



# Discontinuous shear thickening in dense suspensions: Mechanisms, force networks, and fluctuations

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## ARTICLE INFO

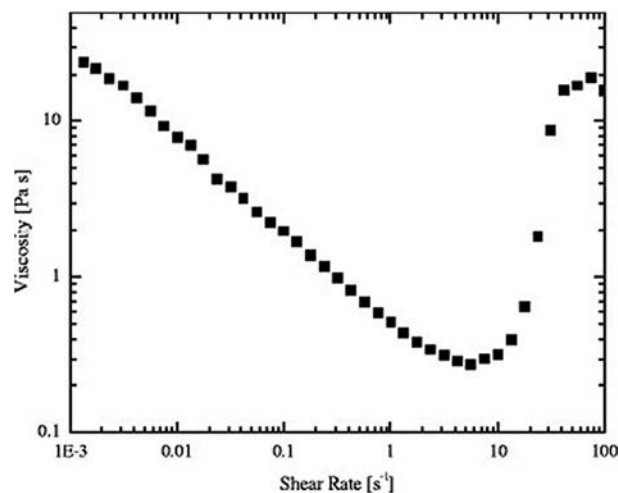
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## ABSTRACT

Dense suspensions of particles in a liquid, with industrial examples including coatings or precursors to solid ceramics and cements, can be quite difficult to process because their flow properties are very sensitive to particle surface interactions. We focus on the extreme rate dependence known as “discontinuous shear thickening” (DST) where the viscosity undergoes a finite and typically large discontinuous jump in viscosity at some shear rate. Simultaneous with DST, there is a large increase in the normal stress, including the nonequilibrium osmotic pressure, or ‘particle pressure’, leading to the historical name of ‘dilatancy’ for shear thickening. Our computational simulations inclusive of the three ingredients of i) lubrication hydrodynamics, ii) repulsive interparticle forces (e.g. due to surface charge) and iii) contact with friction have been shown to reproduce the primary features of DST found experimentally; this is called lubricated-to-frictional (LF) rheology. We describe the main features of the shear thickening transition in the LF scenario, including the observation of extreme fluctuations. Using our simulation results, we explore the microscopic basis for the LF transition in the force network developed under flow.

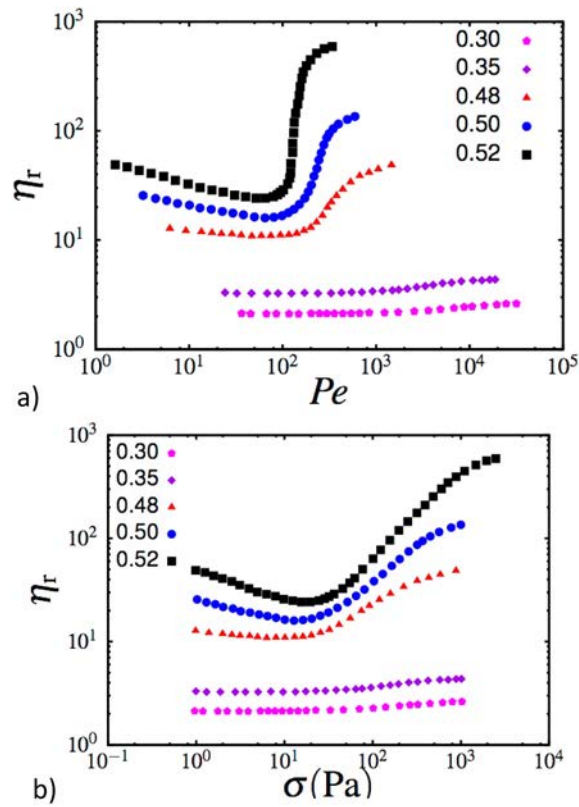
Videos to this article can be found online at <https://doi.org/10.1016/j.sctalk.2022.100031>. **Figures and tables**



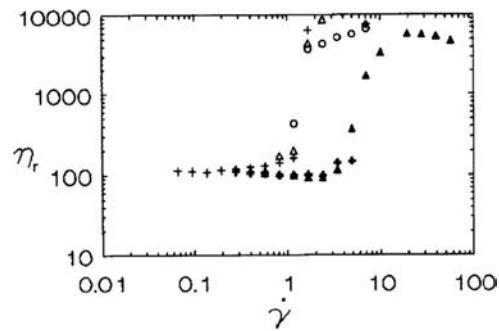
**Fig. 1.** Flow curve for a concentrated cornstarch-in-water dispersion. Figure taken from Khandavalli & Rothstein [1].

