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**Key Points:**

- A method to concoct non-stationary data series is proposed
- Eddy covariance and wavelet analysis methods underestimate turbulent momentum flux under non-stationary condition by about 50%
- Mexican hat wavelet method has the potential to accurately calculate flux of non-stationary turbulence after correction

**Supporting Information:**

Supporting Information may be found in the online version of this article.

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# Quantitative Evaluation of Wavelet Analysis Method for Turbulent Flux Calculation of Non-Stationary Series

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**Abstract** This study evaluates the uncertainties of turbulent flux calculation using eddy covariance (EC) and wavelet analysis (WA) methods. First, a non-stationary data set is concocted by adding periodic waves and random perturbations which mimic the large eddies, turbulent intermittency, and asymmetry into an observational stationary data set, and the theoretical "true" fluxes are used to quantitatively evaluate the accuracy of these methods. Results show that EC and Morlet-wavelet generate biases ranging 50%–100% of the "true" values at different non-stationarity grades, whereas the Mexican hat (Mexhat) wavelet has a bias of about half of them. Furthermore, there is a high correlation of the Mexhat-derived fluxes to the benchmark values, the regression slopes of the values of these two can be improved to almost 1 by adding a correction coefficient. The results suggest the potential of using the Mexhat-wavelet method to calculate turbulent fluxes with high accuracy under non-stationary conditions.

**Plain Language Summary** Eddy covariance (EC) method is the well-accepted technique to calculate turbulent flux under stationary conditions. However, the observational turbulence data sometimes show non-stationarity, and in this case, the EC method is not applicable and wavelet analysis (WA) is frequently used. However, because turbulent fluxes are calculated values, and there are no true flux measurements, the accuracy of WA-calculated fluxes remains unknown. In this study, we constructed a non-stationary data set and used their theoretical true values to evaluate the accuracy of EC and WA methods in flux calculation under non-stationary conditions. It is found that EC and Morlet-wavelet bias 50%–100% of the true values at different non-stationarity grades, while the Mexican hat (Mexhat) wavelet has the bias about half of them. Besides, there is a high correlation of the Mexhat-derived fluxes to the true values, and Mexhat-derived fluxes can be corrected to near true values by adding a correction coefficient. Therefore, the Mexhat-wavelet method has the potential to be used to calculate turbulent fluxes under non-stationary conditions.

## 1. Introduction

An accurate estimate of turbulent fluxes is essential to weather, climate, and environmental studies (e.g., Biermann et al., 2014; Bourassa et al., 2013; Helbig et al., 2021; Li et al., 2006; Natali et al., 2022; J. A. Zhang et al., 2008). To date, the eddy covariance (EC) is the most commonly used technique in quantifying turbulent fluxes (e.g., Aubinet et al., 2012; Baldocchi, 2020; Müller et al., 2010), and is the standard method used by major flux measurement networks around the world (e.g., AmeriFlux, AsiaFlux, CarboEuroFlux, etc.).

Several important assumptions need to be met when using the EC algorithm, such as stationary airflow and horizontal homogeneity (Foken & Napo, 2008; Foken et al., 2004; Lenschow et al., 1994; Thomas & Foken, 2007). However, in reality the earth surface is always more or less inhomogeneous and undulate, especially in urban and mountainous areas, where non-stationary turbulence frequently appears (e.g., Arnfield, 2003; Babić et al., 2016; Panin et al., 1998; Stiperski & Rotach, 2016; Turnipseed et al., 2004). Other complications, such as time-varying surface heat fluxes, individual cloud elements, and horizontal pressure gradients, can also cause non-stationary turbulence (e.g., Andreas et al., 2008; Angevine et al., 2020; Cava et al., 2014; Donateo et al., 2013; Mahrt & Bou-Zeid, 2020; Momen & Bou-Zeid, 2017; Rannik & Vesala, 1999; Sunuararajan & Tjernström, 2000), Moreover, the chaotic turbulence itself may exhibit intermittent, stochastic, and non-linear characteristics, causing

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turbulence to be severely non-stationary (e.g., Cullen et al., 2007; Deventer et al., 2018; Mahrt, 2007; Riederer et al., 2014).

