Compensator-Based Output Feedback Stabilizers for a Class of Planar Systems With Unknown Structures and Measurements

Chunjiang Qian, *Fellow, IEEE*, Shuaipeng He, and Yunlei Zou, *Member, IEEE*

***Abstract*—This article considers the problem of output feedback stabilization for a class of nonlinear planar systems with unknown structures and measurements, which prevent the construction of conventional state observers. By taking advantage of the stability-increasing capability of a lead compensator, we propose a dynamic output feedback controller to globally stabilize the uncertain planar systems. For the special case of linear planar systems with unknown coefficients, a finite-time output feedback stabilizer based on a nonlinear compensator is constructed for a faster convergence rate.**

***Index Terms*—Lead compensator, planar system, unknown measurement, unknown structure.**

I. Introduction

This article considers the problem of output feedback stabilization for a class of nonlinear planar systems described by

$$\begin{aligned}\dot{x}_1 &= g(x_2)\\ \dot{x}_2 &= u\\ y &= h(x_1)\end{aligned} \tag{1}$$

where $[x_1, x_2]^\top \in \mathbb{R}^2$ is the system state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the system output, and $g(s)$ and $h(s)$ are *unknown nonlinear* functions with $g(0) = 0$ and $h(0) = 0$. Our objective is to design an output feedback (i.e., via the output y) controller, which globally asymptotically stabilizes the nonlinear planar system (1).

Planar systems are widely used to describe dynamics of various practical systems in circuit analysis, mechanical and thermal processes, image processing, data transmission, and digital filtering, etc. [1]–[3]. Global output feedback stabilization of nonlinear systems is a very practical but challenging problem in the nonlinear control field [4]–[6]. The difficulty of output feedback stabilization problem for nonlinear systems mainly stems from the inapplicability of separation principle for nonlinear systems due to the finite escape phenomenon [7].

One common assumption for most of existing output feedback control results is that the relationship between the output and the state, i.e., $y = h(x)$, is explicitly known. However, in practice, sometimes it is difficult to obtain the exact structures of $h(\cdot)$ [8]– [11]. To deal with this practical issue, Zhai and Qian [12] proposed an output feedback design approach for nonlinear systems with uncertain output function $y = h(x_1)$ by using homogeneous domination approach [13]. A critical assumption in [12] is that $h(x_1)$ is differentiable and bounded by two linear functions. Chen *et al.* [14] proposed a dual-domination approach to design an output feedback controller for a class of nonlinear systems with unknown measurement sensitivity (i.e., $y = \theta(t)x_1$, the unknown function $\theta(t)$ is bounded by two positive constants). For systems with parametric uncertainty in both state and output equations, Zhang and Lin [15], [16] studied their adaptive and robust output feedback control problems, respectively. By employing the idea of $\mathcal{K}$-filter proposed in [17], the problem of adaptive output feedback control of nonlinear systems with output parametric uncertainty (i.e., $y = \theta x_1$ with an unknown constant θ) is studied in [18].

However, some sensors in the real world might not have the linear relationship between the measurement and the real state. For instance, as shown in [19], the voltage output from an infrared distance sensor is a nonlinear function. A typical nonlinear infrared sensor for the real distance d will only output d^p where the constant p is around 0.8 but its precise value is varying from product to product. In this case, when all the system states can be measured through nonlinear sensors, a robust full-state feedback stabilizer has been designed in [20]. If not all the states can be measured and the measurement function is unknown, all the aforementioned results are inapplicable. To see this point more clearly, consider the following planar system

$$\dot{x}_1 = x_2,\ \ \dot{x}_2 = u,\ \ y = \text{sign}(x_1)|x_1|^q \tag{2}$$

where q is an unknown positive constant. Since q is unknown in $\text{sign}(x_1)|x_1|^q$, it is impossible to use the measured output to design an observer to estimate the unknown state. Moreover, the output function is not even continuously differentiable when $q < 1$.

In addition to the unknown measurement, another challenge for the output feedback stabilization of system (1) is the unknown structure. For example, consider

$$\dot{x}_1 = \text{sign}(x_2)|x_2|^p,\ \ \dot{x}_2 = u,\ \ y = x_1 \tag{3}$$

where p is an unknown positive constant. In practice, the power p could be unknown with different values varying from system to system. As shown in [21], for different boiler-turbine units, the dynamic models may have different power $p's$ as they are identified from the operational data obtained from power plants. Recently, Su *et al.* [22] developed an interval homogeneity-based control scheme to solve the global state feedback stabilization problem of system (3) under the assumption that the power p is in a known interval. Chen *et al.* [23] investigated global asymptotic stabilization problem for a class of nonlinear systems with time-varying powers, which are assumed to be known. However, when p is unknown, the output feedback stabilization problem of (3) is still unsolved.

