

Control-Oriented Data-Driven and Physics-Based Modeling of Maximum Pressure Rise Rate in Reactivity Controlled Compression Ignition Engines

Behrouz Khoshbakht Irdmoussa,¹ L. N. Aditya Basina,² Jeffrey Naber,² Javad Mohammadpour Velni,³ Hoseinali Borhan,⁴ AQ01
and Mahdi Shahbakhti⁵

¹United States

²Michigan Technological University, Mechanical Engineering-Engineering Mechanics Department, USA

³University of Georgia, School of Electrical and Computer Engineering, USA

⁴Cummins Inc, USA

⁵University of Alberta, Department of Mechanical Engineering, USA

Abstract

Reactivity controlled compression ignition (RCCI) is a viable low-temperature combustion (LTC) regime that can provide high indicated thermal efficiency and very low nitrogen oxides (NOx) and particulate matter (PM) emissions compared to the traditional diesel compression ignition (CI) mode [1]. The burn duration in RCCI engines is generally shorter compared to the burn duration for CI and spark-ignition (SI) combustion modes [2, 3]. This leads to a high pressure rise rate (PRR) and limits their operational range. It is important to predict the maximum pressure rise rate (MPRR) in RCCI engines and avoid excessive MPRRs to enable safe RCCI operation over a wide range of engine conditions. In this article, two control-oriented models are presented to predict the MPRR in an RCCI engine. The first approach includes a combined physical and empirical model that uses the first principle of thermodynamics to estimate the PRR inside the cylinder, and the second approach estimates MPRR through a machine learning method based on kernelized canonical correlation analysis (KCCA) and linear parameter-varying (LPV) methods. The KCCA-LPV approach proved to have higher prediction accuracy compared to physics-based modeling while requiring less amount of calibration. The KCCA-LPV approach could estimate MPRR with an average error of 47 kPa/CAD while the physics-based approach's average estimation error was 87 kPa/CAD.

History

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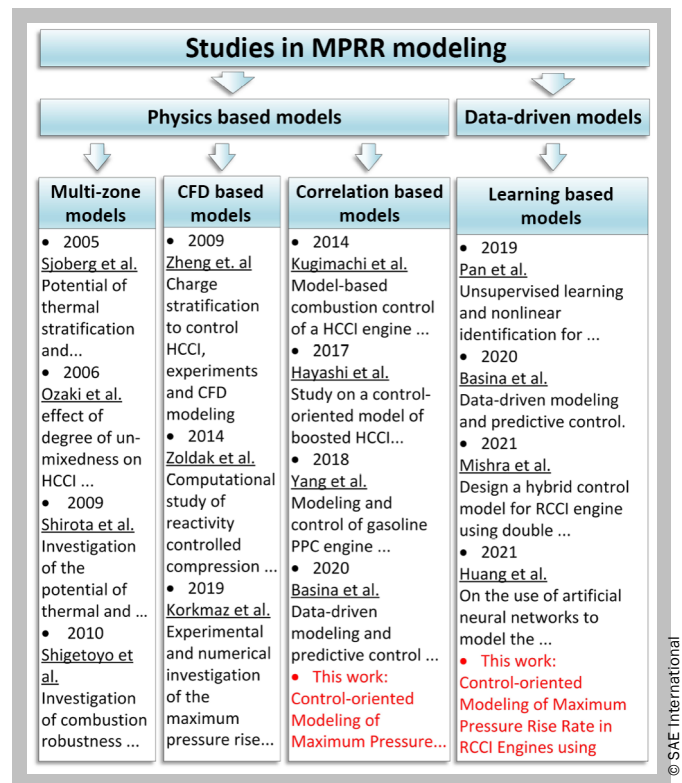
Introduction

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RCCI combustion is a form of LTC regime, which is designed to operate with a mixture of two fuels with different reactivities. These fuels are injected by two sets of injectors to adjust reactivity levels and control the combustion process inside the combustion chamber. Port fuel injectors (PFI) are used to deliver a low reactivity fuel to the intake air flow while a high reactivity fuel is directly injected inside the combustion chamber. Combustion in RCCI engines initiates at high reactivity fuel–air mixture pockets inside the combustion chamber and then advances to burn the low reactive regions [4]. Since high reactivity pocket formation depends on fuel concentration and local temperature and in-cylinder pressure, it is challenging to control combustion in RCCI engines to achieve low emission and high thermal efficiency benefits. Inadequately controlled RCCI engines may generate high carbon monoxide (CO) and unburned hydrocarbon (UHC) emissions and run with high cyclic variability (COV_{IMEP}) [5]. Control of combustion in RCCI and other LTC regimes have been an active area of research in the past two decades [6, 7, 8, 9, 10, 11]. Moreover, running RCCI engines at high loads poses another challenge regarding high maximum pressure rise rate (MPRR) values. High MPRR occurs at high loads due to fuel mixture homogeneity, which causes the formation of simultaneous autoignitions in regions inside the combustion chamber [12]. This issue generates high heat release rates and results in high MPRR values, which creates high combustion noise and high ringing intensity (RI) [5].

Several studies have been conducted to model MPRR in internal combustion (IC) engines. Representation of sample of these methods are presented in Figure 1. These studies can be categorized into physics-based modelings and data-driven modelings. Physics-based models describe IC engines' cycle through the first law of thermodynamics and use calibrated physical models to represent heat transfer, fuel injection, and the combustion process. In contrast, data-driven models use machine learning algorithms to model the relation between measurable IC engine outputs such as load, combustion timing, and controllable engine inputs such as fuel quantity and injection timings [13, 14]. Physics-based modeling studies can be subcategorized into multizone models, computational fluid dynamic (CFD) models, and correlation-based models. Sjöberg et al. [15] developed a multizone model of an HCCI engine and studied the effects of thermal stratification and combustion retardation on MPRR. They found that the high-load operating limit of HCCI engines can be extended by adjusting the thermal stratification. Other researchers including Ozaki et al. [16], Shirota et al. [17], and Jung et al. [18] also estimated MPRR for HCCI engines through a multizone model and studied the effects of thermal and mixing stratification. They confirmed the positive effect of thermal stratification on reducing MPRR and expanding the operational range of HCCI engines. Shigetoyo et al. [19] studied MPRR in HCCI engines through developing a multizone model. They found that retarding combustion can reduce

FIGURE 1 Sample of previous studies in maximum pressure rise rate modeling and characterization and contributions of this study highlighted in red. Data taken from Ref. [15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29].



MPRR for HCCI engines, however excessive retardation can reduce combustion robustness.

Other researchers have developed high-fidelity CFD combustion models to study MPRR for IC engines. Zheng et al. [20] used a CFD simulation to model an n-heptane fueled HCCI-like engine and studied the effects of charge stratification on MPRR. They confirmed the conclusions obtained by multizone modeling and found that increasing the ratio of direct injection to premixed injection creates higher stratification and consequently decreases MPRR for HCCI engines. In another CFD study, Zoldak et al. [21] developed a CFD model of an RCCI engine and compared it with conventional diesel operation while considering MPRR as operational constraints for both of them. They found that by satisfying MPRR constrain, RCCI engines can provide 24% decrease in fuel consumption compared to conventional diesel operation when operated at the same rated power, air–fuel ratio (AFR), and exhaust gas recirculation (EGR) rate. Korkmaz et al. [22] also investigated MPRR in RCCI engines using a validated CFD model. They studied to reduce MPRR to less than 1 bar/CAD in RCCI engines and expand the operational range by implementing optimum injection strategy.