Under non-stationary conditions, EC algorithm may introduce large biases in flux calculation (Göckede et al., 2019; L. Liu et al., 2022; Vitale et al., 2020), and such data are usually discarded when performing EC analyses in previous studies (e.g., Foken & Napo, 2008; Foken et al., 2004). Recently, the wavelet analysis (WA), which does not require the assumptions of stationarity and horizontal homogeneity, becomes an alternative method to analyze turbulence under non-stationary conditions (e.g., Kumar & Foufoula-Georgiou, 1997). The WA method decomposes a series into time–frequency space through fitting the turbulent series with varying amplitudes and frequencies of a selected wavelet function, which effectively extracts time and frequency information from the turbulent signal (Daubechies, 1990; Farge, 1992; Maraun & Kurths, 2004; Torrence & Compo, 1998; N. Zhang et al., 2019). In an early study, Argoul et al. (1989) used the WA method to reveal the cascade of turbulent eddy scales from a tunnel experiment. Mahrt (1991) found that WA was useful to distinguish the dominant scales of asymmetric eddies shown in aircraft observations. Hudgins et al. (1993) used the WA method to analyze the data collected from the boundary layer over the ocean and found intermittency related to small-scale turbulent mixing. Treviño and Andreas (1996) applied the WA method to the non-stationary turbulence, and discussed its limitation on resolving linear and non-linear behaviors in the turbulence signal. Thomas and Foken (2005, 2007) used the WA method to calculate the turbulent statistics contributed by coherent eddy structures and found that the determined peak in the calculated wavelet variance spectrum was consistent with the durations of the characteristic events. Saito and Asanuma. (2008) derived wavelet co-spectra of fluxes over a rice and a larch forest. Zhu et al. (2010) used WA to analyze the turbulences in the unsteady, inhomogeneous hurricane surface layer, and revealed the roles of eddies with different scales in generating fluxes and turbulent kinetic energy. Schaller et al. (2017) compared the fluxes of stationary turbulence data calculated by the WA and EC methods and found an excellent agreement between the two methods. Schaller et al. (2019) used WA to evaluate Methane flux from Arctic permafrost ecosystem, and concluded that WA was a suitable method for resolving flux events on the order of minutes for non-stationary turbulence. Göckede et al. (2019) investigated the impact of non-stationarity on turbulent fluxes by comparing EC calculations against WA references, and found substantial uncertainties associated with short-term fluxes.

Despite the ability of WA to process non-stationary signals, issues regarding the accuracy of WA method on flux calculation under non-stationary condition have yet to be thoroughly addressed. In this study, by using an artificial data set, which is concocted from observations and idealized waves, we aim to provide a quantitative evaluation on the accuracy of WA method in determination of turbulent fluxes by comparing with the theoretical "true" values. Section 2 describes the data and methods used in the study. Section 3 presents the evaluation results followed by the summary and discussions in Section 4.

## 2. Data and Methods

### 2.1. Data

A year-long high-frequency wind data collected at a site in Shouxian county, Anhui Province (116.78°E, 32.57°N) from 1 January 2017 to 31 December 2017 are used in this study. The surrounding areas of the site are generally flat, and are covered mostly by rice-wheat farmland with some dispersed trees. A three-dimensional ultrasonic anemometer (CSAT3A, Campbell Scientific Incorporation, USA) and a gas analyzer (EC150, Campbell Scientific Incorporation, USA) were installed on 2.5 m height, and their sampling frequencies were 10 Hz. The 1-year data were divided into a total of 17,520 30-min subsets. Quality control of wild point removal and coordinate rotation was applied to the data following C. Liu et al. (2020), after that 10,360 subsets of data remained. Figure S1 in Supporting Information S1 shows the temporal variability of the observed wind, temperature, humidity, precipitation, and barometric pressure over the study period.

### 2.2. Method

#### 2.2.1. Stationarity Classification

A method proposed by Foken and Wichura (1996) commonly used to categorize the stationarity of turbulent data series is adopted here. The relative non-stationarity ($RN_{COV}$) of a 30-min turbulent time series is assessed by,

$$\mathrm{RN}_{\mathrm{COV}} = \left| \frac{\overline{C_{xy5\,\mathrm{min}}} - C_{xy30\,\mathrm{min}}}{C_{xy30\,\mathrm{min}}} \right|, \tag{1}$$

where a 30-min series is cut into six 5-min sections. $C_{xy30\,\mathrm{min}}$ and $\overline{C_{xy5\,\mathrm{min}}}$ are the covariance of the 30-min series and the average of covariances of the six 5-min sections, respectively. Foken et al. (2004) categorized the stationarity state of a series into nine grades according to the values of $\mathrm{RN}_{\mathrm{COV}}$ (Table S1 in Supporting Information S1). When calculating fluxes using the EC method, data in grade 1 and 2 are defined as stationary while those in other grades are treated as non-stationary and usually discarded. In the original Shouxian data set, among 10,360 subsets of 30-min data sections in total, 2,533 subsets out of them are identified as non-stationary data sections (i.e., grade 3–9, Table S1 in Supporting Information S1), which account for 24.5% of the total data. Noted that the Shouxian site has a relatively flat underlying surface. One would expect the non-stationary data portion to be larger in areas with heterogeneous underlying surfaces.

### 2.2.2. Turbulent Flux Calculation Methods

#### *2.2.2.1. EC Method*

The EC method is based on the Reynolds decomposition and Taylors' frozen turbulence hypothesis (Foken et al., 2012; Stull, 1988). Reynolds decomposition theory divides the time series of variable $x$ into two parts, a mean value $\overline{x}$ indicating the mean flow, and perturbations $x'$ denoting the turbulence. The turbulent fluxes are calculated as the covariance of two variables, for example, the momentum turbulent flux $\overline{w'u'}$ in the surface layer may be expressed as

$$\overline{w'u'} = \frac{1}{N}\sum_{i=1}^{N}\left[\left(w_i - \overline{w}\right)\cdot\left(u_i - \overline{u}\right)\right], \tag{2}$$

Where $w$ and $u$ are the vertical velocity and horizontal wind component in the $x$-axis direction of a local Cartesian coordinate, respectively; Overbar and prime represent the mean and perturbation away from the mean, respectively; and $N$ is the number of observations in a data series.

