

# Compensator-based current sensorless control for PWM-based DC-DC buck converter system with uncertain voltage measurement

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**Abstract**—This paper investigates the output feedback control problem for a PWM-based DC-DC buck converter system with uncertain voltage measurement. Due to the uncertain output measurement, the conventional state observers are not feasible. Inspired by the idea of a lead compensator in classic linear control theory, we propose a compensator-based current sensorless controller by designing a novel dynamic estimator which only uses the uncertain voltage measurement. A Lyapunov-based stability analysis is provided and the numerical simulation studies demonstrate the effectiveness of the proposed method.

**Index Terms**—current sensorless control, buck converter, compensator-based control, voltage regulation

## I. INTRODUCTION

PWM-based DC-DC buck power converter, as one of the most important kind of power supply technologies, steps down voltage from its supply input to its output. As Fig. 1. (a) shows, a PWM-based DC-DC buck power converter typically contains at least two semiconductors (a diode and a transistor) and at least two energy storage elements (a capacitor and an inductor). Compared with linear voltage regulators, switching-mode buck converters inherently dissipate much less power and have high reliability. Hence, it is widely used in industrial systems, such as communication systems [1], renewable energy systems [2], [3], motor driving systems [4], [5] and computer systems, etc. Since modern electronic systems require high-quality, reliable and efficient power supplies, the control algorithms of DC-DC buck converter systems have been widely investigated [6], [7], [8], [9]. However, the dynamics of parameters like magnetic characteristics and the uncertainties of sensor measurements in the DC-DC converter systems increase instabilities and uncertainties of the system voltage output.

Aiming to improve the voltage regulation precision, system dynamic response and robustness, different control approaches have been proposed by researchers. Backstepping control as an efficient technique for regulation is applied in DC-DC converters [10], [11], and has better performance than PI controller.

Adaptive control [12], [13] can also solve the problem in buck converter but requires heavy calculations and significant hardware resources. Conventional sliding mode control causes steady state error which is improved in [14], [15], [16] [17], but the region of existence of sliding mode needs specific parameters tuning. The implementation of optimal control in [18], [19] provides a closed-loop system with desirable dynamical properties and is capable of withstanding voltage disturbance, but the modeling of the system and designing of the controller requests algorithms and extensive computation.

Among the aforementioned control approaches, there is an interesting category named current sensorless control. Without current feedback, a current sensorless controller uses only voltage measurement to regulate the converter systems. The applications of current sensorless control decrease the size and cost of the converter systems and avoid the potential negative impacts of the current sensors such as measurement delay when the converter is operated at high switching frequencies [20]. The current sensorless control has been extensively applied in many power electronic systems. For example, it is implemented with a single loop and multiple loops for boost converters in [21], [22], and [23] and achieves remarkable steady-state and transient performances. In [24] and [20], the current sensorless control is used in half-bridge and full-bridge system. The current sensorless control approaches normally adopt some observation/estimation techniques to estimate the current value or its related system state, whose performances are largely rely on the effectiveness of their estimation principles. For example, in [6] the authors proposed an reduced-order observer to estimate the system state only by using the voltage measurement. However, these conventional observation/estimation techniques fail when the output measurement is uncertain.

In this note, we focus on designing a current sensorless controller for a PWM-based DC-DC buck converter subject to uncertain voltage measurement by introducing a novel estimation method. Motivated by the idea of lead compensator controllers which are used to stabilize linear systems [25], [26], and to improve the stability of systems with unknown

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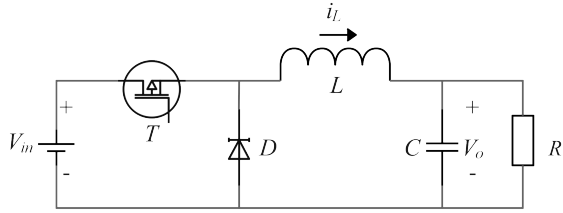
output coefficients, we propose a novel compensator-based dynamic estimator. With the proposed estimator, we can use the uncertain voltage measurement to regulate the DC-DC buck converter without using any additional electronic components. We will also give a rigorous Lyapunov based stability analysis to guarantee the effectiveness of our controller.

