Convexifying Market Clearing of SoC-Dependent Bids From Merchant Storage Participants

Cong Chen, *Student Member, IEEE*, and Lang Tong, *Fellow, IEEE*

***Abstract*—State-of-charge (SoC) dependent bidding allows merchant storage participants to incorporate SoC-dependent operation and opportunity costs in a bid-based market clearing process. However, such a bid results in a non-convex cost function in the multi-interval economic dispatch and market clearing, limiting its implementation in practice. We show that a simple restriction on the bidding format removes the non-convexity, making the multi-interval dispatch of SoC-dependent bids a standard convex piece-wise linear program.**

***Index Terms*—Multi-interval economic dispatch, SoC dependent bid, convexification.**

I. Introduction

RECENT proposals [1] have allowed merchant storage participants in the wholesale electricity market to submit state-of-charge (SoC) dependent offers and bids to capture more accurately the operation and opportunity costs of the energy storage [2], [3], [4]. With such bids, an economic dispatch program tends to schedule the battery SoC within a range favorable to the battery's health and the storage's ability to capture future opportunities under uncertainty.

However, a multi-interval economic dispatch with SoC-dependent bids involves integer variables [5], making the market clearing process computationally expensive for practical implementations. The nonconvexity of SoC-dependent bids also brings pricing challenges and the need for out-of-the-market uplift payments.

In this paper, we propose a simple restriction to the SoC-dependent bidding, referred to as the equal decremental-cost ratio (EDCR) condition, that transforms the nonconvex economic dispatch optimization into a convex piece-wise linear program compatible with the standard market clearing process. A procedure to produce bids satisfying the EDCR condition from the true bid-in cost functions is also proposed.

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The authors are with the Cornell University, Ithaca, NY 14853 USA (e-mail: cc2662@cornell.edu; lt35@cornell.edu).

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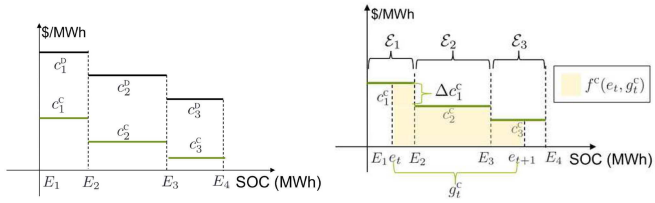

Fig. 1. Left: The SoC-dependent bid and offer format when $K = 3$. Right: Cost of charging the storage by g_t^{C} from e_t to e_{t+1}.

II. SoC-Dependent Bid and Dispatch Models

A. Storage and SoC-Dependent Cost Models

We assume the standard imperfect storage model. In the scheduling interval t, let e_t be the storage SoC, g_t^{C} the charging power, and g_t^{D} the discharging power, respectively. The storage SoC evolves according to

$$e_{t+1} = e_t + g_t^{\mathrm{C}}\eta^{\mathrm{C}} - g_t^{\mathrm{D}}/\eta^{\mathrm{D}},\ g_t^{\mathrm{C}} g_t^{\mathrm{D}} = 0, \tag{1}$$

where $\eta^{\mathrm{C}}, \eta^{\mathrm{D}} \in (0, 1]$ are charging/discharging efficiencies.

A standard piecewise-linear SoC-dependent bid model [1] is illustrated in Fig. 1 (left). Without loss of generality, we partition the SoC axis into K consecutive segments. Within each segment $\mathcal{E}_k := [E_k, E_{k+1}]$, a pair of bid-in *marginal cost/benefit* parameters $(c_k^{\mathrm{C}}, c_k^{\mathrm{D}})$ is defined. The marginal discharging (bid-in) costs (to the grid) $b^{\mathrm{D}}(e_t; \mathbf{c}^{\mathrm{D}}, \mathbf{E})$ and marginal charging (bid-in) benefits (from the grid) $b^{\mathrm{C}}(e_t; \mathbf{c}^{\mathrm{C}}, \mathbf{E})$ are functions of battery SoC e_t. In particular, using the indicator function[1] $\mathbb{1}$,