#### *2.2.2.2. WA Method*

The wavelet transformation of a discrete time series $x(t)(t = 1 \cdots N)$ is

$$W_n(s) = \sum_{n'=1}^{N} x_{n'}(t) \times \psi^*\left(\frac{(n'-n)\delta t}{s}\right), \tag{3}$$

where $W_n(s)$ is the wavelet coefficient, $\psi^*$ is the complex conjugate of the wavelet function $\psi$, $s$, $n$, and $\delta t$ are the wavelet scale, translation index, and data sampling interval of a signal series, respectively. Following previous studies (e.g., Schaller et al., 2017; Torrence & Compo, 1998; Zhu et al., 2010), two frequently used wavelet functions Mexican hat wavelet and Morlet-wavelet, are examined here. The Morlet-wavelet may be written as,

$$\psi^{\mathrm{Morlet}}(\mu) = \pi^{-\frac{1}{4}} \cdot e^{-i\omega_0\mu} \cdot e^{-\frac{\mu^2}{2}}. \tag{4}$$

where $\mu$ is a time parameter. Following Farge (1992) and Schaller et al. (2017), $\omega_0$ has a value of 6. The Mexhat-wavelet may be expressed as,

$$\psi^{\mathrm{Mexhat}}(\mu) = \frac{2\cdot\left(1-\frac{\mu^2}{\sigma^2}\right)}{\sqrt{3\cdot\sigma}\cdot\pi^{\frac{1}{4}}} \cdot e^{-\frac{\mu^2}{2\sigma}}, \tag{5}$$

with $\sigma = 1$. Then, the turbulent flux at a given scale $i$ integrating over the whole data series may be calculated as,

$$E_{wu}(j) = \frac{\delta t}{C}\cdot\frac{1}{N}\cdot\sum_{n=1}^{N}\left[W_n^{w}(s_j)\cdot W_n^{u*}(s_j)\right]. \tag{6}$$

where, $C$ is the reconstruction factor, and following Torrence and Compo (1998), it takes a value of 0.776 for Morlet-wavelet and 3.541 for Mexhat-wavelet, respectively. The wavelet scale is taken as,

$$s_i = s_0 \cdot 2^{j \cdot \delta j}, \quad j = 0, 1, 2 \ldots, J, \tag{7}$$

where, $\delta i$ is the spacing between scales, and it is set as 0.25 here following Schaller et al. (2017) to ensure a sufficient resolution and affordable computation capacity, and $J$ takes

$$J = \delta j^{-1} \cdot \log_2 \left( \frac{N \cdot \delta t}{s_0} \right). \tag{8}$$

The smallest scale $s_0$ here is set to $2\delta t$. Finally, the total turbulent momentum flux over all scales ($E_{wu}$) can be calculated by

$$E_{wu} = \delta j \cdot \sum_{i=0}^{J} \frac{E_{wu}(j)}{s_j}. \tag{9}$$

For the stationary turbulence, $E_{wu}$ should be theoretically equal to $\overline{w'u'}$ determined by EC.

#### 2.2.3. Concoction of Non-Stationary Data Series

Turbulent fluxes cannot be observed directly but have to be calculated mathematically (e.g., Aubinet et al., 2012; Rebmann et al., 2012). Under stationary conditions, EC is a commonly accepted method for calculating turbulent fluxes (e.g., Hammerle et al., 2007). However, for non-stationary conditions, no well-accepted flux calculation method is available at present. While WA method is frequently used in processing non-stationary turbulent data (e.g., Schaller et al., 2017, 2019; Zhu et al., 2010), the applicability and uncertainty of this method have yet been thoroughly addressed and evaluated (Treviño & Andreas, 1996). Here, we concoct non-stationary data series by adding various periodic waves into the observed stationary data. In this way, the perturbations in the concocted non-stationary data series are known, and "true" fluxes can be calculated and used to evaluate the WA method. It is noteworthy that, various factors can cause non-stationary turbulences, for example, air flow through the heterogeneous surface (e.g., Arnfield, 2003; Babić et al., 2016; Panin et al., 1998; Stiperski & Rotach, 2016; Turnipseed et al., 2004); the variation of the radiation (e.g., Andreas et al., 2008; Cullen et al., 2007; Dias et al., 2004; Gluhovsky & Agee, 1994); non-local transportation and the intermittency of turbulence itself (e.g., Cullen et al., 2007; Klipp & Mahrt, 2004; Mahrt, 1998). And the non-stationarity of the turbulences can be different by different driving forces. To make the added fluctuations "physically plausible," we added periodic waves with periods ranging from 1 to 150 min to represent the relatively larger eddies in the atmosphere (Finnigan, 2000; Wyngaard, 1992), and added sections of 2-min random perturbations to mimic the intermittent turbulences (Van de Wiel et al., 2003), and further modified the fluctuations to let them sometime be asymmetric (Agostini et al., 2016). The detailed method to concoct new non-stationary data series is as follows.