In this article, we focus on designing an output feedback controller to globally asymptotically stabilize the nonlinear planar system (1). Since the presence of unknown measurements and unknown structures

Manuscript received December 31, 2020; revised March 23, 2021; accepted May 1, 2021. Date of publication May 11, 2021; date of current version March 29, 2022. This work was supported by the U.S. National Science Foundation under Grant 1826086. Recommended by Associate Editor A. Chaillet. *(Corresponding author: Chunjiang Qian.)*

Chunjiang Qian and Shuaipeng He are with the Department of Electrical and Computer Engineering, The University of Texas at San Antonio, San Antonio, TEXAS 78249 USA (e-mail: chunjiang.qian@utsa.edu; shuaipeng.he@utsa.edu).

Yunlei Zou is with the School of Mathematical Sciences, Yangzhou University, Yangzhou 225002, China (e-mail: zouyl0903@163.com).

Digital Object Identifier 10.1109/TAC.2021.3079360

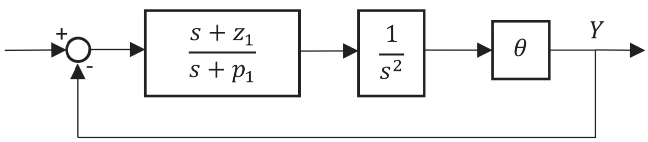

Fig. 1. Block diagram of system (4) under a first-order compensator.

has made the conventional observer-based methods inapplicable, a new method is needed. In this article, we present a new design method, which is inspired by the lead compensator controller for linear systems. In classic control theory, the lead compensator has been known for its capability of increasing system stability [24], [25]. We also discover that the lead compensator can increase stability of some systems with unknown output coefficients. Consider a double-integrator system $\ddot{x} = u$ with the output $y = \theta x$ for an unknown positive constant θ. The transfer function of this system is described by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\theta}{s^2}. \tag{4}$$

We want to see if we can design a stabilizing output feedback controller for the transfer function (4) without identifying the unknown constant θ. If we use a PID controller, the denominator of the closed-loop system contains $1 + \frac{K_I + K_P s + K_D s^2}{s} G(s)$, which implies that the poles of the closed-loop system are the roots of

$$s^3 + \theta k_1 s^2 + \theta k_2 s + \theta k_3 = 0. \tag{5}$$

According to Routh–Hurwitz criterion [26], the third order polynomial $s^3 + a_2 s^2 + a_1 s + a_0 = 0$ has all roots in the open left half plane if and only if a_2, a_0 are positive and $a_2 a_1 > a_0$. Therefore, to ensure stability of the closed-loop system, the PID control coefficients need to satisfy $\theta k_1 k_2 > k_3$, from which k_1, k_2, and k_3 cannot be determined unless we know the bound of θ. When we apply a first-order compensator $\frac{s+z_1}{s+p_1}$ to system (4), as shown in Fig. 1 the characteristic equation of the closed-loop system is

$$s^3 + p_1 s^2 + \theta s + \theta z_1 = 0. \tag{6}$$

It is clear that (6) is Hurwitz stable as long as $p_1 > z_1 > 0$, which implies that $\frac{s+z_1}{s+p_1}$ is a lead compensator. Here, we have just shown that a lead compensator can stabilize system (4) even when the measurement is not precisely known.

In this article, we will adopt the same idea of using a lead compensator to design a stabilizing output feedback controller for a class of uncertain nonlinear systems such as (2) and (3). By taking advantage of the stability-increasing capability of the lead compensator, a global stabilizer can be designed to solve the output feedback control problem for system (1), even in the presence of unknown structures and measurements. Moreover, in the case when system (1) becomes a linear system with unknown parameters, we can design a nonlinear compensator and a finite-time controller to improve the convergence rate of the closed-loop system.

II. Main Results

In this section, we first solve the output feedback stabilization problem of the nonlinear system (1). Then, for a special class of linearly parameterized systems, we show that our proposed control scheme can be used to handle perturbations and improve the convergence rate of the closed-loop system.

A. Compensator-Based Stabilizers for the Nonlinear System (1)

To solve the global output feedback stabilization problem of (1), the following conditions are imposed on the nonlinear functions g and h.

Assumption 1: The function $g(s)$ with $g(0) = 0$ is strictly increasing.

Assumption 2: The function $h(s)$ with $h(0) = 0$ satisfies the following:

(i) $h(s) \neq 0$ when $s \neq 0$;
(ii) $\int_0^x h(s)ds > 0$ when $x \neq 0$;
(iii) $\lim_{|x| \to +\infty} \int_0^x h(s)ds = +\infty$.

