(e,1)} \rangle$ depends on $\mathbf{V}_k$ only through its magnitude, this is seen by noting that the inner product of two vectors is unchanged when an orthogonal transformation is applied to both arguments, and the distribution of $\mathbf{X}_k^{(e,1)}$ is unchanged under any orthogonal transformation. Thus, assuming $\mathbf{V}_k = (\|\mathbf{V}_k\|, 0, \dots, 0)$ and following the same arguments as in [12, Appendix E], we prove that

$$\mathbb{P}[\mathbf{X}_k^{(U)} \in \mathcal{D}_k] \geq \max\{\mathcal{L}_{k,1}, \mathcal{L}_{k,2}\} \quad (37)$$

where $\mathcal{L}_{k,1}$ and $\mathcal{L}_{k,2}$ are defined in (20) and (21), respectively. Since the $\lfloor 2^{n_U R_v} \rfloor$ codewords are generated independently,

$$\mathbb{P}[\mathcal{E}_{k,v}] \leq (1 - \max\{\mathcal{L}_1, \mathcal{L}_2\})^{\lfloor 2^{n_U R_v} \rfloor}. \quad (38)$$

See [21] for the detailed proof.

2) *Analyzing $\mathbb{P}[\mathcal{E}_k^{(U)} | \mathcal{E}_{k,v}]$:* To evaluate this error event, we use the threshold bound for maximum-metric decoding. I.e.

$$\begin{aligned} \mathbb{P}[\mathcal{E}_k^{(U)} | \mathcal{E}_{k,v}] & \leq \mathbb{P}[i(\mathbf{V}_k; \mathbf{Y}_{k,1}) \leq \gamma_U] \\ & + L_U \lfloor 2^{n_U R_v} \rfloor \cdot \mathbb{P}[i(\bar{\mathbf{V}}_k; \mathbf{Y}_{k,1}) > \gamma_U] \end{aligned} \quad (39)$$

for any γ_U , where $\bar{\mathbf{V}}_k \sim f_{\mathbf{V}_k}$ and is independent of $(\mathbf{V}_k, \mathbf{Y}_{k,1})$. We start by calculating $\mathbb{P}[i(\bar{\mathbf{V}}_k, \mathbf{Y}_{k,1}) > \gamma_U]$:

$$\mathbb{P}[i(\bar{\mathbf{v}}_k, \mathbf{y}_{k,1}) > \gamma_U | \mathbf{Y}_{k,1} = \mathbf{y}_{k,1}] \quad (40)$$

$$\begin{aligned} &= \int_{\bar{\mathbf{v}}_k} \mathbb{1}\{i(\bar{\mathbf{v}}_k, \mathbf{y}_{k,1}) > \gamma_U\} \\ & \times \exp(-i(\bar{\mathbf{v}}_k, \mathbf{y}_{k,1})) f_{\mathbf{V}_k | \mathbf{Y}_{k,1}}(\bar{\mathbf{v}}_k | \mathbf{y}_{k,1}) d\bar{\mathbf{v}}_k \end{aligned} \quad (41)$$

$$\begin{aligned} &\leq \int_{\bar{\mathbf{v}}_k} \frac{f_{\mathbf{Y}_{k,1} | \mathbf{V}_k}(\mathbf{y}_{k,1} | \bar{\mathbf{v}}_k)}{f_{\mathbf{Y}_{k,1}}(\mathbf{y}_{k,1})} e^{-\gamma_U} \\ & \times \exp(-i(\bar{\mathbf{v}}_k, \mathbf{y}_{k,1})) f_{\mathbf{V}_k | \mathbf{Y}_{k,1}}(\bar{\mathbf{v}}_k | \mathbf{y}_{k,1}) d\bar{\mathbf{v}}_k \end{aligned} \quad (42)$$