1. Mean values of the stationary 30-min subsets are removed and only the perturbations are kept.
2. Six periodic sinusoidal waves with periods randomly selected from 1 to 10, 11–20, 21–30, 31–60, 61–90, and 91–150 min, respectively, are added to the original data to represent the relatively larger eddies in the atmosphere (e.g., Finnigan, 2000; Wyngaard, 1992). The amplitudes of these waves are also randomly selected from 0 to 1.35 m $s^{-1}$ for the vertical wind $w$ and 0–2.32 m $s^{-1}$ for the horizontal wind component $u$. The chosen value of 1.35 (2.32) is the average of the maximum perturbations of the 10,360 subsets of 30-min non-stationary data for $w$ ($u$). The phases of these periodic waves are again set as random. The sinusoidal waves added to $w$ and $u$ are independent of each other.
3. A number of sections of 2-min random perturbations are then inserted into the data. The number of sections is randomly chosen in 1–10. These random perturbations are used to mimic the intermittent turbulences. Their amplitudes are set as a random number between 0 and 0.68 (0–1.16) for $w$ ($u$), half of the values in the second step, considering that the intermittent turbulences are generally weaker than the continuous turbulences (e.g., Van de Wiel et al., 2003).
4. Starting from the data generated from step 3, randomly select 1–10 1-min data segments in the 30-min data subset, and then find the maximum (or minimum, by randomly choice) 25% of the disturbances in these 1-min data segments, and then reduce (or enlarge) the values larger than the 75% (or smaller than 25%) quantile value to the 75% (or 25%) quantile value in the corresponding 1-min data segments. This is to introduce asymmetries in the concocted data series (Agostini et al., 2016).

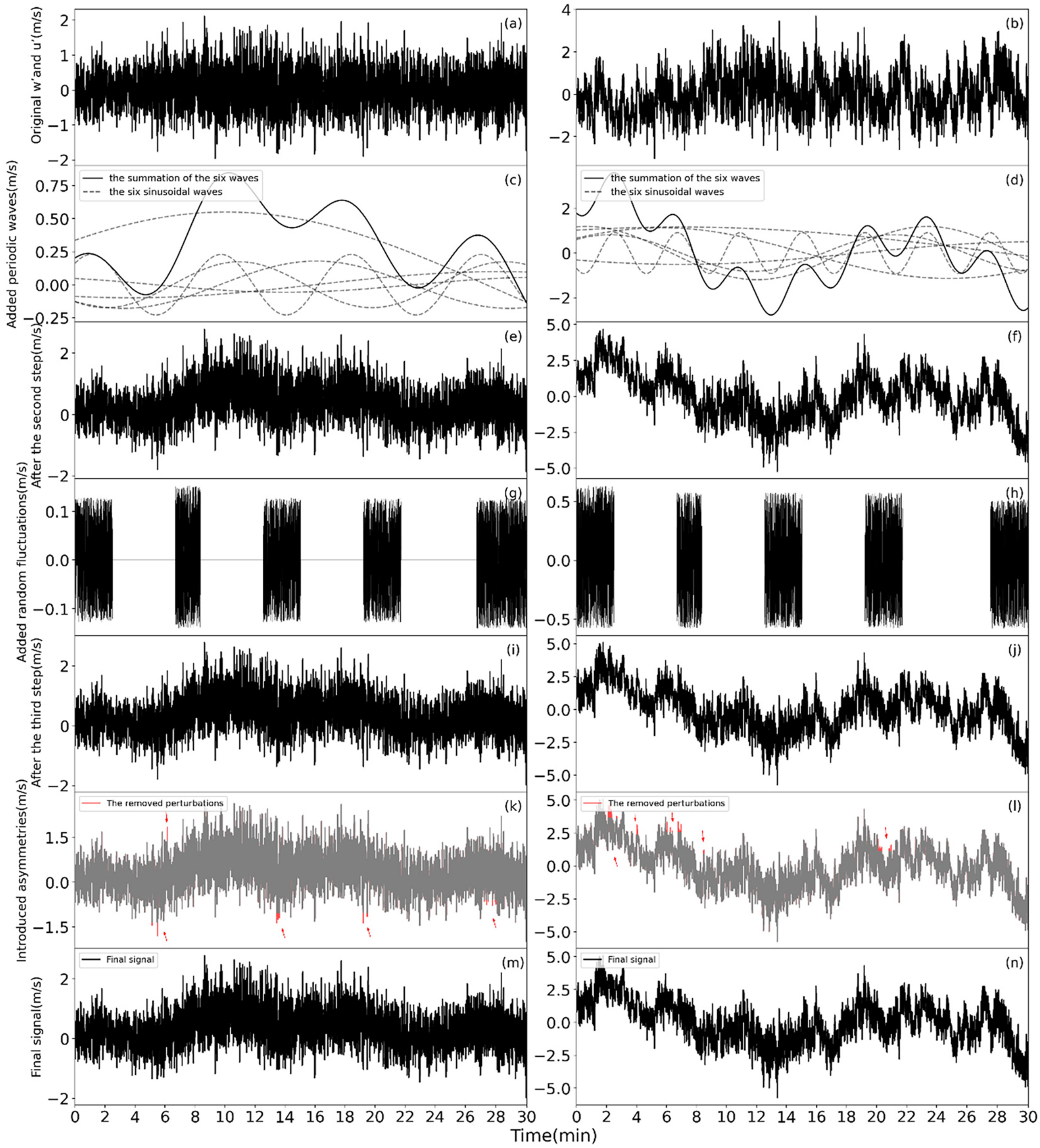


**Figure 1.** Illustration of concocting data series for vertical velocity *w* (left column) and *x*-direction wind component *u* (right column), respectively. (a, b): Original data series after removing the mean. (c, d): The six added sinusoidal waves and their summations described in step 2. (e, f): The signals after step-2 concoction. (g, h): The added random perturbations described in step 3. (i, j): The signals after step-3 concoction. (k, l): The introduced asymmetry as described in step 4, the removed extreme disturbances are marked as red and are pointed by red arrows. (m, n): The final signals after all concoction steps.