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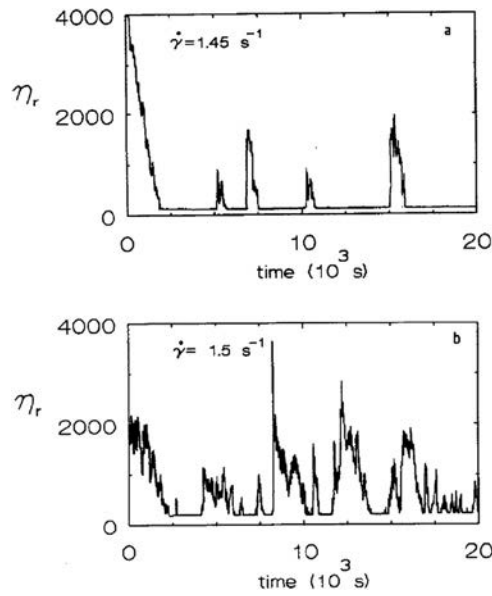
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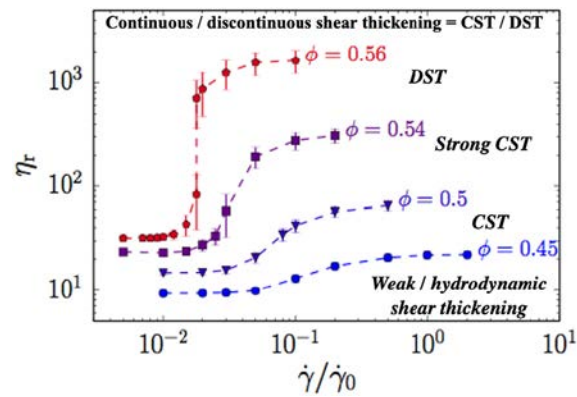
**Fig. 2.** The relative viscosity of a suspension of 520 nm diameter silica particles in nearly-index matched (hence negligible van der Waals forces) 200 MW poly-ethylene glycol as function of a) dimensionless shear rate,  $Pe$ ; and b) shear stress. Figure taken from Cwalina & Wagner [3].



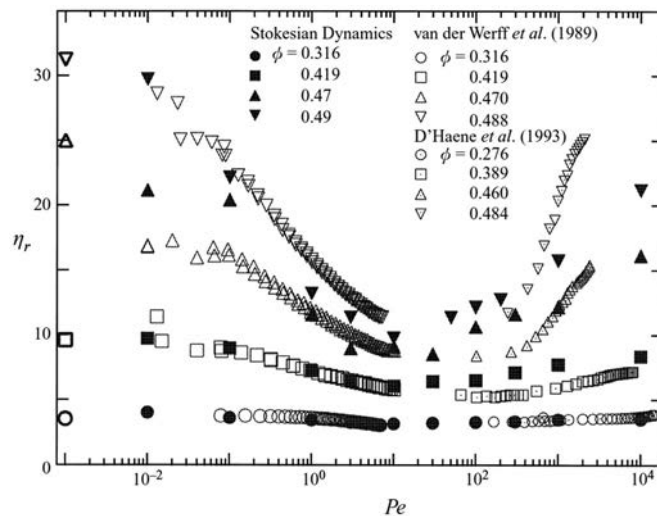
**Fig. 3.** The shear rate dependence of the relative viscosity of polyvinylchloride particles of diameter 1.4  $\mu\text{m}$  in dioctylphtalate (also studied in Hoffman's seminal work [2]), at two temperatures, 20 °C (left curve) and 50 °C. Figure taken from Boersma et al. [4].