$$\leq e^{-\gamma_U}. \quad (43)$$

Now we calculate $\mathbb{P}[i(\mathbf{V}_k, \mathbf{Y}_{k,1}) \leq \gamma_U]$. Note that $\mathbf{Y}_{k,1}$ and $\mathbf{Y}_{k,1} | \mathbf{V}_k$ do not follow a Gaussian distribution. We however define equivalent Gaussian distributions $Q^{(U)}(\mathbf{y}_{k,1}) = \mathcal{N}(\mathbf{y}_{k,1}; \mathbf{0}, I_n \sigma_1^2)$ and $W^{(U)}(\mathbf{y}_{k,1} | \mathbf{v}_k) = \mathcal{N}(\mathbf{y}_{k,1}; h_{k,k} \mathbf{V}_k, I_n \sigma_{y|v}^2)$ where σ_1^2 is defined in (24a) and $\sigma_{y|v}^2 = 1$. We also introduce

$$\tilde{i}(\mathbf{v}_k; \mathbf{y}_{k,1}) := \ln \frac{W^{(U)}(\mathbf{y}_{k,1} | \mathbf{v}_k)}{Q^{(U)}(\mathbf{y}_{k,1})}. \quad (44)$$

Lemma 1: It holds that

$$\frac{i(\mathbf{v}_k; \mathbf{y}_{k,1})}{\tilde{i}(\mathbf{v}_k; \mathbf{y}_{k,1})} \geq J_U \quad (45)$$

where

$$J_U := (n_U - 2) \ln(2a_1 a_2) - 2n_U P(a_1^2 \beta_{e,1} + a_2^2 \beta_e) - \frac{e^{c_T} a_2^2 \beta_e P}{\sqrt{2\pi a_1^2 \beta_{e,1} P}} - \kappa, \quad (46)$$

where $a_1 = h_{k,k}(1 - \alpha_{k,1})$, $a_2 = h_{k-1,k} - h_{k,k}\alpha_{k,2}$, $\kappa = \ln(\frac{1}{2}) + c_T + \ln(\sqrt{\frac{\pi}{8}}) - 2\ln(h_{k,k})$ with $c_T \leq 2$.

Proof: See [21]. ■

As a result, we have:

$$\mathbb{P}[i(\mathbf{V}_k; \mathbf{Y}_{k,1}) \leq \gamma_U] \quad (47)$$

$$\leq \mathbb{P}[\tilde{i}(\mathbf{V}_k; \mathbf{Y}_{k,1}) \leq \frac{\gamma_U}{J_U}] \quad (48)$$

$$= \mathbb{P}\left[\ln \frac{\frac{1}{(\sqrt{2\sigma_{y|v}^2 \pi})^{n_U}} \exp\left(-\frac{\|\mathbf{Y}_{k,1} - h_{k,k}\mathbf{V}_k\|^2}{2\sigma_{y|v}^2}\right)}{\frac{1}{(\sqrt{2\pi\sigma_1^2})^{n_U}} \exp\left(-\frac{\|\mathbf{Y}_{k,1}\|^2}{2\sigma_1^2}\right)} \leq \frac{\gamma_U}{J_U}\right] \quad (49)$$

$$\begin{aligned} &\leq \mathbb{P}\left[h_{k,k}^2 n_U \beta_U P + \|\mathbf{Z}_{k,1}\|^2 \right. \\ &\quad + 2h_{k,k}^2 n_U P \sqrt{\beta_U \beta_{e,1}} + 2h_{k,k} h_{k-1,k} n_U P \sqrt{\beta_U \beta_e} \\ &\quad + 2h_{k,k} \sqrt{n_U P \beta_U} \cdot \|\mathbf{Z}_{k,1}\| + 2h_{k,k} h_{k-1,k} n_U P \sqrt{\beta_{e,1} \beta_e} \\ &\quad + 2h_{k,k} \sqrt{n_U P \beta_{e,1}} \|\mathbf{Z}_{k,1}\| + 2h_{k-1,k} \sqrt{n_U P \beta_e} \|\mathbf{Z}_{k,1}\| \\ &\quad + \frac{2h_{k,k}^2 \sigma_1^2}{\sigma_1^2 - \sigma_{y|v}^2} n_U P \sqrt{\beta_U r_k} + \frac{2h_{k,k}^2 \sigma_1^2}{\sigma_1^2 - \sigma_{y|v}^2} n_U P \sqrt{\beta_{e,1} r_k} \\ &\quad \left. + \frac{2h_{k,k} \sigma_1^2}{\sigma_1^2 - \sigma_{y|v}^2} \left(h_{k-1,k} n_U P \sqrt{\beta_e r_k} + \sqrt{n_U P r_k} \|\mathbf{Z}_{k,1}\|\right) \right] \\ &\geq \tilde{\gamma}_U - n_U P(h_{k,k}^2 \beta_{e,1} + h_{k-1,k}^2 \beta_e) \end{aligned} \quad (50)$$