The concoction procedures listed above successfully create a controllable non-stationary data series from a real observed stationary data series and yet avoid much complexity. An example of 30-min data of $w$ and $u$ going through the concoction procedures stated above is shown in Figure 1. The $RN_{COV}$ value starts from grade 1 of the original data series from observations and ends up with grade 6 ($RN_{COV}$ = 1.52) after concocting. We also tested different concocting choices in these steps and found that the main conclusions of the study kept unchanged (Please refer to the Table S2 and Figure S3 in Supporting Information S1). The observational stationary data set and the Python scripts for the non-stationary data concocting can be found in Wu et al. (2022).

## 3. Results

### 3.1. The Original Stationary Data

In the stationary state (i.e., grade 1–2 in Table S1 in Supporting Information S1), the turbulent fluxes are commonly calculated by the EC method. Here, we compare the fluxes by the Morlet-wavelet and Mexhat-wavelet methods to those by EC method using the 7,827 subsets of the grade 1–2 data (Figure S2 in Supporting Information S1). It is found that both the Morlet-wavelet and Mexhat-wavelet methods produce results very close to those by the EC method with coefficients of determination (which is the square of Pearson correlation coefficient) of 0.997 and 0.998, respectively. However, Morlet-wavelet method is inclined to overestimate slightly and has a regression slope of 1.05, whereas Mexhat-wavelet method is inclined to underestimate with a regression slope of 0.91. These results are consistent with the theoretical result that in a strict stationary state the EC determined turbulent fluxes should be identical to those determined by WA and agree with Held (2014) and Schaller et al. (2017), who found that WA produces turbulent fluxes similar to those of EC, and Schaller et al. (2017), who also showed that Mexhat-wavelet method tended to generate slightly smaller fluxes than Morlet-wavelet.

### 3.2. The Concocted Non-Stationary Data

The WA method has been frequently used to analyze turbulences in the non-stationary condition. However, the uncertainty of the fluxes derived by WA has not yet been thoroughly evaluated because no benchmark flux measurements are available under non-stationary conditions. By using the method described in Section 2.2.3, the 7,827 subsets of stationary data are concocted, and 6,483 out of them become non-stationary (i.e., categorization higher than or equal to grade 3) (Table S1 in Supporting Information S1). For these non-stationary subsets, the perturbations are the summation of those in the original stationary data and those in the added known signals. In other words, the added signals are all treated as perturbations, and by adding them to the original perturbations, we have the exact perturbations. Therefore, by adding the perturbations caused by each part (i.e., the original stationary data and the added known signals), we can have the exact perturbations ($u'_{\text{exact}}$ and $w'_{\text{exact}}$) for $u$ and $w$, respectively:

$$x'_{\text{exact}} = x'_{\text{stationary}} + x'_{\text{sin}} + x'_{\text{random}} + x'_{\text{asym}}. \tag{10}$$

Here, $x$ can be replaced by $u$ or $w$, $x'_{\text{stationary}}$ represents the perturbation in the original stationary data, $x'_{\text{sin}}$ is the perturbation of the added six periodic sinusoidal waves in the second step of non-stationary data concocting. $x'_{\text{random}}$ and $x'_{\text{asym}}$ are the perturbations introduced in the third and fourth step of non-stationary data concocting, respectively.

And the fluxes of the concocted non-stationary data can be obtained by calculating $\overline{u'_{\text{exact}}w'_{\text{exact}}}$ (the overbar means averaging over the 30 min):

$$\overline{u'_{\text{exact}}w'_{\text{exact}}} = \overline{\left(u'_{\text{stationary}} + u'_{\text{sin}} + u'_{\text{random}} + u'_{\text{asym}}\right)\left(w'_{\text{stationary}} + w'_{\text{sin}} + w'_{\text{random}} + w'_{\text{asym}}\right)}. \tag{11}$$

It should be mentioned that the inclusion of the flux from these added frequencies is not linear, that is, not simply by adding $\left(u'_{\text{sin}} + u'_{\text{random}} + u'_{\text{asym}}\right)\left(w'_{\text{sin}} + w'_{\text{random}} + w'_{\text{asym}}\right)$ to the original stationary flux, but including the interaction between the stationary and non-stationary parts of the perturbations as shown in Equation 11.

The fluxes calculated by Equation 11 are further taken as the benchmark to quantitatively evaluate the accuracy of EC, Morlet-wavelet and Mexhat-wavelet flux-calculation methods, as shown in Figures 2a1–2c1. It can be seen that values from all the three methods deviate from the benchmark values. The EC and Morlet-wavelet methods have a regression slope of 0.40 and 0.23, and coefficient of determination of only 0.415 and 0.427, respectively. Mexhat-wavelet method has a regression slope of 0.54 which is also an indication of underestimation, but it shows a large coefficient of determination of 0.889, suggesting that its accuracy may be improved by adding a correction coefficient.