$$\begin{cases} b^{\mathrm{C}}\left(e_t; \mathbf{c}^{\mathrm{C}}, \mathbf{E}\right) := \sum_{k=1}^{K} c_k^{\mathrm{C}} \mathbb{1}_{\{e_t \in \mathcal{E}_k\}} \\ b^{\mathrm{D}}\left(e_t; \mathbf{c}^{\mathrm{D}}, \mathbf{E}\right) := \sum_{k=1}^{K} c_k^{\mathrm{D}} \mathbb{1}_{\{e_t \in \mathcal{E}_k\}} \end{cases} \tag{2}$$

with $\mathbf{c}^{\mathrm{C}} := (c_k^{\mathrm{C}})$, $\mathbf{c}^{\mathrm{D}} := (c_k^{\mathrm{D}})$ and $\mathbf{E} := (E_k)$ as parameters.

For the longevity of the battery and the ability to capture profit opportunities, it is more costly to discharge when the SoC is low, and the benefit of charging is small when the SoC is high. Therefore, typical bid-in discharge costs (c_k^{D}) and charging benefits (c_k^{C}) are monotonically decreasing. Furthermore, the storage participant is willing to discharge only if the selling price is higher than the buying price. Hence, the storage participant's willingness to sell by discharge (adjusted to the discharging efficiency) must be higher than its willingness to purchase (adjusted to the charging efficiency), *i.e.*, $c_{\mathrm{K}}^{\mathrm{D}}\eta^{\mathrm{D}} > c_1^{\mathrm{C}}/\eta^{\mathrm{C}}$. Together, SoC-dependent bids and offers satisfy the following.

Assumption 1: The SoC-dependent cost/benefit parameters $\{(c_k^{\mathrm{C}}, c_k^{\mathrm{D}}), \eta^{\mathrm{C}}, \eta^{\mathrm{D}}\}$ satisfy the following monotonicity conditions

[1] $\mathbb{1}_{\{s \in \mathcal{E}_i\}}$ equals to 1 when $s \in \mathcal{E}_i$.

$\forall k = 1, \cdots, K-1$:

$$\begin{cases} c_k^{\mathrm{C}} \geq c_{k+1}^{\mathrm{C}} \\ c_k^{\mathrm{D}} \geq c_{k+1}^{\mathrm{D}} \end{cases} \text{ and } c_1^{\mathrm{C}}/\eta^{\mathrm{C}} < c_{\mathrm{K}}^{\mathrm{D}}\eta^{\mathrm{D}}.$$

B. Cost Function of SoC-Dependent Bids

SoC-dependent bids and offers induce SoC-dependent scheduling costs involving the (*ex ante*) SoC e_t in scheduling stage t before the dispatch and the (*ex post*) SoC e_{t+1} after the dispatch that may be in a different SoC partitioned segment. Specifically, the stage cost $f(g_t^{\mathrm{C}}, g_t^{\mathrm{D}}, e_t)$ in interval t is given by[2]

$$f\left(g_t^{\mathrm{C}}, g_t^{\mathrm{D}}, e_t\right) := f^{\mathrm{D}}\left(e_t, g_t^{\mathrm{D}}\right) - f^{\mathrm{C}}(e_t, g_t^{\mathrm{C}}), \tag{3}$$

where f^{D} is the discharging cost, and f^{C} is the charging benefit. In particular, for every $e_t \in \mathcal{E}_m$ and $e_{t+1} \in \mathcal{E}_n$,

$$f^{\mathrm{C}}\left(e_t, g_t^{\mathrm{C}}\right) := \mathbb{1}_{\{n \geq m\}} g_t^{\mathrm{C}} c_n^{\mathrm{C}} + \mathbb{1}_{\{n > m\}} \sum_{k=m}^{n-1} \frac{\Delta c_k^{\mathrm{C}}}{\eta^{\mathrm{C}}} \left(E_{k+1} - e_t\right),$$