Remark 1: It is obvious that any function satisfying Assumption 1 also satisfies Assumption 2, but not vice versa. In fact, Assumption 2 includes more general functions such as $y = h_1(x_1) = \begin{cases} \theta, & x_1 \geq \theta \\ x_1, & -\theta < x_1 < \theta \\ -\theta, & x_1 \leq -\theta \end{cases}$ and $y = h_2(x_1) = \frac{\theta x_1}{1+x_1^2}$ for an unknown positive constant θ, neither of which is strictly increasing. For systems (2) and (3), it is easy to verify that the function $\text{sign}(s)|s|^\theta$ for any unknown $\theta > 0$ satisfies Assumption 1 (also Assumption 2).

The functions $g(x)$ and $h(x)$ in (1) are not only nonlinear but also unknown. Therefore, it is impossible to construct a traditional observer to estimate system states for system (1). For example, the state matrix A of the linearized system of (3) around the origin is either a zero matrix for $p > 1$ or nonexistent for $p < 1$. In either case, we are not able to design a conventional Luenberger observer. Therefore, the problem of using output feedback to globally stabilize the uncertain system (1) is very challenging and remains largely unsolved. Motivated by the superior stability-increasing capability of the lead compensator implemented on (4), we propose a new compensator-based stabilizer to tackle the unknown system structure and unknown measurement. Consider a lead compensator

$$U(s) = \frac{s+z_1}{s+p_1} E(s),\ E(s) = R(s) - Y(s)$$

for two constants $p_1 > z_1 > 0$. For the stabilization problem where $R(s) = 0$, defining $Z(s) = \frac{-1}{s+p_1} Y(s)$, the state-space realization of the lead compensator is

$$u(t) = -y(t) - (p_1 - z_1)z(t),\ \dot{z}(t) = u(t) - z_1 z(t). \tag{7}$$

Based on (7), we design a compensator-based output feedback controller in the following theorem.

Theorem 1: Under Assumptions 1 and 2, the following compensator-based controller:

$$u = -b(y + z) \tag{8}$$

$$\dot{z} = u - c \cdot z \tag{9}$$

with two positive constants b and c, globally asymptotically stabilizes the uncertain system (1).

Proof: Defining $e = x_2 - z$, the derivative of e can be obtained as

$$\dot{e} = cx_2 - ce. \tag{10}$$

In addition, the output feedback control law (8) can be rewritten as

$$u = -b(y + z) = -bh(x_1) - bx_2 + be. \tag{11}$$

Substituting (11) into (1), together with the error dynamic (10), we have the following closed-loop system:

$$\dot{x}_1 = g(x_2)$$

$$\dot{x}_2 = -bh(x_1) - bx_2 + be$$

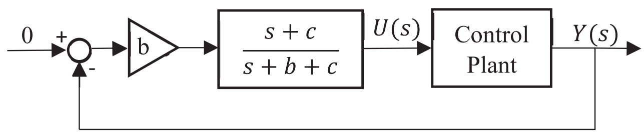

Fig. 2. Block diagram of a closed-loop system with a lead compensator.

$$\dot{e} = cx_2 - ce. \tag{12}$$

Construct a Lyapunov function

$$V(x_1, x_2, e) = b\int_0^{x_1} h(s)ds + \int_0^{x_2} g(s)ds + \frac{b}{c}\int_0^{e} g(s)ds. \tag{13}$$

Under Assumptions 1 and 2, it is straightforward to verify that the Lyapunov function V defined in (13) is positive definite and radially unbounded. Taking the derivative of the Lyapunov function (13) along the trajectories of the closed-loop system (12), we have

$$\begin{aligned}\dot{V}|_{(12)} &= bh(x_1)g(x_2) + g(x_2)(-bh(x_1) - bx_2 + be)\\ &\quad + bg(e)(x_2 - e)\\ &= -b(g(x_2) - g(e))(x_2 - e).\end{aligned} \tag{14}$$

Due to the strict monotone property of g, it can be concluded that $\dot{V}$ in (14) is seminegative definite. Moreover, $\dot{V} = 0$ implies that $x_2 = e$. By LaSalle's invariance principle [27], all the trajectories will converge to the invariant set

$$M = \{(x_1, x_2, e)|x_2 - e = 0\}.$$

Together with

$$\frac{d(x_2 - e)}{dt} = -bh(x_1) - (c + b)(x_2 - e)$$

we can conclude $x_1 = 0$ in M. Then, based on the equation $\dot{x}_1 = g(x_2)$, it is clear that $x_2 = 0$ in M. Since $x_2 - e = 0$ in M, we can consequently conclude that in the invariant set M, we have $(x_1, x_2, e) = (0, 0, 0)$. Therefore, the closed-loop system (12) is globally asymptotically stable. ∎

Remark 2: Based on the output feedback controller (8)-(9), we have $Z(s) = \frac{1}{s+c}U(s)$ and $U(s) = -bY(s) - bZ(s)$. The transfer function of dynamical compensator is $U(s) = \frac{b(s+c)}{s+c+b}(-Y(s))$, which is a lead compensator with the pole $-b - c$ and zero $-c$. The block-diagram of the system and controller is depicted in Fig. 2 .