The rest of the paper is organized as follows. In section II, we give four subsections: The first subsection describes the physical and mathematical model of a PWM-based DC-DC buck converter. The second subsection introduces the lead compensator. Then we give our main theorem and stability analysis in the third subsection. And the last subsection is dedicated to simulation study. Finally, section III draws the conclusion.

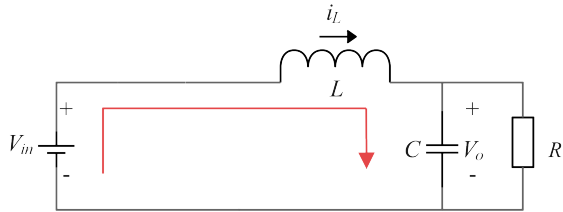
## II. MAIN RESULTS

### A. Model description of PWM-based DC-DC buck converter

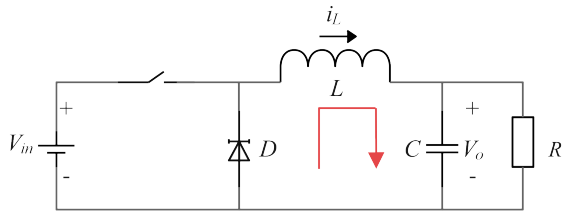
Fig.1 shows the circuit of a typical PWM-based DC-DC buck converter and the structure of its on/off mode. Here  $V_{in}$  is the input voltage source,  $T$  is the switching device,  $D$  is the diode,  $L$  is the inductor,  $C$  is the capacitor,  $R$  is the resistor and  $V_o$  is the output voltage. The PWM signal  $\mu \in [0, 1]$  controls the switching device to work in ON/OFF status. When the switching device is on, the current flow to



(a) Circuit of a DC-DC buck converter



(b) Switching device ON



(c) Switching device OFF

Fig. 1. Average model circuit of a PWM-based DC-DC buck converter.

the output capacitor and resistor. The physical model can be described as

$$\begin{aligned} V_{in} &= L \frac{di_L}{dt} + V_o, \\ C \frac{dV_o}{dt} &= i_L - \frac{V_o}{R}. \end{aligned} \quad (1)$$

When the switching device is off, the inductor creates a voltage across it since the current in an inductor cannot change suddenly. This voltage is allowed to charge the capacitor and power the load through the diode when the switch is turned off, maintaining output current throughout the switching cycle. The physical model is represented by

$$\begin{aligned} L \frac{di_L}{dt} &= -V_o, \\ C \frac{dV_o}{dt} &= i_L - \frac{V_o}{R}. \end{aligned} \quad (2)$$

According to (1) and (2), the average model of a PWM-based DC-DC buck converter can be described as

$$\begin{aligned} \frac{di_L}{dt} &= \frac{1}{L} \mu V_{in} - \frac{1}{L} V_o, \\ \frac{dV_o}{dt} &= \frac{1}{C} i_L - \frac{1}{RC} V_o. \end{aligned} \quad (3)$$

Let  $x_1 = V_o - V_{ref}$  where  $V_{ref}$  is the reference output voltage, and  $x_2 = \frac{1}{C} i_L - \frac{1}{RC} V_o$ . Then the average model (3) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= u + f_2, \end{aligned} \quad (4)$$

where  $f_2 = -\frac{1}{LC} x_1 - \frac{1}{RC} x_2$  and the control input  $u$  is denoted as  $u = \frac{\mu V_{in} - V_{ref}}{LC}$ . It's easy to verify that  $f_2$  satisfies linear growth condition

$$|f_2| \leq c_1(|x_1| + |x_2|), \quad (5)$$

where  $c_1$  is a constant related to system parameters  $R, L, C$ .