Multizone and CFD model-based MPRR models are computationally expensive and cannot be used for high-speed applications such as feedback controller design. Therefore, researchers have developed another class of MPRR models,

which is the correlation-based model. Kugimachi et al. [23] represented MPRR for an HCCI engine as a correlation based upon charge composition, temperature, and volume at the end of combustion. They also developed a discrete control-oriented model that used a linear quadratic regulator (LQR) strategy to build a combustion controller. Hikita et al. [24] also built a control-oriented model for the HCCI engine and developed a correlation for MPRR based on the thermodynamic properties of charge at the end of combustion. They used this model to build a feedforward load controller for the HCCI engine. Yang et al. [25] developed a control-oriented model for a gasoline-fueled partially premixed combustion (PPC) engine and used it to structure a PI and an MPC controller to control load and combustion phasing while considering MPRR and emission constraints. They presented a correlation for MPRR based on the first law of thermodynamics and used a double Wiebe function to characterize heat release. A similar approach for representing MPRR is used by Basina et al. [26] while developing an LPV-MPC controller for load and combustion phasing in an RCCI engine. Their controller was able to track CA50 and IMEP without violating allowable MPRR values of 5.8 bar/CAD.

The physics-based MPRR models require characterization and calibration of a model to represent thermodynamic process inside the combustion chamber, which can be time-

consuming. Researchers recently employed a new data-driven approach to model MPRR, which will either reduce modeling time or increase model accuracy. Pan et al. [27] developed one of the first data-driven MPRR models and used an unsupervised learning process to estimate pressure trace of a diesel engine during the combustion rate shaping (CRS) process. In another work, Basina et al. [26] developed a data-driven MPRR model using data obtained from an RCCI engine model. They built their model using linear parameter-varying (LPV) approach and could estimate transient MPRR with 60 kPa/CAD accuracy. Shin et al. [30] used a deep learning approach to train and estimate maximum cylinder pressure, the crank angle at maximum cylinder pressure, and MPRR for a gasoline engine. Their model consisted of seven hidden layers and could estimate MPRR with 2 kPa/CAD as the estimation error. Liu et al. [31] used a backpropagation artificial neural network model to estimate maximum in-cylinder pressure and pressure rise rate (PRR) for an optical gasoline engine. Their goal was to avoid testing conditions with unsafe maximum pressure and MPRR values for an optical engine. They found that a well-trained artificial neural network model can provide fast and consistent results, making it an easy-to-use tool for designing future experimental testings for the optical engine. Huang et al. [28] used artificial neural networks to model peak cylinder pressure and its location, MPRR, and indicated mean effective pressure for a heavy-duty natural gas spark-ignition engine. Mishra et al. used random forest machine learning (RFML) approach along a parametrized double Wiebe function to predict MPRR in an RCCI engine. Their model could estimate MPRR with 1.8 kPa/CAD as the mean estimation error [29]. These developed MPRR models can estimate MPRR very accurately, however, they are computationally expensive. Moreover, these models are trained based

on steady-state data and plant dynamics was not considered in their modeling approach. Consequently, they are not appropriate for fast control applications due to their high computational load and lack of plant dynamic identification [28, 30, 31]. Recent works by Maldonado et al. on real-time implementation of spiking neural networks for SI engine control show promising results. This approach is a good candidate for real-time MPRR control of RCCI engines due to its low computational load [32, 33, 34].

Extension of high load operation in RCCI engines requires an active MPRR control strategy based on a fast control-oriented MPRR model. This article proposes two new control-oriented MPRR models in RCCI engines that are trained with a wide range of experimental transient data. These models can be used to control MPRR and extend the high load operation of RCCI engines.

This article is organized as follows. The experimental RCCI engine setup is described in Section II. Then, the first control-oriented MPRR model using Wiebe-based modeling is described in Section III. Subsequently, the algorithm to develop a data-driven KCCA-LPV model for MPRR estimation is given in Section IV. Finally, a summary of results and comparison between the two MPRR models is provided.

Experimental Setup

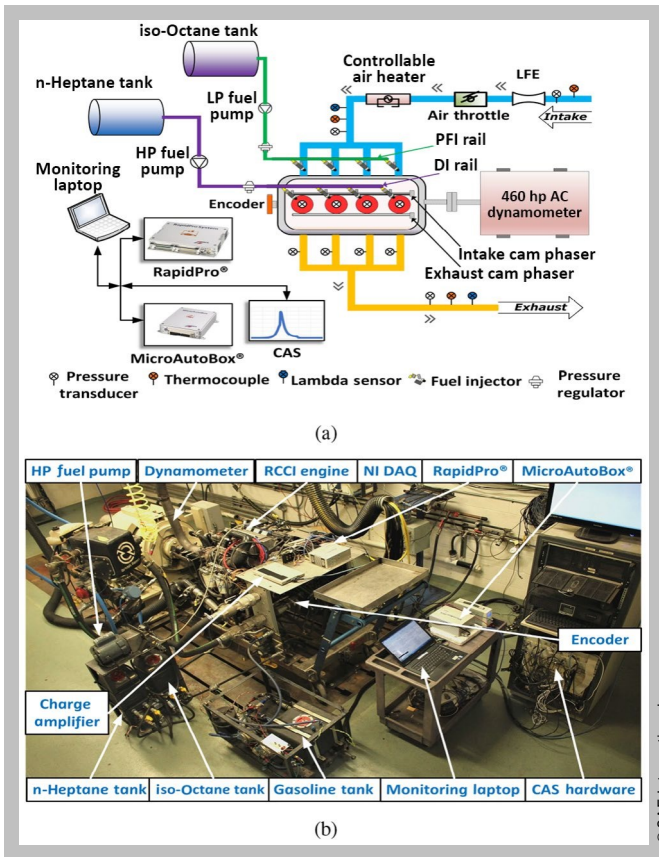
This research utilized a 2.0L GM Ecotec engine in the Energy Mechatronics Laboratory (EML) at the Michigan Tech's Advanced Propulsion Systems (APS) Research Center. The experimental setup layout and view are presented in Figure 2. The specifications of the engine are given in Table 1. The engine is equipped with PFI and DI injectors to enable RCCI operation. The PFI fuel system is used to deliver iso-octane as the low reactivity fuel while the DI fuel system is used to deliver n-heptane as the high reactivity fuel. The ratio between low reactive fuel and high reactivity fuel is characterized by the premixed ratio (PR). It is defined based on chemical energy from the low reactive fuel divided by the total chemical energy delivered by both fuels and calculated according to Equation 1.

$$PR = \frac{m_{\text{iso-octane}} LHV_{\text{iso-octane}}}{m_{\text{iso-octane}} LHV_{\text{iso-octane}} + m_{\text{n-heptane}} LHV_{\text{n-heptane}}}. \quad \text{Eq. (1)}$$

where LHV represents lower heating value of fuels and m represents injected fuel mass per cylinder.