The lead compensator is commonly used to increase system stability mainly for second-order processes in classic control theory [24], [25]. Theorem 1 shows that the lead compensator can also be used to handle the uncertain nonlinear system (1). But due to the lack of a proper higher dimensional lead compensator, it is challenging to extend Theorem 1 to the higher dimensional case.

As stated in Remark 1, the function $\text{sign}(s)|s|^\theta$ for any unknown $\theta > 0$ satisfies Assumption 1. Therefore, the motivating examples (2) and (3) are special cases of (1) satisfying Assumptions 1 and 2. Consequently, the output feedback controller (8) and (9) can also globally stabilize these two systems. The specific results for those two systems are described in the following corollary.

Corollary 1: The output feedback controller (8), (9) with positive constants b and c globally asymptotically stabilizes both system (2) and system (3).

In addition to systems (2) and (3), we can see that Theorem 1 can be used to handle the case even when the output is bounded, such as the output functions $h_1(x_1)$ and $h_2(x_1)$ described in Remark

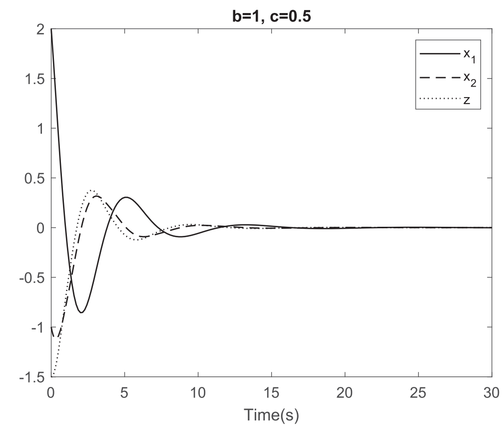

Fig. 3. State trajectories of (16) with compensator-based stabilizer (8), (9).

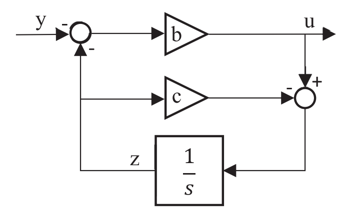

Fig. 4. Implementation of the lead compensator.

1. The boundedness of outputs is not desirable for the conventional observer-based method since two different displacements could yield the same reading, for example, $h_1(\theta) = h_1(10\theta) = \theta$. However, the compensator-based controller proposed in Theorem 1 is able to handle those bounded output functions of planar systems. For instance, by Theorem 1, it is now possible to design a compensator-based output feedback controller for the following system:

$$\dot{x}_1 = x_2, \;\; \dot{x}_2 = u, \;\; y = \frac{\theta x_1}{1 + x_1^2} \tag{15}$$

for an unknown positive constant θ.

In what follows, we use an example to demonstrate how a system with unknown structure and measurement can be globally stabilized by the proposed output feedback controller.

Example 1: Consider a rigid body plant driven by a force on a smooth surface, i.e., $\ddot{x}_1 = u$ where x_1 is the position displacement. Assume we have used a position sensor which is $y = \text{sign}(x_1)|x_1|^\theta$ with an unknown constant θ and the mass M is unknown. Then, the mathematical model of the system can be written as

$$\dot{x}_1 = Mx_2, \;\; \dot{x}_2 = u, \;\; y = \text{sign}(x_1)|x_1|^\theta \tag{16}$$

where $x_2 := \dot{x}_1/M$. The numerical simulation result for (16) with stabilizer (8), (9) is shown in Fig. 3 with $M = 2, \theta = 1.2, b = 1, c = 0.5$, and $[x_1(0), x_2(0), z(0)] = [2, -1, -1.5]$.

Remark 3: Theorem 1 holds for any parameters $b > 0$ and $c > 0$. Therefore, we have the flexibility in adjusting these two parameters to meet certain performance requirements. The controller can be implemented as shown in Fig. 4 with one integrator and two gain knobs. This is a typical lead compensator which was originally developed for linear systems. However, as stated in Theorem 1 or Corollary 1, the same compensator works perfectly for planar systems with uncertain structures and measurements. This is achieved by constructing the new

Lyapunov function (13) based on the uncertain nonlinear functions and pairing it seamlessly with the lead compensator (8), (9).

The previous result can be easily extended to the following nonlinear planar system:

$$\begin{aligned}\dot{x}_1 &= g(x_2) + f(x_1)\\ \dot{x}_2 &= u\\ y &= h(x_1)\end{aligned} \tag{17}$$

for some special functions $f(x_1)$.

Theorem 2: Under Assumptions 1 and 2, the output feedback controller (8), (9) with positive constants b and c globally asymptotically stabilizes system (17) if $h(x_1)f(x_1) \le 0, \forall x_1$.