Assume that the voltage sensor has a drift such that the output  $y = \delta x_1$  with an unknown positive constant  $\delta$ .

### B. The lead compensator

In classic control theory, the lead compensator has been known for its capability of increasing system stability [25], [26]. We also discover that the lead compensator can increase stability of some systems with unknown output coefficients. Consider a double-integrator system  $\ddot{x} = u$  with the output  $y = \theta x$  for an unknown positive constant  $\theta$ . The transfer function of this system is described by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\theta}{s^2}. \quad (6)$$

We want to see if we can design a stabilizing output feedback controller for the transfer function (6) without identifying the unknown constant  $\theta$ . If we use a PID controller, the denominator of the closed-loop system contains  $1 + \frac{K_I + K_P s + K_D s^2}{s} G(s)$ , which implies that the poles of the closed-loop system are the roots of

$$s^3 + \theta k_1 s^2 + \theta k_2 s + \theta k_3 = 0. \quad (7)$$

According to Routh-Hurwitz criterion [27], the third order polynomial  $s^3 + a_2s^2 + a_1s + a_0 = 0$  has all roots in the open left half plane if and only if  $a_2, a_0$  are positive and  $a_2a_1 > a_0$ . Therefore, to ensure stability of the closed-loop system, the PID control coefficients need to satisfy  $\theta k_1 k_2 > k_3$ , from which  $k_1, k_2$  and  $k_3$  cannot be determined unless we know the bound of  $\theta$ . When we apply a first-order compensator  $\frac{s+z_1}{s+p_1}$  to system (6), as shown in Fig. 2,

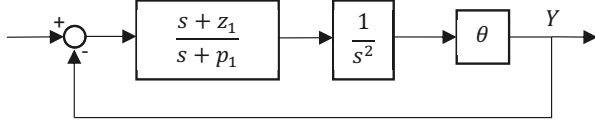


Fig. 2. Block diagram of system (6) under a first-order compensator.

the characteristic equation of the closed-loop system is

$$s^3 + p_1s^2 + \theta s + \theta z_1 = 0. \quad (8)$$

It is clear that (8) is Hurwitz stable as long as  $p_1 > z_1 > 0$ , which implies that  $\frac{s+z_1}{s+p_1}$  is a lead compensator. Here we have just shown that a lead compensator can stabilize system (6) even when the measurement is not precisely known.

In this note, we will adopt the same idea of using a lead compensator to design a stabilizing output feedback controller for system (4). By taking advantage of the stability-increasing capability of the lead compensator, a global stabilizer can be designed to solve the output feedback control problem for system (4), even in the presence of unknown measurement. Consider a lead compensator

$$U(s) = \frac{s+z_1}{s+p_1} E(s), \quad E(s) = R(s) - Y(s)$$

for two constants  $p_1 > z_1 > 0$ . For the stabilization problem where  $R(s) = 0$ , defining  $Z(s) = \frac{-1}{s+p_1} Y(s)$ , the state-space realization of the lead compensator is

$$u(t) = -y(t) - (p_1 - z_1)z(t), \quad \dot{z}(t) = u(t) - z_1z(t). \quad (9)$$

### C. Compensator-based current sensorless controller design

In this subsection, we present a new design method which is inspired by the lead compensator controller for linear systems. Based on (9), we design a compensator-based current sensorless controller in the following theorem. Under Assumption II-A, the following compensator-based current sensorless controller

$$u = -\lambda^2(y + z), \quad (10)$$

$$\dot{z} = \frac{u}{\lambda} - \lambda k z, \quad (11)$$

with a positive constant  $k$  and a large enough positive constant  $\lambda$ , globally asymptotically stabilizes the system (4).