The engine is connected to a 460 hp AC dynamometer. A dSPACE MicroAutoBox unit is used as an engine control unit while a dSPACE RapidPro unit provides power and control signals to actuators. Due to the sensitivity of RCCI operation to intake air temperature, a controllable air heater is included to warm the intake air flow to desired temperatures. The pressure trace from the engine is collected by a combustion analysis system (CAS) at 0.1 CAD resolution and then provided to a Xilinx programmed Spartan-6 field FPGA unit to compute real-time MPRR. The experimental pressure traces

FIGURE 2 RCCI engine experimental setup: (a) layout, (b) real setup overview.



from this engine are used to obtain MPRR based on PRRs at 0.1 crank angle intervals. The computed MPRRs are later utilized for MPRR model development and validation. Figure 3 presents a sample of experimental data set obtained by operating the RCCI engine with a series of input fuel quantity (FQ), the start of injection (SOI), and PR. Iso-octane and n-heptane were injected based on corresponding FQ, PR, and SOI values for each cycle, and pressure trace and encoder output were recorded and used to compute MPRR for the cycle.

Empirical!Wiebe-Based!MPRR!Model

Empirical Wiebe-driven MPRR modeling is a physics-based method used to develop a control-oriented MPRR correlation [25]. This article develops the first MPRR model in RCCI engines using transient experimental data, which can be used to estimate MPRR in RCCI engines based on the control actions. This model requires the development of an in-cylinder PRR model. Pressure rise rate can be computed using the first law of thermodynamics as presented in Equation 2 [35]

$$\frac{dP}{d\theta} = \frac{\gamma - 1}{V} \left(\frac{dQ}{d\theta} - \frac{dQ_w}{d\theta} \right) - \frac{\gamma P}{V} \cdot \frac{dV}{d\theta} \quad \text{Eq. (2)}$$

TABLE 1 Engine specifications.

Make	General Motors
Model	Ecotec 2.0!L Turbocharged
Engine type	4 stroke, 4 cylinders
Fuel system	Direct and port fuel injection
Displaced volume	1998 [cc]
Bore	86 [mm]
Stroke	86 [mm]
Compression ratio	9.2:1
Max. engine power	164 at 5300 [kW at rpm]
Max. engine torque	353 at 2400 [Nm at rpm]
Firing order	1-3-4-2
IVO	25.5/-24.5 [°CAD bTDC]
IVC	2/-48 [°CAD bBDC]
EVO	36/-14 [°CAD bBDC]
EVC	22/-28 [°CAD bTDC]
Valve lift	10.3 [mm]

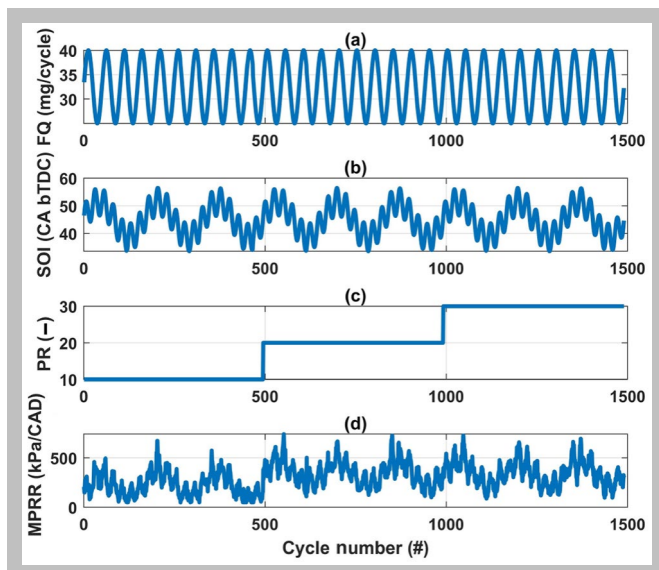
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where $\frac{dQ}{d\theta}$ represents the rate of heat release, γ is the ratio of specific heats, P is the instantaneous pressure, V is the instantaneous volume, and $\frac{dQ_w}{d\theta}$ is the heat transfer to the wall computed through the Woschni heat transfer model [36].

Heat release from the fuel is linearly correlated with mass fraction burn (MFB) of the fuel. MFB can be modeled using a single-term or a double-term Wiebe function presented at Equations 3 and 4, respectively [35]

$$X_b(\theta) = 1 - \exp \left[- \left(\frac{\theta - \theta_{soc}}{\alpha \Delta \theta} \right)^m \right] \quad \text{Eq. (3)}$$

FIGURE 3 Experimental data from RCCI engine at $T_{in} = 333$ K, $N = 1200$ RPM, $P_{in} = 96.5$ kPa.



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$$X_b(\theta) = \lambda \left[1 - \exp \left[- \left(\frac{\theta - \theta_{soc}}{\alpha_1 \Delta \theta} \right)^{m_1} \right] \right] + (1 - \lambda) \left[1 - \exp \left[- \left(\frac{\theta - \theta_{soc}}{\alpha_2 \Delta \theta} \right)^{m_2} \right] \right]. \quad \text{Eq. (4)}$$

where X_b is the mass fraction burned, θ is the instantaneous angle, θ_{soc} is the crank angle at the start of combustion, " θ " is the combustion duration, λ is the fraction of the mixture that burns in the fast combustion stage, and α , m , m_1 , m_2 are calibration parameters.

Figure 4 presents PRR values throughout cycle for the transient experimental data presented in Figure 3. Mass fraction burnt data for the same transient data is also presented in Figure 5. It can be observed that MPRR happens less than 15 CAD after the start of combustion (SOC). Based on the comparison between the MFB estimation of a single-term and a double-term Wiebe function, which is presented in Figure 6, the double-term Wiebe function estimates experimental MFB accurately for the entire combustion process. However, the single-term Wiebe function's accuracy is limited to angles close to the SOC.

Since MPRR happens close to SOC, either a single-term or a double-term Wiebe function can be used to represent experimental MFB. The single-term Wiebe function as given in Equation 3 is preferred in this work due to the simplicity of its calibration compared to a double-term Wiebe function.

The Wiebe function combined with the LHV of the fuels is used to present the fuel heat release rate form as

$$\frac{dQ}{d\theta} = LHV_{eff} \frac{m}{\alpha \Delta \theta} \left(\frac{\theta - \theta_{soc}}{\Delta \theta} \right)^{(m-1)} \exp \left[- \left(\frac{\theta - \theta_{soc}}{\alpha \Delta \theta} \right)^m \right] \quad \text{Eq. (5)}$$

The lower heating value of fuels is denoted by LHV_{eff} and is calculated by

$$LHV_{eff} = (1 - PR) \times LHV_{DI} + PR \times LHV_{PFI}, \quad \text{Eq. (6)}$$

FIGURE 4 Experimental pressure rise rate for transient cycles, $N = 1000$ RPM, $T_{in} = 333$!K.