To examine the accuracy of these methods at different non-stationary grades, Figures 2a2–2a8, 2b2–2b8, and 2c2–2c8 further presents the scatterplots of the three methods for different non-stationary grades. Generally, biases increase with the non-stationary grades, in particular for the EC and Morlet-wavelet methods. To quantify the deviations of the fluxes determined by the three methods away from the benchmark values, we used a

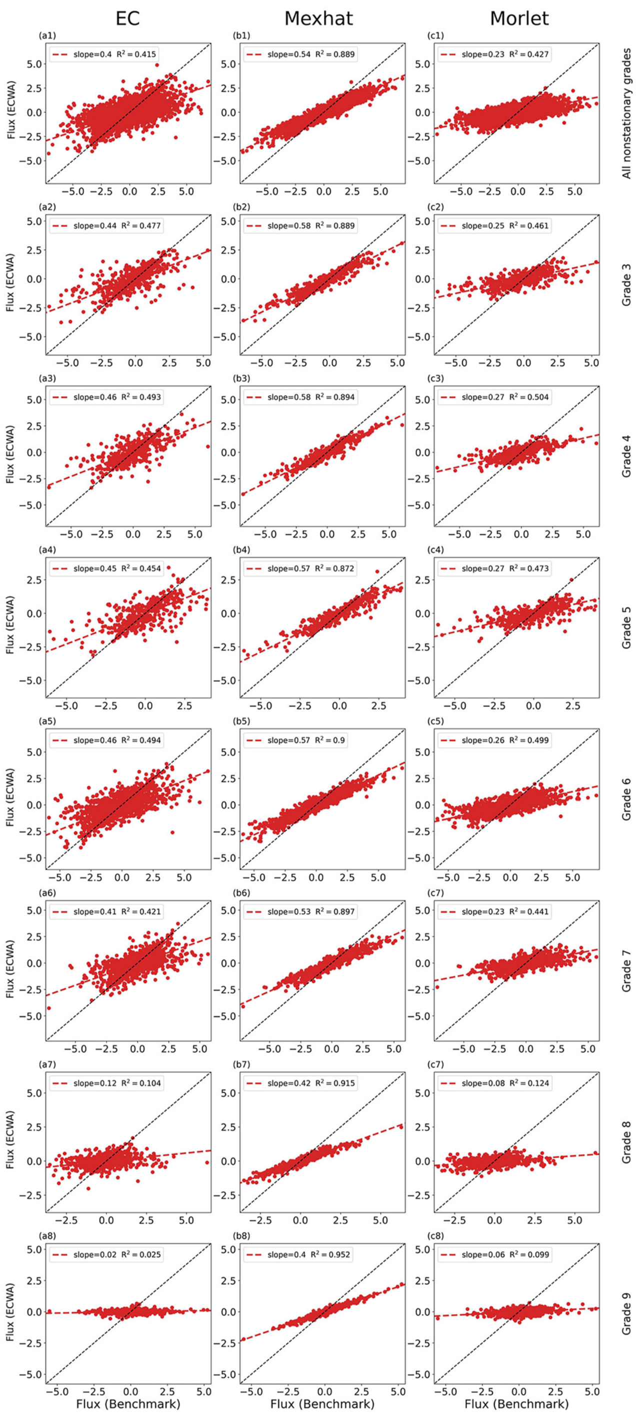


**Figure 2.** Scatterplot of benchmark flux values against the values by (a1–a8) the eddy covariance method, (b1–b8) the Mexhat-wavelet method, and (c1–c8) the Morlet-wavelet method. Rows 1–8 are the plots of non-stationary data with all non-stationary grades and grades of 3–9, respectively. The dashed red and black lines indicate the linear regression line and 1:1 line, respectively.

parameter known as the absolute values of relative error (AVRE), which is defined as,

$$\text{AVRE} = \left| \frac{F_{\text{ECWA}} - F_{\text{benchmark}}}{F_{\text{benchmark}}} \right| \times 100\%, \tag{12}$$

where $F_{\text{ECWA}}$ represents the flux derived by the EC, Morlet-wavelet or Mexhat-wavelet methods, and $F_{\text{benchmark}}$ is the benchmark flux obtained from Equation 11. The results are shown in Figure 3. It can be seen that the AVRE values for the EC and Morlet-wavelet methods are about 50% when the non-stationary grade is below grade 6, but they reach nearly 100% at grade 9. In contrast, the median values of AVRE for the Mexhat-wavelet method are only about half of those of the other two methods. Moreover, although the AVRE values increase and the regression slopes decrease with the increase of non-stationary grades (meaning that underestimation becomes worse), the coefficients of determination of the Mexhat-wavelet method remain large (>0.88) and almost invariant with the grades (Figures 2b2–2b8). These results suggest that the fluxes calculated by the Mexhat-wavelet method are less biased from and more correlated to the benchmark values than the other two methods when processing non-stationary data.