Proof: Using the same Lyapunov function (13), we have

$$\begin{aligned}\dot{V}|_{(17)\&(8)-(9)} &= bh(x_1)f(x_1) - b(g(x_2) - g(e))(x_2 - e)\\ &\le -b(g(x_2) - g(e))(x_2 - e) \le 0.\end{aligned} \tag{18}$$

Following the same line in the proof of Theorem 1, based on (18), we can conclude that the closed-loop system (17) and (8), (9) is globally asymptotically stable. ■

Remark 4: Due to the unknown function $h(x_1)$, it is impossible to obtain x_1 from the output y. Therefore, in general $f(x_1)$ is not precisely known and cannot be used in output feedback controller design. As a result, Theorem 2 can only handle some special functions satisfying $h(x_1)f(x_1) \le 0$. Even if $f(x_1)$ appears as a matched uncertain in $\dot{x}_1 = g(x_2),\ \dot{x}_2 = u + f(x_1)$, we still cannot use the controller to cancel $f(x_1)$ unless it can be represented as a known function of y, i.e., $\tilde{f}(y) = f(x_1)$. The feasible way to handle those unknown functions is to use a domination approach, which will be discussed in the following section.

B. Special Case: Linear Systems With Unknown Parameters

Consider the following linear system:

$$\dot{x}_1 = \theta_1 x_2,\ \ \dot{x}_2 = \theta_2 u,\ \ y = \theta_0 x_1 \tag{19}$$

where θ_i, $i = 0, 1, 2$, are unknown positive constants.

If θ_i's are known, the output feedback control problem of (19) is trivial. When θ_i's are unknown, the output feedback stabilization problem of (19) is still solvable if the bounds of the unknown parameters are known. In the case when the bounds of the parameters are completely unknown, the problem becomes very challenging.[1]

In [18], even when the signs of the parameters are unknown, the output feedback control problem has been solved using a $\mathcal{K}$-filter and a dynamic gain. In this article, we show that the output feedback stabilization problem can be solved using the simple stabilizer (8), (9) for (19) with unknown positive parameters θ_0, θ_1, and θ_2.

Corollary 2: The following controller:

$$u = -y - z,\ \dot{z} = u - cz \tag{20}$$

with a constant $c > 0$, globally asymptotically stabilizes (19).

Proof: This result is a direct application of Theorem 1. In fact, system (19) has a transfer function in the form of (4) with $\theta := \theta_0\theta_1\theta_2$. Therefore, for any positive constant $c > 0$, the output feedback controller (20) globally stabilizes the linear system (19). As a result, the closed-loop system (19), (20) in the following form:

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\z\end{bmatrix} = \begin{bmatrix}0 & \theta_1 & 0\\ -\theta_0\theta_2 & 0 & -\theta_2\\ -\theta_0 & 0 & -(c+1)\end{bmatrix}\begin{bmatrix}x_1\\x_2\\z\end{bmatrix} = \mathcal{A}\begin{bmatrix}x_1\\x_2\\z\end{bmatrix} \tag{21}$$

is globally asymptotically stable for any $c > 0$. It is worth pointing out that the positiveness of the parameters can be relaxed to $\theta_0\theta_1\theta_2 > 0$. ■

Corollary 2 can be used to solve the output feedback stabilization problem of the following system with nonlinear perturbations:

$$\begin{aligned}\dot{x}_1 &= \theta_1 x_2 + f_1(x_1)\\ \dot{x}_2 &= \theta_2 u + f_2(x_1, x_2)\\ y &= \theta_0 x_1\end{aligned} \tag{22}$$

where θ_i, $i = 0, 1, 2$, are unknown positive constants and f_i's are unknown nonlinear functions.

Theorem 3: Assume the bounds of the parameters θ_i's are known and f_1 and f_2 satisfy the following linear growth conditions:

$$f_1(x_1) \le \rho_1|x_1| \tag{23}$$

$$f_2(x_1, x_2) \le \rho_2(|x_1| + |x_2|) \tag{24}$$

for two known constants ρ_1 and ρ_2. Then, there is a large enough L such that the following controller:

$$u = -L^2(y + z),\ \dot{z} = u/L - Lcz \tag{25}$$

with a given positive constant c, globally asymptotically stabilizes (22).

Proof: By defining $x_2 = L\bar{x}_2$, the closed-loop system of (22)–(25) can be rewritten as

$$\frac{d}{dt}\begin{bmatrix}x_1\\\bar{x}_2\\z\end{bmatrix} = L\mathcal{A}\begin{bmatrix}x_1\\\bar{x}_2\\z\end{bmatrix} + \begin{bmatrix}f_1(x_1)\\ \frac{f_2(x_1, L\bar{x}_2)}{L}\\ 0\end{bmatrix} \tag{26}$$

where $\mathcal{A}$ is the same as the one in (21). With the known upper and lower bounds of θ_i's, $\mathcal{A}$ is a Hurwitz matrix with known upper and lower bounds of eigenvalues. On the other hand, the growth condition (24) guarantees the following holds:

$$f_2(x_1, x_2)/L \le \rho_2(|x_1| + |L\bar{x}_2|)/L \le \rho_2(|x_1| + |\bar{x}_2|) \tag{27}$$

for $L \ge 1$. Note ρ_1 and ρ_2 are known constants in (23) and (27). Therefore, by Lyapunov Robust Theorem, when L is large enough, the system (26) is globally asymptotically stable. ■