*Proof:* Substituting controller (10) into system (4) and together with (11), we have the following closed-loop system

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\lambda^2 \delta x_1 - \lambda^2 z + f_2, \\ \dot{z} &= -\lambda \delta x_1 - \lambda(k+1)z. \end{aligned} \quad (12)$$

With a coordinate transformation  $\zeta_1 = x_1, \zeta_2 = x_2/\lambda, \zeta_3 = z$ , the closed-loop system (12) can be rewritten as

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix} &= \begin{bmatrix} 0 & \lambda & 0 \\ -\lambda \delta & 0 & -\lambda \\ -\lambda \delta & 0 & -\lambda(k+1) \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix} + \begin{bmatrix} 0 \\ f_2/\lambda \\ 0 \end{bmatrix} \\ &\triangleq \lambda \mathcal{A} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix} + \phi, \end{aligned} \quad (13)$$

where the system matrix  $\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 \\ -\delta & 0 & -1 \\ -\delta & 0 & -(k+1) \end{bmatrix}$  and the

vector  $\phi = \begin{bmatrix} 0 \\ f_2/\lambda \\ 0 \end{bmatrix}$ . To compute the eigenvalues of the system matrix  $\mathcal{A}$ , we obtain the following characteristic polynomial

$$s^3 + (k+1)s^2 + \delta s + \delta c = 0.$$

By Routh-Hurwitz criterion, it is easy to verify that the eigenvalues of  $\mathcal{A}$  are in the left half plane since  $\delta$  and  $k$  are both positive. Thus, there exist a symmetric positive definite matrix  $P$  such that

$$\mathcal{A}^T P + P \mathcal{A} = -I_{3 \times 3}. \quad (14)$$

Choose a positive definite and proper Laypunov function  $V = \zeta^T P \zeta$ , then taking the derivative along the closed-loop system (13) we have

$$\begin{aligned} \dot{V}|_{(13)} &= \dot{\zeta}^T P \zeta + \zeta^T P \dot{\zeta}, \\ &= \lambda \zeta^T (\mathcal{A}^T P + P \mathcal{A}) \zeta + \phi^T P \zeta + \zeta^T P \phi, \\ &\leq -\lambda \|\zeta\|^2 + 2\|\zeta\| \|P\| \|\phi\|. \end{aligned} \quad (15)$$

Based on (5), it's easy to obtain that

$$\|\phi\| \leq c_2 \|\zeta\|,$$

where  $c_2$  is a constant related to system parameters  $R, L, C$ . Thus, (15) can be rewritten as

$$\dot{V}|_{(13)} \leq -(\lambda - 2c_2 \|P\|) \|\zeta\|^2. \quad (16)$$

Choose a large enough  $\lambda$  such that  $\lambda - 2c_2 \|P\| > 0$  and it can be guaranteed that  $\dot{V}$  is negative definite. Therefore, the closed-loop system (13) is globally asymptotically stable.  $\zeta_1 = 0 \rightarrow x_1 = 0$ , which implies the output voltage  $V_o$  tracks the reference voltage  $V_{ref}$ .

### D. Simulation

The control framework of a PWM-based DC-DC buck converter under controller (10)-(11) is shown in Fig. 3. In this subsection, numerical simulations are studied to verify the effectiveness of the proposed control method. The involved components values are selected as shown in TABLE I. In the simulation, the parameters are selected as  $\delta = 0.8, \lambda = 25$ , and  $k = 2$ . Fig. 4 and 5 show the response curves of the output voltage and the inductor current. It can be observed that the output voltage tracks the reference value in a short settling time. Fig. 6 shows the duty cycle of the PWM control signal.

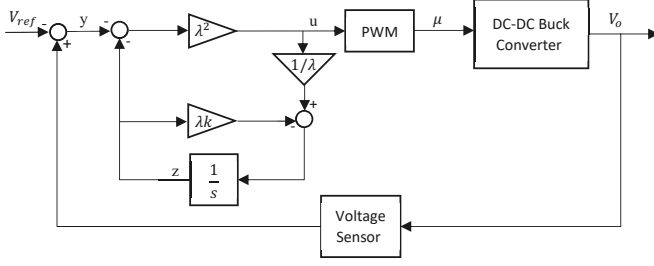


Fig. 3. Control framework of PWM-based DC-DC buck converter.