### 3.3. Correction Based on Mexhat-Wavelet

The above analyses indicate that the Mexhat-wavelet method is the most accurate among the three compared methods, but it still tends to underestimate the benchmark fluxes. The underestimation is mainly caused by its inability to reconstruct the signals with large periods (i.e., the added signals with periods larger than 1 min). As illustrated by Figure S3 in Supporting Information S1, the larger the periods of the added signals, the larger the biases are generated. A spectrum analysis also shows no apparent losses of signals during the WA of the concocted non-stationary series (figure not shown). However, we note that the fluxes calculated by the Mexhat-wavelet method have large coefficients of determination, thus, it is plausible to correct the estimated fluxes by adding a correction coefficients ($C_0$) to the Mexhat-wavelet method based on the benchmark values,

$$E_{wu(\text{modified})} = C_0 E_{wu(\text{original})}. \tag{13}$$

The values of the correction coefficients for different grades are given in Table S1 in Supporting Information S1. These correction coefficients appear to work well for the concocted data constructed in this study. As illustrated by Figure 4, the corrected fluxes from the Mexhat-wavelet method have the regression slopes near 1 for all non-stationary grades. It is also found that the correction coefficients are sensitive to the periods of the sinusoidal waves being added (Figure S3 in Supporting Information S1), suggesting that cautions are needed when correcting the WA derived fluxes for turbulence data that contains large eddies. This issue will be further investigated in our future study. Finally, it should be pointed out that the correction coefficients listed in Table S1 in Supporting Information S1 are derived from the artificial concocted data series. Whether they can be applied to real non-stationary turbulent data is an issue that needs to be further addressed.

## 4. Summary and Discussion

Atmospheric turbulence often shows non-stationary characteristics. Although EC method is taken as the flux measurement technique under stationary conditions, there is no well-accepted flux calculation method

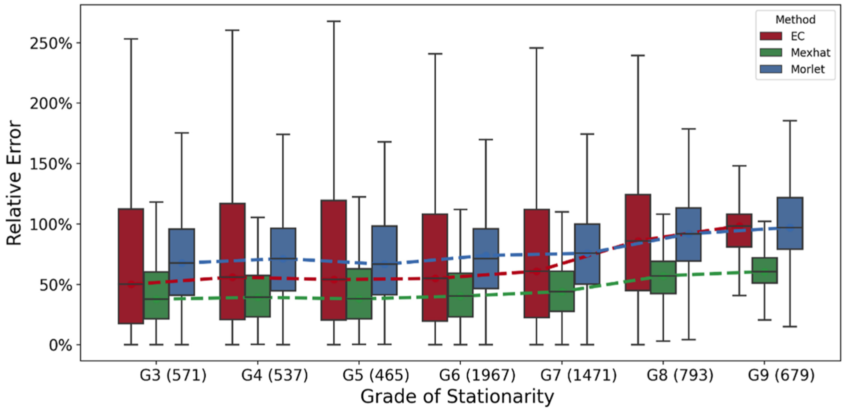


**Figure 3.** Boxplots of absolute values of relative error for the eddy covariance (maroon), Mexhat-wavelet (green), and Morlet-wavelet (blue) methods. The median values are indicated by the black lines within the boxes. The upper and lower edges of the boxes indicate the 25th and 75th percentiles.