$$f^{\mathrm{D}}\left(e_t, g_t^{\mathrm{D}}\right) := \mathbb{1}_{\{n \leq m\}} g_t^{\mathrm{D}} c_n^{\mathrm{D}} + \mathbb{1}_{\{n < m\}} \sum_{k=n+1}^{m} \eta^{\mathrm{D}} \Delta c_{k-1}^{\mathrm{D}} (E_k - e_t),$$

with $\Delta c_k^{\mathrm{C}} := c_k^{\mathrm{C}} - c_{k+1}^{\mathrm{C}}$ and $\Delta c_k^{\mathrm{D}} := c_k^{\mathrm{D}} - c_{k+1}^{\mathrm{D}}$. Fig. 1 (right) illustrates $f^{\mathrm{C}}\left(e_t, g_t^{\mathrm{C}}\right)$ in an example with $K = 3, m = 1$, and $n = 3$. Note that the stage cost $f\left(g_t^{\mathrm{C}}, g_t^{\mathrm{D}}, e_t\right)$ is nonconvex, although it is convex if given e_t.

C. The Multi-Interval Economic Dispatch

We consider a multi-interval dispatch model involving T intervals and M buses. In decision interval t, let g_{it}^{C} and g_{it}^{D} be the charging and discharging decision variables, respectively, and let e_{it} be the SoC of unit i. With the single stage cost in (3), the T-interval operation cost of storage i is given by

$$F_i\left(\boldsymbol{g}_i^{\mathrm{C}}, \boldsymbol{g}_i^{\mathrm{D}}; s_i\right) := \sum_{t=1}^{T} f_i\left(g_{it}^{\mathrm{C}}, g_{it}^{\mathrm{D}}, e_{it}\right), \tag{4}$$

where $\boldsymbol{g}_i^{\mathrm{C}}, \boldsymbol{g}_i^{\mathrm{D}} \in \mathbb{R}^T$ denote the vector of charging and discharging power for storage i over T-interval, respectively.

For the interval t, let d_{it} be the demand at bus i, and we define $\boldsymbol{d}[t] := (d_{1t}, \cdots, d_{Mt})$ as the demand vector for all buses. Let $\boldsymbol{g}^{\mathrm{G}}[t] := (g_{1t}^{\mathrm{G}}, \cdots, g_{Mt}^{\mathrm{G}})$ be the vector of bus generations. Similarly defined are $\boldsymbol{g}^{\mathrm{D}}[t]$ and $\boldsymbol{g}^{\mathrm{C}}[t]$ as the vector of charging and discharging power of the battery storage, respectively. For simplicity, we establish the dispatch model with one generator and one storage at each bus, which is extendable to general cases. Given the convex generator cost $f_i^{\mathrm{G}}(g_{it}^{\mathrm{G}})$, the initial SoC $e_{i1} = s_i$, and the load forecast $(\boldsymbol{d}[t])$ over the T-interval scheduling horizon, the economic dispatch minimizes the system operation costs is given by

$$\begin{aligned} \underset{\{(g_{it}^{\mathrm{G}}, g_{it}^{\mathrm{C}}, g_{it}^{\mathrm{D}}, e_{it})\}}{\text{minimize}} \quad & \sum_{i=1}^{M} \left(F_i\left(\boldsymbol{g}_i^{\mathrm{C}}, \boldsymbol{g}_i^{\mathrm{D}}; s_i\right) + \sum_{t=1}^{T} f_i^{\mathrm{G}}\left(g_{it}^{\mathrm{G}}\right) \right) \\ \text{subject to} \quad & \forall t \in \{1, ..., T\}, \forall i \in \{1, ..., M\} \\ \boldsymbol{\mu}[t]: \quad & \boldsymbol{S}\left(\boldsymbol{g}^{\mathrm{G}}[t] + \boldsymbol{g}^{\mathrm{D}}[t] - \boldsymbol{g}^{\mathrm{C}}[t] - \boldsymbol{d}[t]\right) \leq \boldsymbol{q} \end{aligned}$$