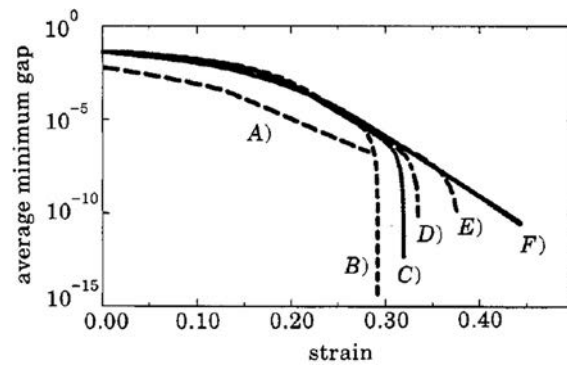
**Fig. 4.** The time series of the relative viscosity at 20 °C from Fig. 3, showing very large fluctuations returning to a relatively constant baseline. Figure taken from Boersma et al. [4].



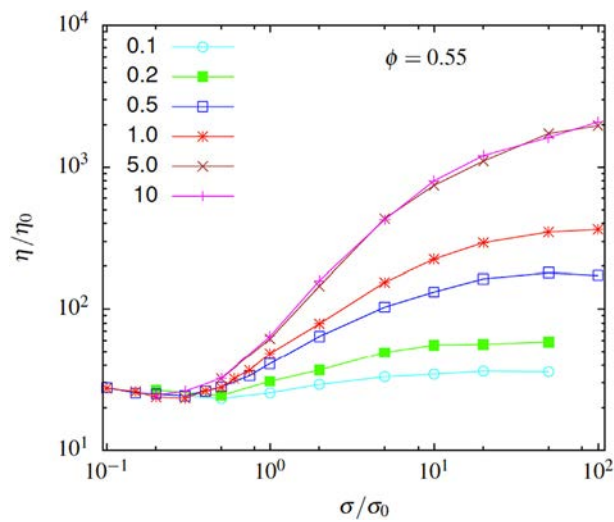
**Fig. 5.** Data from simulation method of Mari et al. [10] illustrating shear thickening regimes.



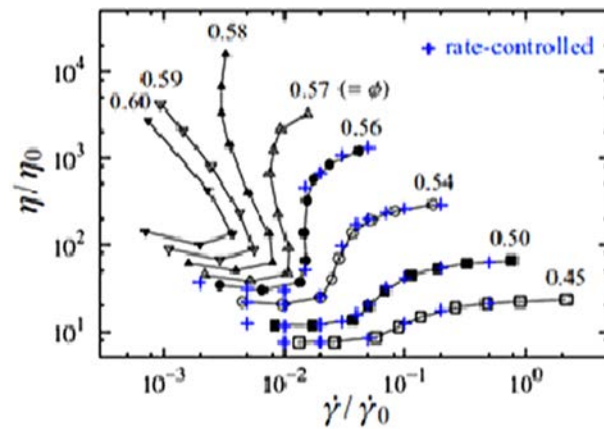
**Fig. 6.** Shear rate dependence from Stokesian Dynamics simulations of Brownian hard sphere suspensions at the volume fractions  $\phi$  shown, in comparison with experimental data [7,8]. Figure from Foss & Brady [5] (similar to Phung et al. [6]).