$$= \mathbb{P}[\|\mathbf{Z}_{k,1}\|^2 + b_1 \|\mathbf{Z}_{k,1}\| \geq \tilde{\gamma}_U] \quad (51)$$

$$= 1 - F\left(\sqrt{\tilde{\gamma}_U + \frac{b_1^2}{4}} - \frac{b_1}{2}\right) + F\left(-\sqrt{\tilde{\gamma}_U + \frac{b_1^2}{4}} - \frac{b_1}{2}\right) \quad (52)$$

where

$$\tilde{\gamma}_U := \frac{\sigma_1^2}{\sigma_1^2 - 1} \left(n_U \ln(\sigma_1^2) - \frac{2\gamma_1}{J_U} - h_{k,k}^2 r_k n_U P\right) \quad (53a)$$

$$\bar{\gamma}_U := \tilde{\gamma}_U - n_U P c_2^2 - \frac{2h_{k,k} \sigma_1^2 n_U P \sqrt{r_k}}{\sigma_1^2 - 1} c_2 \quad (53b)$$

$$b_1 := 2\sqrt{n_U P} \left(c_2 + \frac{h_{k,k} \sqrt{r_k} \sigma_1^2}{\sigma_1^2 - 1}\right) \quad (53c)$$

where c_2 is defined in (24f). Note that $\|\mathbf{Z}_{k,1}\|$ follows a chi distribution of degree n_U and $F(\cdot)$ is its corresponding CDF.

By defining $u_{k,1} := \frac{b_1}{2}$ and $u_{k,2} := \sqrt{\tilde{\gamma}_U + \frac{b_1^2}{4}}$ and combining this bound with the bound in (43), we proved upper bound (25).