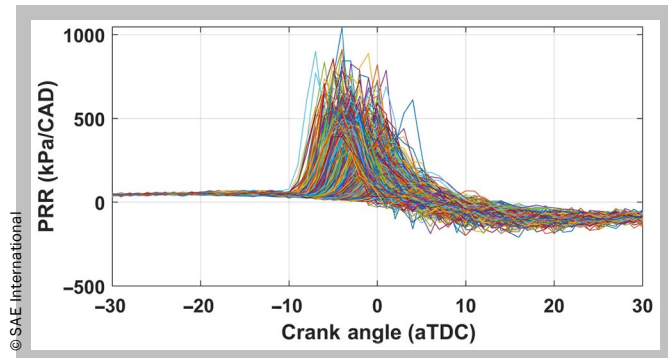
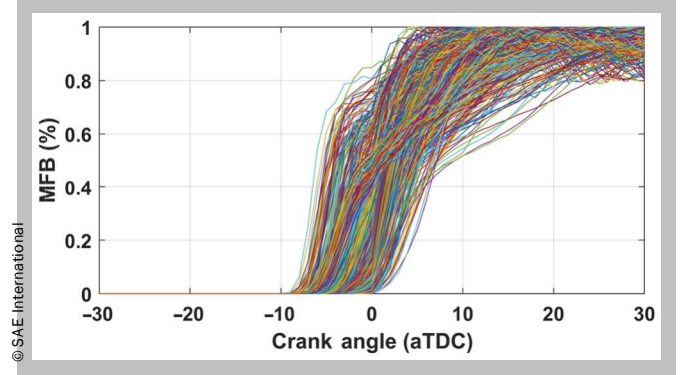


FIGURE 5 Experimental mass burnt fraction for transient cycles, $N = 1000$ RPM, $T_{in} = 333$!K.



where LHV_{DI} represents the lower heating value of the DI's injected fuel, in this case, n-heptane, and LHV_{PFI} is the LHV of the PFI injected fuel, in this case, iso-octane.

Accurate MPRR estimation requires calibration of Equation 3 as the Wiebe function used for MFB estimation. Calibration parameters are m as the shape factor and " θ " as the burn duration, which are described by the following equations [26]

$$m = C_1 + C_2 \times (1 + K) \times SOI + C_3 \times K \quad \text{Eq. (7)}$$

$$\Delta \theta = C_1 + K \left(C_2 \times \phi_{air} + C_3 \times \phi_{pfi} + C_4 \times SOI + C_5 \right) + C_4 \times SOI \quad \text{Eq. (8)}$$

in which K is as follows:

$$K = \exp \left[- (SOI - C_6)^{C_7} \right]. \quad \text{Eq. (9)}$$

Equations 7 to 9 are parameterized using transient experimental data collected from the RCCI engine with 1500 consecutive cycles; 65% of these points, i.e., 975 data points were used to parametrize the Wiebe function while the rest, i.e., 525 data points, were used to test the calibrated Wiebe function.

FIGURE 6 Experimental mass fraction burn estimation, $FQ = 17$ mg/cycle, $SOI = 40$ CAD bTDC, $N = 1000$ RPM, $T_{in} = 333$!K.

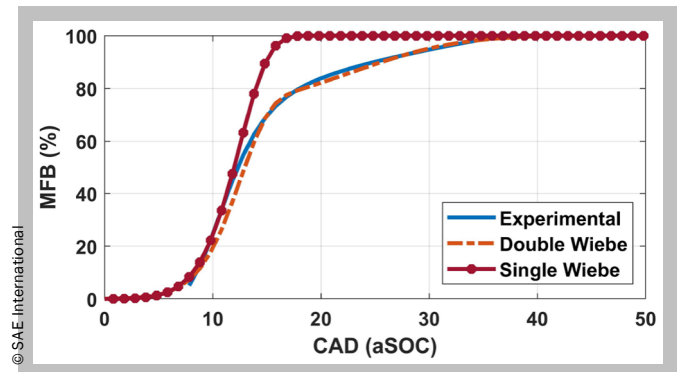
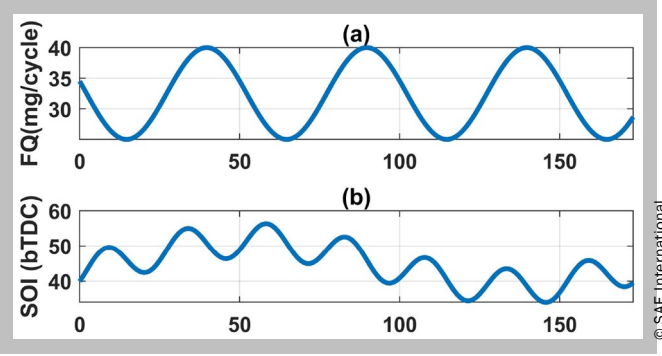


FIGURE 7 Time histories of FQ and SOI for the test data at $N = 1000$ RPM, $T_{in} = 333$ K.



Calibrated MPRR model was then used to estimate MPRR for the test dataset. Figure 7 presents time history of FQ and SOI at test data set. Experimental test dataset and modeled MPRR for three experimental datasets are presented in Figure 8. It can be observed that the MPRR model was able to predict experimental MPRRs with the root mean squared error (RMSE) of 87 kPa/CAD. The Wiebe-based MPRR model could only estimate the general trend of test data. This is due to the static structure of this model, which does not consider the effect of engine inputs at previous cycles on the current cycle MPRR value. This problem will be addressed in the next section by implementing a dynamic state-space KCCA-LPV structure.

FIGURE 8 Performance of the Wiebe-based approach to model MPRR for the test data at $N = 1000$ RPM, $T_{in} = 333$ K, (a) $PR = 12$, (b) $PR = 18$, (c) $PR = 24$.

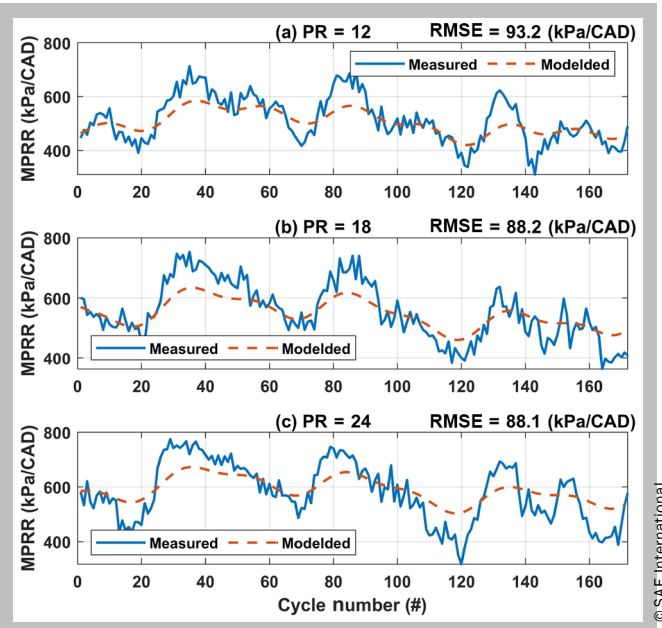
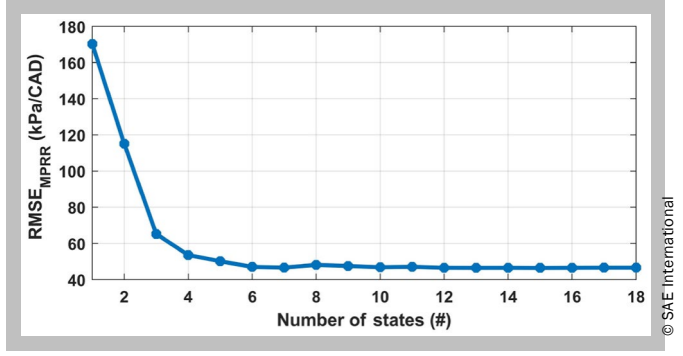


FIGURE 9 Effect of unknown states number on KCCA-driven MPRR model prediction accuracy.