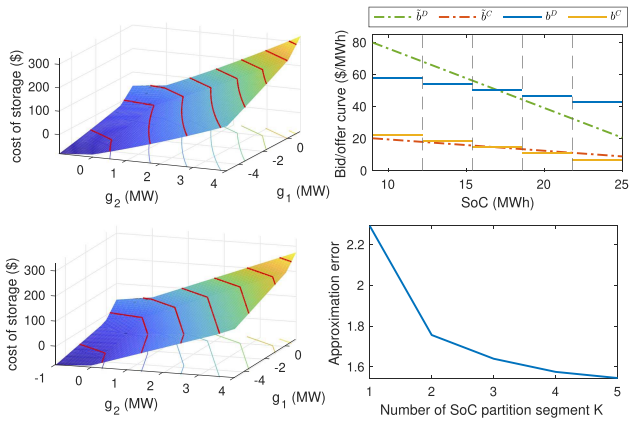

Fig. 2. Top left: nonconvex true storage cost in 2-interval. Top right: true SoC-dependent marginal cost and optimal EDCR approximation bids ($K = 5$). Bottom left: optimal EDCR approximation cost in 2-interval. Bottom right: EDCR approximation error.

$$\begin{aligned} \lambda_t: \quad & \mathbf{1}^{\intercal}\left(\boldsymbol{g}^{\mathrm{G}}[t] + \boldsymbol{g}^{\mathrm{D}}]t] - \boldsymbol{g}^{\mathrm{C}}[t]\right) = \mathbf{1}^{\intercal}\boldsymbol{d}[t] \\ & e_{it} + g_{it}^{\mathrm{C}}\eta^{\mathrm{C}} - g_{it}^{\mathrm{D}}/\eta^{\mathrm{D}} = e_{i(t+1)} \\ & 0 \leq g_{it}^{\mathrm{G}} \leq \bar{g}_i^{\mathrm{G}} \\ & 0 \leq g_{it}^{\mathrm{C}} \leq \bar{g}_i^{\mathrm{C}} \\ & 0 \leq g_{it}^{\mathrm{D}} \leq \bar{g}_i^{\mathrm{D}} \\ & \underline{E}_i \leq e_{i(t+1)} \leq \bar{E}_i \\ & g_{it}^{\mathrm{C}} g_{it}^{\mathrm{D}} = 0 \\ & e_{i1} = s_i, \end{aligned} \tag{5}$$

where the DC power flow model is considered with the shift-factor matrix $\boldsymbol{S} \in \mathbb{R}^{2B \times M}$ for a network with B branches and the branch flow limit $\boldsymbol{q} \in \mathbb{R}^{2B}$. The system operation constraints include power balance constraints, SoC state-transition constraints, charging/discharging capacity limits, and SoC limits. The bilinear constraint, $g_{it}^{\mathrm{C}} g_{it}^{\mathrm{D}} = 0, \forall i, t$, prevents the simultaneous charging and discharging decisions.

Note that (5) is nonconvex for two reasons. First, the objective function is nonconvex and subdifferentiable because the nonconvex multi-stage storage operation cost (4), as is shown in Fig. 2 (top left). Second, the equality constraint banning simultaneous charging/discharging decisions in (5) is bilinear. In the following section, we remove these two forms of nonconvexities.

III. Convexifying Market Clearing

We now convexify the objective function and relax the bilinear equality constraints of the market clearing problem (5). Theorem 1 below gives a condition on the bid-in cost parameters that convexifies the objection function[3].