Remark 5: By utilizing a scaling gain, we are able to use the stable linear part to dominate those nonlinearities with constant growth rates. If the growth rates ρ_1 and ρ_2 are polynomials of the output or the bounds of θ_i's are unknown, the lead compensator with a constant scaling gain will no longer work. In those cases, the possible solution is to use a dynamic gain as proposed in [15] and [16].

For faster convergence, we can design a finite-time output feedback controller for (19) even in the case when the parameters are unknown.

Theorem 4: The planar system (19) can be globally stabilized in a finite time by the following output feedback controller:

$$u = -a\left(y^{1-2\alpha} + z^{\frac{1-2\alpha}{1-\alpha}}\right) - b\left(y^{1+2\beta} + z^{\frac{1+2\beta}{1+\beta}}\right) \tag{28}$$

$$\dot{z} = u - c\left(az^{\frac{1-2\alpha}{1-\alpha}} + bz^{\frac{1+2\beta}{1+\beta}}\right) \tag{29}$$

where $\alpha \in (0, 1/2)$ and $\beta \ge 0$ are ratios of an even integer and an odd integer, and a, b, c are positive constants.

[1] Since the system matrix A, input matrix B, and output matrix C of (19) are unknown, it is impossible to construct a Luenberger observer for (19).

Proof: Construct a Lyapunov function

$$V(x_1, x_2, z) = \frac{a y^{2-2\alpha}}{(2-2\alpha)\theta_0} + \frac{b y^{2+2\beta}}{(2+2\beta)\theta_0} + \frac{\theta_1 x_2^2}{2\theta_2} + \frac{\theta_1 (x_2 - \theta_2 z)^2}{2c\theta_2} \tag{30}$$

which is obviously positive definite and radially unbounded.

Taking the derivative of V along the trajectories of closed-loop system (19) and (28), (29) yields

$$\begin{aligned}\dot{V}|_{(19)\&(28)-(29)} &= (a y^{1-2\alpha} + b y^{1+2\beta})\theta_1 x_2 - \theta_1 x_2 \Big(a y^{1-2\alpha} \\ &\quad + b y^{1+2\beta} + a z^{\frac{1-2\alpha}{1-\alpha}} + b z^{\frac{1+2\beta}{1+\beta}} \Big) \\ &\quad + \theta_1 (x_2 - \theta_2 z) \left(a z^{\frac{1-2\alpha}{1-\alpha}} + b z^{\frac{1+2\beta}{1+\beta}} \right) \\ &= -\theta_1 \theta_2 z \left(a z^{\frac{1-2\alpha}{1-\alpha}} + b z^{\frac{1+2\beta}{1+\beta}} \right) \le 0. \end{aligned} \tag{31}$$

Similar to the proof of Theorem 1, from (31) global asymptotic stability of the closed-loop system can be obtained based on LaSalle's invariance principle. In addition, by selecting the homogeneous weights $(r_1, r_2, r_3) = (1, 1-\alpha, 1-\alpha)$, we can verify that the closed-loop system

$$\begin{aligned}\dot{x}_1 &= \theta_1 x_2 \\ \dot{x}_2 &= -\theta_2 a \left((\theta_0 x_1)^{1-2\alpha} + z^{\frac{1-2\alpha}{1-\alpha}} \right) \\ &\quad - \theta_2 b \left((\theta_0 x_1)^{1+2\beta} + z^{\frac{1+2\beta}{1+\beta}} \right) \\ \dot{z} &= -a(\theta_0 x_1)^{1-2\alpha} - (a + ca) z^{\frac{1-2\alpha}{1-\alpha}} \\ &\quad - b(\theta_0 x_1)^{1+2\beta} - (b + bc) z^{\frac{1+2\beta}{1+\beta}} \end{aligned} \tag{32}$$

is locally homogeneous with a negative degree $-\alpha$ [28]. Therefore, by [28, Lemma 3] the closed-loop system (32) is globally asymptotically stable and locally finite-time stable. ■

Remark 6: Theorem 4 can be extended to the nonlinear system (22) to achieve finite-time stabilization if the nonlinearities satisfy the following growth conditions similar to those in [29]:

$$f_1(x_1) \le \rho_1 |x_1|^{1-\alpha}$$

$$f_2(x_1, x_2) \le \rho_2 \left(|x_1|^{1-2\alpha} + |x_2|^{\frac{1-2\alpha}{1-\alpha}} \right)$$

for three positive constants $\alpha \in (0, 1/2)$, ρ_1, and ρ_2.