KCCA-Driven MPRR Model

This section presents the second approach to model MPRR in RCCI engines using a data-driven approach. Data-driven modeling approaches can provide a reasonably accurate estimation without prior knowledge. This provides significant modeling flexibility and significantly accelerates the modeling process for an unknown phenomenon compared to physics-based models, which require accurate physical understanding and modeling. Therefore, a data-driven approach based on kernelized canonical correlation analysis (KCCA) is employed in this section to obtain data-driven modeling benefits. Moreover, the KCCA method enables plant states to be independent of the plant outputs, which allows states' numbers to be adjusted to reach the highest accuracy of the data-driven model. This study adopts the state estimation approach developed by Rizvi et al. [37] and generates KCCA combustion model based on implemented engine inputs and measured engine outputs. A state-space dynamic model for a LPV system can be presented as

$$X_{k+1} = A(p_k)X_k + B(p_k)U_k + K(p_k)E_k, \quad \text{Eq. (10a)}$$

$$Y_k = C(p_k)X_k + E_k, \quad \text{Eq. (10b)}$$

where U_k and Y_k denotes the inputs and outputs, respectively, and X_k represents unknown states vector at moment k . Matrices $A(p_k)$, $B(p_k)$, $K(p_k)$, and $C(p_k)$ represent LPV state-space matrices that are generally dependent on scheduling variables p_k ; E_k denotes additive Gaussian white noise. Equation 10 can be rewritten as $E_k = Y_k - C(p_k)X_k$ to form the state-space model as follows

$$X_{k+1} = \hat{A}(p_k)X_k + \hat{B}(p_k)U_k + K(p_k)E_k, \quad \text{Eq. (11a)}$$

$$Y_k = C(p_k)X_k + E_k, \quad \text{Eq. (11b)}$$

where $\hat{A}(p_k) = A(p_k) - K(p_k)C(p_k)$ and $\hat{B}(p_k) = B(p_k) - K(p_k)C(p_k)$. Identification of $\hat{A}(p_k)$, $\hat{B}(p_k)$, $K(p_k)$, and

$C(p_k)$ requires estimation of states (X_k) associated with measured data, i.e., U_k, Y_k . State-space LPV (LPV-SS) formulation of Equation 11 can be used to obtain future outputs as

$$\begin{bmatrix} Y_k \\ Y_{k+1} \\ \vdots \\ Y_{k+d+1} \end{bmatrix} = \begin{bmatrix} O_f^d \diamond p \end{bmatrix}_k X_k + \begin{bmatrix} H_f^d \diamond p \end{bmatrix}_k \begin{bmatrix} U_k \\ U_{k+1} \\ \vdots \\ U_{k+d+1} \end{bmatrix} + \begin{bmatrix} L_f^d \diamond p \end{bmatrix}_k \begin{bmatrix} Y_k \\ Y_{k+1} \\ \vdots \\ Y_{k+d+1} \end{bmatrix} + \begin{bmatrix} e_k \\ e_{k+1} \\ \vdots \\ e_{k+d+1} \end{bmatrix}, \quad \text{Eq. (12)}$$

in which $(O_f^d \diamond p)$ is the observability matrix at time instant k along with the scheduling trajectory p , $(H_f^d \diamond p)$ is a forward Toeplitz matrix, and $(L_f^d \diamond p)$ is a lower triangle matrix. Future measured outputs, inputs, noise, and scheduling parameter vectors for time instant k are collected to form the following matrices

$$\bar{Y}_{k+d}^d := [Y_k^\top \dots Y_{k+d-1}^\top]^\top, \quad \text{Eq. (13a)}$$

$$\bar{U}_{k+d}^d := [U_k^\top \dots U_{k+d-1}^\top]^\top, \quad \text{Eq. (13b)}$$

$$\bar{E}_{k+d}^d := [e_k^\top \dots e_{k+d-1}^\top]^\top, \quad \text{Eq. (13c)}$$

$$\bar{P}_{k+d}^d := [p_k^\top \dots p_{k+d-1}^\top]^\top, \quad \text{Eq. (13d)}$$

where d denotes the future data window size. Equation 12 can be rewritten using Equation 13 as

$$\bar{Y}_{k+d}^d = (O_f^d \diamond p)_k X_k + (H_f^d \diamond p)_k \bar{U}_{k+d}^d + (L_f^d \diamond p)_k \bar{Y}_{k+d}^d + \bar{E}_{k+d}^d. \quad \text{Eq. (14)}$$

Unknown states at time step k based on future inputs and outputs are computed from Equation 14 and given as

$$X_k = (O_f^d \diamond p)_k^\dagger \left((I - (L_f^d \diamond p)_k) \bar{Y}_{k+d}^d - (H_f^d \diamond p)_k \bar{U}_{k+d}^d - (Q^d \diamond p)^\dagger \bar{E}_{k+d}^d \right). \quad \text{Eq. (15)}$$

Since E is an independent zero-mean process noise, which is identically distributed for our experimental data, $(O_f^d \diamond p)^\dagger \bar{E}_{k+d}^d$ is expected to be zero and can be eliminated. State estimation represented by Equation 15 is simplified to Equation 16 by defining $Z_{k+d}^d = \begin{bmatrix} \bar{U}_{k+d}^d \\ \bar{Y}_{k+d}^d \end{bmatrix}$ as the collection of future inputs and outputs:

$$X_k = (O_f^d \diamond p)_k^\dagger \begin{bmatrix} -(H_f^d \diamond p)_k & I - (L_f^d \diamond p)_k \end{bmatrix} \bar{Z}_{k+d}^d. \quad \text{Eq. (16)}$$

Future mapping matrix can be defined as

$$\Phi_f(\bar{p}_{k+d}^d) = (O_f^d \diamond p)_k^\dagger \begin{bmatrix} -(H_f^d \diamond p)_k & I - (L_f^d \diamond p)_k \end{bmatrix}, \quad \text{Eq. (17)}$$

and state estimation at time step k can be simply expressed as

$$X_k = \Phi_f(\bar{p}_{k+d}^d) \bar{Z}_{k+d}^d. \quad \text{Eq. (18)}$$

This approach is also applicable to past measurements to estimate unknown states at time step k based on step-wise output calculation from past d step measurements as

$$X_k = (X_p^d \diamond p)_k X_{k-d} + (R_p^d \diamond p)_k \begin{bmatrix} U_{k-d} \\ U_{k-d+1} \\ \vdots \\ U_{k-1} \end{bmatrix} + (V_p^d \diamond p)_k \begin{bmatrix} Y_{k-d} \\ Y_{k-d+1} \\ \vdots \\ Y_{k-1} \end{bmatrix}. \quad \text{Eq. (19)}$$

Past measured outputs, inputs, noise, and scheduling parameter vectors for time instant k are denoted by Y^d, U^d, E^d and P^d and can be computed similar to future data defined at Equation 13. These definitions can be used to rewrite state estimation as

$$X_k = (X_p^d \diamond p)_k X_{k-d} + (R_p^d \diamond p)_k \bar{U}_k^d + (V_p^d \diamond p)_k \bar{Y}_k^d. \quad \text{Eq. (20)}$$