Theorem 1: If a storage participant's bid-in parameters satisfy the equal decremental-cost ratio (EDCR) condition,

$$\frac{c_k^{\mathrm{C}} - c_{k-1}^{\mathrm{C}}}{c_k^{\mathrm{D}} - c_{k-1}^{\mathrm{D}}} = \eta^{\mathrm{C}}\eta^{\mathrm{D}}, \forall k, \tag{6}$$

[2]For simplicity, indexes and ramping costs for storage are ignored here.

[3]Storage index i is omitted in Theorem 1 and Section IV for simplicity.

under Assumption 1, the multi-interval storage operation cost in (4) is piecewise linear convex given by

$$F\left(\boldsymbol{g}^{\mathrm{C}}, \boldsymbol{g}^{\mathrm{D}}; s\right) = \max_{j\in\{1,\ldots,K\}} \left\{\alpha_j(s) - c_j^{\mathrm{C}}\mathbf{1}^{\intercal}\mathbf{g}^{\mathrm{C}} + c_j^{\mathrm{D}}\mathbf{1}^{\intercal}\mathbf{g}^{\mathrm{D}}\right\} \quad (7)$$

with $\alpha_j(s) := -\sum_{k=1}^{j-1} \frac{\Delta c_k^{\mathrm{C}}(E_{k+1}-E_1)}{\eta^{\mathrm{C}}} - \frac{c_j^{\mathrm{C}}(s-E_1)}{\eta^{\mathrm{C}}} + h(s)$ and $h(s) := \sum_{i=1}^{K} \mathbb{1}_{\{s\in\mathcal{E}_i\}} \left(\frac{c_i^{\mathrm{C}}(s-E_1)}{\eta^{\mathrm{C}}} + \sum_{k=1}^{i-1} \frac{\Delta c_k^{\mathrm{C}}(E_{k+1}-E_1)}{\eta^{\mathrm{C}}}\right)$.

See the proof in Appendix [6]. Note that if bid-in costs are derived from the value function of the stochastic storage optimization based on price forecasts as in [4], [5], the derived bids satisfy (6)[4]. The following lemma supports the exact relaxation of $g_{it}^{\mathrm{C}} g_{it}^{\mathrm{D}} = 0, \forall i, t$.

Lemma 1: Under Assumption 1 and EDCR condition, if the locational marginal prices (LMPs) from the relaxed economic dispatch are non-negative, the relaxation of the bilinear constraints $g_{it}^{\mathrm{C}} g_{it}^{\mathrm{D}} = 0, \forall i, t$ in (5) is exact.

See the proof in Appendix [6]. The computation of LMP (after relaxation of the bilinear constraint) is standard. Specifically, we define the LMP by $\pi_{it} := \lambda_t^* - \boldsymbol{S}(:, i)^{\intercal}\boldsymbol{\mu}^*[t]$ for bus i and interval t with the optimal dual solutions of (5) after relaxing the bilinear equality constraints.

The non-negative assumption on LMP has been considered in [7], [8] for the exact relaxation of bilinear constraint in (5) for differentiable objective functions. Here we have a slight generalization for a convex piecewise linear objective function by deploying the subgradient measure [[9], p. 281]. See the proof in Appendix [6].

IV. Optimal EDCR Approximatkon

In constructing the SoC-dependent storage bids and offers in (2), the true marginal costs (or true marginal cost $\tilde{b}^{\mathrm{D}}(e_t)$ and marginal benefit $\tilde{b}^{\mathrm{C}}(e_t)$) may not satisfy the EDCR condition. The following optimization aims at finding the optimal approximation of $\tilde{b}^{\mathrm{C}}(e_t)$ and $\tilde{b}^{\mathrm{D}}(e_t)$ with the EDCR condition satisfied by parameters $\boldsymbol{\theta} = \{\mathbf{c}^{\mathrm{C}}, \mathbf{c}^{\mathrm{D}}, \boldsymbol{E}\}$,

$$\underset{\boldsymbol{\theta}\in\boldsymbol{\Theta}}{\text{minimize}} \quad ||b^{\mathrm{C}}(\cdot|\boldsymbol{\theta}) - \tilde{b}^{\mathrm{C}}(\cdot)||_2^2 + ||b^{\mathrm{D}}(\cdot|\boldsymbol{\theta}) - \tilde{b}^{\mathrm{D}}(\cdot)||_2^2. \quad (8)$$