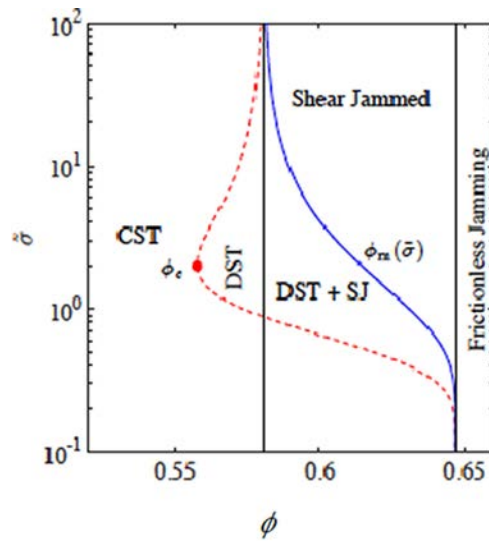
**Fig. 7.** Lubrication breakdown: the minimum gap for a  $\phi = 0.51$  suspension of monodisperse hard spheres interacting only by Stokes flow, using different integration schemes for the particle motion (A-F), showing the inability of lubrication to keep surfaces from reaching scales for which continuum theory breaks down in  $O(1)$  strain. Figure from Melrose & Ball [9].



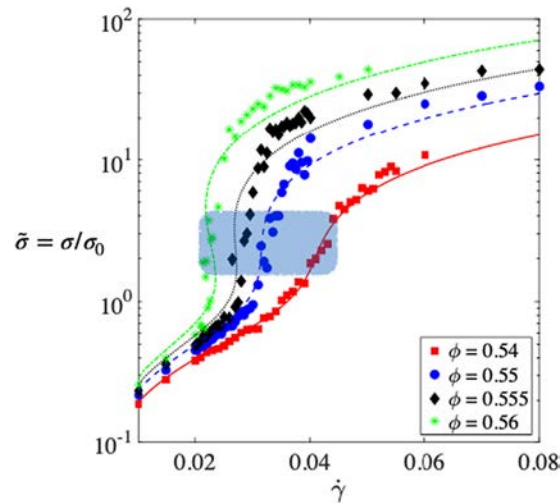
**Fig. 8.** Unpublished work showing the relative viscosity response for different interparticle friction coefficients shown by the legend as a function of the imposed stress  $\sigma$ , normalized by the repulsive force and particle size,  $\sigma_0 = F_R/a^2$ . For  $\sigma/\sigma_0 \ll 1$ , no effect of friction is seen.



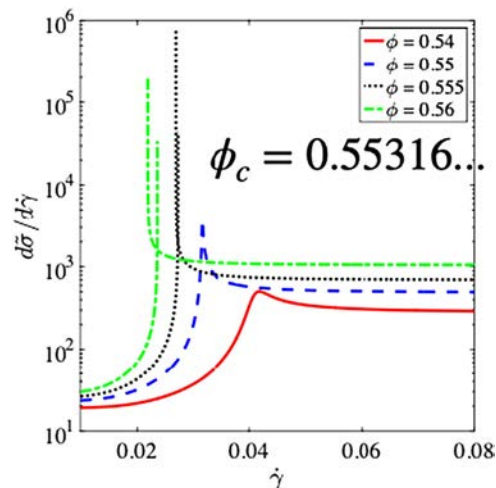
**Fig. 9.** Summary of flow curves obtained by the simulation model LF-DEM [10]: relative viscosity as a function of dimensionless shear. Blue + points are at controlled shear rate and others at controlled stress, with  $\phi$  values labeled.



**Fig. 10.** Flow state diagram [11,12] on volume fraction-dimensionless stress axes for a friction coefficient of 1 and bidisperse suspension of half large and small particles at radius ratio 1.4. The solid vertical lines at  $\phi \sim 0.58$  and  $0.65$  are the frictional (large stress) and frictionless jamming (low stress) fractions.



**Fig. 11.** Dimensionless stress as a function of shear rate from simulation [12,13] and fitting to the Wyart & Cates [11]. The shaded region shows a transition from single-valued to multivalued behavior as a function of shear rate with varying stress and is analyzed below.



**Fig. 12.** Stress susceptibility defined as the rate of variation of stress with shear rate, deduced from the Wyart & Cates model [11] fitted to simulation data (figure taken from [13]). The susceptibility diverges first at the  $\phi \sim 0.553$  value noted on the plot and is multivalued for larger  $\phi$ .