Example 2: For system (19), we choose $\theta_0 = 1.2$, $\theta_1 = 2$, $\theta_2 = 1$ in the simulation study. For demonstration, we choose different parameters in controller (28), (29) to compare different control performances.

First, we choose $a = 0$, $b = 1$, $c = 0.5$, $\alpha = 2/7$ and $\beta = 0$, which results in the following linear controller:

$$u = -y - z, \; \dot{z} = u - 0.5z. \tag{33}$$

The simulation result shown in Fig. 5 is conducted for the initial conditions $[x_1(0), x_2(0), z(0)] = [2 \; -1 \; -1.5]$.

Then, we add the lower order term by choosing $a = 1$, $b = 1$, $c = 0.5$, $\alpha = 2/7$, and $\beta = 0$, which gives

$$u = -y^{3/7} - y - z^{3/5} - z, \; \dot{z} = u - 0.5z^{3/5} - 0.5z. \tag{34}$$

The simulation result shown in Fig. 6 is conducted for the same initial conditions $[x_1(0), x_2(0), z(0)] = [2 \; -1 \; -1.5]$. It is clear that with the lower order term, the trajectories of the closed-loop system converge to zero in a shorter time.

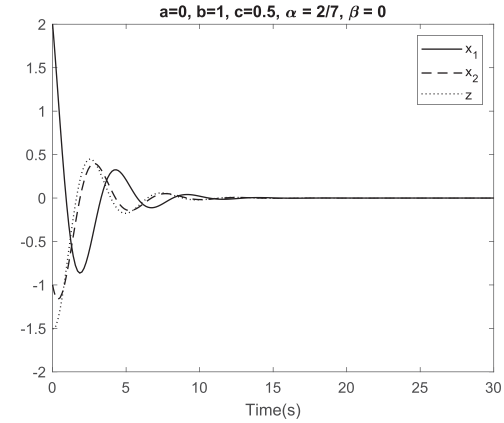

Fig. 5. State trajectories of (19) with controller (33).

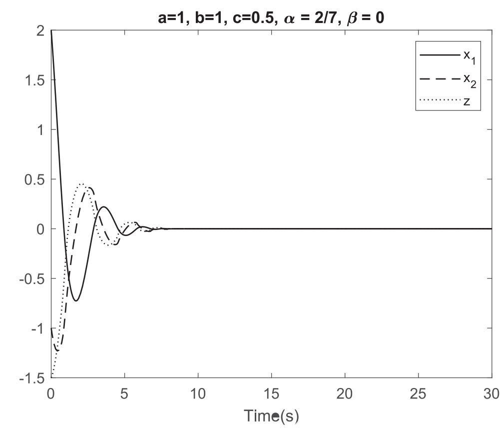

Fig. 6. State trajectories of (19) with controller (34).

III. Conclusion

This article has shown that the lead compensator commonly used in classic control theory can be utilized to solve the global output feedback stabilization problem for a class of nonlinear planar systems with unknown structures and measurements. Several variants of compensator-based controllers have been showcased for different kind of planar systems and for different performances. Although the compensator-based method has demonstrated effectiveness in the planar case, the problem of how to apply this method to higher dimensional systems is still open and worth further investigation.

References

[1] W.-S. Lu, *Two-Dimensional Digital Filters*, vol. 80. Boca Raton, FL, USA: CRC Press, 1992.

[2] Y. B. Shtessel, I. A. Shkolnikov, and A. Levant, "Smooth second-order sliding modes: Missile guidance application," *Automatica*, vol. 43, no. 8, pp. 1470–1476, 2007.

[3] N. Yeganefar, N. Yeganefar, M. Ghamgui, and E. Moulay, "Lyapunov theory for 2-D nonlinear roesser models: Application to asymptotic and exponential stability," *IEEE Trans. Autom. Control*, vol. 58, no. 5, pp. 1299–1304, May 2013.

[4] X.-H. Xia and W.-b. Gao, "On exponential observers for nonlinear systems," *Syst. Control Lett.*, vol. 11, no. 4, pp. 319–325, 1988.

[5] V. Andrieu and L. Praly, "A unifying point of view on output feedback designs for global asymptotic stabilization," *Automatica*, vol. 45, no. 8, pp. 1789–1798, 2009.

[6] H. Du, C. Qian, S. Yang, and S. Li, "Recursive design of finite-time convergent observers for a class of time-varying nonlinear systems," *Automatica*, vol. 49, no. 2, pp. 601–609, 2013.