The objective fuction measures the distance between the original *true marginal cost* and the approximation bids/offers, and $\boldsymbol{\theta}$ is restricted in a set $\boldsymbol{\Theta}$ satisfying Assumption 1 and the EDCR condition from Theorem 1. With N data samples $(S_n, B_n^{\mathrm{C}}, B_n^{\mathrm{D}})_{n=1}^N$ from the *true marginal cost*, the objective is $\frac{1}{N}\sum_{n=1}^{N}((b^{\mathrm{C}}(S_n|\boldsymbol{\theta}) - B_n^{\mathrm{C}})^2 + (b^{\mathrm{D}}(S_n|\boldsymbol{\theta}) - B_n^{\mathrm{D}})^2)$.

Optimization (8) for the optimal EDCR approximation is in general nonconvex. By fixing $\boldsymbol{E}$ while solving for $(\mathbf{c}^{\mathrm{C}}, \mathbf{c}^{\mathrm{D}})$, or fixing $(\mathbf{c}^{\mathrm{C}}, \mathbf{c}^{\mathrm{D}})$ while solving for $\boldsymbol{E}$, we can iteratively approach the (local) optimal solution by solving a convex problem in each iteration.

V. Example

Consider an ideal storage with the initial SoC at 15.5 MWh, $T = 2$ and the original nonconvex multi-interval storage cost shown in the top left of Fig 2. The axis labels use the notation $g_t = g_t^{\mathrm{D}} - g_t^{\mathrm{C}}, t = 1, 2$, for storage's net-producing power. The true SoC-dependent bids, $\tilde{b}^{\mathrm{C}}(e_t)$ and $\tilde{b}^{\mathrm{D}}(e_t)$, are shown in Fig. 2 (top right). From the EDCR approximation in (8) with even SoC partitions[5], we can approximate the true SoC-dependent bids and achieve the convex cost function shown in Fig. 2 (bottom left and top right). In this ideal storage which has $\eta^{\mathrm{C}} = \eta^{\mathrm{D}} = 1$, the EDCR condition in Theorem 1 decreased to a special case, $c_k^{\mathrm{C}} - c_{k-1}^{\mathrm{C}} = c_k^{\mathrm{D}} - c_{k-1}^{\mathrm{D}}, \forall k$ (shown in top right of Fig. 2).

The bottom right part of Fig. 2 illustrates the approximation error between the original SoC-dependent bids, $\tilde{b}^{\mathrm{C}}(e_t)$ and $\tilde{b}^{\mathrm{D}}(e_t)$, and the optimal EDCR approximation bids, $b^{\mathrm{C}}(e_t)$ and $b^{\mathrm{D}}(e_t)$. It is observed that, with more SoC partition segments, a smaller approximation error can be achieved.

VI. Conclusion

It's essential to remove non-convexities for a large-scale deployment of storage. This paper convexifies the market clearing process by imposing a condition on the SoC-dependent bidding. We propose a sufficient condition—the equal decremental-cost ratio (EDCR) condition—to convexify the market clearing of multi-interval economic dispatch with SoC-dependent bids from merchant storage participants. And an optimal EDCR approximation method is proposed to compute the SoC-dependent bid from the true cost of storage.

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[5] $\mathcal{E}_k$ satisfies $E_k = \underline{E} + \frac{(k-1)(\bar{E}-\underline{E})}{K}$ based on the SoC upper bound $\bar{E} = 25$ MWh and lower bound $\underline{E} = 9$MWh for all $k \in \{1, \ldots, K\}$.