TABLE I  
COMPONENT VALUES OF DC-DC BUCK CONVERTER.

Descriptions	Parameters	values
Input voltage	$V_{in}$	30(V)
Reference output voltage	$V_{ref}$	15(V)
Inductance	$L$	3.3(mH)
Capacitance	$C$	1000( $\mu F$ )
Load resistance	$R$	100( $\Omega$ )

### III. CONCLUSION

This paper studies the problem of current sensorless control for PWM-based DC-DC buck converter system with uncertain voltage measurement. By taking advantage of the stability-increasing capability of a lead compensator, we proposed a dynamic output feedback controller to regulate the output voltage of a DC-DC buck converter with uncertain voltage measurement. The numerical simulation studies has demonstrated the effectiveness of the proposed method.

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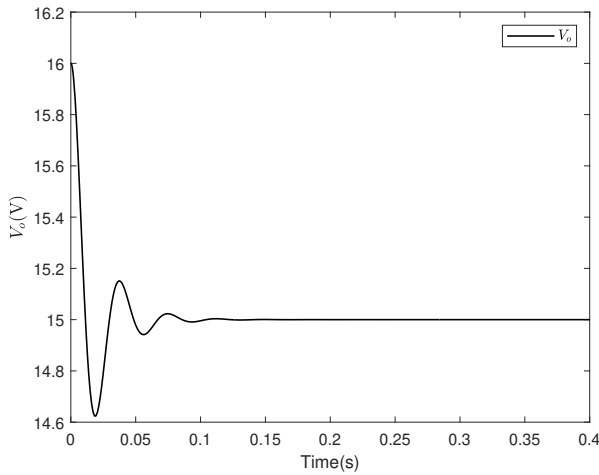


Fig. 4. Response curves of the output voltage.

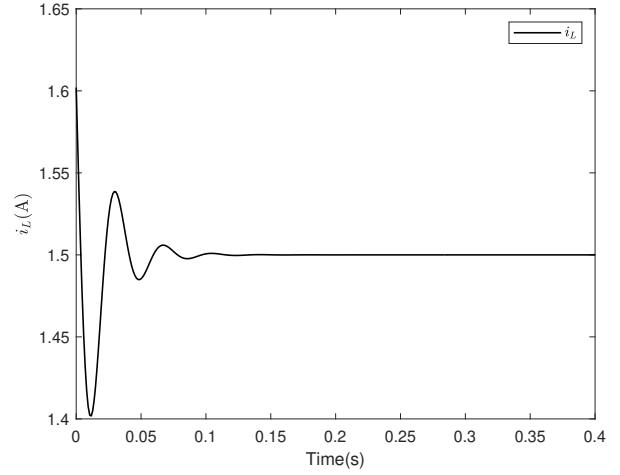


Fig. 5. Response curves of the Inductor current.

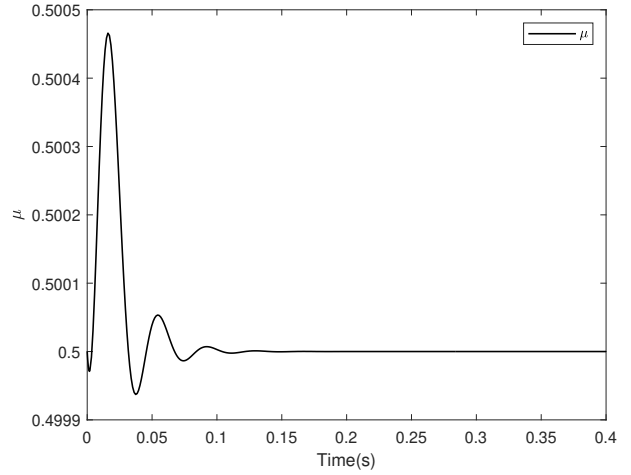


Fig. 6. Control signal - PWM duty cycle.

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