Choosing d such that $(X_p^d \diamond p)_k \approx 0$ and defining $Z_k^d = \begin{bmatrix} \bar{U}_k^d \\ \bar{Y}_k^d \end{bmatrix}$, state estimation in Equation 20 is expressed as

$$X_k = \begin{bmatrix} (R_p^d \diamond p)_k & (V_p^d \diamond p)_k \end{bmatrix} \bar{Z}_k^d. \quad \text{Eq. (21)}$$

State estimation at time step k based on the past data is simplified to the following by defining $\Phi_p(\bar{p}_k^d)$ as the past mapping matrix

$$\Phi_p(\bar{p}_k^d) = \begin{bmatrix} (R_p^d \diamond p)_k & (V_p^d \diamond p)_k \end{bmatrix}, \quad \text{Eq. (22)}$$

$$X_k = \Phi_p(\bar{p}_k^d) \bar{Z}_k^d. \quad \text{Eq. (23)}$$

The past data-based state estimation approach shown in Equation 18 can be employed to obtain a collection of all estimated states at all time steps. This collection is named $\#_p$ and defined as

$$\Phi_p := \begin{bmatrix} \Phi_p(\bar{p}_1^d) \bar{Z}_1^d & \Phi_p(\bar{p}_2^d) \bar{Z}_2^d & \dots & \Phi_p(\bar{p}_N^d) \bar{Z}_N^d \end{bmatrix}^\top. \quad \text{Eq. (24)}$$

Similarly, $\#_f$ is defined as the collection of estimated states at all time steps based on the future data estimation method as

$$\Phi_f = \begin{bmatrix} \Phi_f(\bar{p}_{1+d}^d) \bar{Z}_{1+d}^d & \Phi_f(\bar{p}_{2+d}^d) \bar{Z}_{2+d}^d & \dots & \Phi_f(\bar{p}_{N+d}^d) \bar{Z}_{N+d}^d \end{bmatrix}^\top. \quad \text{Eq. (25)}$$

Past data-based estimated states ($\#_p$) and future data-based estimated states ($\#_f$) should ideally be identical. However, due to measurement uncertainties and modeling estimations, they tend to be different. Maximizing correlation

between $\#_p$ and $\#_f$ can be accomplished by canonical correlation analysis (CCA) method. The CCA problem for $\#_p$ and $\#_f$ can be formulated as

$$\max_{v_j, w_j} v_j^T \Phi_f^T \Phi_p w_j \quad \text{s.t.} \quad v_j^T \Phi_f^T \Phi_f v_j = w_j^T \Phi_p^T \Phi_p w_j = 1 \quad \text{Eq. (26)}$$

Equation 27 presents the CCA optimization problem in a regularized setting based on an LS-SVM formulation.

$$\max_{v_j, w_j, s, r} J(v_j, w_j, s, r) = \gamma \sum_{k=1}^N \left(s_k r_k - v_j^T \frac{1}{2} z_k - v_j^T \frac{1}{2} z_{k+d} \right) - \frac{1}{2} v_j^T v_j - \frac{1}{2} w_j^T w_j$$

$$\text{s.t.} \quad s_k = v_j^T \Phi_f (\bar{P}_{k+d}^d) \bar{Z}_{k+d}^d \quad r_k = w_j^T \Phi_p (\bar{P}_k^d) \bar{Z}_k^d \quad \text{Eq. (27)}$$

Solution for the regularized CCA problem can be obtained by forming Lagrangian as

$$L(v_j, w_j, s, r) = J(v_j, w_j, s, r) - \sum_{k=1}^N \eta_j^k (s_k - v_j^T \Phi_f (\bar{P}_{k+d}^d) \bar{Z}_{k+d}^d) - \sum_{k=1}^N \kappa_j^k (r_k - w_j^T \Phi_p (\bar{P}_k^d) \bar{Z}_k^d), \quad \text{Eq. (28)}$$

where $\eta_j = [\eta_j^1 \dots \eta_j^N]^T$ and $\kappa_j = [\kappa_j^1 \dots \kappa_j^N]^T$ are Lagrange multipliers. Since the problem is in a convex form, the global minimum is computed where derivatives for Lagrangian function variables are zero. The problem presented at Equation 26 can be converted to the following generalized eigenvalue problem

$$K_{pp} p \kappa_j = \lambda_j (v_j K_{ff} f + I) \eta_j, \quad \text{Eq. (29a)}$$

$$K_{ff} f \eta_j = \lambda_j (v_j K_{ff} f + I) \kappa_j, \quad \text{Eq. (29b)}$$

where $K_{pp} = \Phi_p \Phi_p^T$ and $K_{ff} = \Phi_f \Phi_f^T$. Lagrangian multipliers are the solution of the generalized eigenvalue problem below

$$\begin{bmatrix} 0 & K_{pp} \\ K_{ff} & 0 \end{bmatrix} \begin{bmatrix} \eta_j \\ \kappa_j \end{bmatrix} = \lambda_j \begin{bmatrix} v_j K_{ff} f + I & 0 \\ 0 & v_j K_{pp} f + I \end{bmatrix} \begin{bmatrix} \eta_j \\ \kappa_j \end{bmatrix} \quad \text{Eq. (30)}$$

Finally, the computed Lagrangian multipliers are used to calculate the estimated states as

$$x_k^j = \kappa_j \begin{bmatrix} (\bar{Z}_1^d)^T \bar{k} (\bar{P}_1^d, \bar{P}_{k+d}^d) \\ (\bar{Z}_2^d)^T \bar{k} (\bar{P}_2^d, \bar{P}_k^d) \\ \vdots \\ (\bar{Z}_N^d)^T \bar{k} (\bar{P}_N^d, \bar{P}_k^d) \end{bmatrix} = \eta_j \begin{bmatrix} (\bar{Z}_{1+d}^d)^T \bar{k} (\bar{P}_{1+d}^d, \bar{P}_{k+d}^d) \\ (\bar{Z}_{2+d}^d)^T \bar{k} (\bar{P}_{2+d}^d, \bar{P}_{k+d}^d) \\ \vdots \\ (\bar{Z}_{N+d}^d)^T \bar{k} (\bar{P}_{N+d}^d, \bar{P}_{k+d}^d) \end{bmatrix} \quad \text{Eq. (31)}$$