[7] F. Mazenc, L. Praly, and W. Dayawansa, “Global stabilization by output feedback: Examples and counterexamples,” *Syst. Control Lett.*, vol. 23, no. 2, pp. 119–125, 1994.
[8] J. J. Carr, *Sensors and Circuits: Sensors, Transducers, and Supporting Circuits for Electronic Instrumentation, Measurement, and Control*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1993.
[9] A. A. Prasov and H. K. Khalil, “A nonlinear high-gain observer for systems with measurement noise in a feedback control framework,” *IEEE Trans. Autom. Control*, vol. 58, no. 3, pp. 569–580, Mar. 2013.
[10] H. Wang and Q. Zhu, “Adaptive output feedback control of stochastic nonholonomic systems with nonlinear parameterization,” *Automatica*, vol. 98, pp. 247–255, 2018.
[11] Q. Zhu and H. Wang, “Output feedback stabilization of stochastic feedforward systems with unknown control coefficients and unknown output function,” *Automatica*, vol. 87, pp. 166–175, 2018.
[12] J. Zhai and C. Qian, “Global control of nonlinear systems with uncertain output function using homogeneous domination approach,” *Int. J. Robust Nonlinear Control*, vol. 22, no. 14, pp. 1543–1561, 2012.
[13] C. Qian, “A homogeneous domination approach for global output feedback stabilization of a class of nonlinear systems,” in *Proc. IEEE Amer. Control Conf.*, 2005, pp. 4708–4715.
[14] C.-C. Chen, C. Qian, Z.-Y. Sun, and Y.-W. Liang, “Global output feedback stabilization of a class of nonlinear systems with unknown measurement sensitivity,” *IEEE Trans. Autom. control*, vol. 63, no. 7, pp. 2212–2217, Jul. 2018.
[15] X. Zhang and W. Lin, “Nonidentifier-based adaptive control for nonlinearly parameterized systems with measurement uncertainty,” *Int. J. Robust Nonlinear Control*, vol. 30, no. 8, pp. 3055–3072, 2020.
[16] X. Zhang and W. Lin, “Robust output feedback control of polynomial growth nonlinear systems with measurement uncertainty,” *Int. J. Robust Nonlinear Control*, vol. 29, no. 13, pp. 4562–4576, 2019.
[17] G. Kreisselmeier, “Adaptive observers with exponential rate of convergence,” *IEEE Trans. Autom. Control*, vol. 22, no. 1, pp. 2–8, Feb. 1977.
[18] X. Zhang and W. Lin, “A k-filter-based adaptive control for nonlinear systems with unknown parameters in state and output equations,” *Automatica*, vol. 105, pp. 186–197, 2019.
[19] J.-C. Zufferey, “Application Note for an Infrared, Triangultation-Based Distance Sensor With an Analog, Non-Linear Output.” 2004, [Online]. Available: http://zuff.info/SharpGP2D12_E.html
[20] W. Zha, C. Qian, J. Zhai, and S. Fei, “Robust control for a class of nonlinear systems with unknown measurement drifts,” *Automatica*, vol. 71, pp. 33–37, 2016.
[21] J.-Z. Liu, S. Yan, D.-L. Zeng, Y. Hu, and Y. Lv, “A dynamic model used for controller design of a coal fired once-through boiler-turbine unit,” *Energy*, vol. 93, pp. 2069–2078, 2015.
[22] Z. Su, C. Qian, and J. Shen, “Interval homogeneity-based control for a class of nonlinear systems with unknown power drifts,” *IEEE Trans. Autom. Control*, vol. 62, no. 3, pp. 1445–1450, Mar. 2017.
[23] C.-C. Chen, C. Qian, X. Lin, Z.-Y. Sun, and Y.-W. Liang, “Smooth output feedback stabilization for a class of nonlinear systems with time-varying powers,” *Int. J. Robust Nonlinear Control*, vol. 27, no. 18, pp. 5113–5128, 2017.
[24] L. Shaw, “Pole placement: Stability and sensitivity of dynamic compensators,” *IEEE Trans. Autom. Control*, vol. AC-16, no. 2, pp. 210–210, Apr. 1971.
[25] H. Seraji, “Pole assignment using dynamic compensators with prespecified poles,” *Int. J. Control*, vol. 22, no. 2, pp. 271–279, 1975.
[26] A. Hurwitz, “Ueber die bedingungen, unter welchen eine gleichung nur wurzeln mit negativen reellen theilen besitzt,” *Mathematische Annalen*, vol. 46, no. 2, pp. 273–284, 1895.
[27] J. LaSalle, “Some extensions of liapunov’s second method,” *IRE Trans. Circuit Theory*, vol. CTT-7, no. 4, pp. 520–527, 1960.
[28] Y. Hong, J. Huang, and Y. Xu, “On an output feedback finite-time stabilization problem,” *IEEE Trans. Autom. Control*, vol. 46, no. 2, pp. 305–309, Feb. 2001.
[29] H. Wang and Q. Zhu, “Finite-time stabilization of high-order stochastic nonlinear systems in strict-feedback form,” *Automatica*, vol. 54, pp. 284–291, 2015.