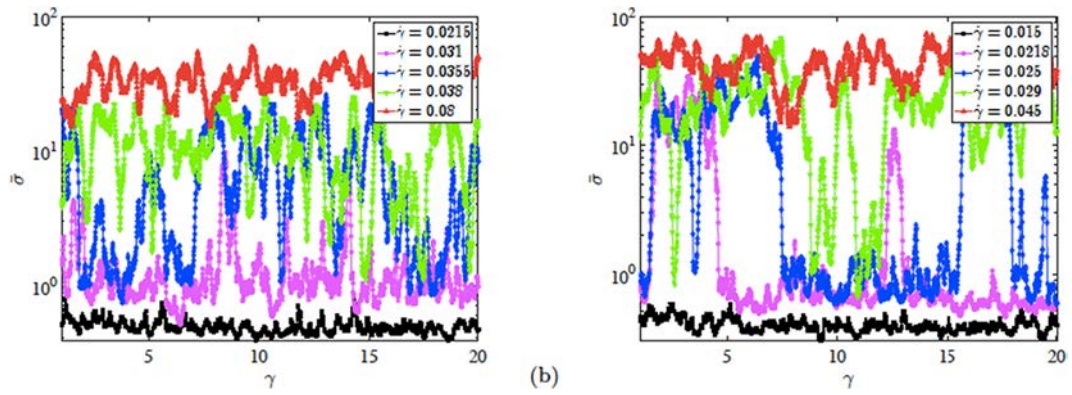


Fig. 13. Dimensionless stress fluctuations as a function of strain (dimensionless time) at fixed shear rates. (left)  $\phi = 0.55$  (DST onset) and (right)  $\phi = 0.56$  (fully in DST). Figure taken from [13].

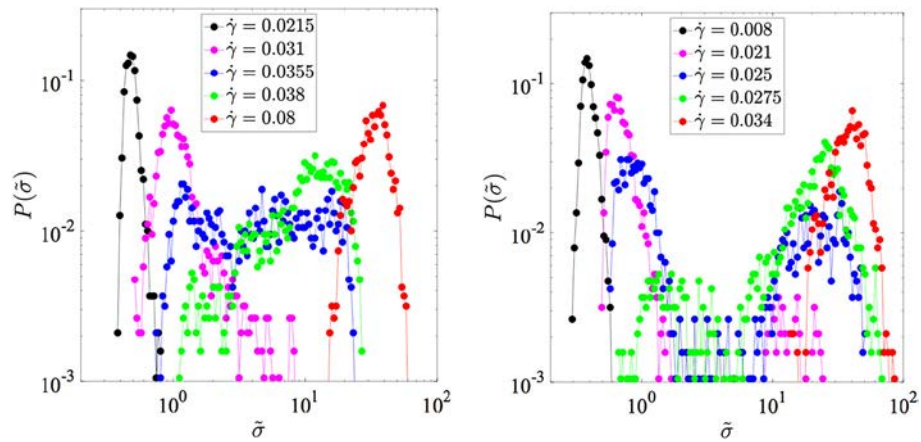


Fig. 14. Probability of dimensionless stress corresponding to the time series of Fig. 13: (left)  $\phi = 0.55$  (DST onset) and (right)  $\phi = 0.56$  (fully in DST). Note the flat distribution at intermediate shear rates for  $\phi = 0.55$ , and bimodal distributions for  $\phi = 0.56$  [14]. Figure taken from [14].