The second step in data-driven MPRR modeling is to use estimated states through the KCCA method along with measured inputs and outputs to obtain a state-space dynamic model of the RCCI engine through a LPV identification. This

article utilizes least-squared SVM (LS-SVM) to determine matrices $\hat{A}(p_k)$, $B(p_k)$, $C(p_k)$, and $K(p_k)$ in Equation 11. These state-space matrices are defined based on support vector weighting matrices and feature maps using the equations presented in Equation 32

$$\hat{A}(p_k) = W_1 \Phi_1(p_k); \quad B(p_k) = W_2 \Phi_2(p_k), \quad \text{Eq. (32a)}$$

$$K(p_k) = W_3 \Phi_3(p_k); \quad C(p_k) = W_4 \Phi_4(p_k), \quad \text{Eq. (32b)}$$

where unknown support vector weighting matrices are shown by $W_{1;2;3;4}$ and unknown feature maps are represented by $\Phi_{1;2;3;4}$. An SVM-based discrete-time state-space model can be represented by

$$X_{k+1} = W_1 \Phi_1(p_k) X_k + W_2 \Phi_2(p_k) U_k + W_3 \Phi_3(p_k) Y_k, \quad \text{Eq. (33a)}$$

$$Y_k = W_4 \Phi_4(p_k) X_k. \quad \text{Eq. (33b)}$$

State-space matrices can be obtained from feature maps and weighting matrices as follows based on the approach presented by Rizivi et al. [38]

$$\hat{A}_e(x) = W_1 \Phi_1(x) = \sum_{k=1}^N \alpha_k x_k^T \bar{k}^1(p_k, x), \quad \text{Eq. (34a)}$$

$$B_e(x) = W_2 \Phi_2(x) = \sum_{k=1}^N \alpha_k u_k^T \bar{k}^2(p_k, x), \quad \text{Eq. (34b)}$$

$$K_e(x) = W_3 \Phi_3(x) = \sum_{k=1}^N \alpha_k y_k^T \bar{k}^3(p_k, x), \quad \text{Eq. (34c)}$$

$$C_e(x) = W_4 \Phi_4(x) = \sum_{k=1}^N \beta_k x_k^T \bar{k}^3(p_k, x), \quad \text{Eq. (34d)}$$

where α and β are Lagrangian multipliers of the LS-SVM optimization problem, and $\bar{k}^i(p, p_k)$ is Gaussian kernel function defined as

$$\bar{k}_i(p_j, p_k) = \exp \left(-\frac{\|p_j - p_k\|_2^2}{2\sigma_i^2} \right), \quad \text{Eq. (35)}$$

where σ_i denotes the standard deviation and $\|\cdot\|_2$ is the l_2 norm. The defined Gaussian function is used in our LPV modeling approach to perform the so-called kernel trick. Computed matrices $\hat{A}_e(\cdot)$, $B_e(\cdot)$, $K_e(\cdot)$, and $C_e(\cdot)$ in Equation 34 can be substituted into Equation 3 to represent the data-driven state-space dynamic model of the plant.

The described data-driven method was implemented on the experimental data collected from the RCCI engine. The input PR, FQ, and n-heptane SOI were varied and MPRR was computed from acquired pressure traces data. Figure 3 presents a sample of the experimental data obtained from the RCCI engine. The collected experimental data were divided into training and test datasets. The training dataset, which consists of 65% of data, is used to train the KCCA-LPV model and the remaining 35% is reserved as the test data. In this work, the effect of the number of states at the dynamic

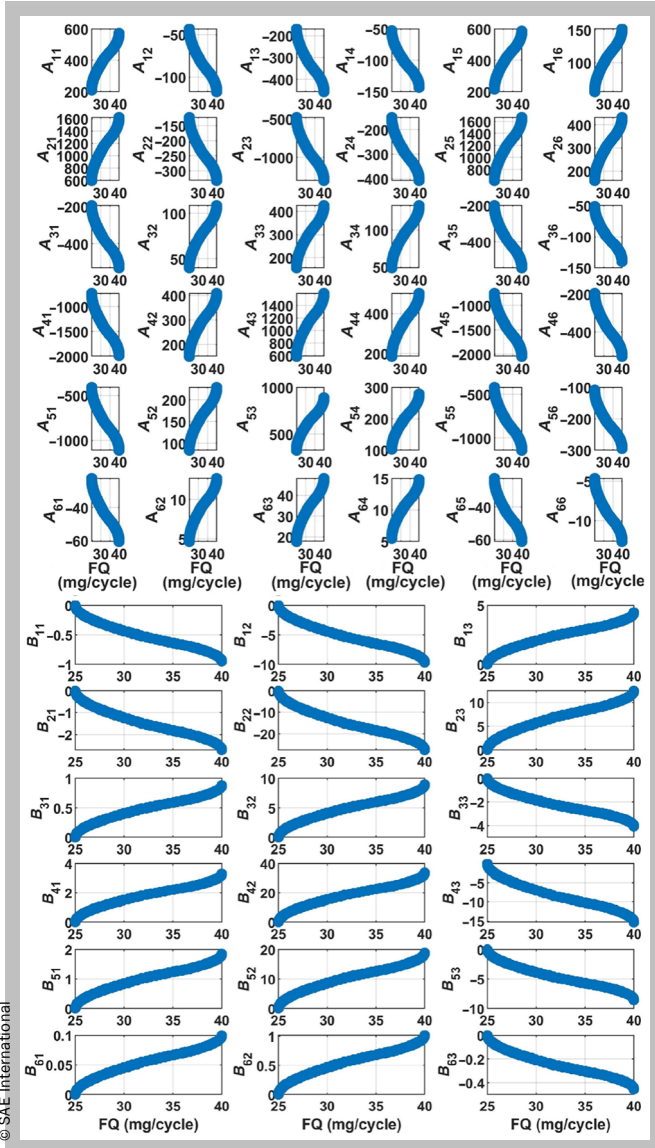
state-space model on MPRR estimation accuracy is also studied. Figure 9 presents the effect of unknown state numbers on estimation accuracy. It can be observed that by increasing the number of states, estimation error decreases and reaches an almost constant value. In this article, the number of unknown states is selected to be six where high estimation accuracy can be achieved while avoiding the high computational cost associated with using higher state numbers. The state-space dynamic model of the MPRR is structured and presented in Equation 36. Dependency of the identified A_e and B_e matrices on the FQ as the scheduling parameter is shown in Figure 10. Elements of these matrices represent an almost linear trend per FQ variation. They also have a relatively large range of variation. These characteristics demonstrate that

the identified model is a LPV representation of the MPRR dynamics in the RCCI engine.

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix}_{(k+1)} = \hat{A}_e(FQ) \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix}_{(k)} + B_e(FQ) \begin{bmatrix} FQ \\ SOI \\ PR \end{bmatrix}_{(k)} \quad \text{Eq. (36a)}$$

$$[MPRR]_{(k+1)} = C(FQ) [X_1 X_2 X_3 X_4 X_5 X_6]_{(k+1)}^T, \quad \text{Eq. (36b)}$$

FIGURE 10 Dependency of the elements of the identified A and B matrices (of the learned LPV model) on the scheduling parameter (FQ).



The developed KCCA-LPV model is then used to estimate the experimental MPRR values. Figure 11 presents estimation results for the learned KCCA model for the test data at three PR values. It demonstrates that the LPV model can estimate the experimental MPRRs with an average estimation error of 45 kPa/CAD. Figure 12 presents the comparison between MPRR estimation of the two models at PR = 14, PR = 20, and PR = 18. The root mean square error of the Wiebe-based model is 64 kPa/CAD, 67 kPa/CAD, and 87 kPa/CAD for PR = 14, PR = 20, and PR = 28, respectively. On the other hand, the data-driven model could estimate the experimental data at PR = 14, PR = 20, and PR = 28 with an average error of 28 kPa/CAD, 29 kPa/CAD, and 48 kPa/CAD.

It can be observed that the data-driven method can estimate the measured MPRR values with better accuracy compared to the Wiebe-based method while requiring less development time due to its flexible modeling characteristics.