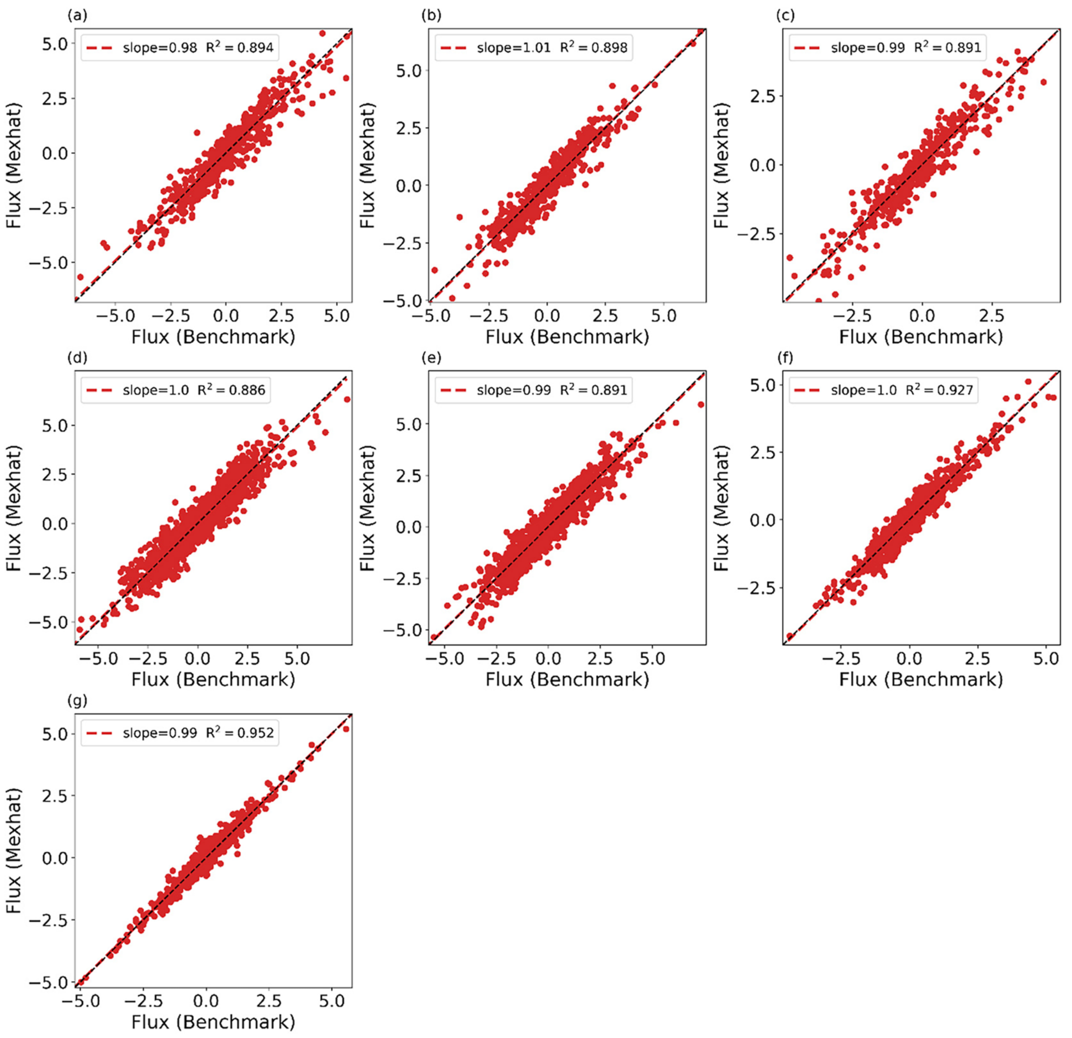


**Figure 4.** Same as Figure 2, but for Mexhat-wavelet method only. (a–g) Are the plots of non-stationary data with grades of 3–9, respectively.

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for non-stationary turbulence. While WA is a frequently used method for turbulence decomposition under non-stationary conditions, issues regarding the accuracy of the fluxes calculated by WA have not been thoroughly addressed.

A 1-year long 10-Hz high frequency data set observed over a rice-wheat paddy is used in this study. It is found that, for the 10,360 legs of 30-min data, 24.5% of them are non-stationary based on the categorization criterion proposed by Foken et al. (2004). The stationary observational data are used to concoct the fabricated non-stationary data by adding six periodic waves and random perturbations, which mimic the large eddies, turbulent intermittency, and asymmetry, respectively (Agostini et al., 2016; Finnigan, 2000; Van de Wiel et al., 2003; Wyngaard, 1992). The fluxes derived from the concocted data series are taken as the benchmark to quantitatively evaluate the accuracy of the fluxes calculated by the EC and WA methods. The results show that the EC and Morlet-wavelet methods generate large biases about 50% of the benchmark values when the non-stationary grades are below grade 6, but reach nearly 100% at grade 9, whereas the errors generated by the Mexhat-wavelet method are only about half of those of the other two methods. In addition, the calculated fluxes from the EC and Morlet-wavelet methods are less correlated to the benchmark values than those from the Mexhat-wavelet method that has a coefficient of determination of 0.889 for the entire concocted non-stationary data. It is further found that the underestimation of fluxes by the WA methods is mainly caused by the added turbulent components with large periods, suggesting that such flux underestimation may be relaxed by adding a correction coefficient. This correction method works particularly well for the Mexhat-wavelet method. Our results show that after the correction the regression slopes between the fluxes determined by the Mexhat-wavelet method and the benchmark values can reach nearly 1 for all non-stationary grades.

It should be noted that in the methods of adding random perturbations, the vertical wind component $w$ and the horizontal wind component $u$ are treated separately, in other words, the added perturbations of $w$ and $u$ are independent from each other. However, in reality, it is plausible that the $w$ and $u$ fluctuations are correlated. Therefore, to simulate this situation, additional experiments are performed in which $w$ and $u$ are generated at the same frequencies but with different phases to mimic the correlated $w$ and $u$. The rest steps of adding fluctuations remain the same as those described in the baseline experiments. After the same processing technique with EC, Mexhat-wavelet and Morlet-wavelet methods, similar results as in Figure 2 are found: EC and Morlet-wavelet methods are not capable of calculating the fluxes particularly for non-stationarity grade 8 and 9 where the correlation coefficients to the benchmark values drops below 0.2. In contrast, the calculated fluxes of Mexhat-wavelet method show high correlation with the benchmark values for all non-stationarity grades, although the degree of correlation to the benchmark values and the regression slopes change slightly depending on how $w$ is correlated to $u$ (Figure not shown). The above results show that whether the added $w$ and $u$ fluctuations are correlated or not, the main conclusions here are not affected.

This study provides a quantitative evaluation of the accuracy of the fluxes determined by the WA method under non-stationary conditions. The characteristics of biases generated by the WA method are revealed, and the correction coefficients for relaxing the biases generated by the Mexhat-wavelet method are derived. However, it should be cautious when applying these coefficients to process the real observational data, since they are derived purely from a concocted data set. How to correct the fluxes of real non-stationary turbulent flow determined by the WA methods is an issue that requires further investigation. Nonetheless, this study suggests that the Mexhat-wavelet provides a promising method for estimating turbulent fluxes in non-stationary conditions. To further explore this possibility, our future work will focus on the following two areas. First, analyze more observational data to characterize the non-stationary turbulence generated by different atmospheric processes and to create the corresponding concocted datasets in each category, and then, compare the similarities and differences of the correction coefficients derived under different characterized turbulence categories. Second, examine if the linear underestimation of the fluxes of concocted datasets by the Mexhat-wavelet method is common to all non-stationary turbulence with different characteristics and explore the mechanisms underlying the flux underestimation. We believe that these practices may shed a new light on quantifying turbulent fluxes in non-stationary conditions.

## Data Availability Statement

The observational stationary data set and the Python scripts for the non-stationary data concocting are archived on Zenodo (https://doi.org/10.5281/zenodo.7145616, Wu et al., 2022).

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