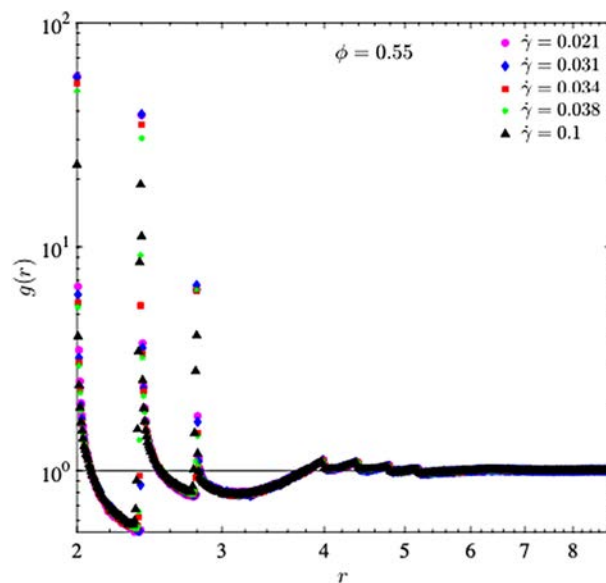
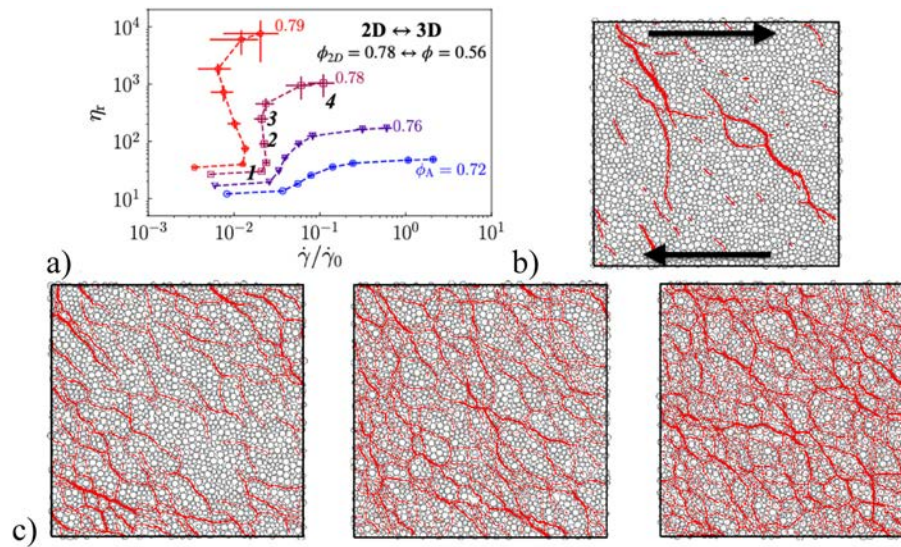


Fig. 15. Unpublished results (similar in [10]) showing that very similar pair distribution function across the DST onset, i.e. a strong shear thickening transition.





**Fig. 16.** a) Frictional contact network in DST response of a 2D suspension at  $\phi_{2D} = 0.78$ , with b) the direction of flow and (red lines) frictional contacts shown for point 1 at  $\sigma/\sigma_0 = 0.5$ , and c) the contacts for points 2–4, at  $\sigma/\sigma_0 = 1, 2$ , and 100 (left to right). Figure adapted from [14].

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## Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Further Reading

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Morris develops constitutive and bulk fluid mechanical descriptions appropriate for complex fluids. Defining questions are: *How are mixture flows intrinsically different from their single-phase counterparts, and why? What is the appropriate predictive framework for these materials accounting for their multiphase nature?*

The focus has been on suspensions, from submicron colloids to sand slurries. Unifying features of these materials are the influence of hydrodynamic interactions and the flow-induced

microstructure on rheology and bulk flow, as well as particle migration. Morris has recently focused on instabilities due to inertia in flows of suspensions and on frictional interactions between particles in viscous liquids, toward understanding of shear thickening.

Morris was elected Fellow of the American Physical Society (APS) in 2013 and of the Society of Rheology in 2019. He was awarded the 2015 and 2020 *J. Rheology* Publication Awards, the 2017 AIChE/Shell Thomas Baron Award for Fluid-Particle Systems, and 2019 Stanley Corrsin Award of the APS, and the 2022 Weissenberg Award of the European Society of Rheology.

Jeff Morris served as the Secretary-Treasurer of APS Division of Fluid Dynamics (2018-2021), and is an Associate Editor of the Journal of Fluid Mechanics. He authored the text *A Physical Introduction to Suspension Dynamics*, with Elisabeth Guazzelli.