REFERENCES

- [1] H. Tataria, M. Shafi, A. F. Molisch, M. Dohler, H. Sjöland, and F. Tufveson, "6G wireless systems: vision, requirements, challenges, insights, and opportunities," *Proceedings of the IEEE*, vol. 109, no. 7, pp. 1166–1199, July 2021.
- [2] P. Popovski, Č. Stefanović, J. J. Nielsen, E. de Carvalho, M. Angelichinoski, K. F. Trillingsgaard, and A. Bana, "Wireless access in ultra-reliable low-latency communication (URLLC)," *IEEE Transactions on Communications*, vol. 67, no. 8, pp. 5783–5801, Aug. 2019.
- [3] A. K. Bairagi et al., "Coexistence mechanism between eMBB and URLLC in 5G wireless networks," *IEEE Transactions on Communications*, vol. 69, no. 3, pp. 1736–1749, March 2021.
- [4] A. Anand, G. de Veciana, and S. Shakkottai, "Joint scheduling of URLLC and eMBB traffic in 5G wireless networks," *IEEE/ACM Transactions on Networking*, vol. 28, no. 2, pp. 477–490, April 2020.
- [5] H. Nikbakht, M. Wigger, M. Egan, S. Shamai (Shitz), J-M. Gorce, and H. V. Poor, "An information-theoretic view of mixed-delay traffic in 5G and 6G," *Entropy*, vol. 24, no. 5, 2022.
- [6] A. Tajer, A. Steiner, and S. Shamai (Shitz), "The broadcast approach in communication networks," *Entropy* 2021, 23, 120.
- [7] K. M. Cohen, A. Steiner, and S. Shamai (Shitz), "The broadcast approach under mixed delay constraints," in *Proceedings of the IEEE International Symposium on Information Theory*, Cambridge (MA), USA, July 1–6, pp. 209–213, 2012.
- [8] H. Nikbakht, M. Wigger, and S. Shamai (Shitz), "Random user activity with mixed delay traffic," in *Proceeding of the IEEE Information Theory Workshop*, Apr. 11–14, 2021.
- [9] A. Cohen, M. Médard, and S. Shamai (Shitz), "Broadcast approach meets network coding for data streaming," Online: *arXiv:2202.03018v1*, Feb. 2022.
- [10] H. Nikbakht, M. Egan, and J-M. Gorce, "Dirty paper coding for consecutive messages with heterogeneous decoding deadlines in the finite blocklength regime," [Research Report] Inria - Research Centre Grenoble – Rhône-Alpes. 2022. Available: <https://hal.inria.fr/hal-03556888>.
- [11] M. H. M. Costa, "Writing on dirty paper (Corresp.)," *IEEE Transactions on Information Theory*, vol. 29, no. 3, pp. 439–441, May 1983.
- [12] J. Scarlett, "On the dispersions of the Gel'fand - Pinsker channel and dirty paper coding," *IEEE Transactions on Information Theory*, vol. 61, no. 9, pp. 4569–4586, Sept. 2015.
- [13] P. H. Lin, S. C. Lin, P-W. Chen, M. Mross, and E. A. Jorswieck, "Gaussian broadcast channels in heterogeneous blocklength constrained networks," Online: *arXiv:2109.07767v1*, Sep. 2021.
- [14] M. Mross, P. H. Lin, and E. A. Jorswieck, "New inner and outer bounds for 2-user Gaussian broadcast channels with heterogeneous blocklength constraints," Online: *arXiv:2202.02110v1*, Feb. 2022.
- [15] R. Kassab, O. Simeone, and P. Popovski, "Coexistence of URLLC and eMBB services in the C-RAN uplink: an information-theoretic study," in *Proceeding of the IEEE Global Communications Conference*, Abu Dhabi, United Arab Emirates, Dec 9–13, 2018.
- [16] H. Nikbakht, M. Wigger, W. Hachem, and S. Shamai (Shitz), "Mixed delay constraints on a fading C-RAN uplink," in *Proceeding of the IEEE Information Theory Workshop*, Visby, Sweden, Aug 25–28, 2019.
- [17] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Transactions on Information Theory*, vol. 49, no. 7, pp. 1691–1706, July 2003.
- [18] T. Erseghe, "Coding in the finite-blocklength regime: Bounds based on Laplace integrals and their asymptotic approximations," *IEEE Transactions on Information Theory*, vol. 62, no. 12, pp. 6854–6883, 2016.
- [19] E. MolavianJazi and J. N. Laneman, "A second-order achievable rate region for Gaussian multi-access channels via a central limit theorem for functions," *IEEE Transactions on Information Theory*, vol. 61, no. 12, pp. 6719–6733, Dec. 2015.
- [20] A. J. Stam, "Limit theorems for uniform distributions on spheres in high-dimensional Euclidean spaces," *Journal of Applied Probability*, vol. 19, no. 1, pp. 221–228, 1982.
- [21] H. Nikbakht, M. Wigger, S. Shamai (Shitz), J-M Gorce, and H. V. Poor, "Joint coding of URLLC and eMBB in Wyner's soft-handoff network in the finite blocklength regime," Online: *arXiv*.
- [22] Y. Liu, B. Clerckx, and P. Popovski, "Network slicing for eMBB, URLLC, and mMTC: An uplink rate-splitting multiple access approach," Online: *arXiv:2208.10841*, Aug. 2022.