FIGURE 11 Performance of the KCCA-driven model to estimate MPRR for the test data at $T_{in} = 333$ K, $N = 1200$ RPM, $P_{in} = 96.5$ kPa and (a) PR = 12, (b) PR = 18, (c) PR = 24.

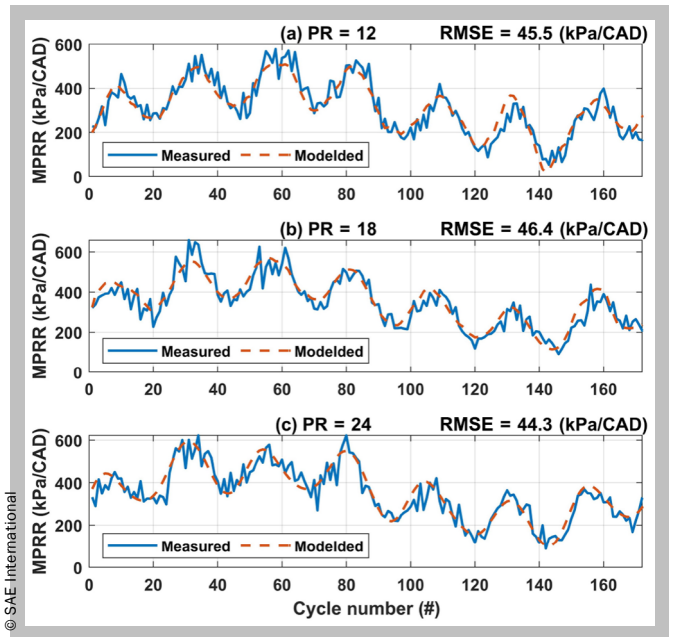
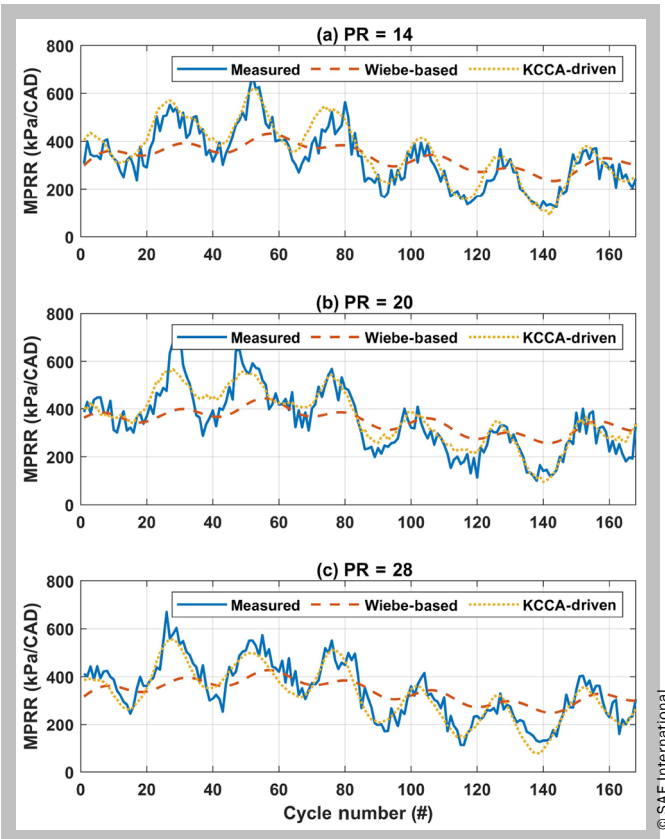


FIGURE 12 Estimation performance of the Wiebe-based and KCCA-driven MPRR models at $T_{in} = 333$ K, $N = 1200$ RPM, $P_{in} = 96.5$ kPa (a) PR = 14, RMSE Wiebe-based = 64 kPa/CAD, RMSE KCCA-Driven = 28 kPa/CAD (b) PR = 20, RMSE Wiebe-based = 67 kPa/CAD, RMSE Data-Driven = 29 kPa/CAD. (c) PR = 28, RMSE Wiebe-based = 88 kPa/CAD, RMSE KCCA-Driven = 48 kPa/CAD.



Moreover, these models are accurate enough to be used in MPRR control applications since MPRR at RCCI engines should be below 10 bar/CAD [39] and estimation errors for Wiebe-based and KCCA-LPV MPRR models are less than 9% and 4.8% of the allowable MPRR value, respectively.

Summary and Conclusions

This article developed and demonstrated two control-oriented MPRR models for a 2-liter 4-cylinder RCCI engine. The first MPRR model is developed based on a physics-based Wiebe method while the second model is obtained through a data-driven KCCA modeling approach. The first MPRR model simulated the in-cylinder PRR through the first law of thermodynamics implementation and calibrated the model by adjusting a Wiebe function, which represents fuel heat release rate. Test results showed that the Wiebe-based MPRR model can estimate experimental MPRR with a mean estimation error of 87 kPa/CAD. In the second MPRR model, unknown

states are estimated by the KCCA method, and state-space representations of the RCCI combustion dynamics are computed by the LPV method. The developed KCCA-driven MPRR model could estimate the experimental MPRR values with an average error of 47 kPa/CAD.

This study demonstrated the cycle-by-cycle MPRR in RCCI engines can be modeled through empirical modeling and machine learning approaches. Both of these methods have a very low computational burden due to their empirical and state-space structure. Modeling efforts and estimation accuracy showed that the machine learning algorithm can provide significantly higher estimation accuracy compared to empirical modeling. The superior performance of the data-driven model is due to the implementation of highly nonlinear support vector machines to learn the stochastic behavior of the MPRR in RCCI engines. This research can be advanced by employing the proposed MPRR models for developing MPRR controller and expanding the high load operation of RCCI engines.

Contact Information

Behrouz Khoshbakht Irdmousa
bkhoshi@mtu.edu

Definitions/Abbreviations

CA50 - Crank angle for 50% fuel burnt (CAD aTDC)

COV_{IMEP} - Coefficient of variation of IMEP (%)

e - Estimation or tracking error

E - Stochastic white noise

EOC - End of combustion (CAD aTDC)

EVC - Exhaust valve closing (CAD aTDC)

FQ - Fuel injected per cycle (mg/cycle)

$\mathcal{H}$ - Forward Toeplitz matrix

IMEP - Indicated mean effective pressure (kPa)

IVC - Inlet valve closing timing (CAD aTDC)

IVO - Inlet valve opening timing (CAD aTDC)

J - Least-square cost function

k - Gaussian kernel function

LHV - Lower heating value (kJ/kg)

m - Injected fuel mass (kg)

MPRR - Maximum pressure rise rate (kPa/CAD)

Q - Heat release (kJ)

PR - Premixed ratio of dual fuels (-)

PRR - Pressure rise rate (kPa/CAD)

$\mathcal{R}$ - Reachability matrix

SOI - n-heptane injection timing (CAD bTDC)

T_{in} - Temperature at inlet valve closing (K)

T_{soc} - Temperature at start of combustion (K)

U - RCCI engine inputs
W - Support vector weighting matrix
X - RCCI engine states
 $\alpha, \beta, \kappa, \eta$ - Lagrange multipliers
 θ - Crank angle (degree)
 γ - Heat capacity ratio
 λ - First-stage Wiebe function fraction
 Φ - Feature map function

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